Quantum criticality in condensed matter

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Scientific American 308, 44 (January 2013)

PHYSICS
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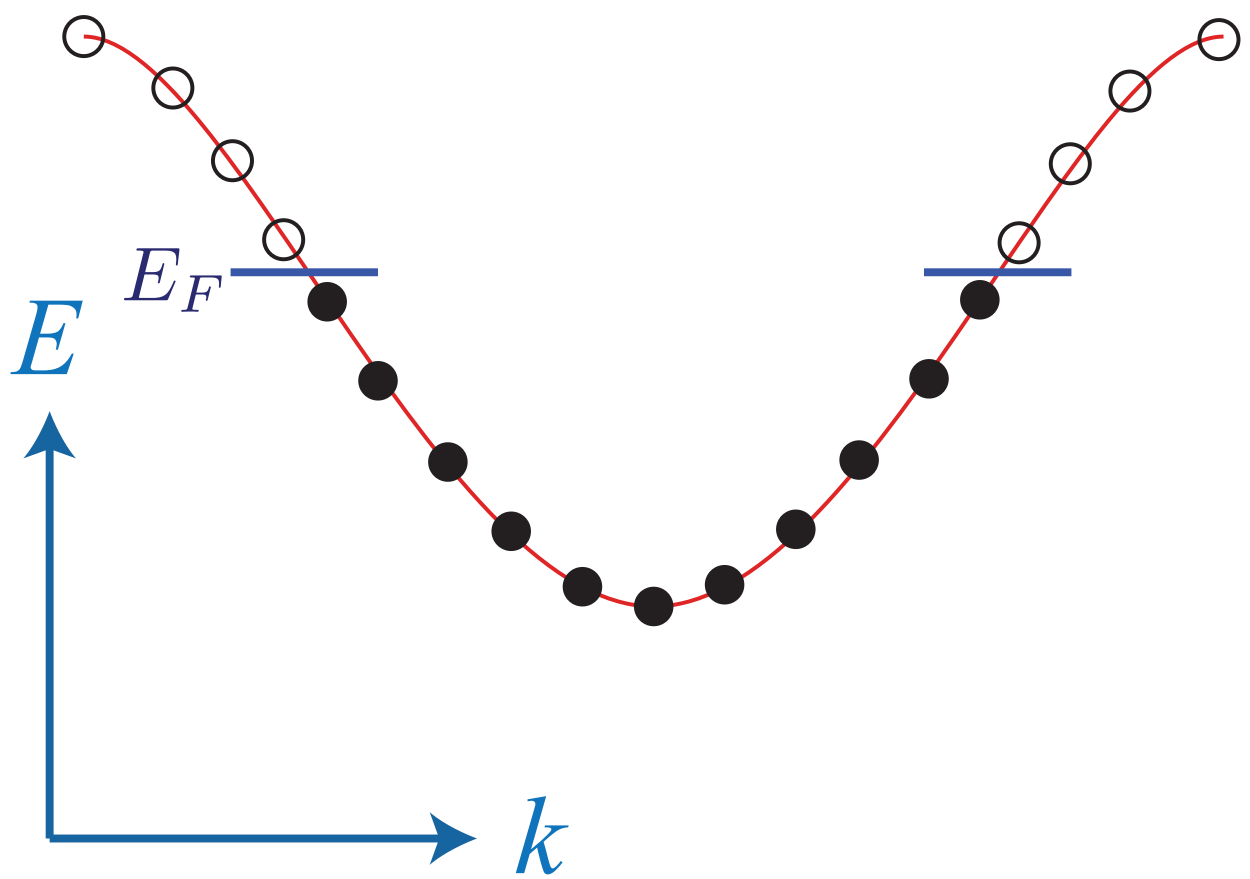
Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

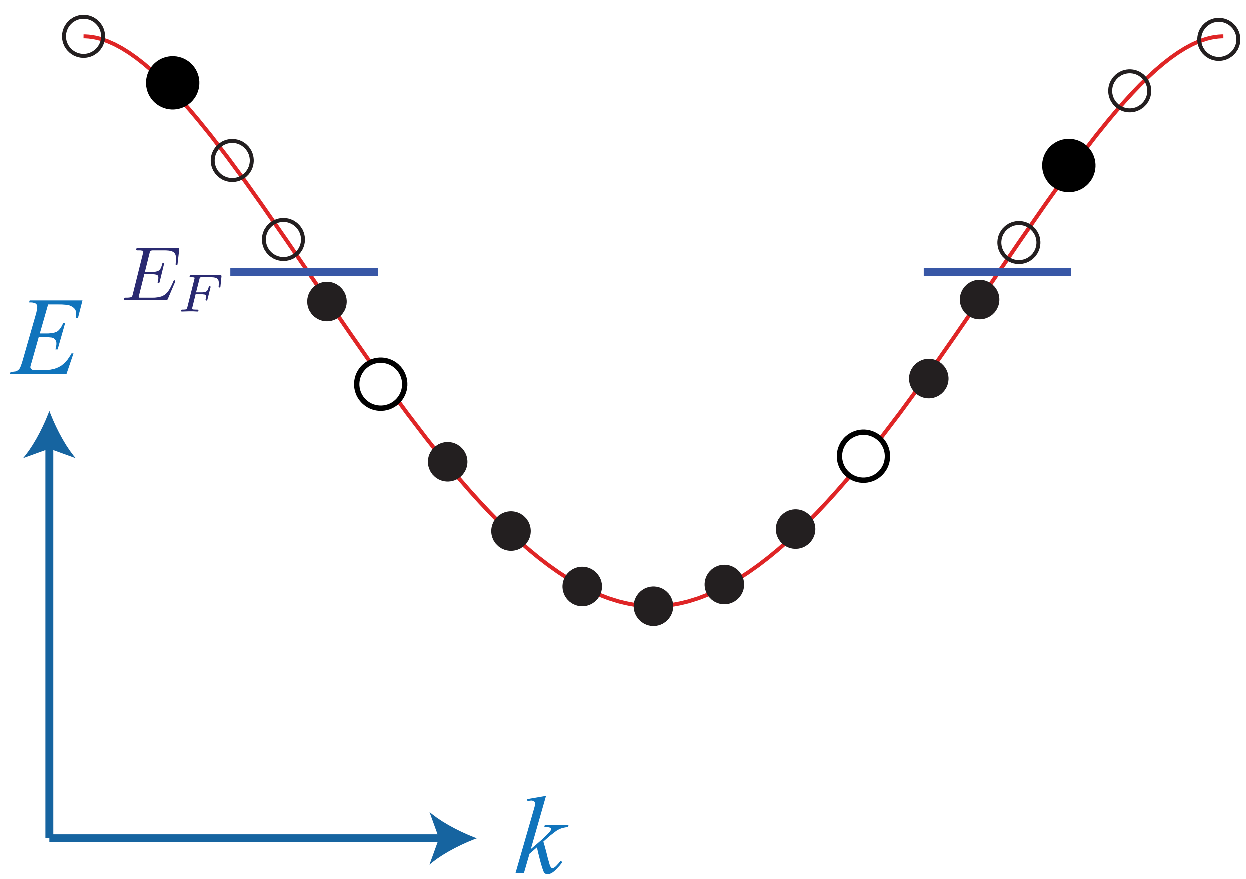
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle

quantum entanglement,

and no quasiparticles

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

   Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity

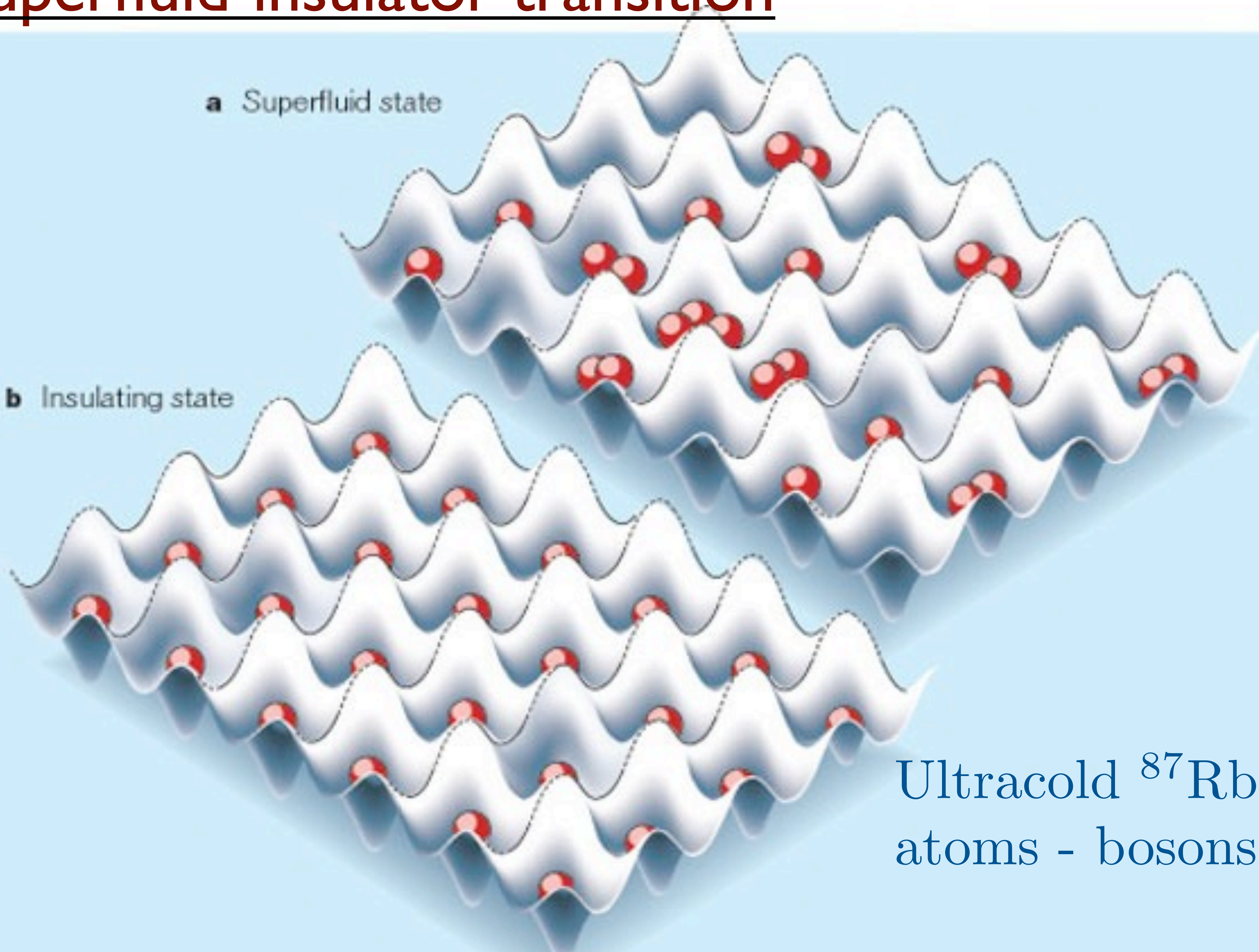
   The pnictides and the cuprates

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
Conformal field theories and gauge-gravity duality

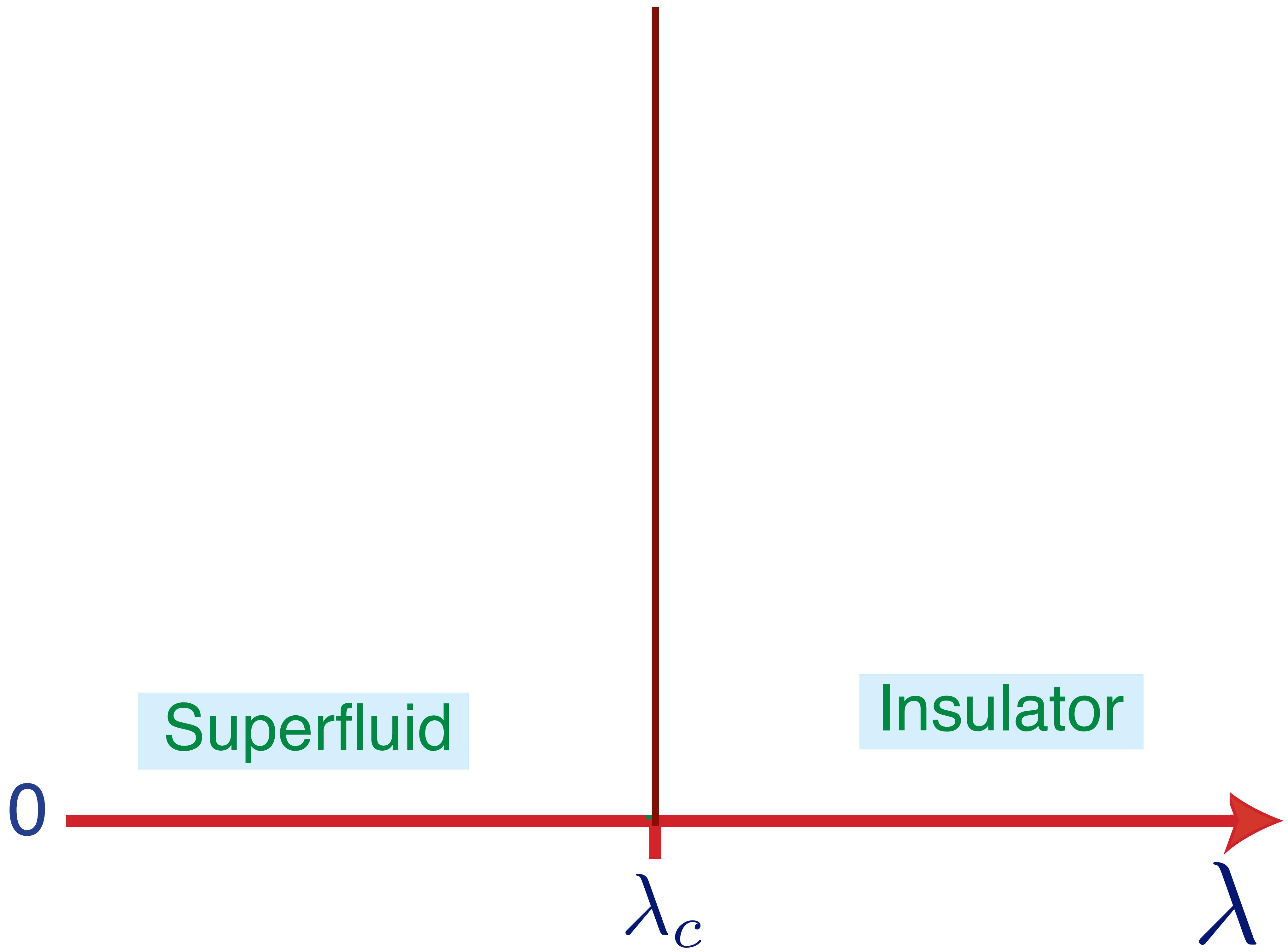
2. Metals with antiferromagnetism, and high temperature superconductivity
The pnictides and the cuprates

Superfluid-insulator transition

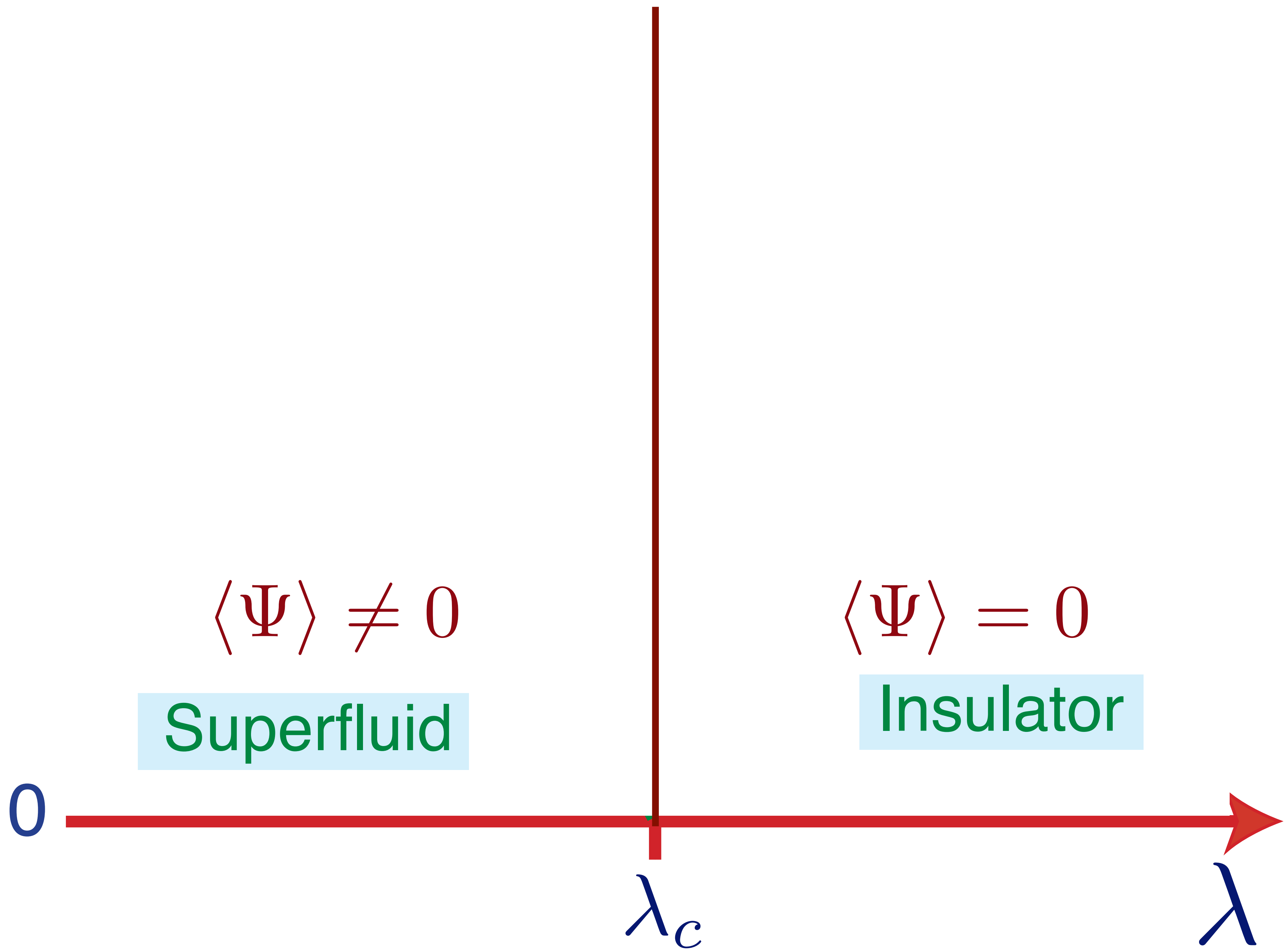


Ultracold ^{87}Rb atoms - bosons

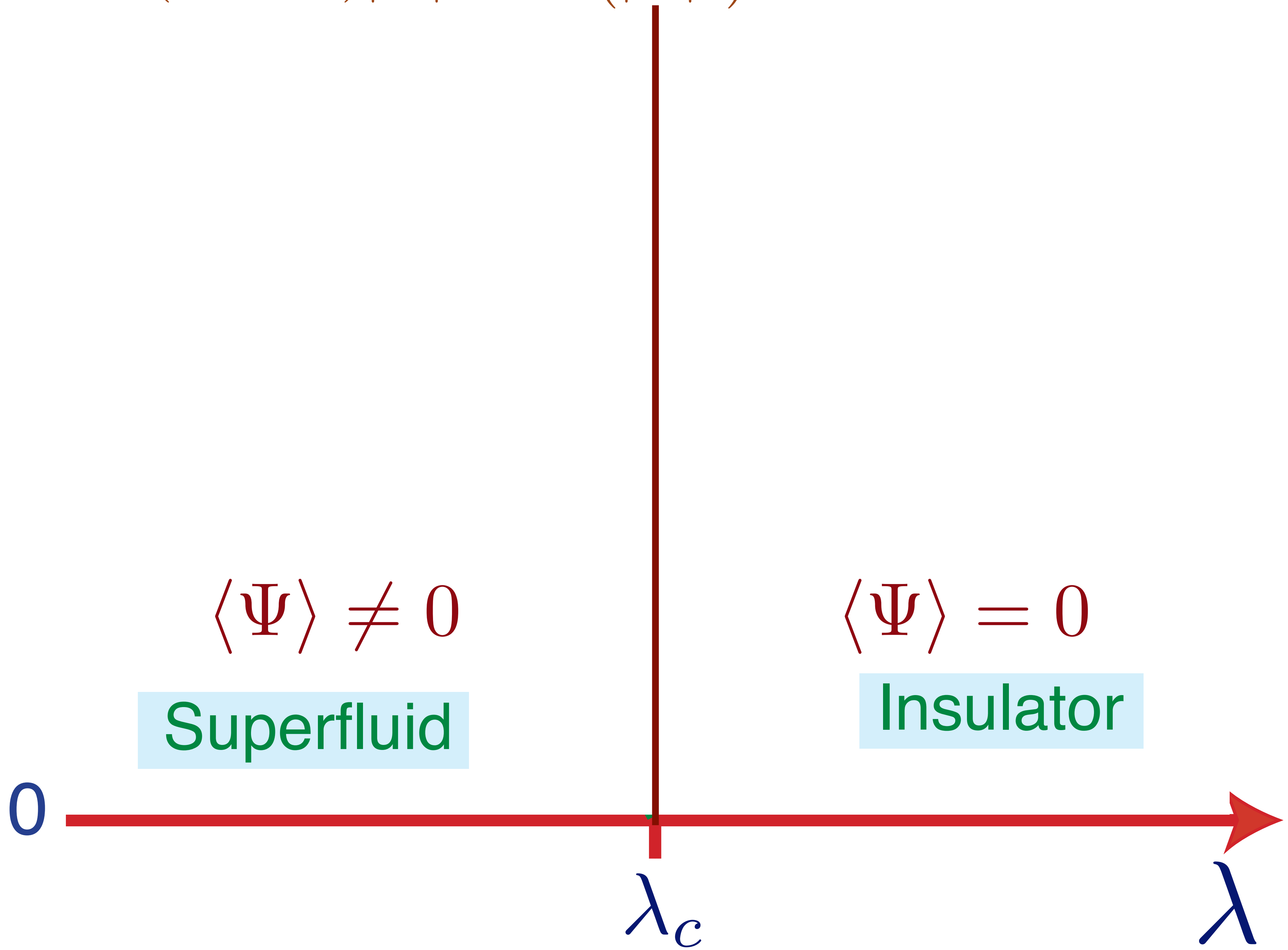
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).


Superfluid
Insulator
0
λ_c
λ

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



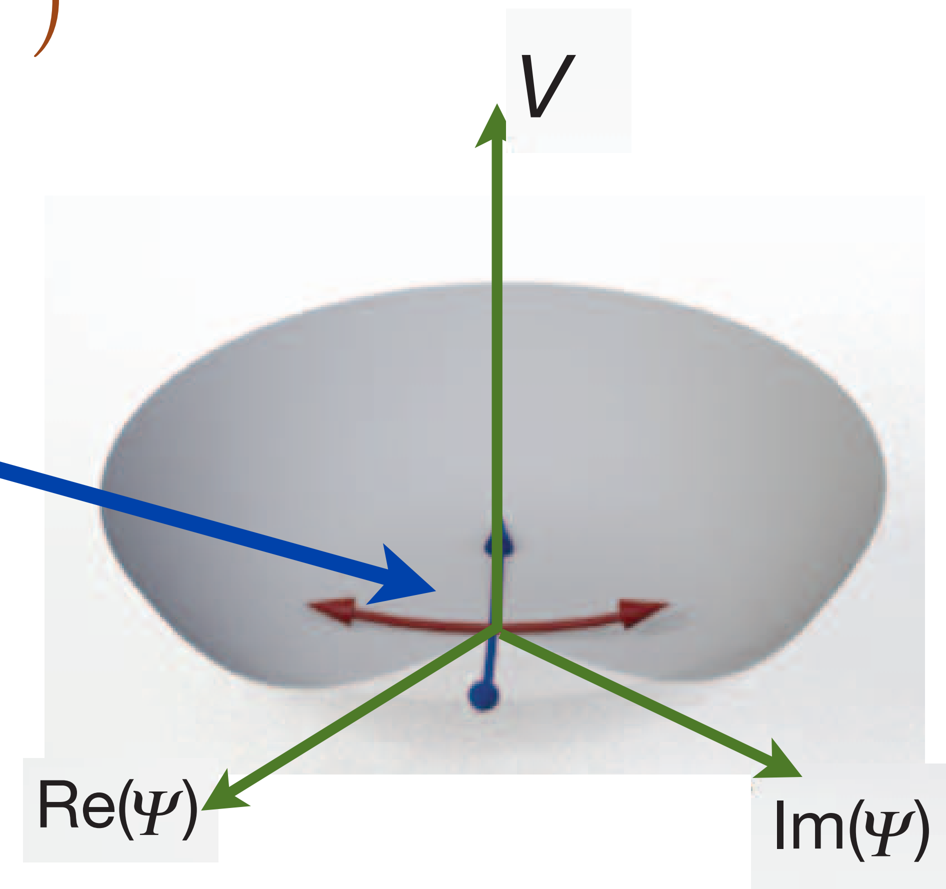
$$
\begin{aligned}
\mathcal{S} &= \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \\
V(\Psi) &= (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2
\end{aligned}
$$



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



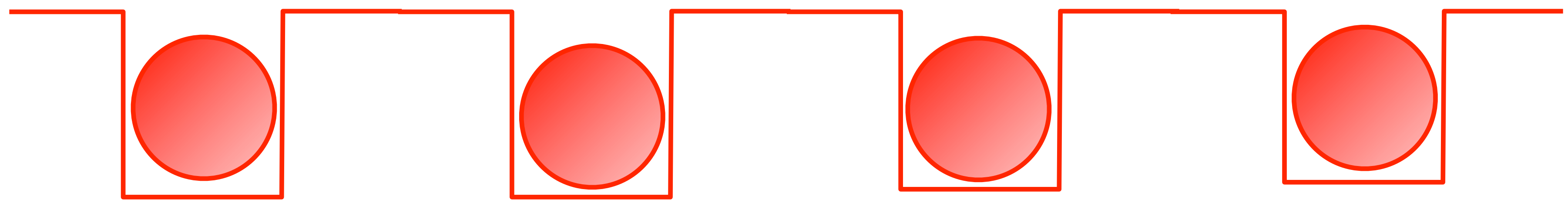
$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

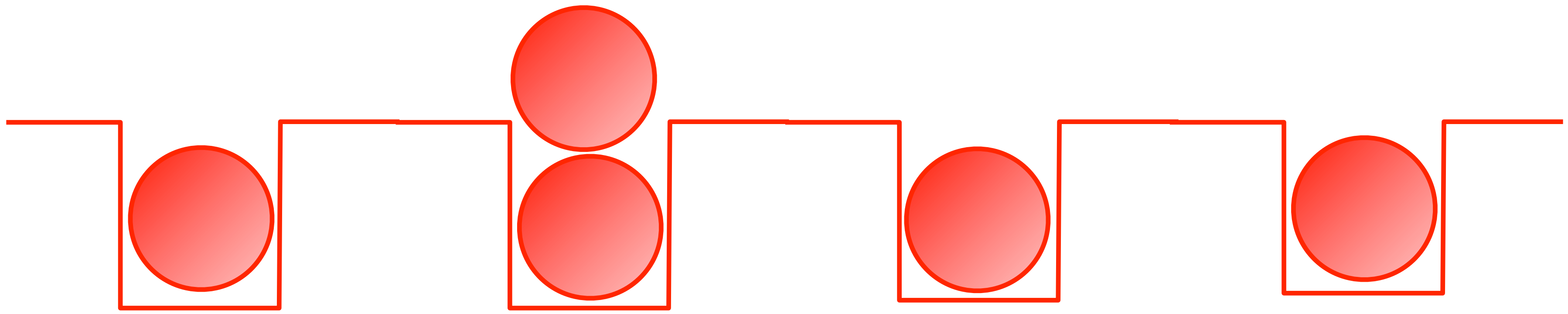
Insulator

0 λ_c λ



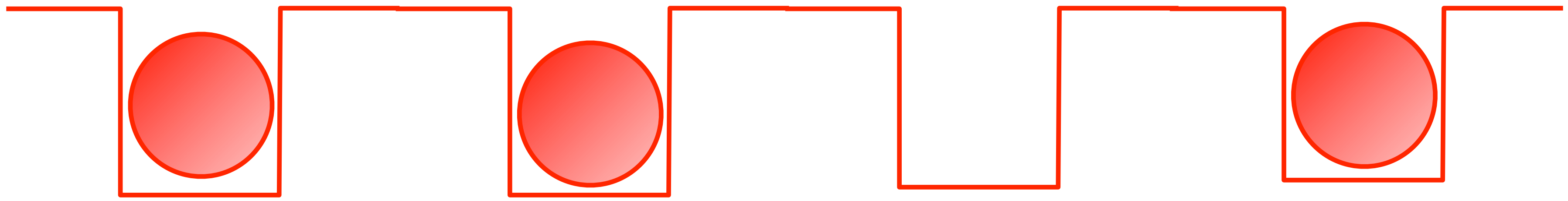
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



Particles $\sim \Psi^\dagger$

Excitations of the insulator:



$$\text{Holes} \sim \Psi$$

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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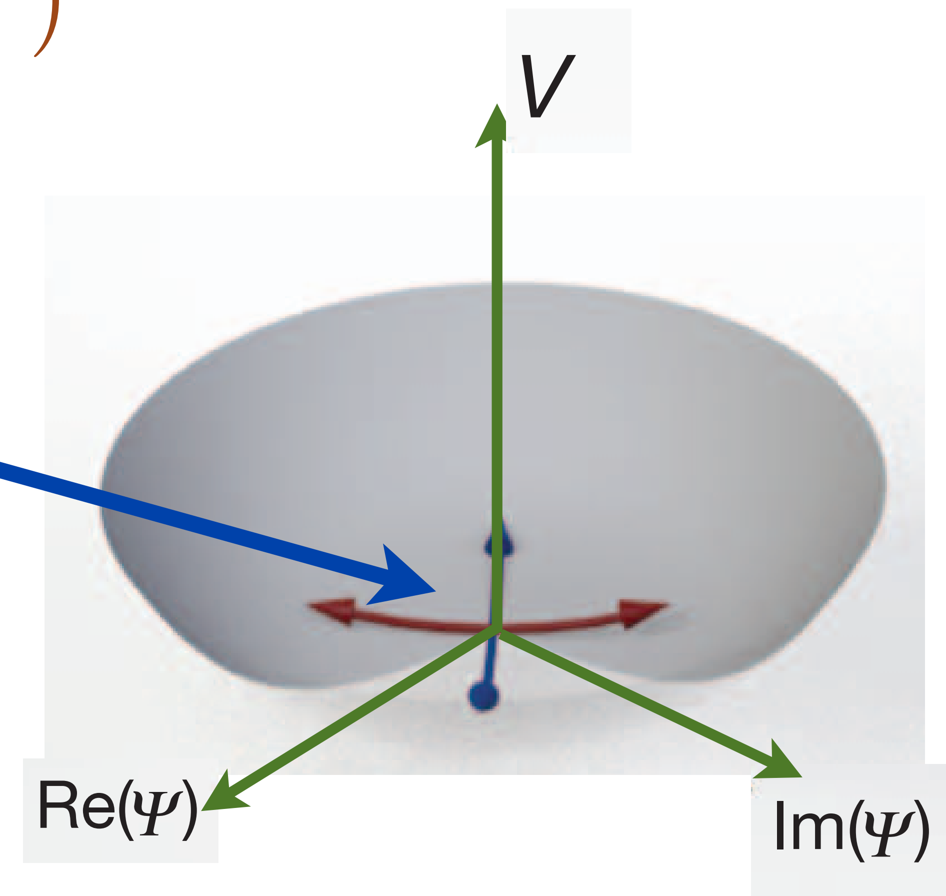

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$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0

λ_c

λ

$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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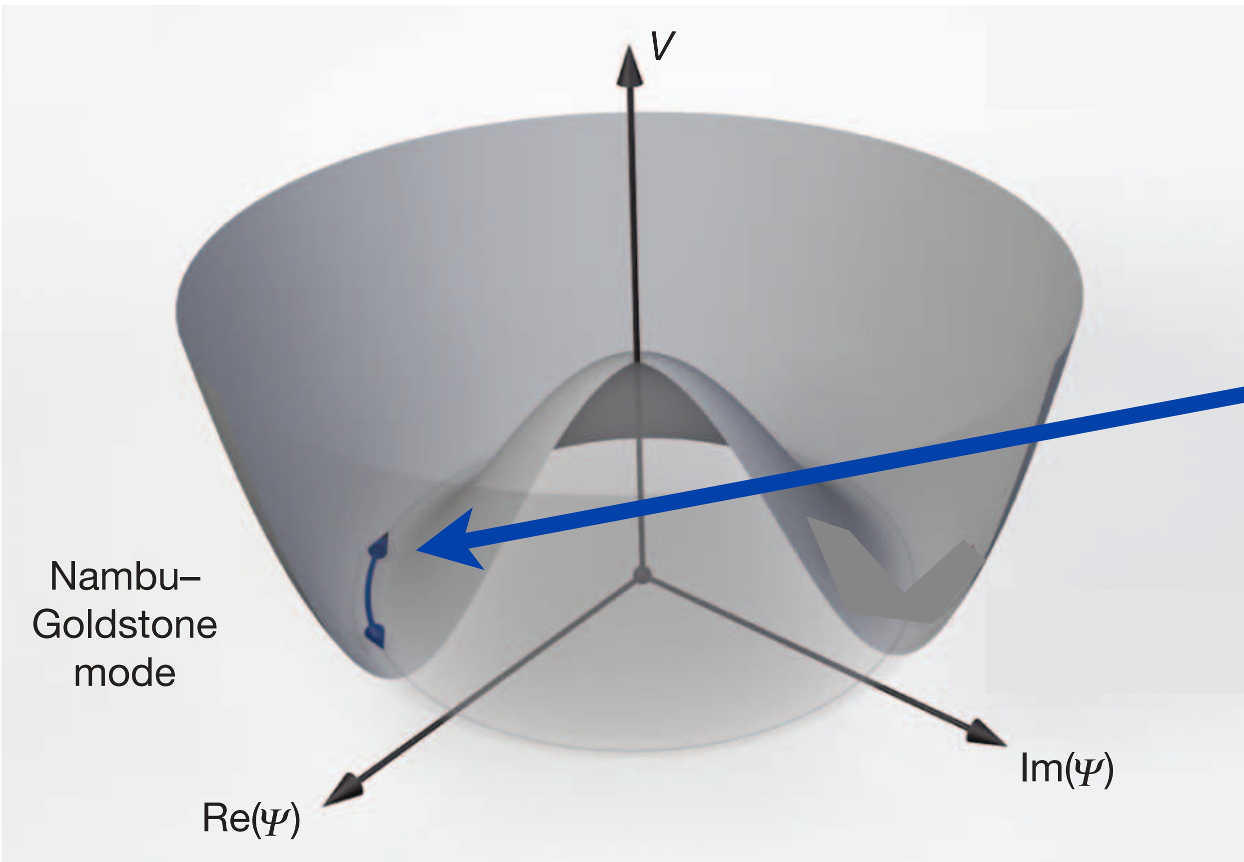

Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 — λ_c — λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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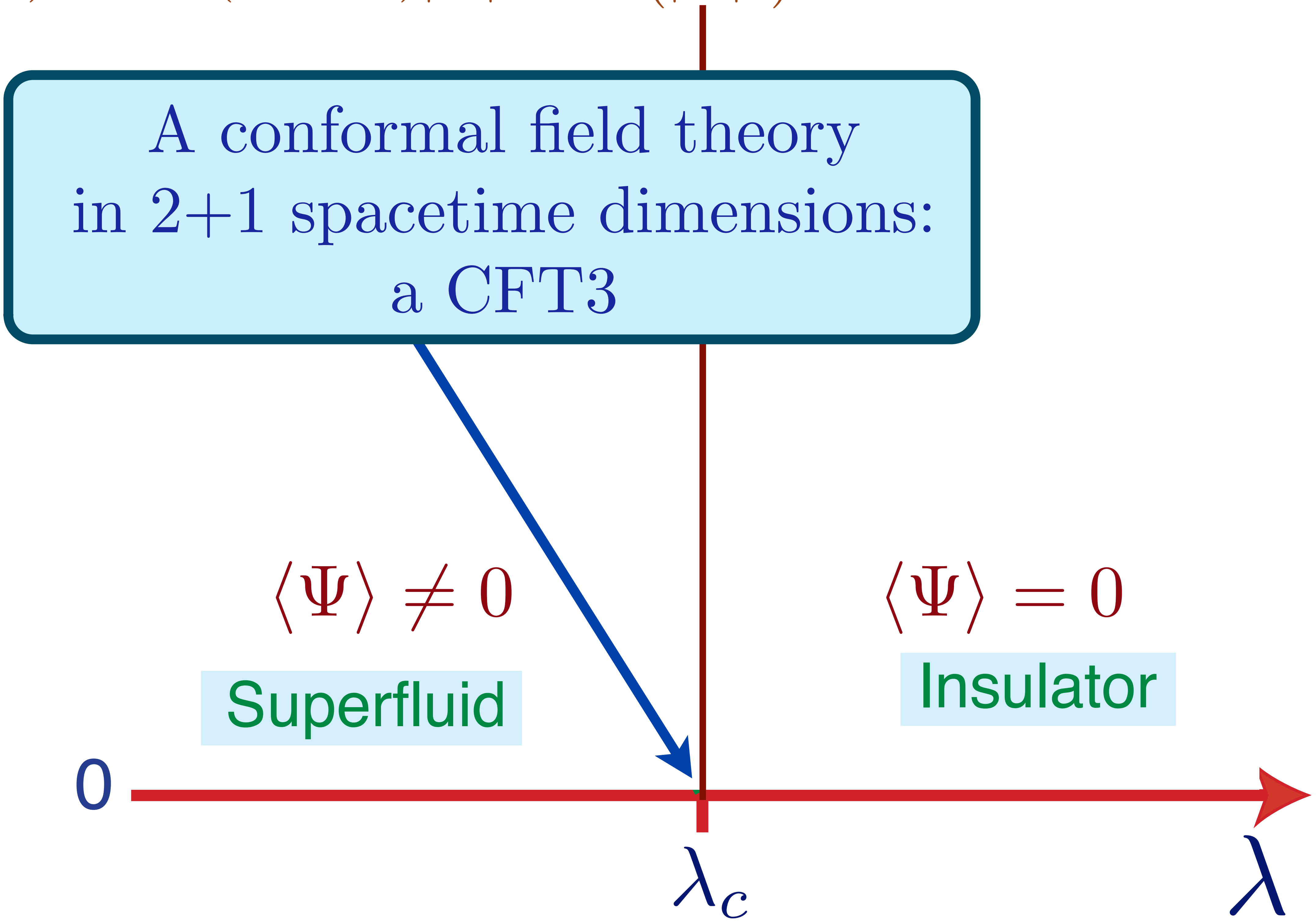

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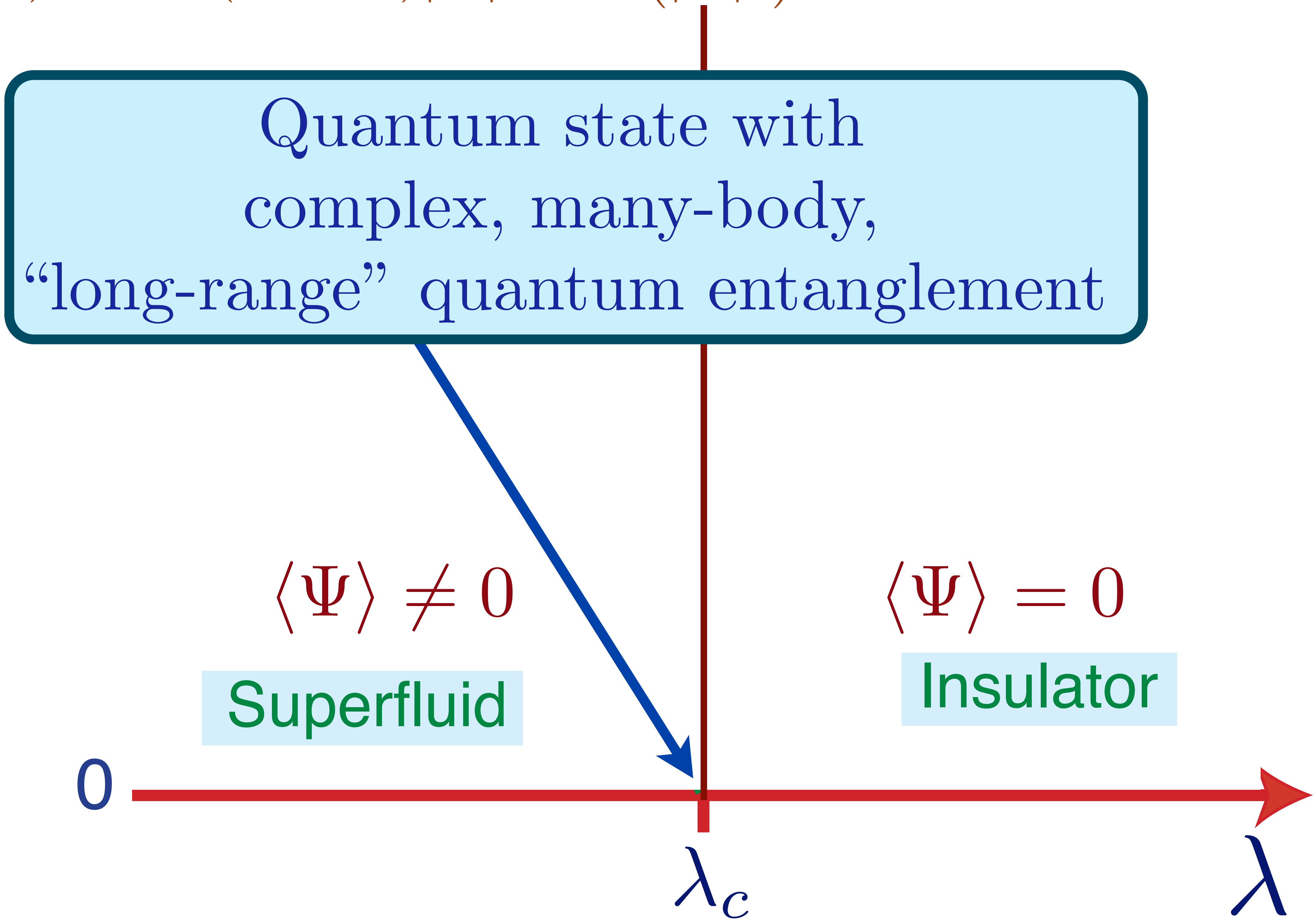

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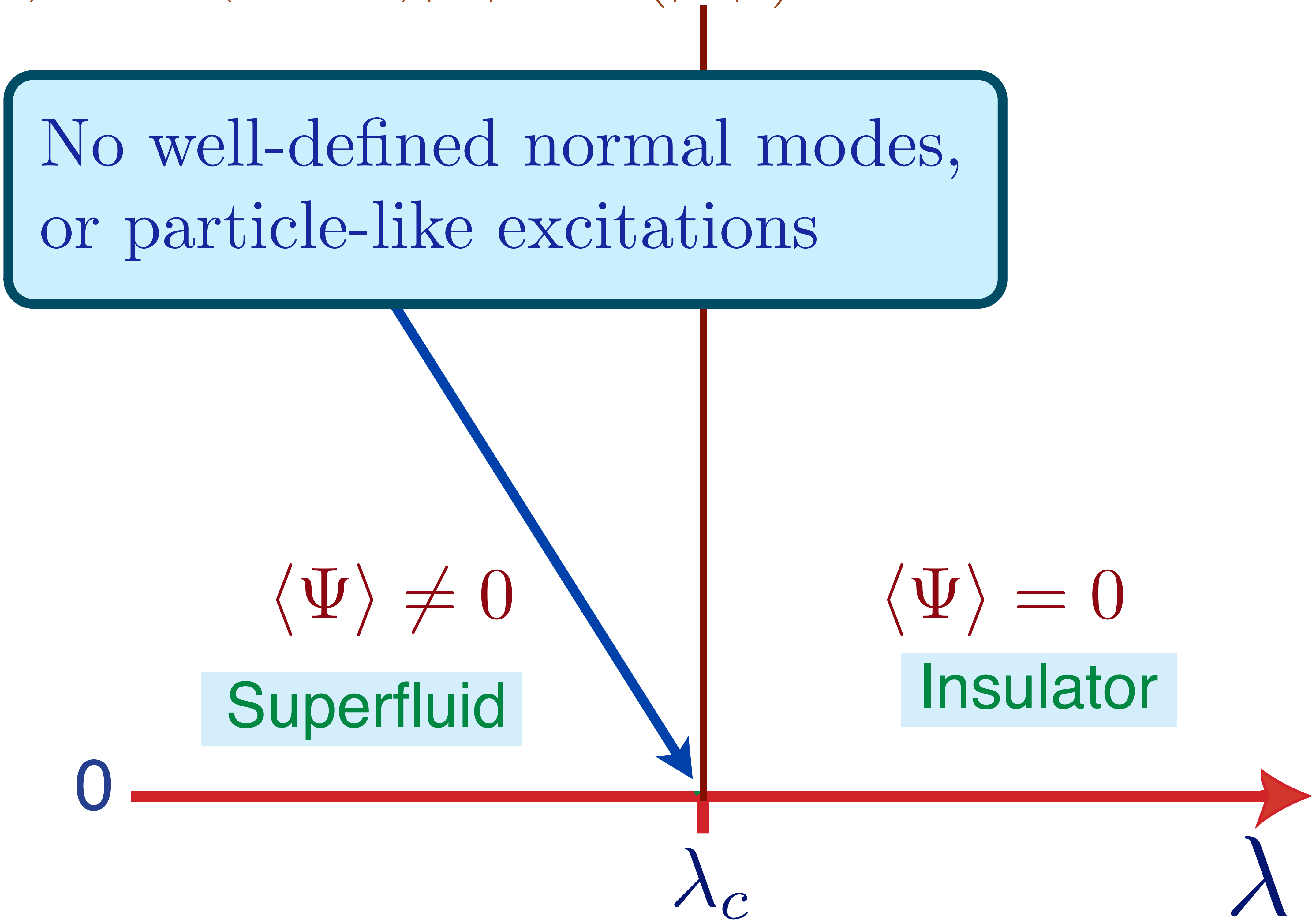

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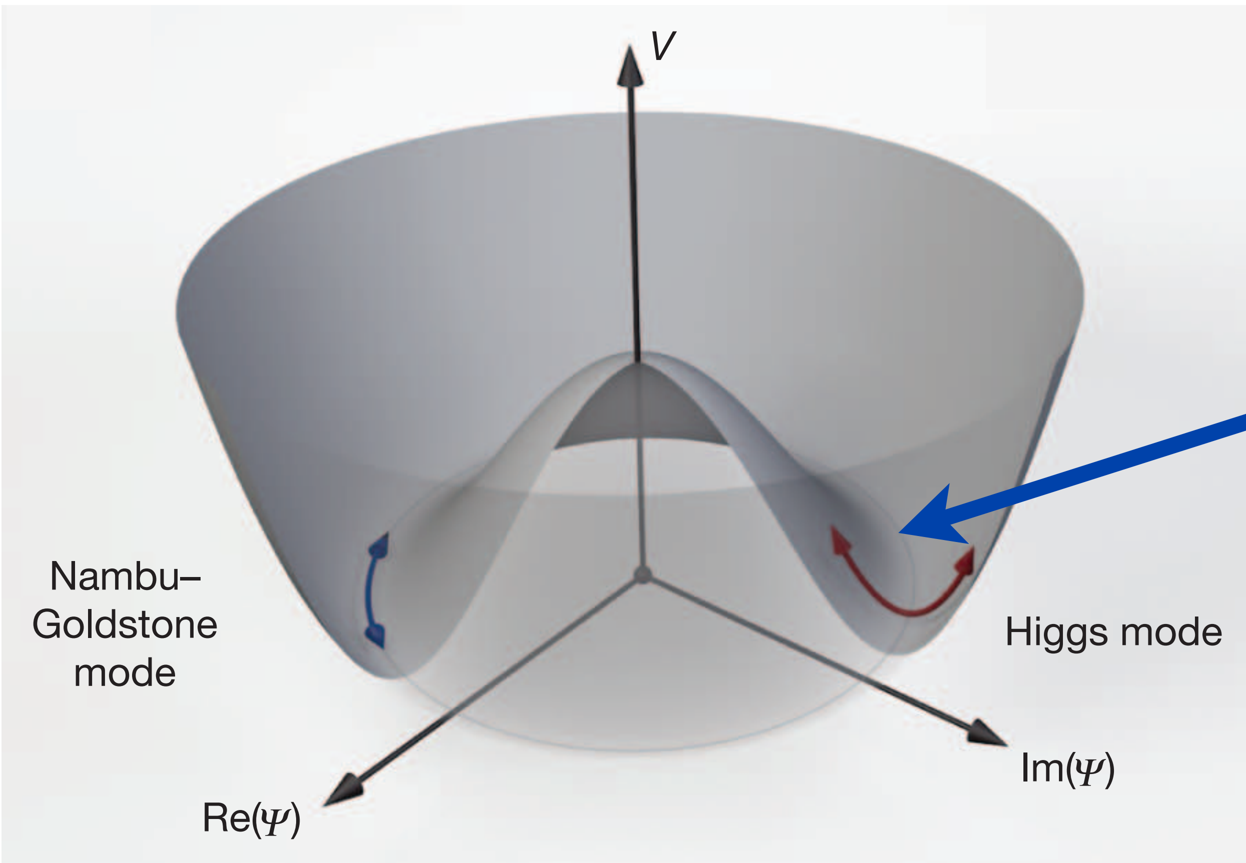


Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$

Superfluid

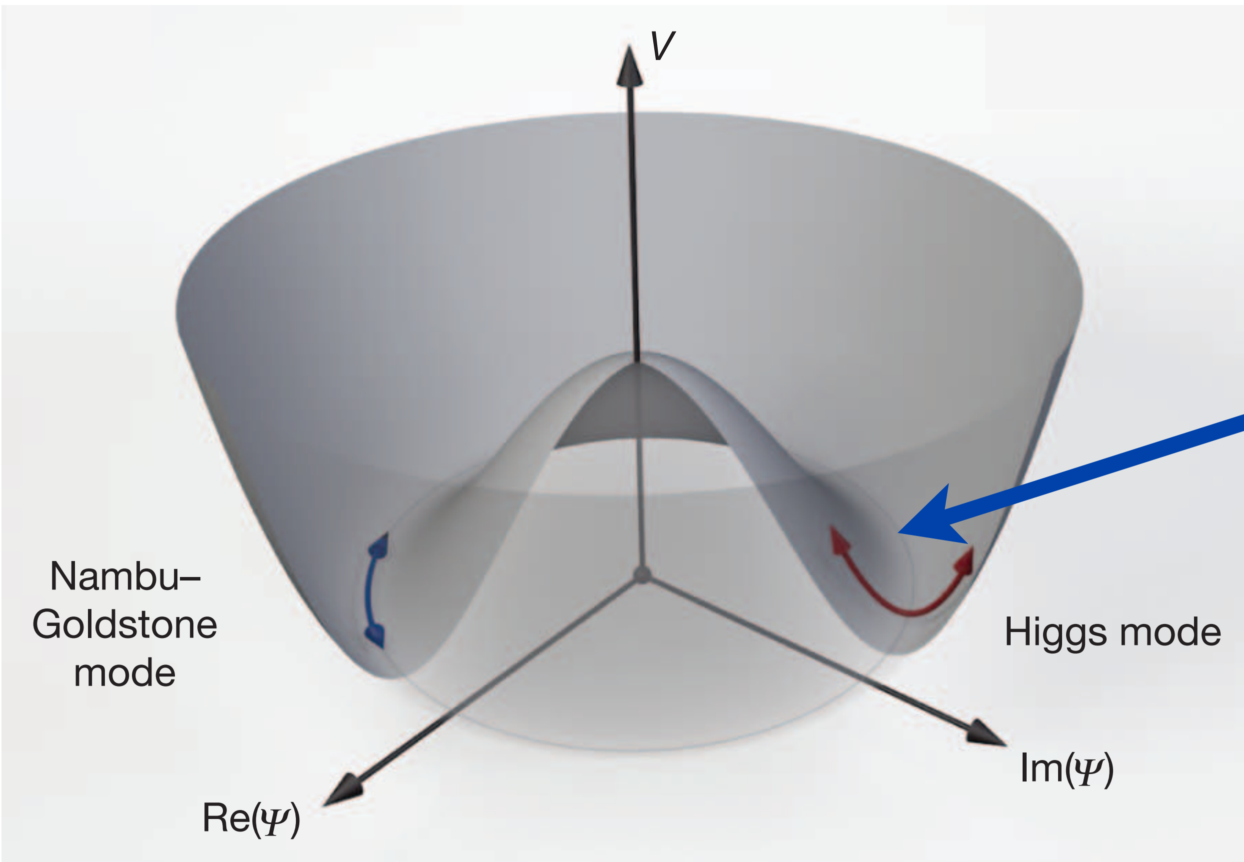
$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Despite rapid decay,
there is a well-defined
Higgs "quasi-normal mode".
This is associated with a pole
in the lower-half
of the complex frequency plane.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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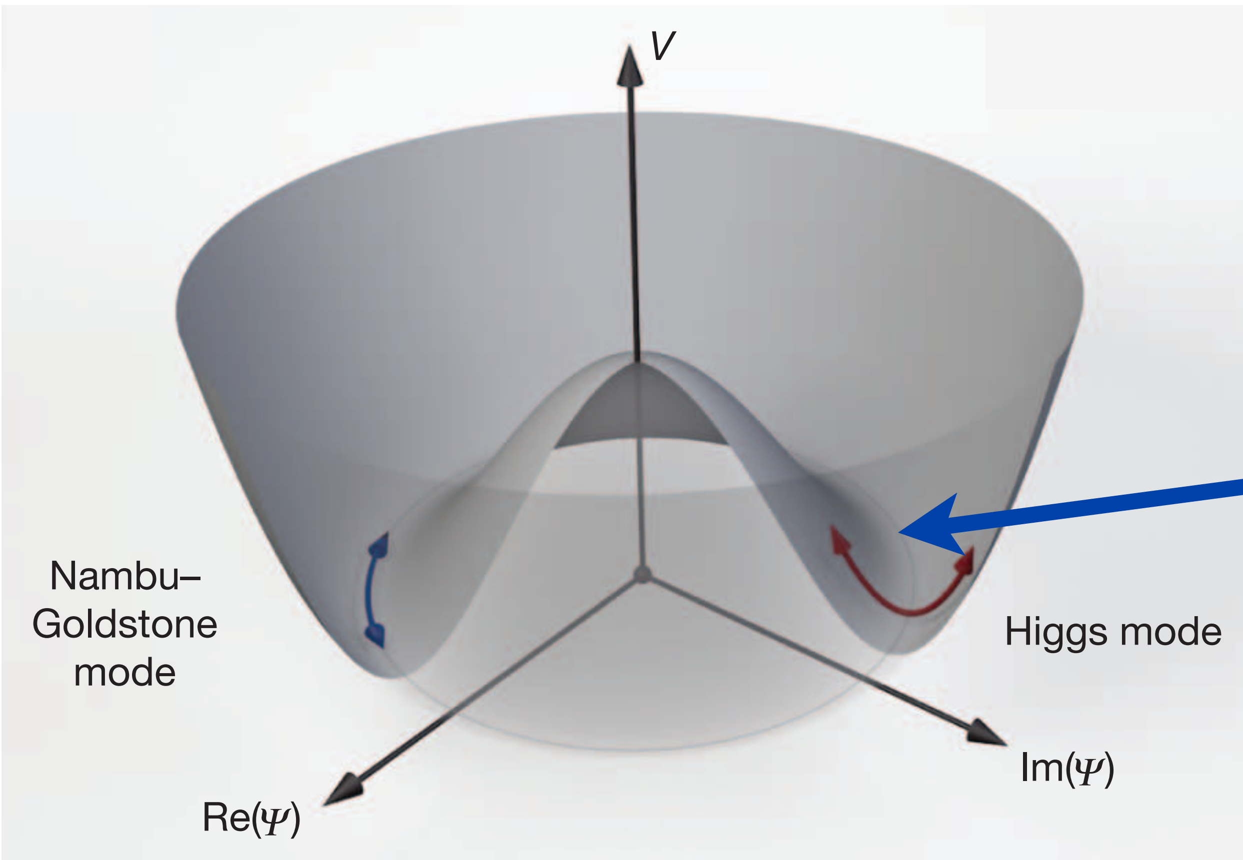
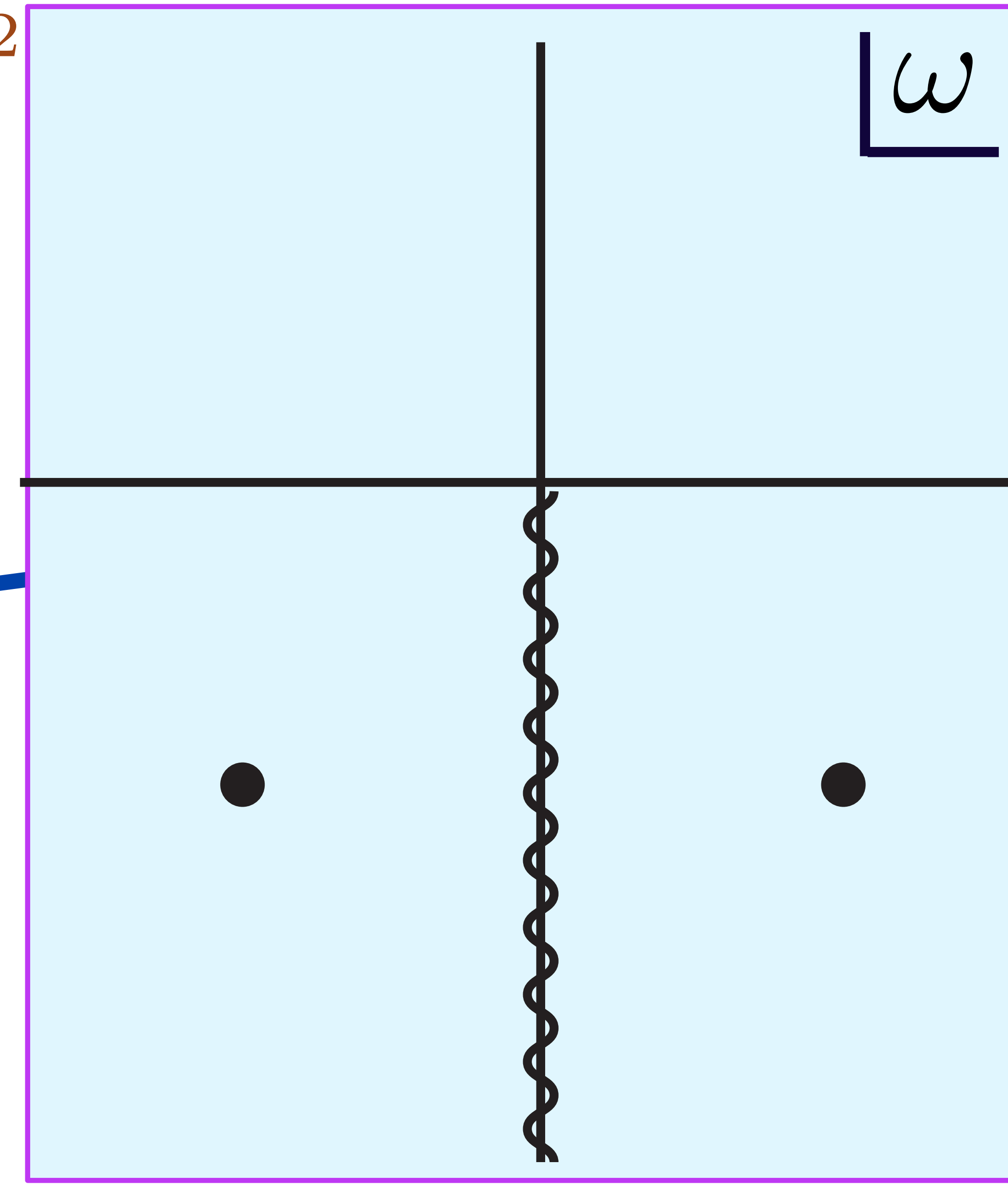




D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

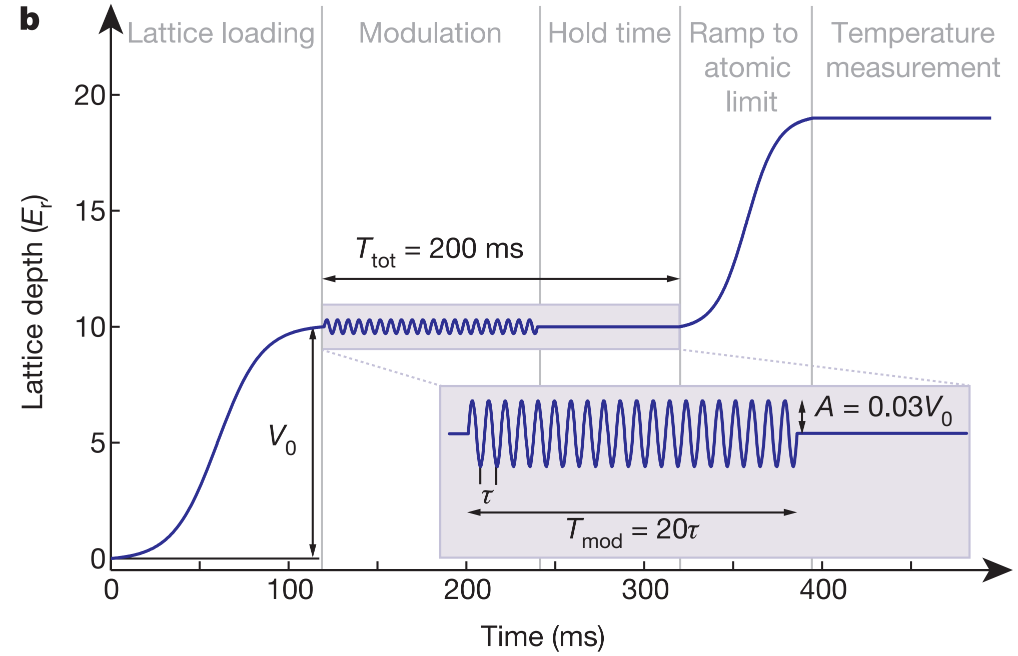
The Higgs quasi-normal mode is at the frequency

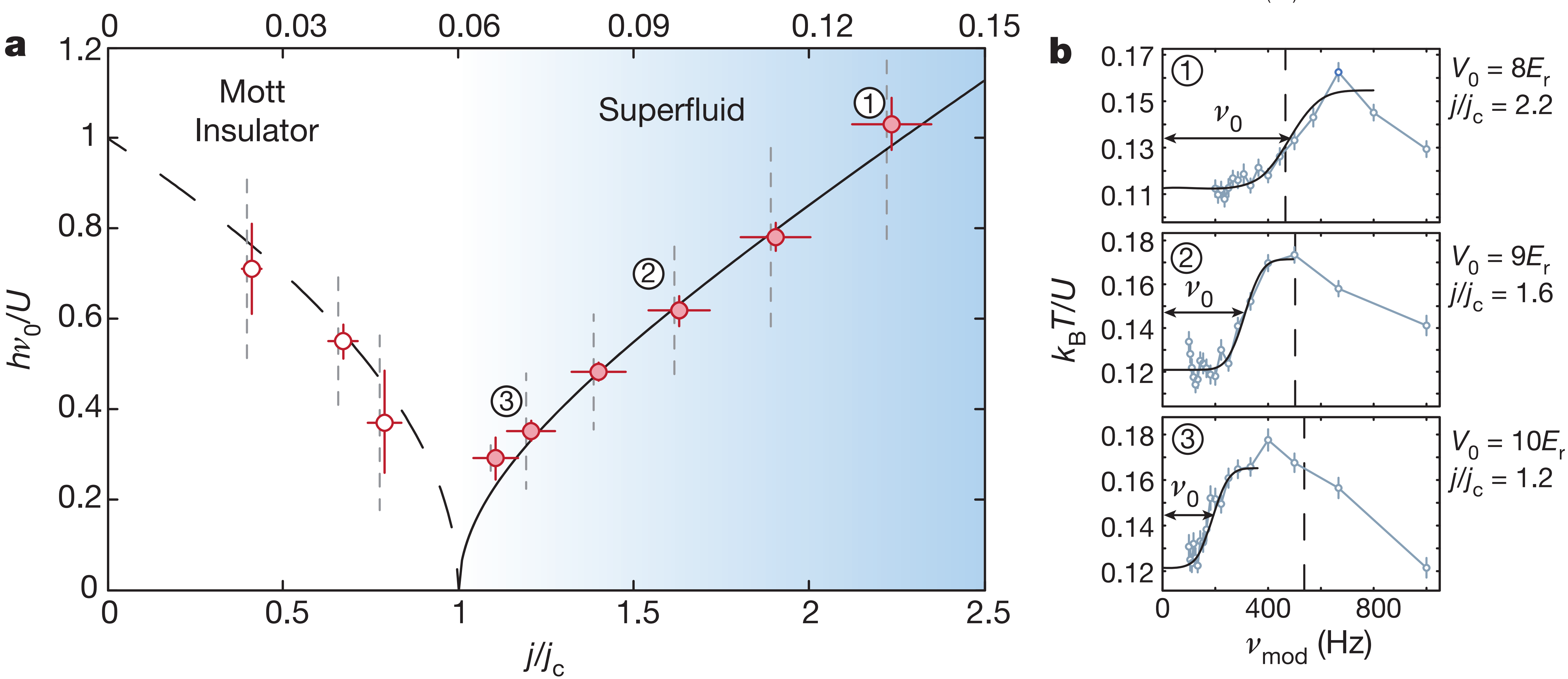
$$\frac{\omega_{\text{pole}}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4 + \sqrt{2}\log\left(3 - 2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\, i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the "paramagnetic" state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ. The universal answer is a consequence of the strong interactions in the CFT3

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

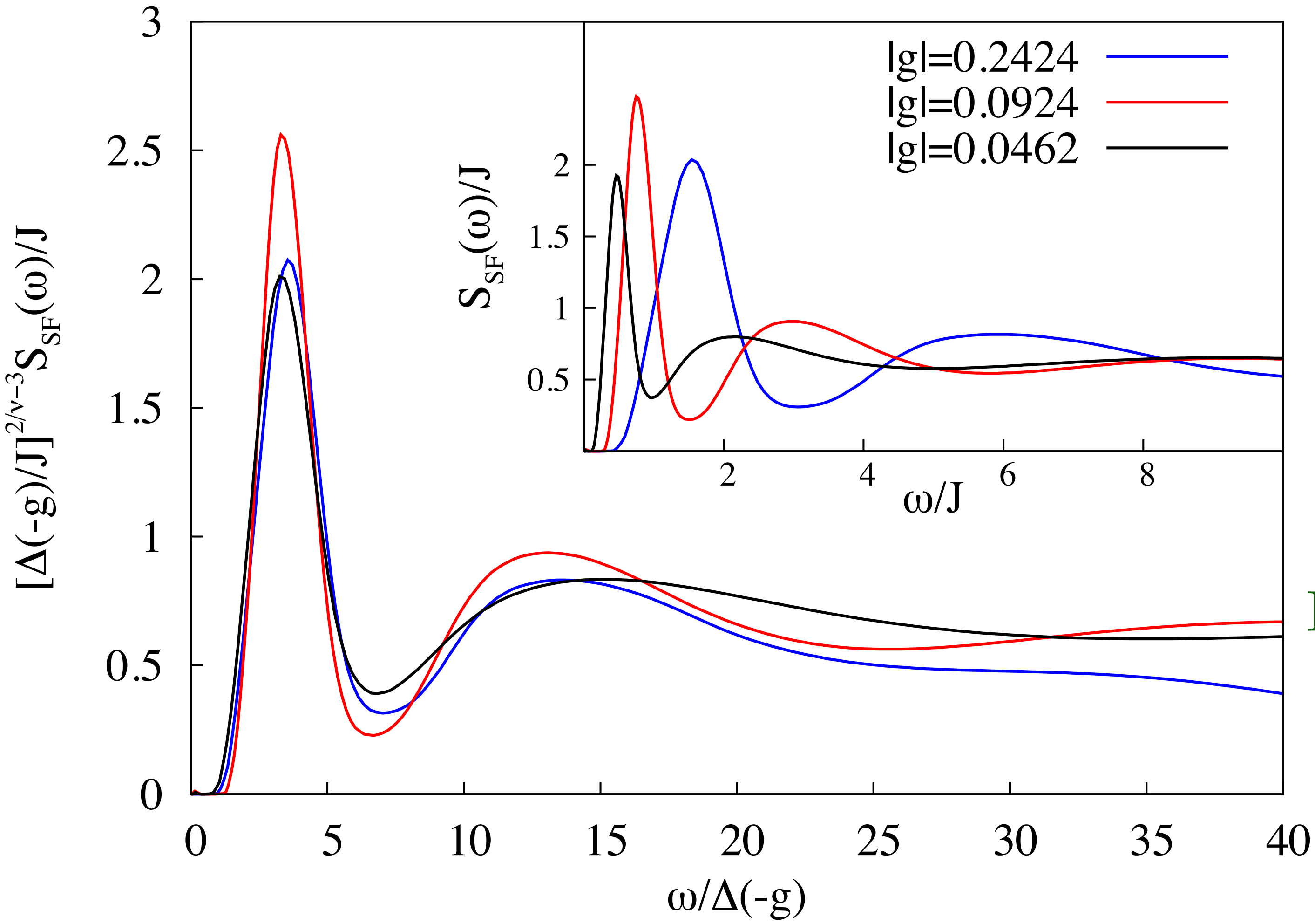
Response to modulation of lattice depth scales as expected from the LHP pole





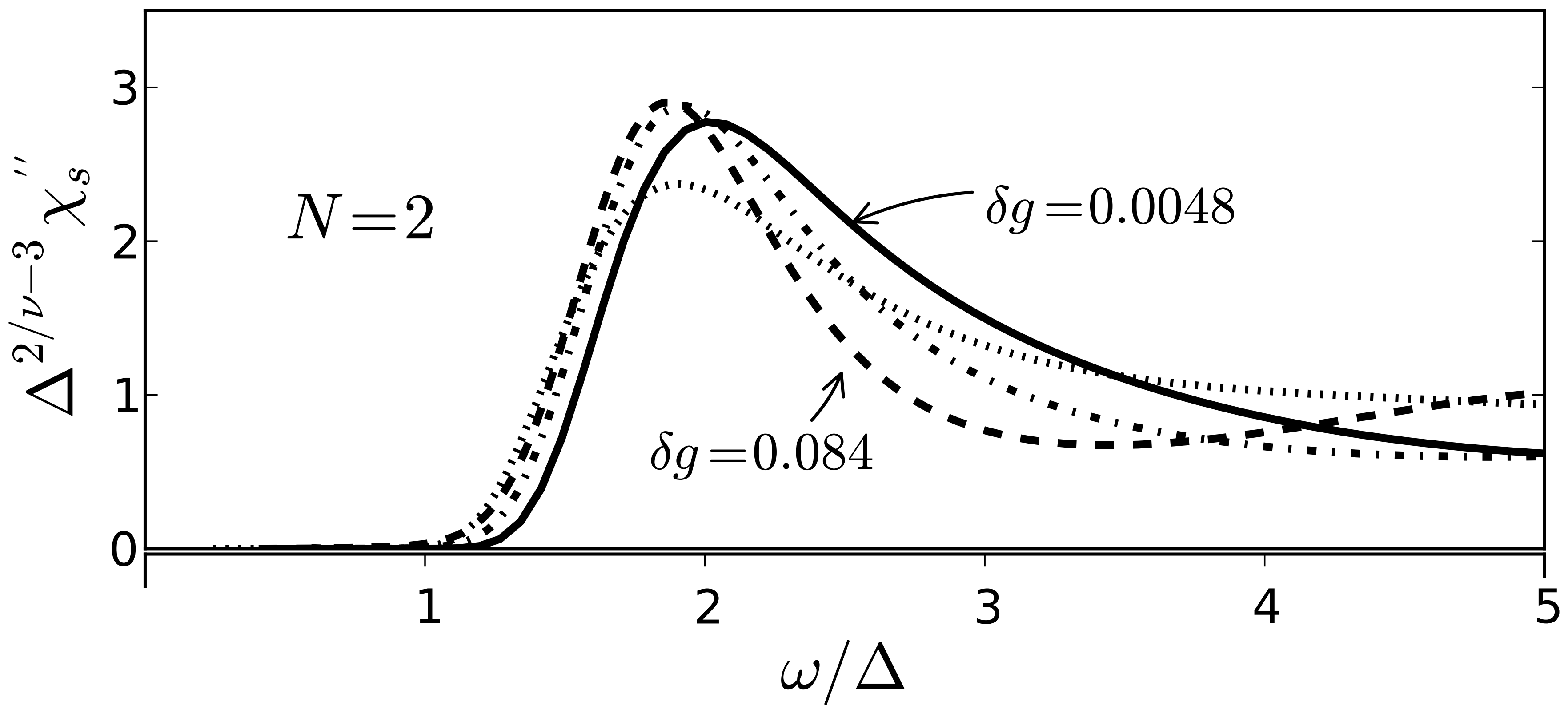
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode in quantum Monte Carlo



Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, and Nikolay Prokof'ev, arXiv:1301.3139

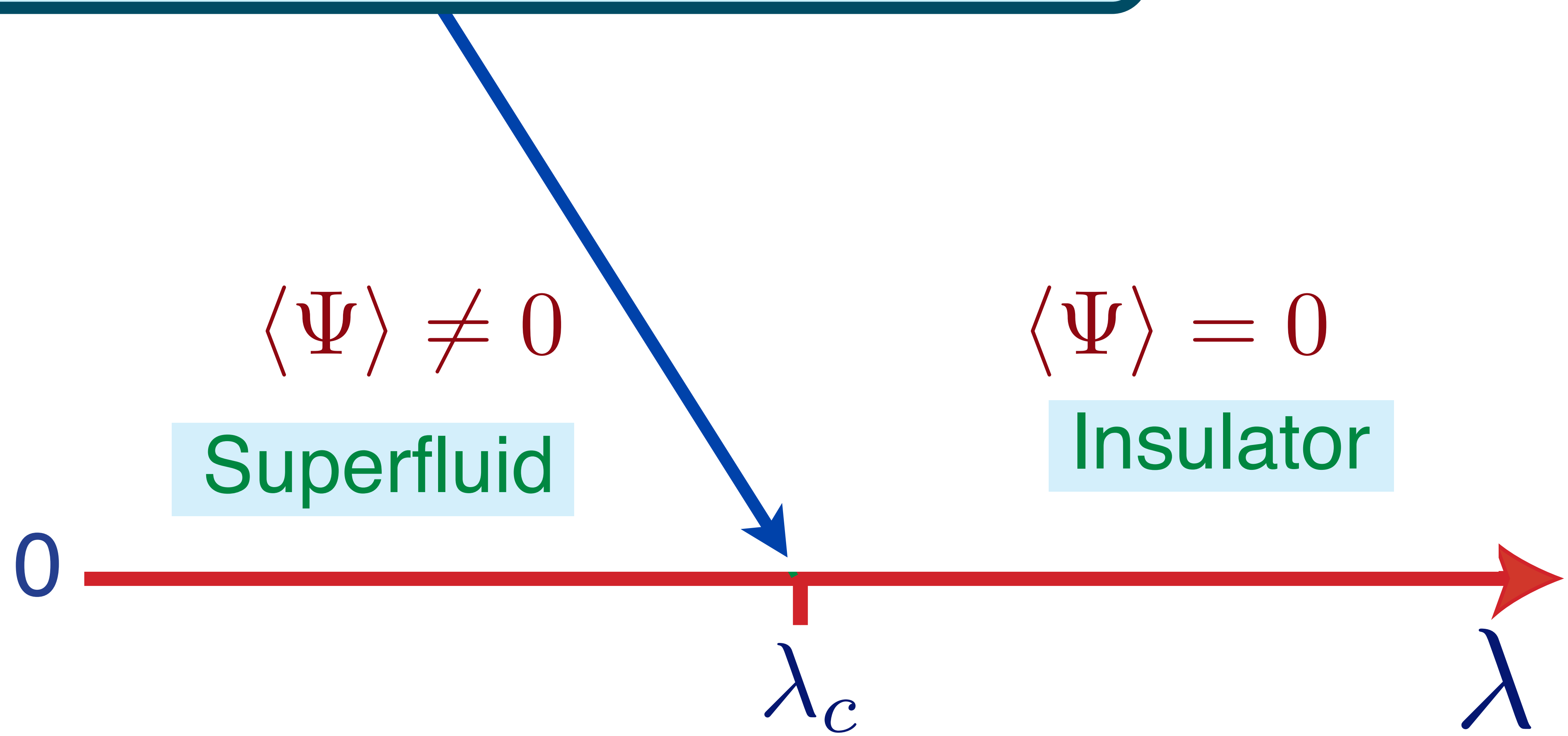


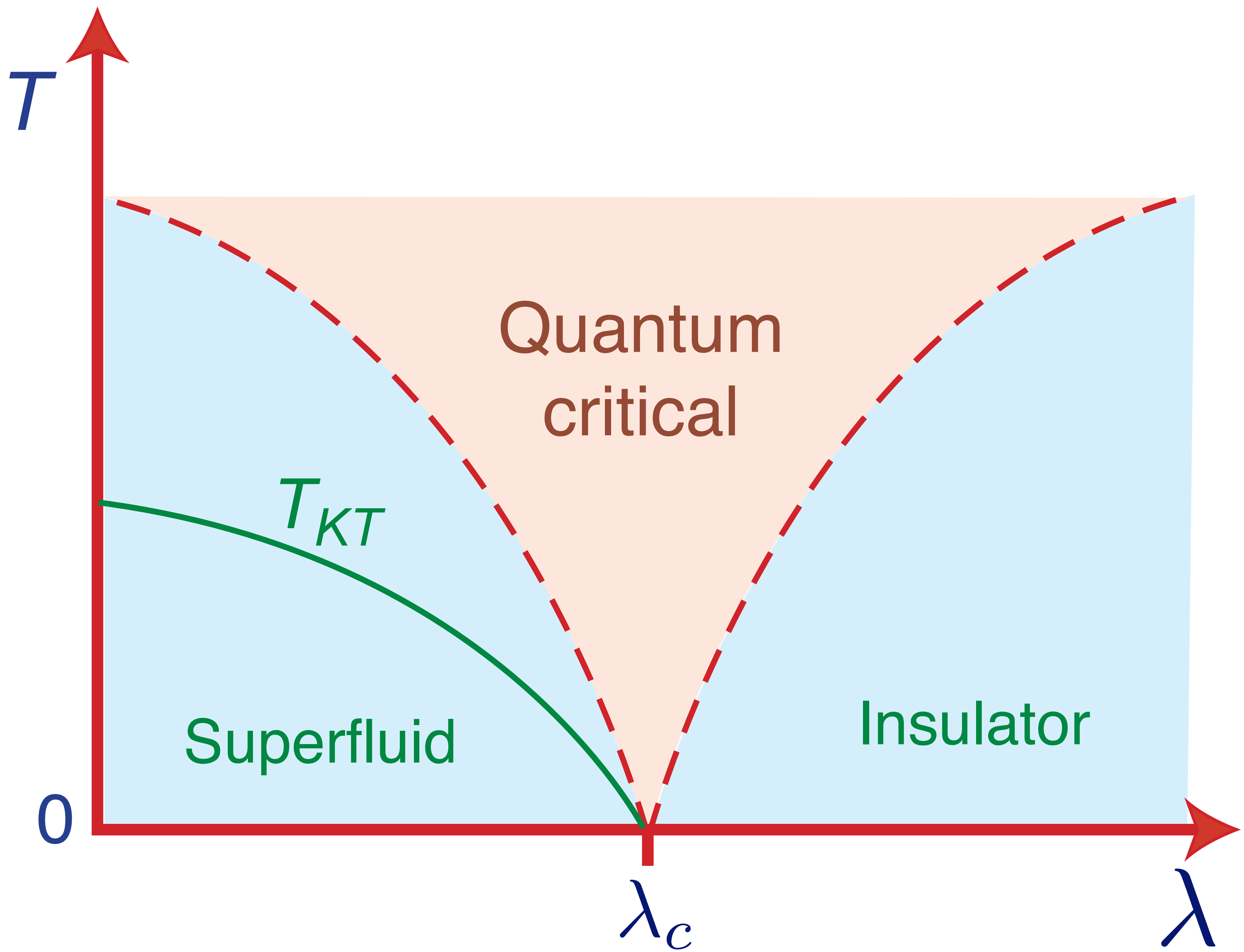
Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759

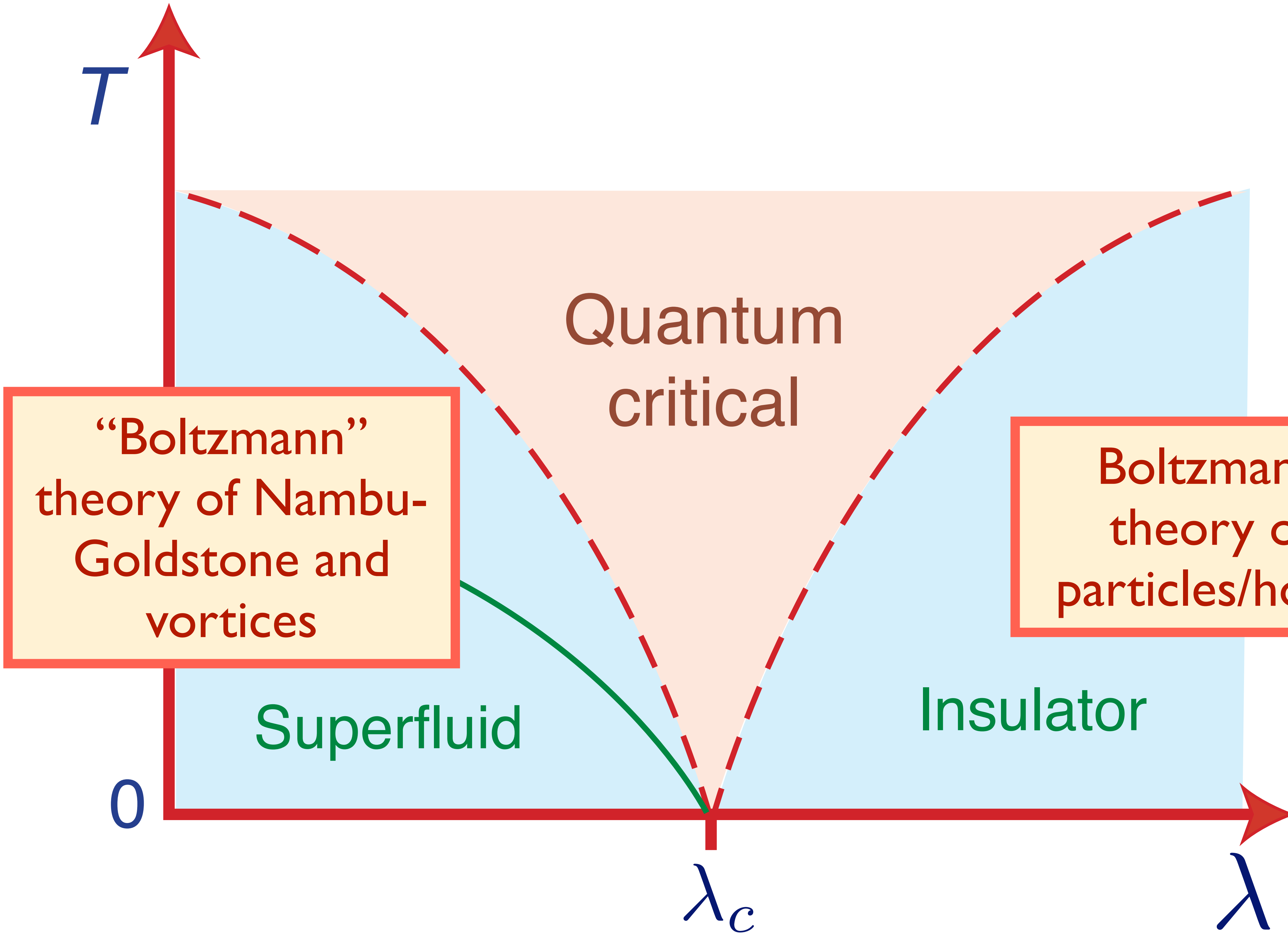
$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

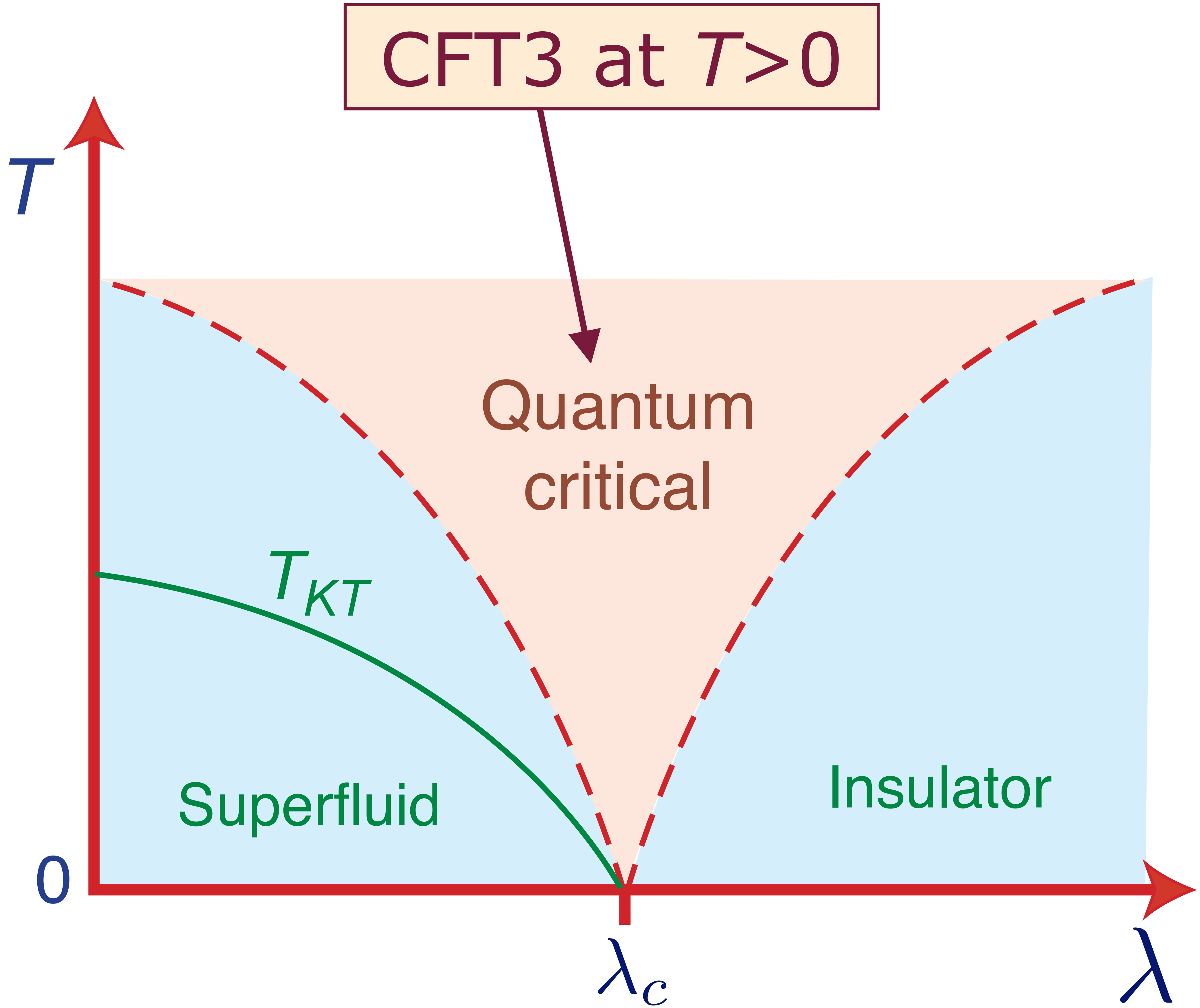
$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

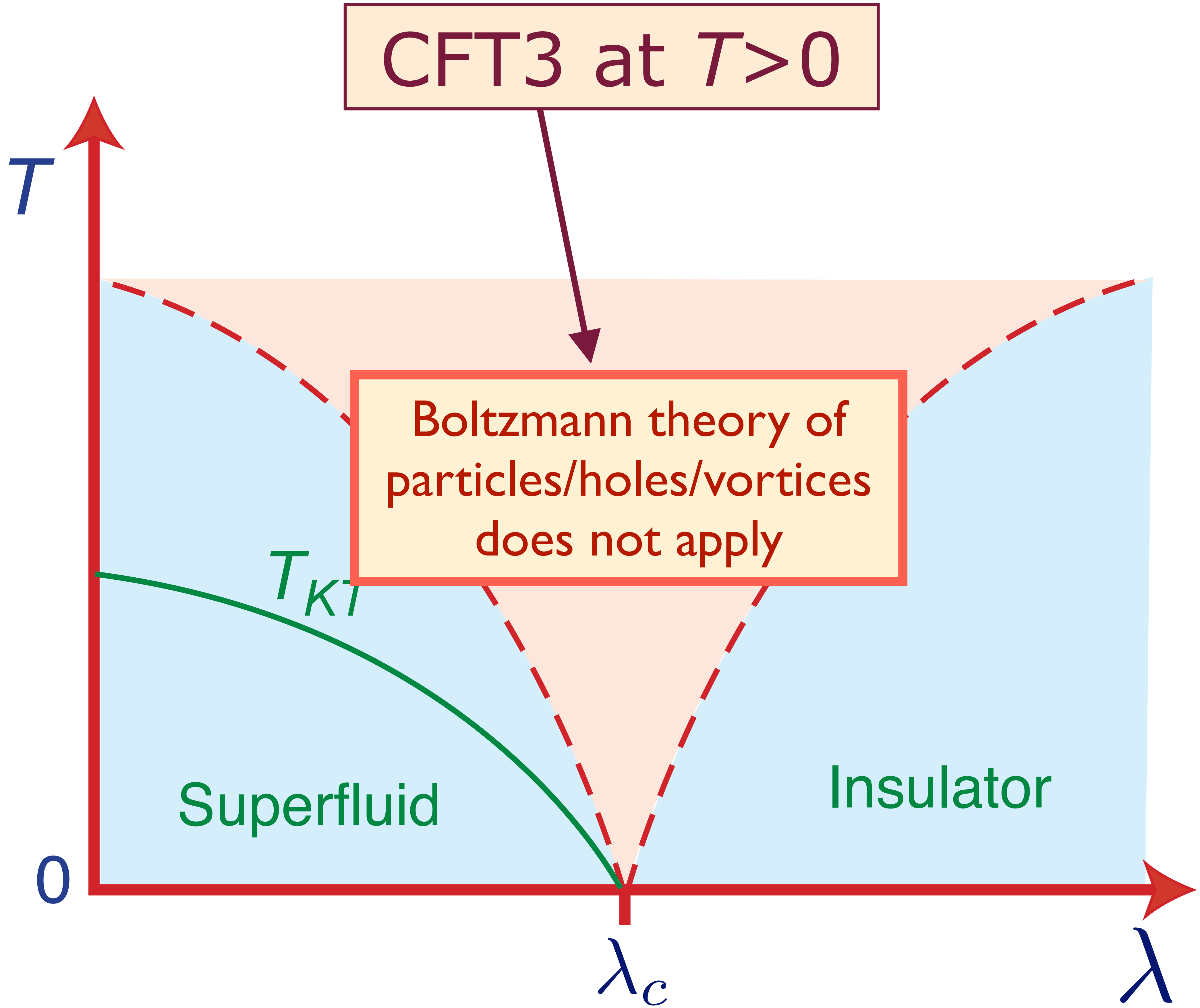
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3

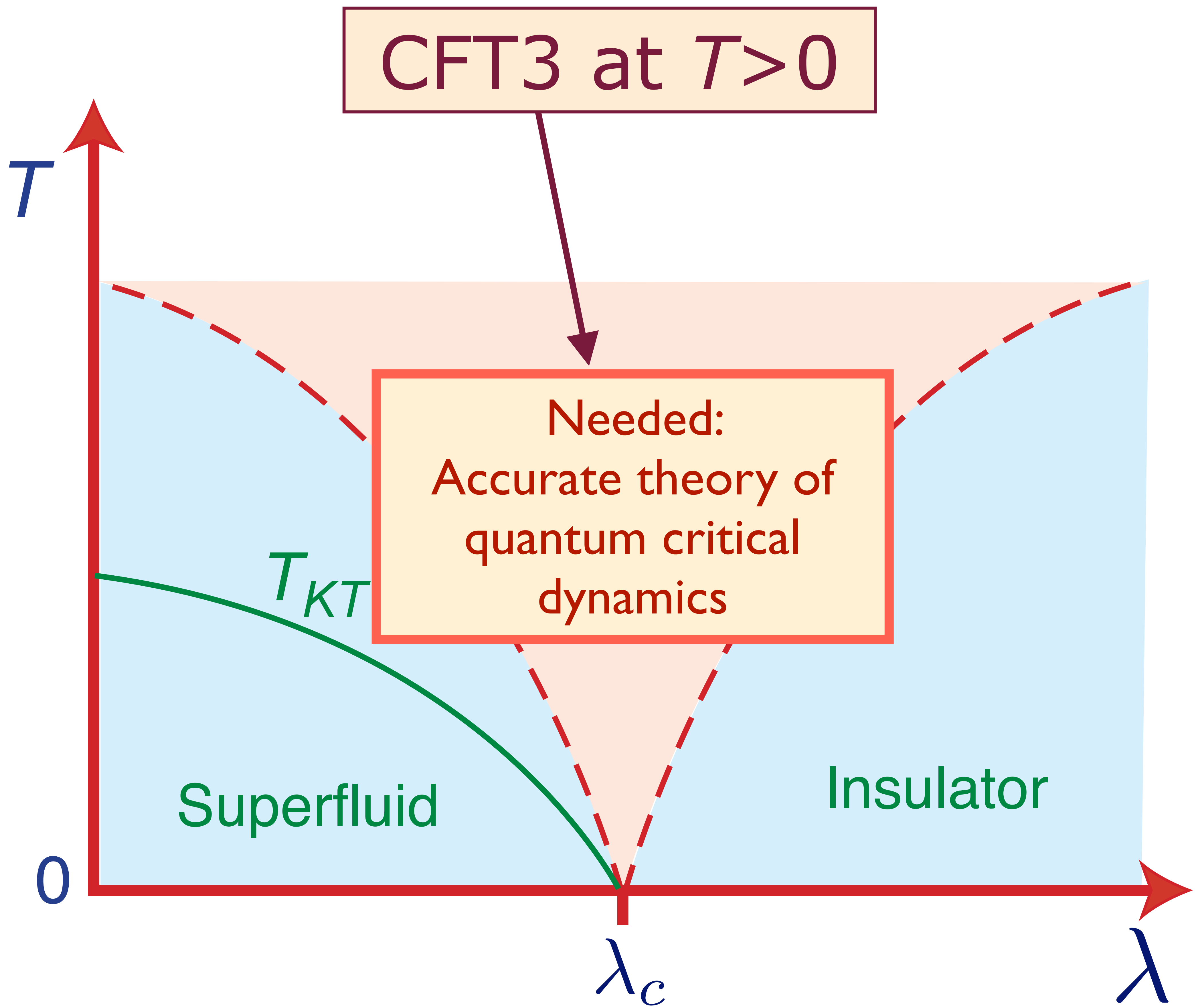



T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


T
Quantum
critical
"Boltzmann"
theory of Nambu-
Goldstone and
vortices
Boltzmann
theory of
particles/holes
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λc
λ


CFT3 at $T>0$
T
Boltzmann theory of
particles/holes/vortices
does not apply
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Needed:
Accurate theory of
quantum critical
dynamics
T KT
Superfluid
Insulator
0
λc
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C} \frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

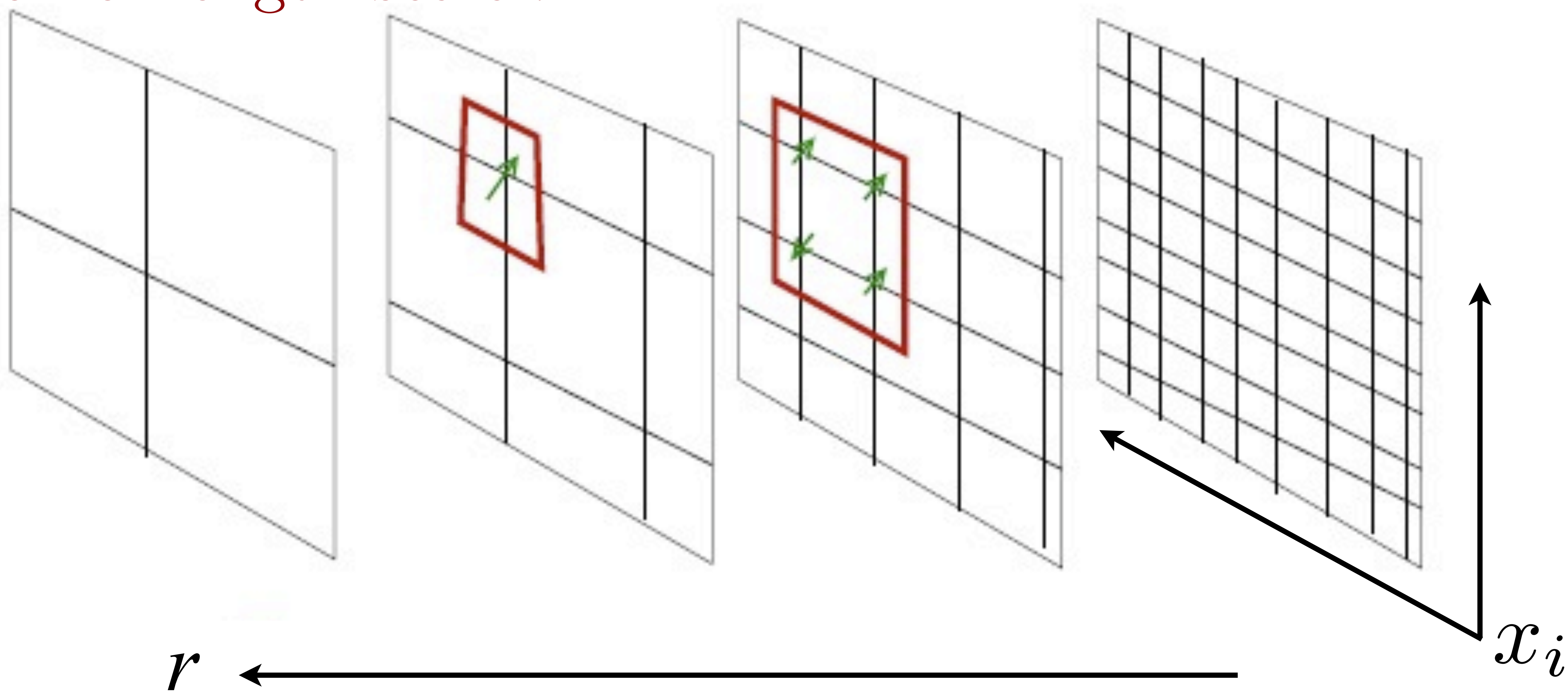
Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

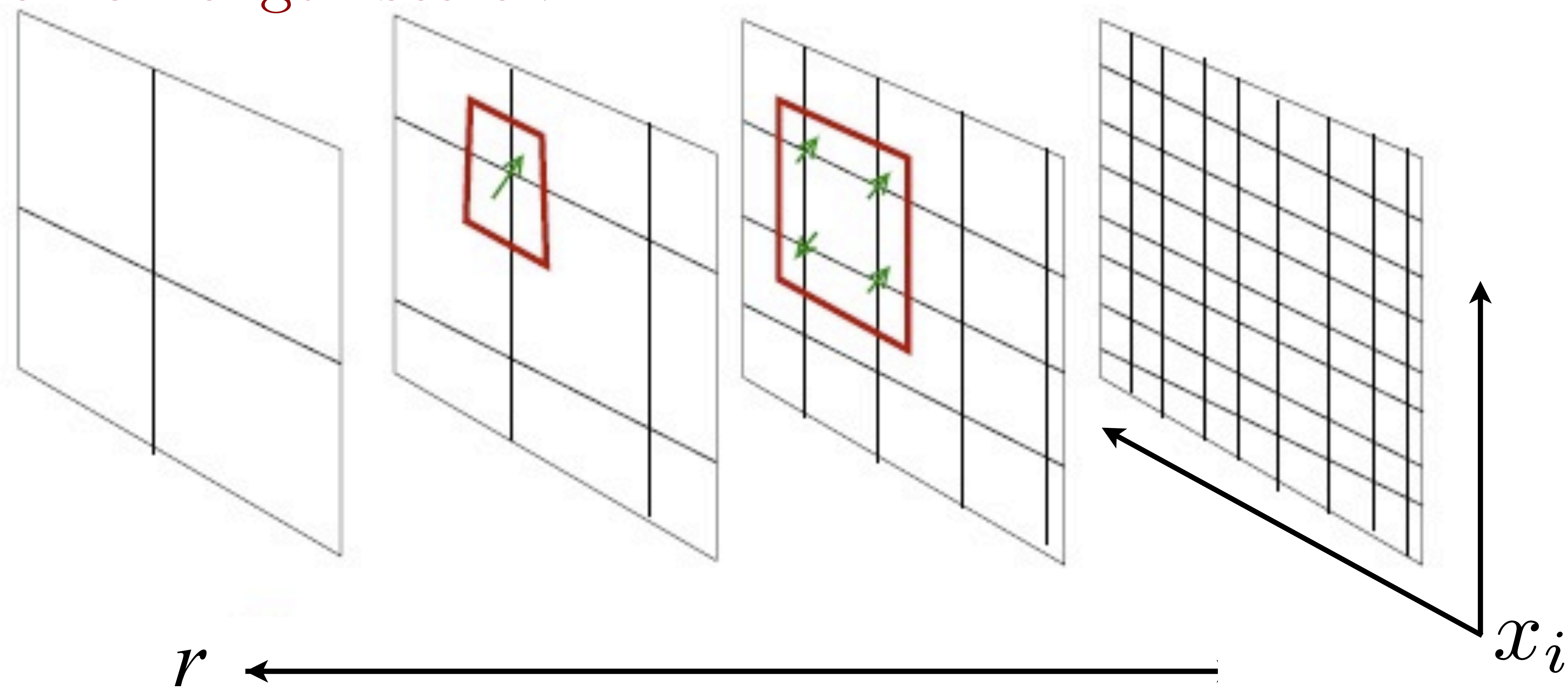
M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



J. McGreevy, arXiv0909.0518

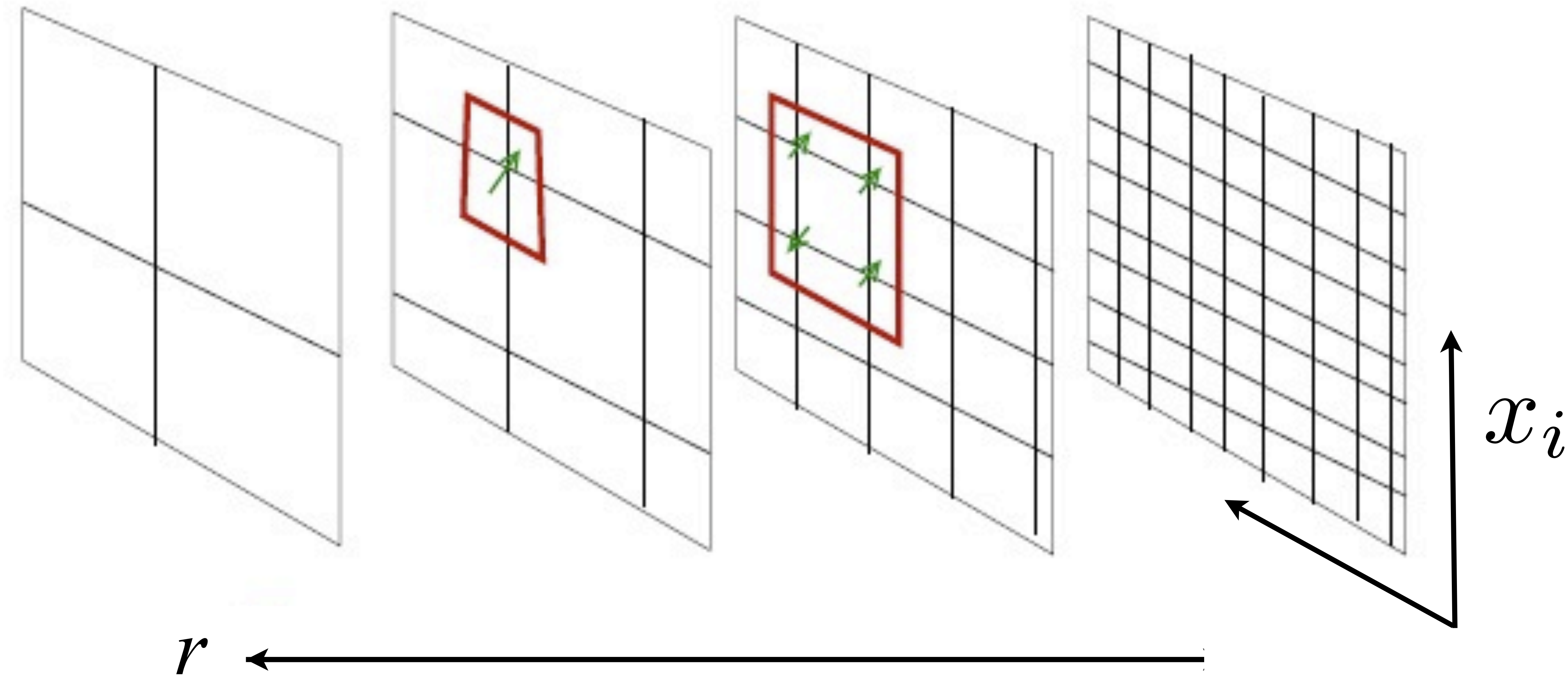
Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518

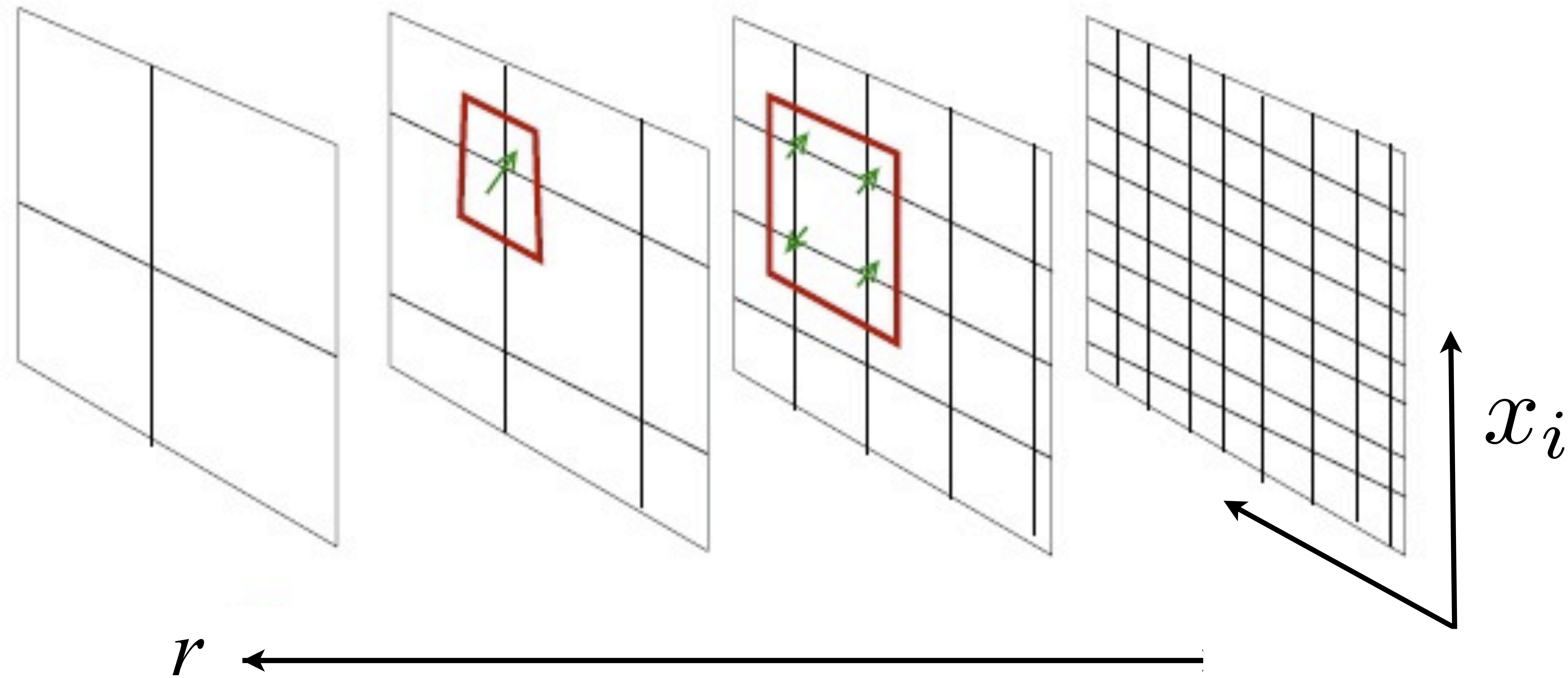
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

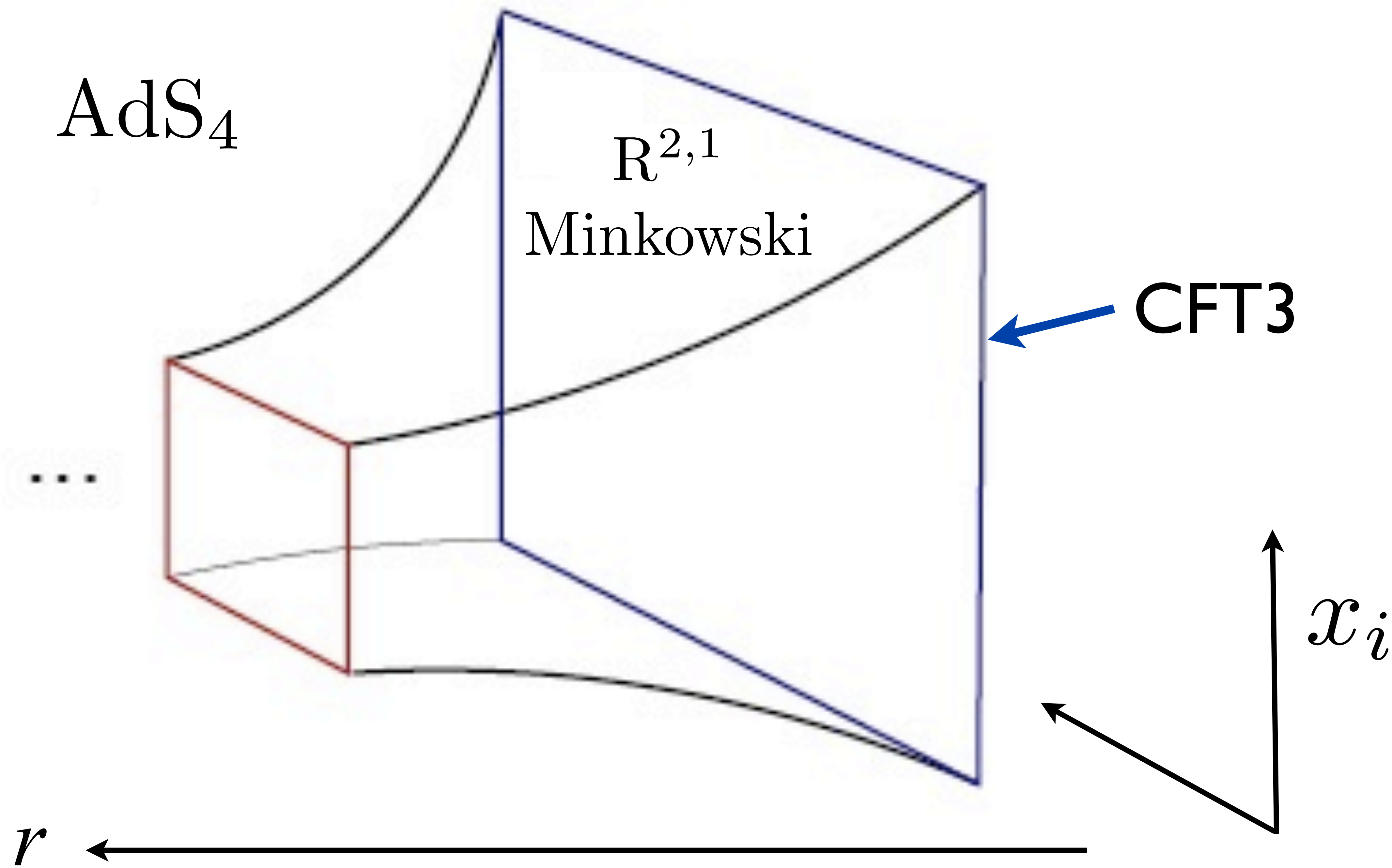


This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

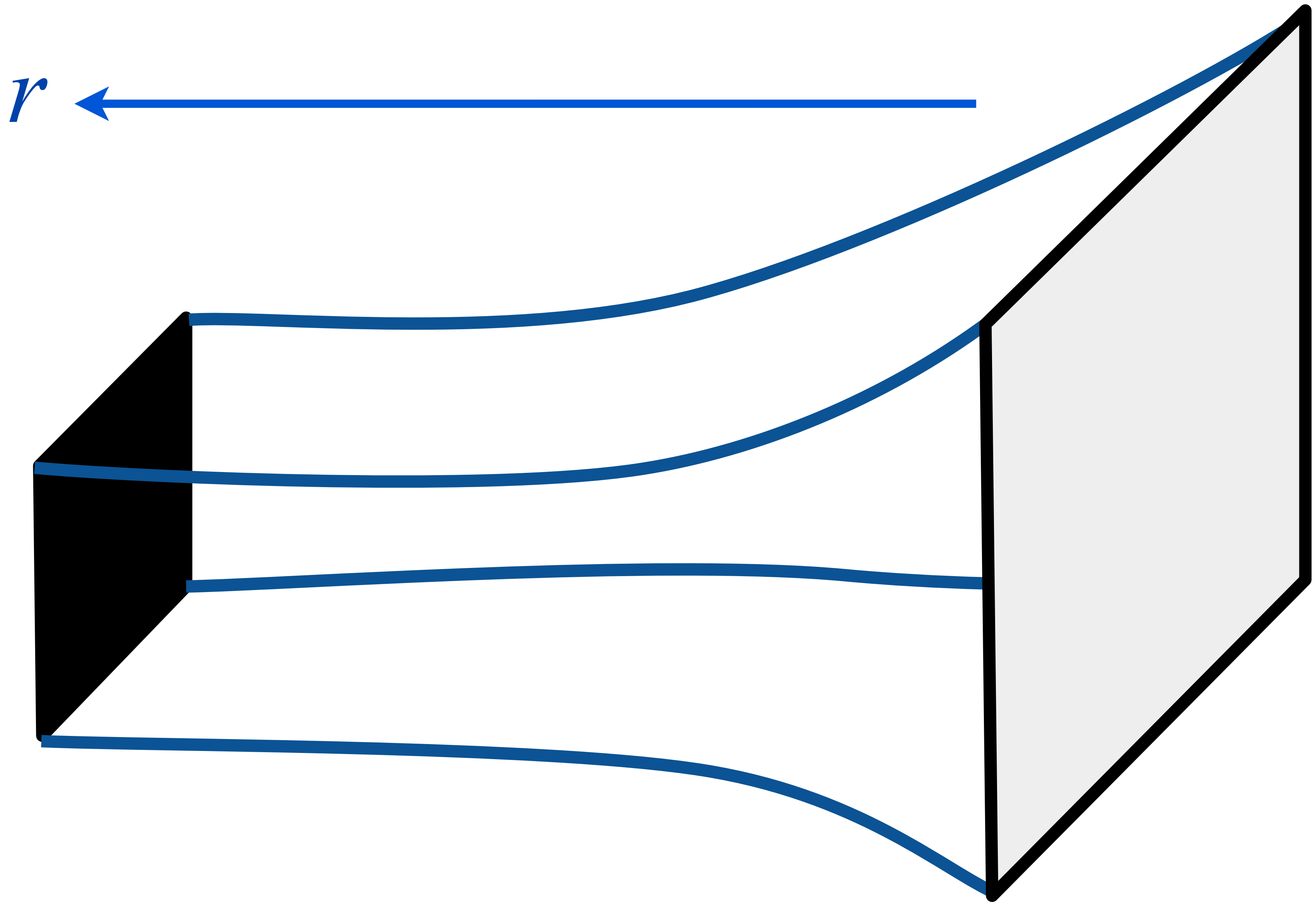
AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

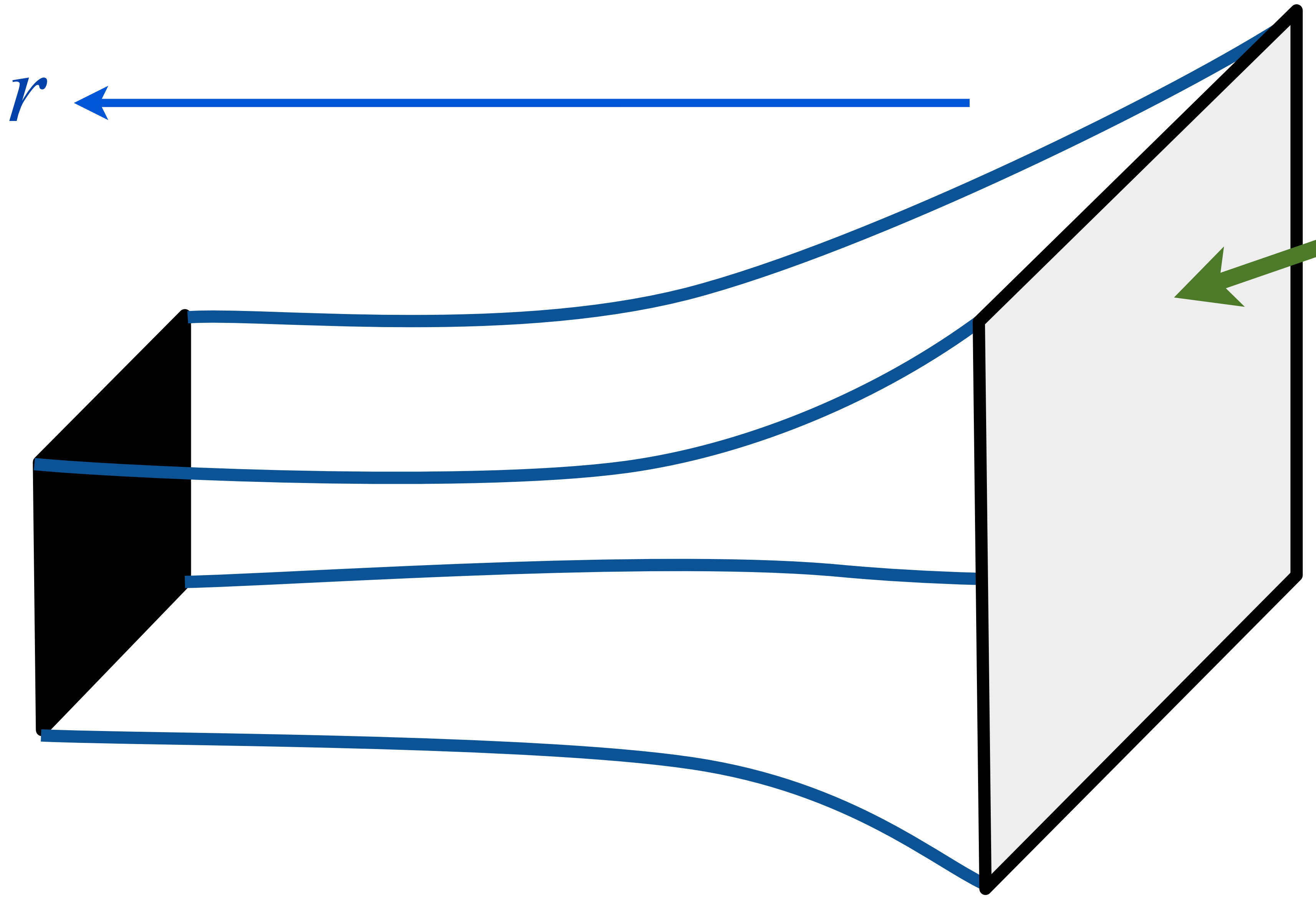
Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

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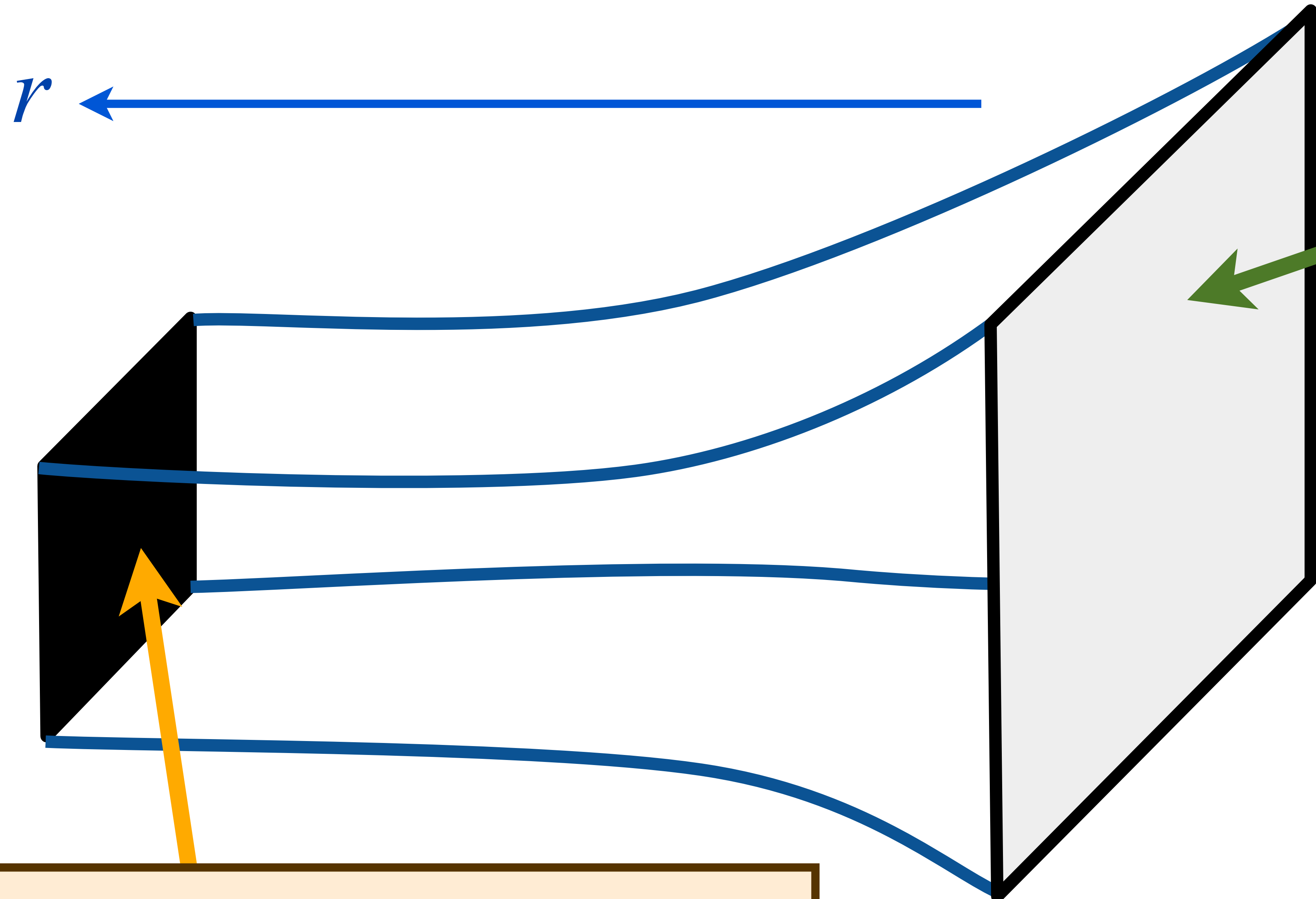
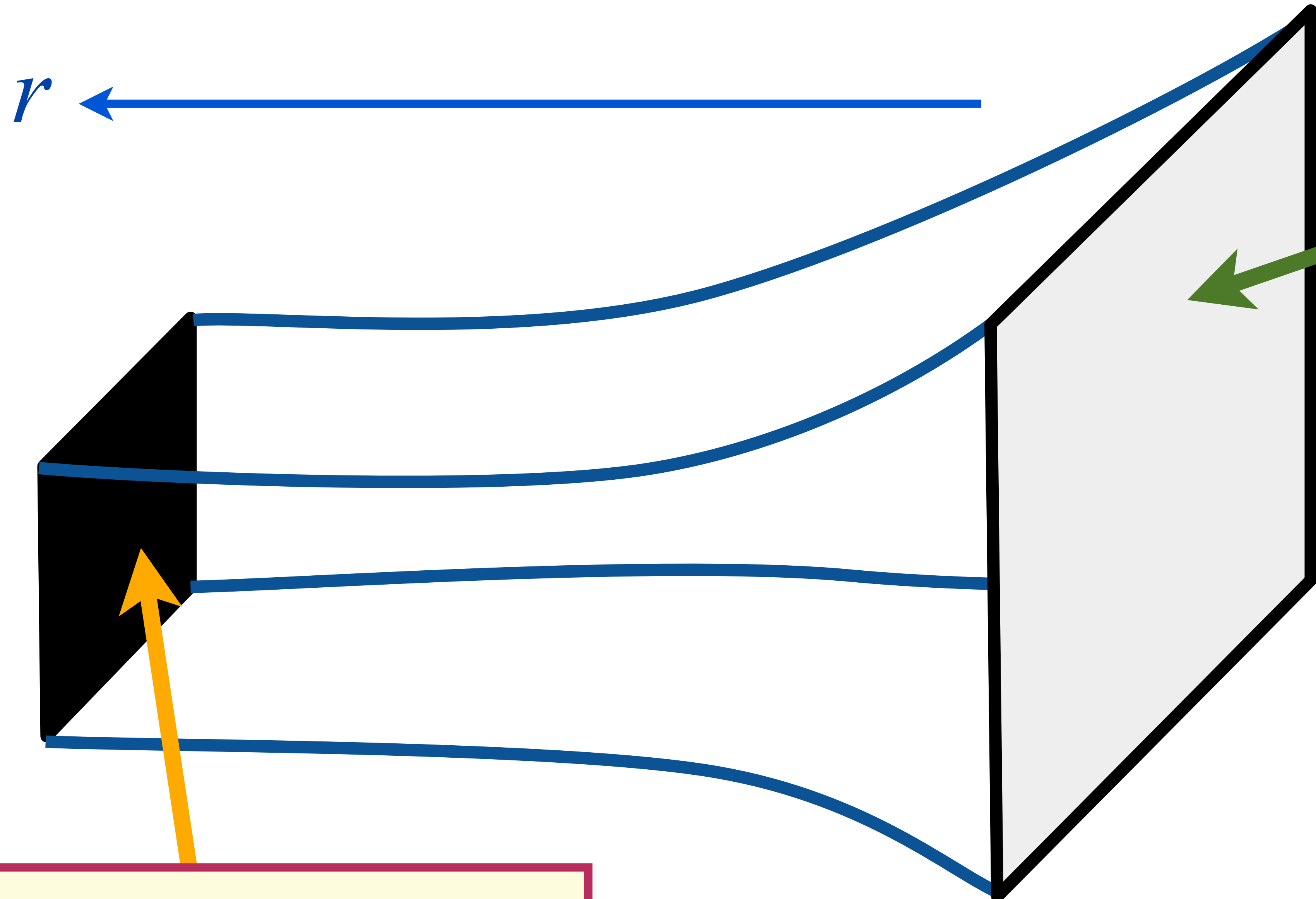


A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

A "horizon", similar to the surface of a black hole !

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

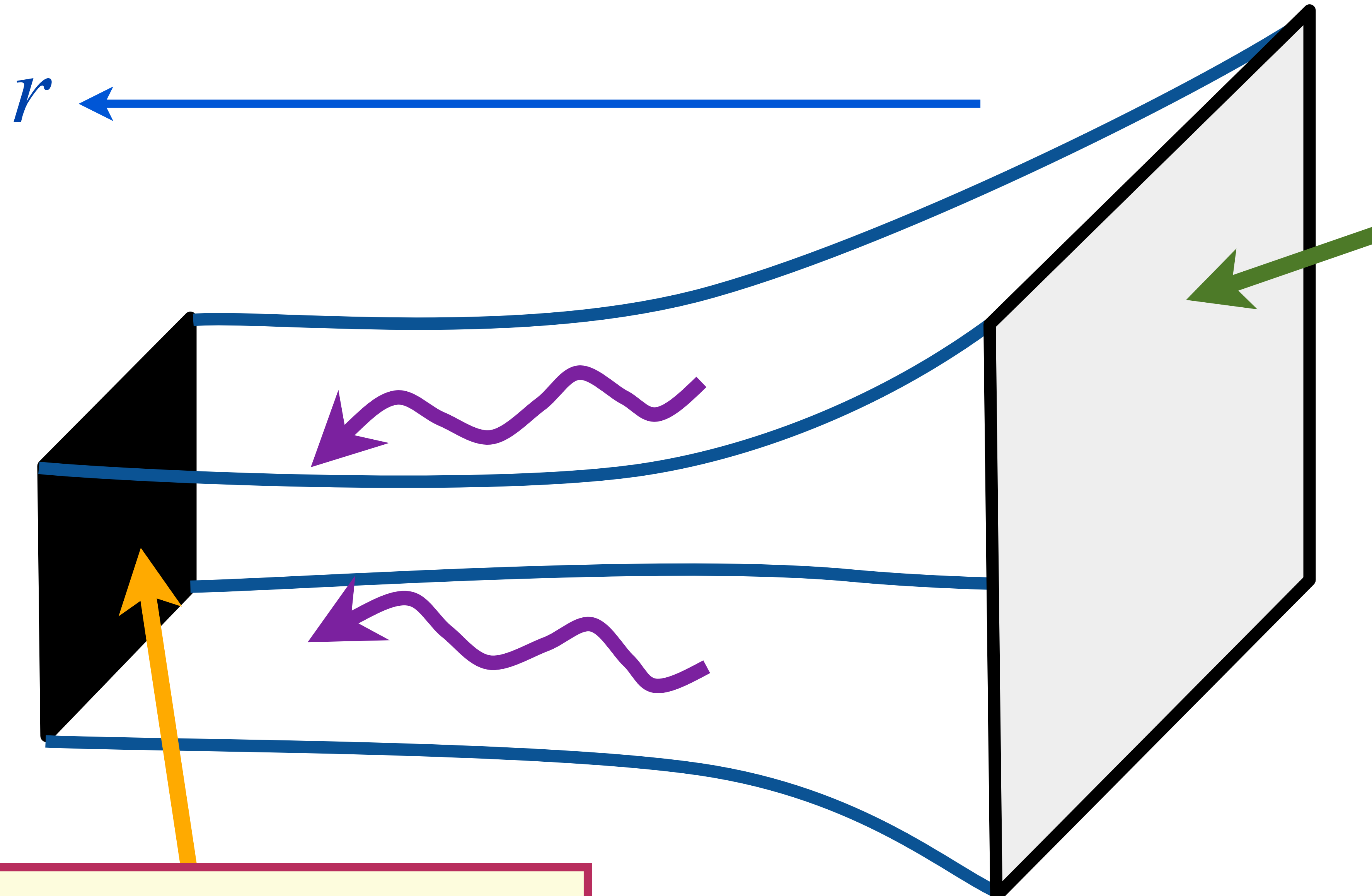
Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Gauge-gravity duality at non-zero temperatures

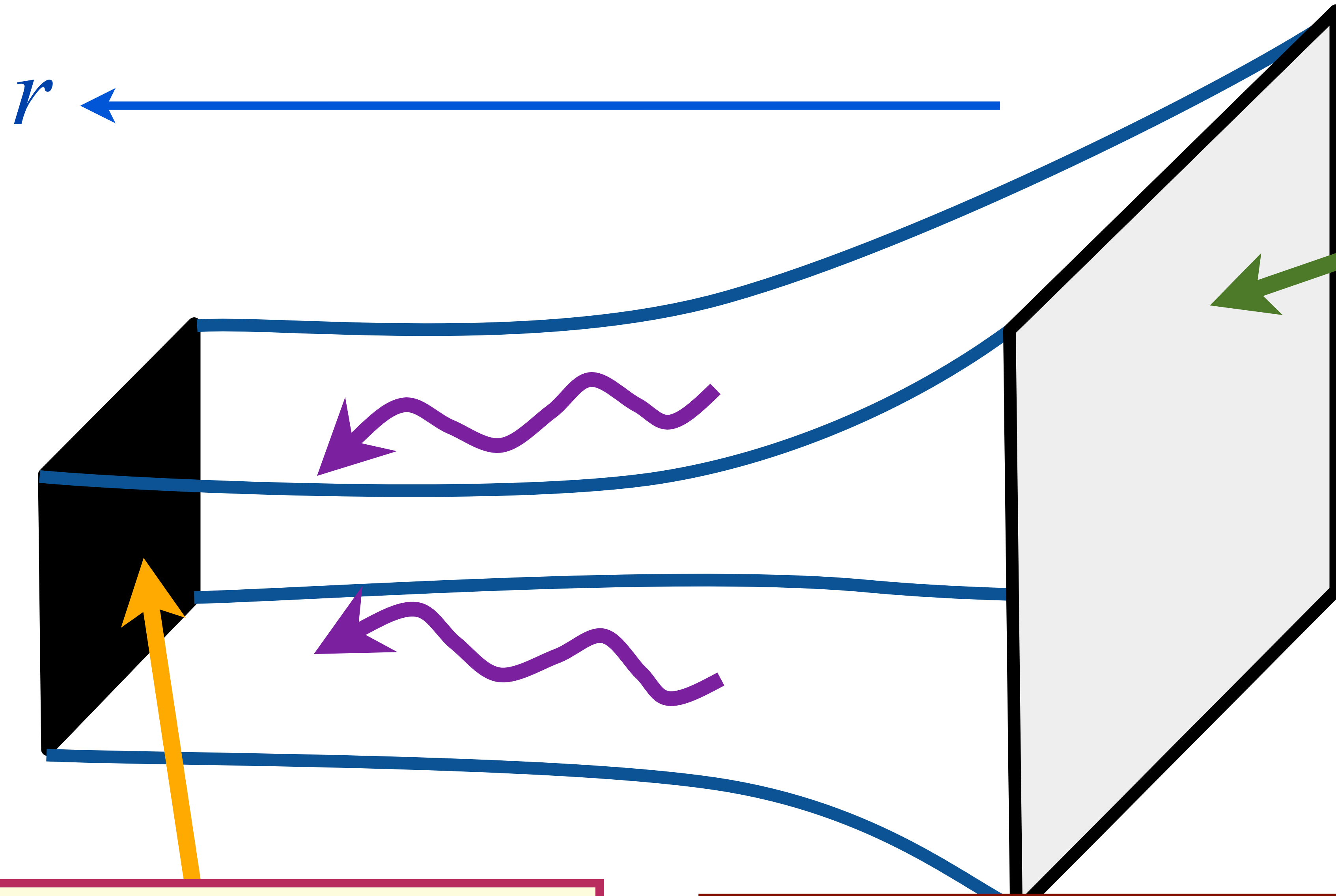


A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Quasi-normal modes of quantum criticality = waves falling into black hole

Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Characteristic damping time of quasi-normal modes: $(k_B/\hbar)\times$ Hawking temperature

AdS$_4$ theory of quantum criticality

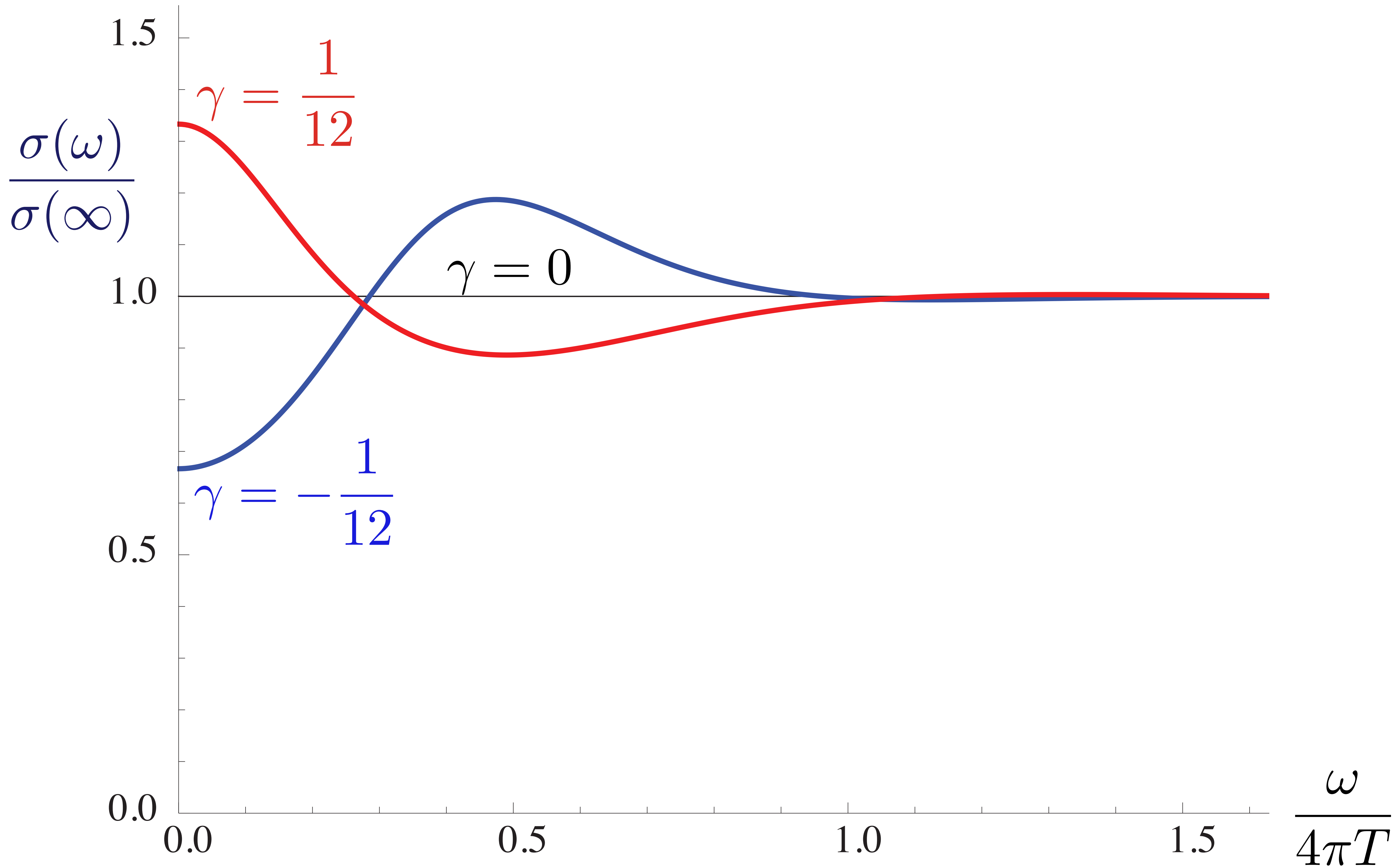
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator.

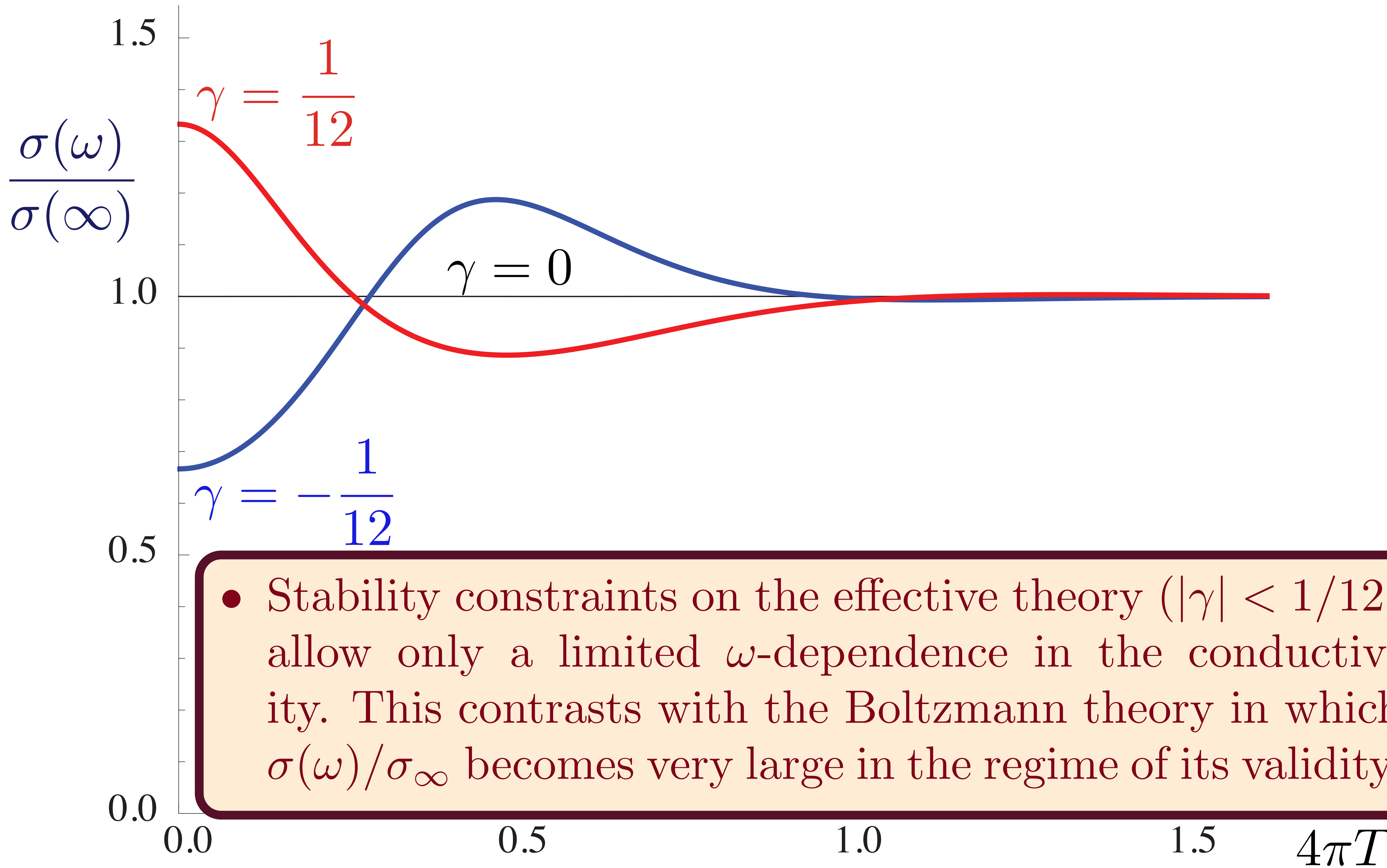
R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS$_4$ theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity. This contrasts with the Boltzmann theory in which $\sigma(\omega)/\sigma_\infty$ becomes very large in the regime of its validity.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality

PRL **95,** 180603 (2005) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2005

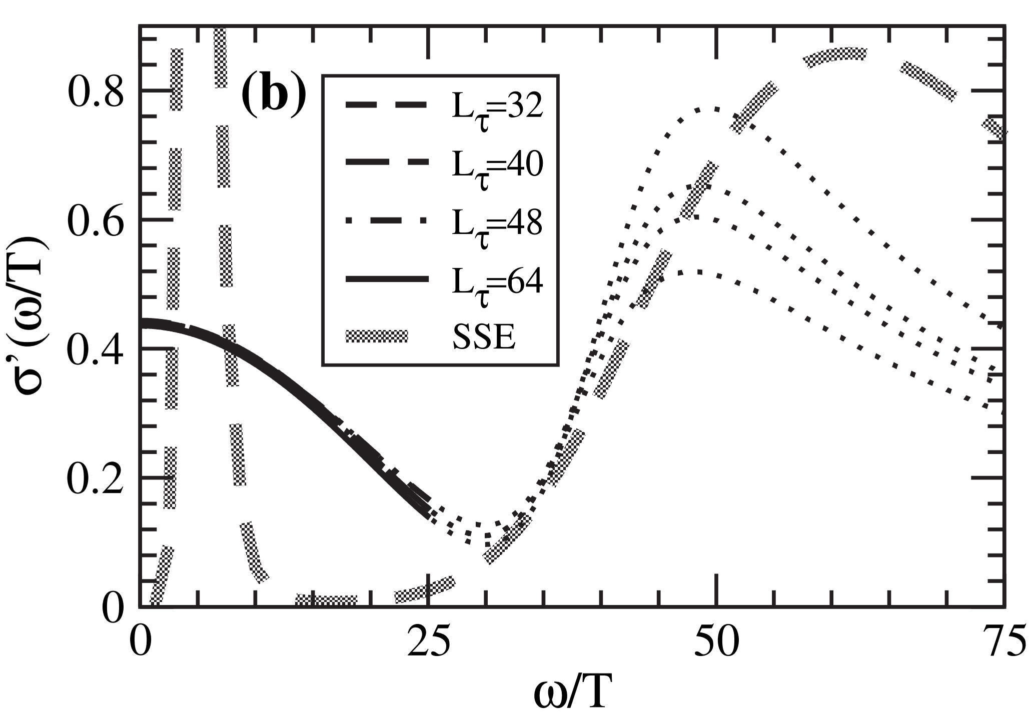
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

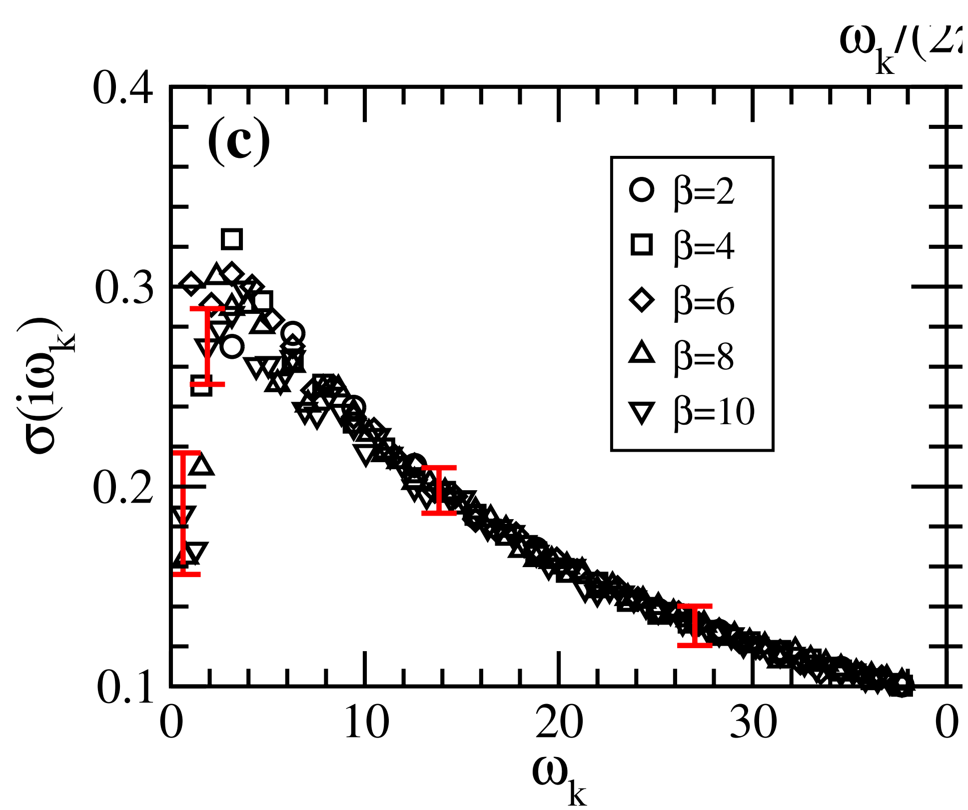
Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T. We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.





AdS$_4$ theory of quantum criticality

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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

   Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity

   The pnictides and the cuprates

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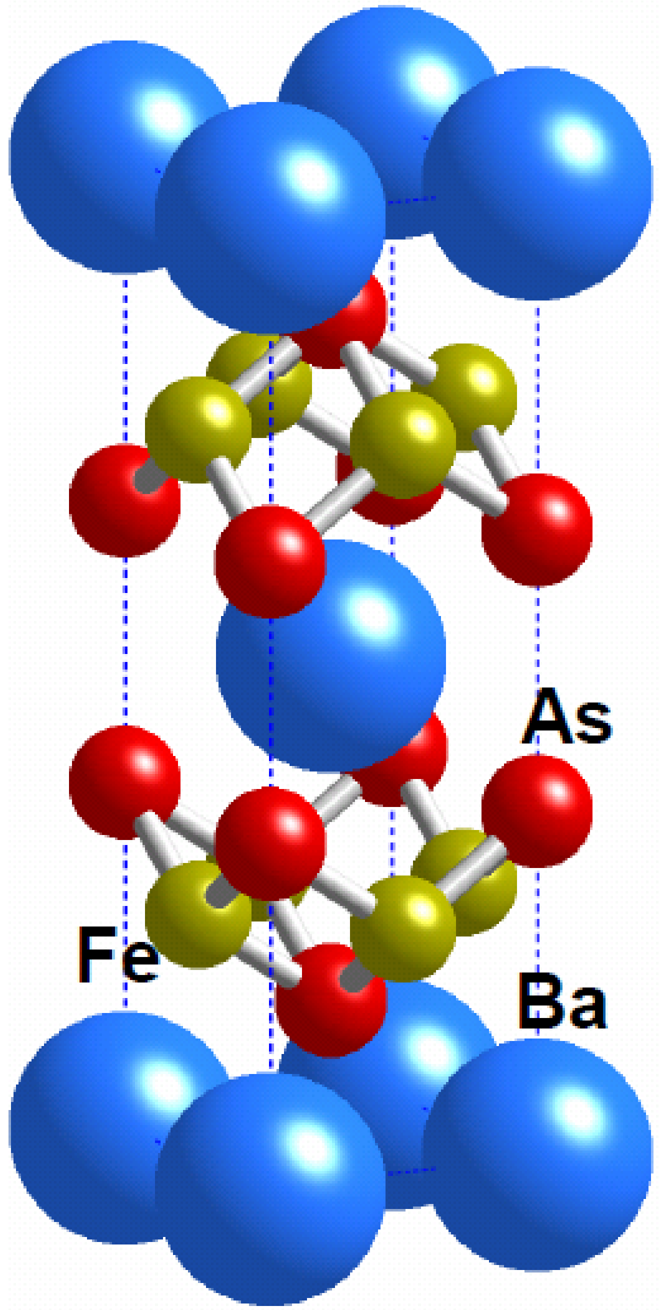
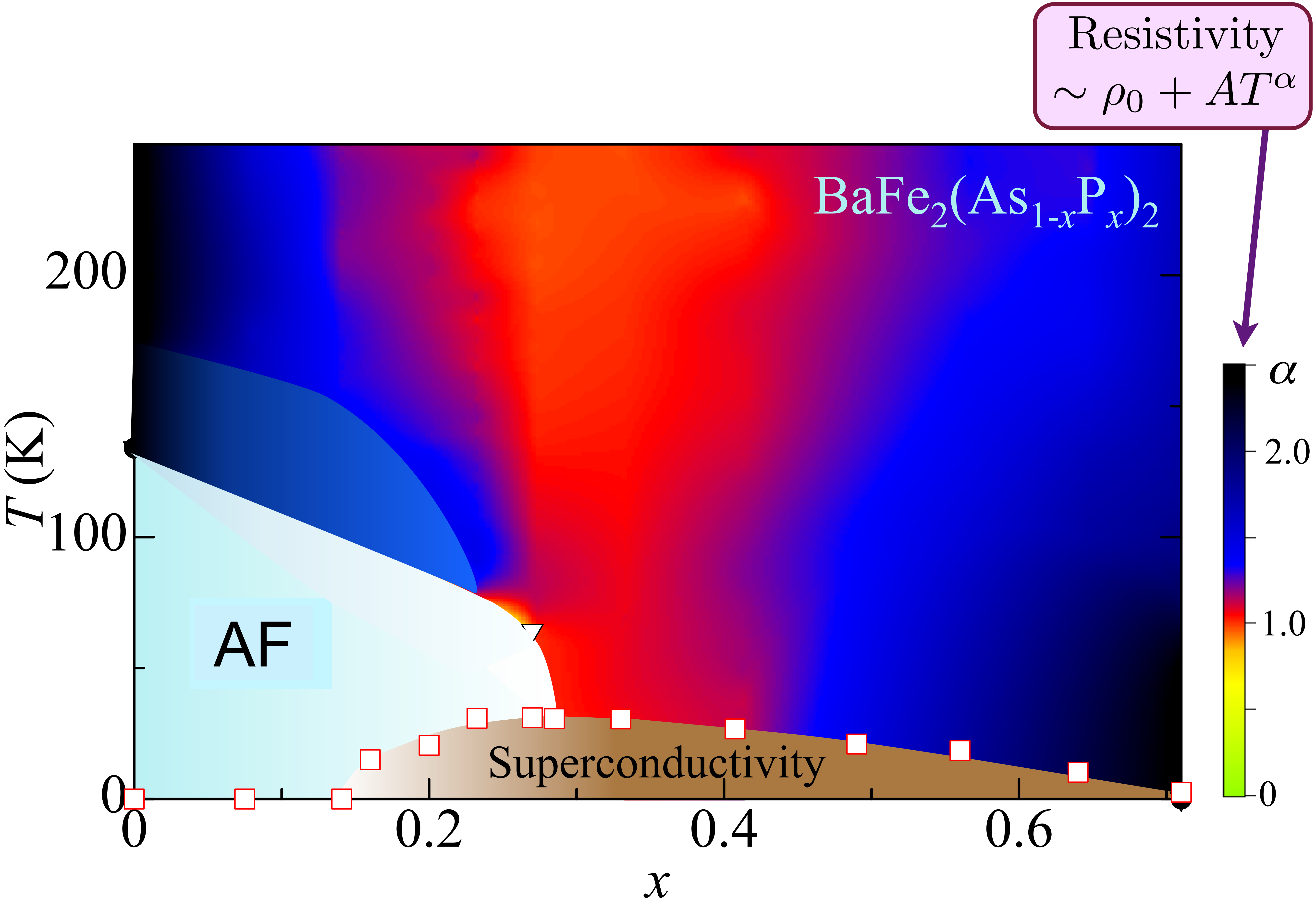
Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity
The pnictides and the cuprates

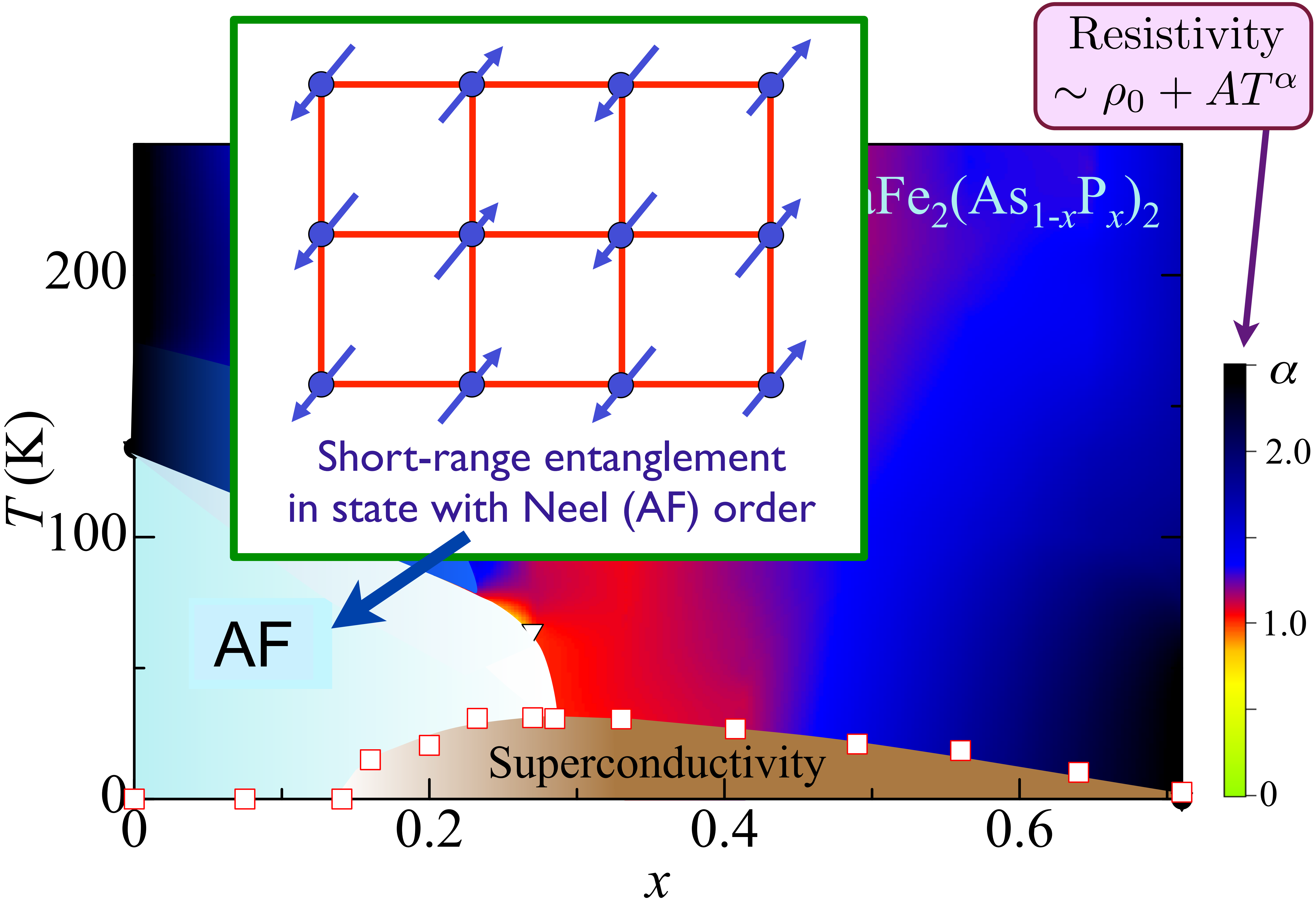
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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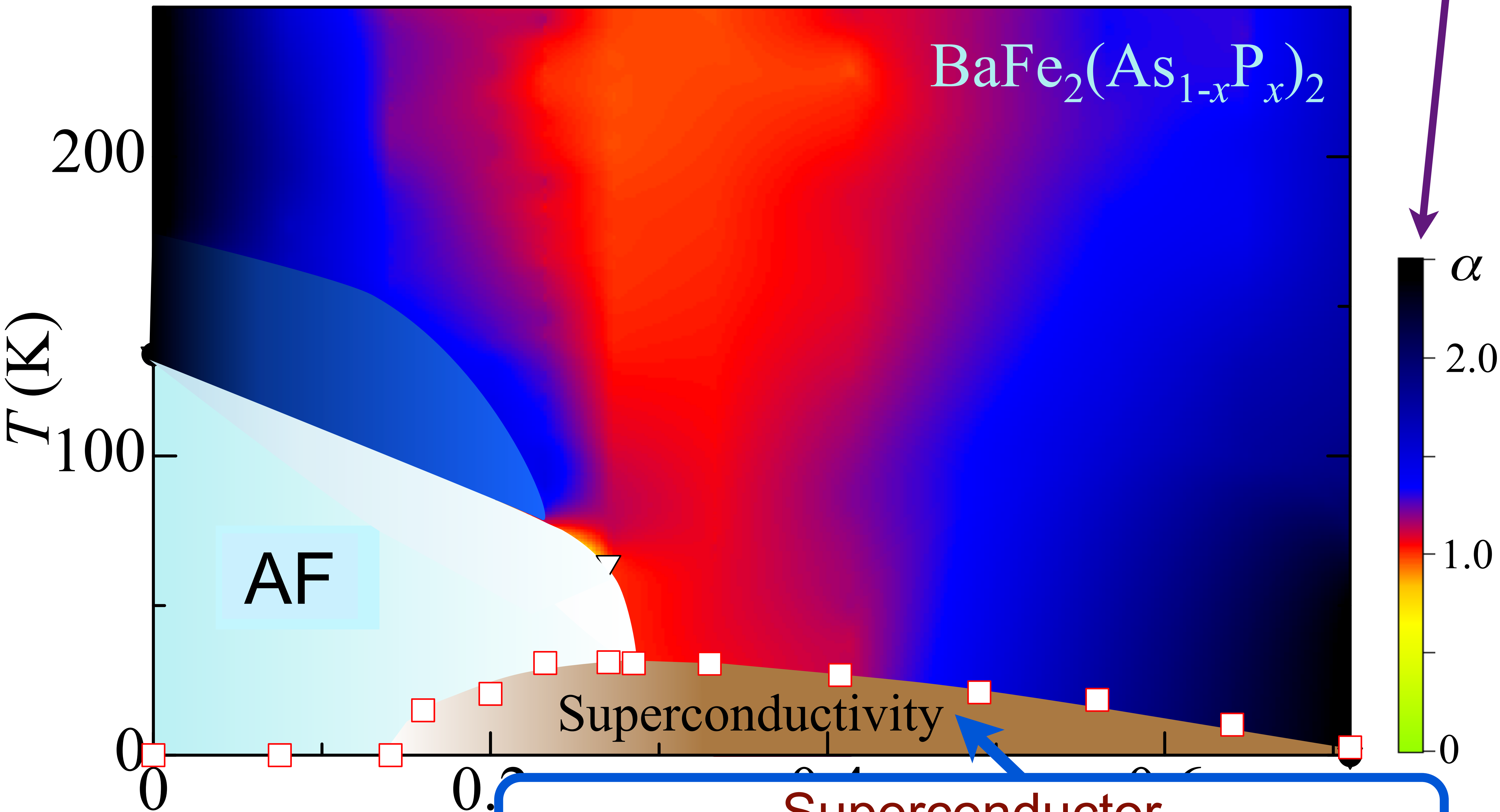
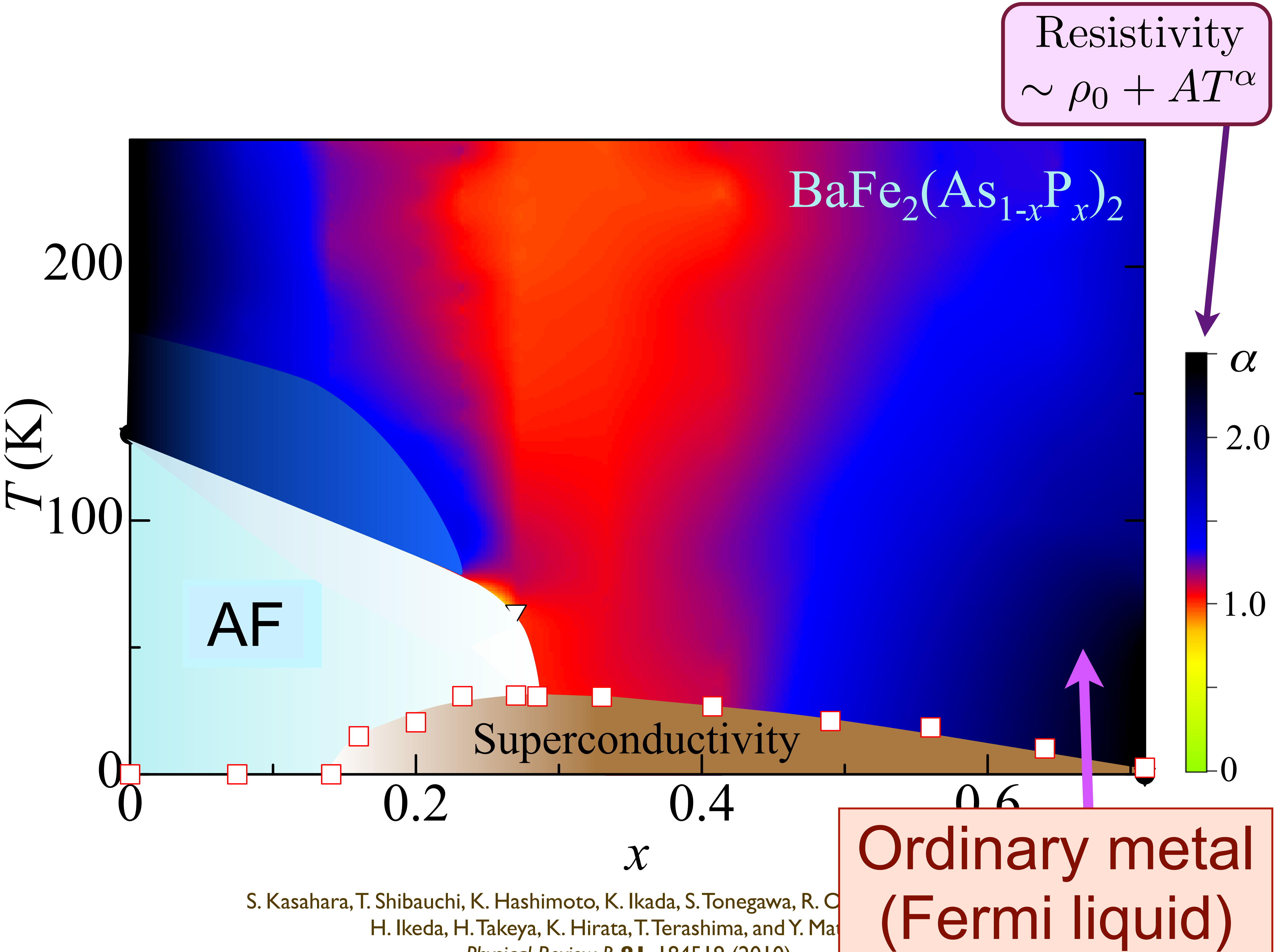
Resistivity
$\sim \rho_0 + AT^\alpha$



Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

S. Kasahara, T. Shiba
H. Ike



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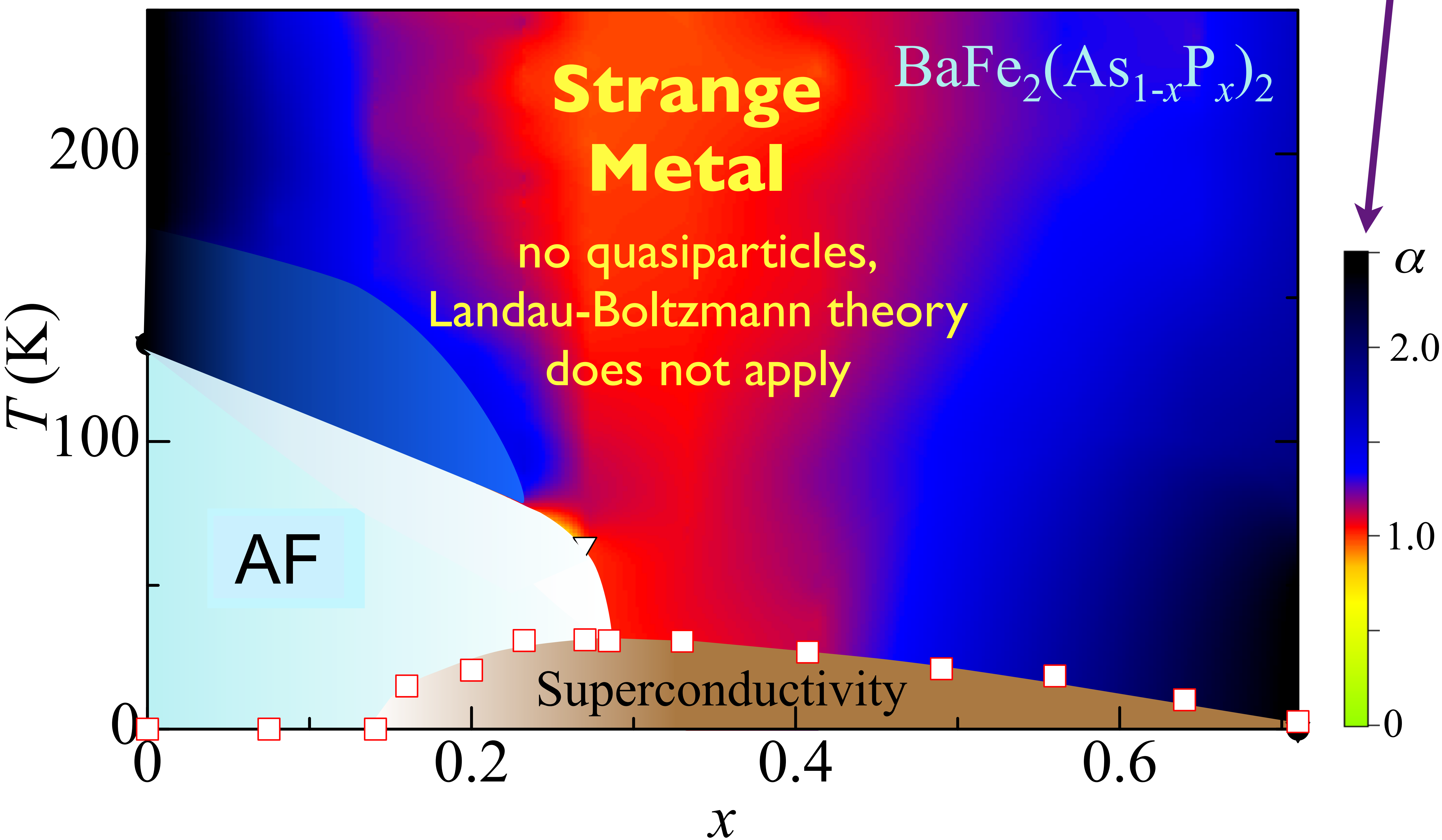

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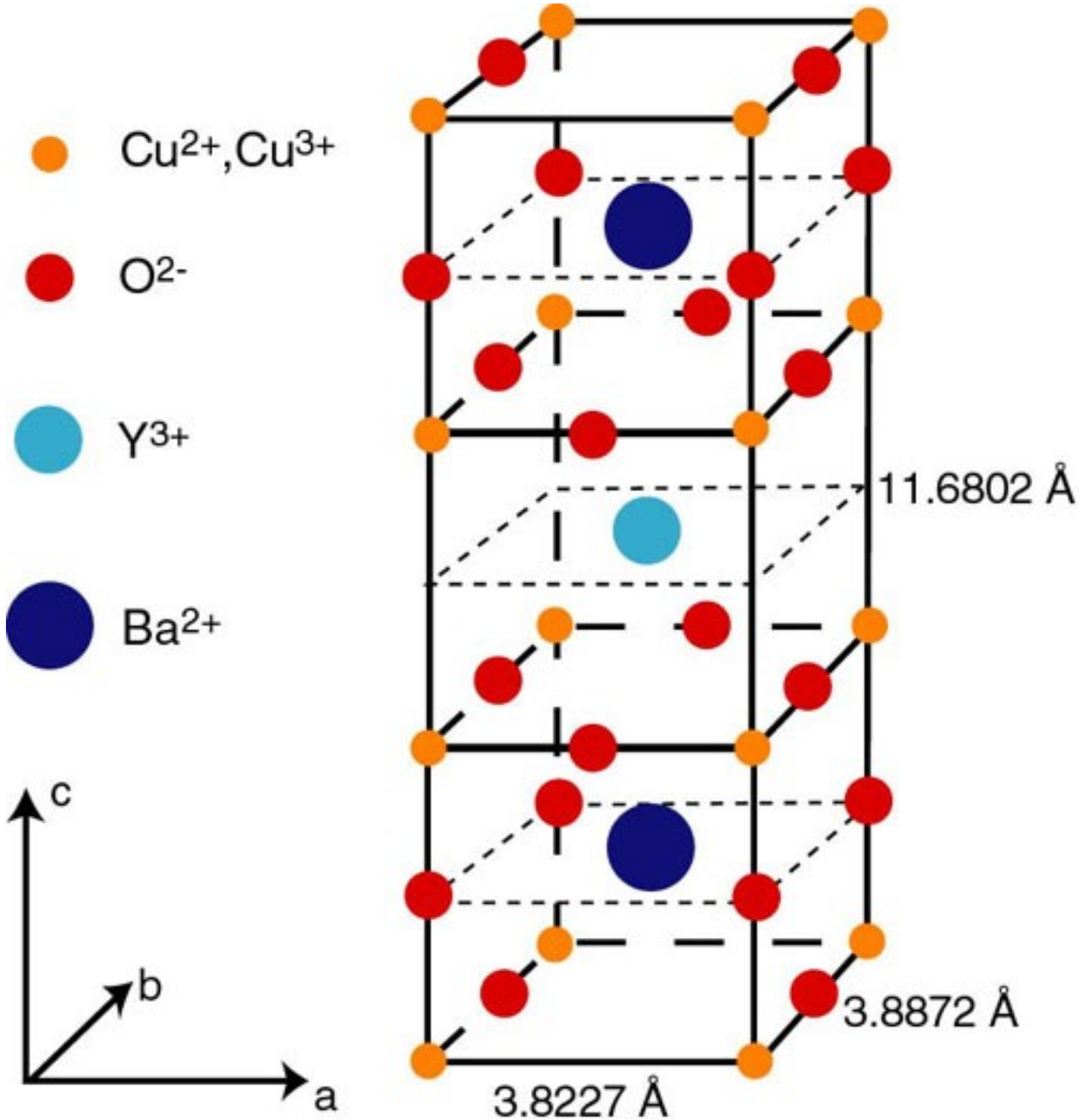
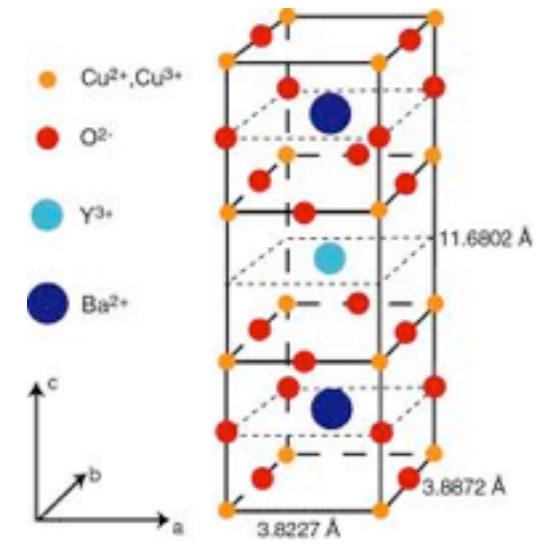
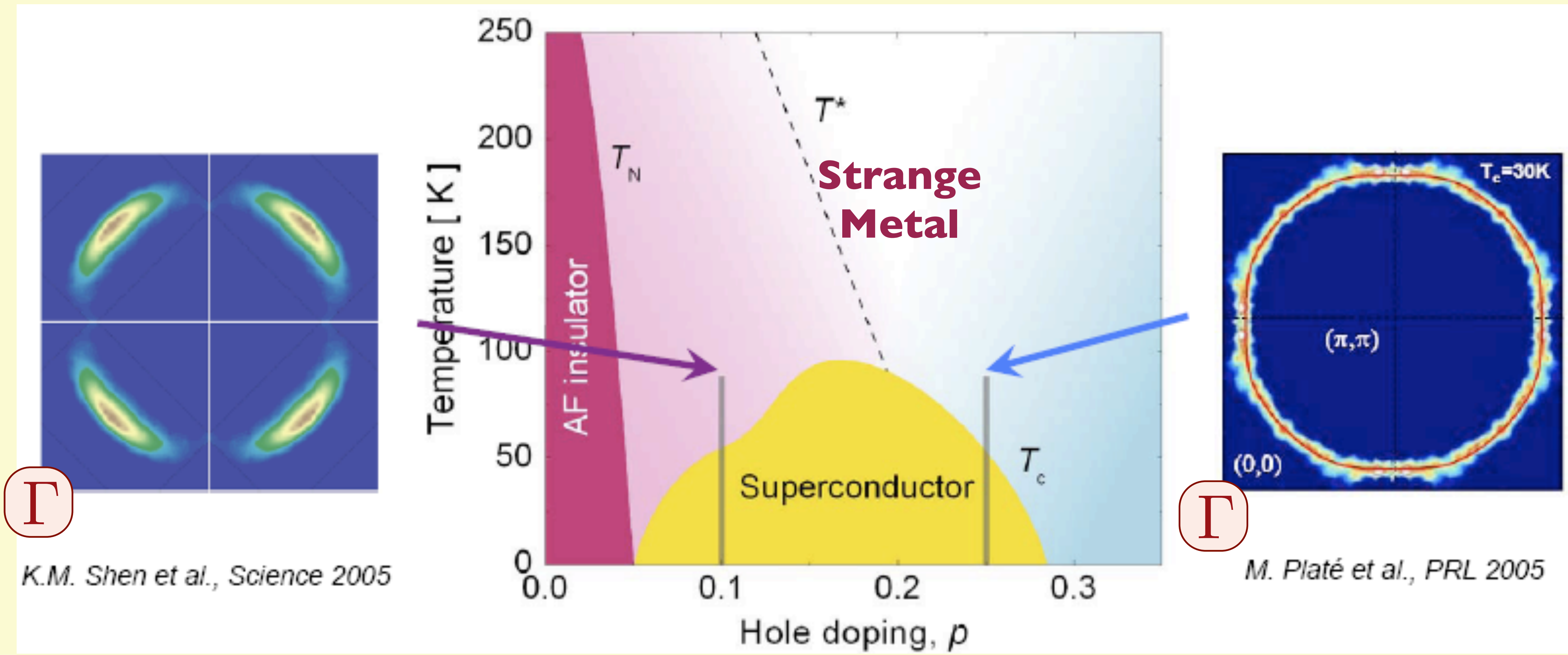
High temperature superconductors



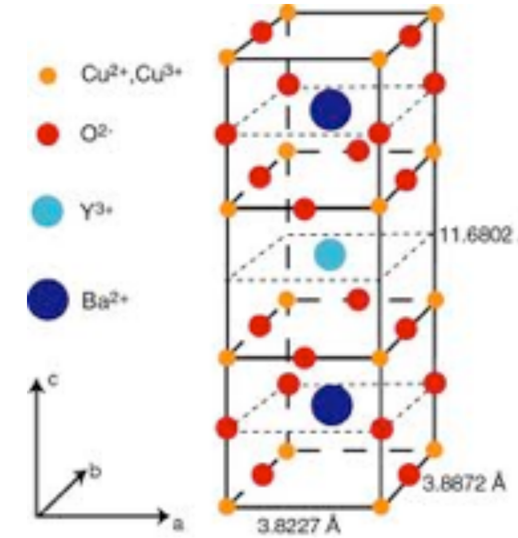
$YBa_2Cu_3O_{6+x}$

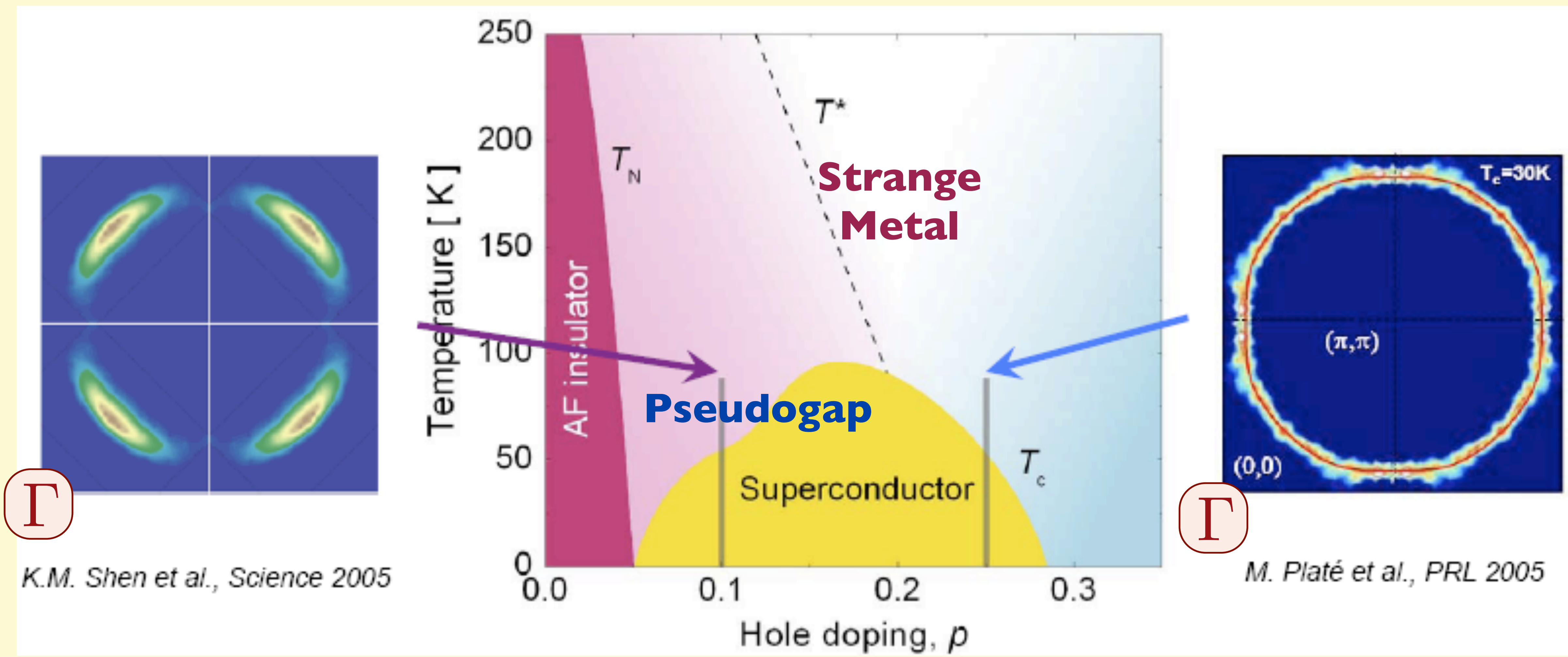




Smaller hole Fermi-pockets

Large hole Fermi surface



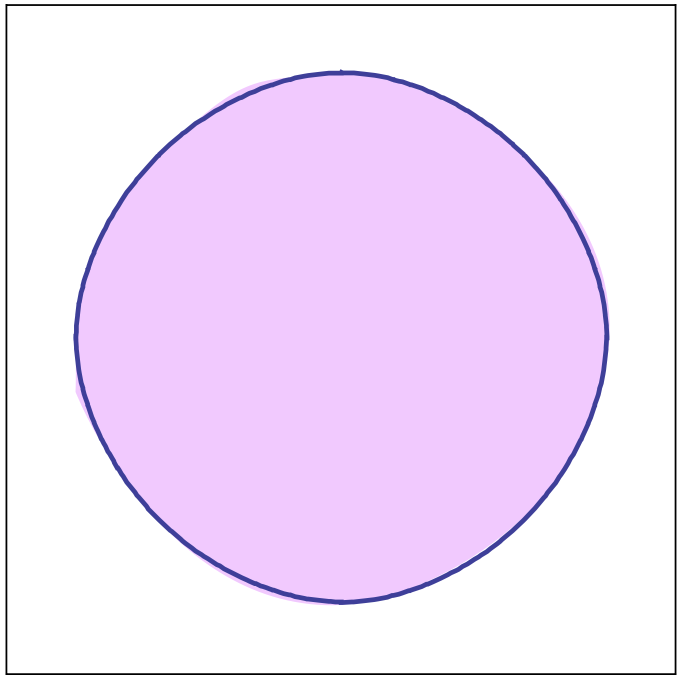


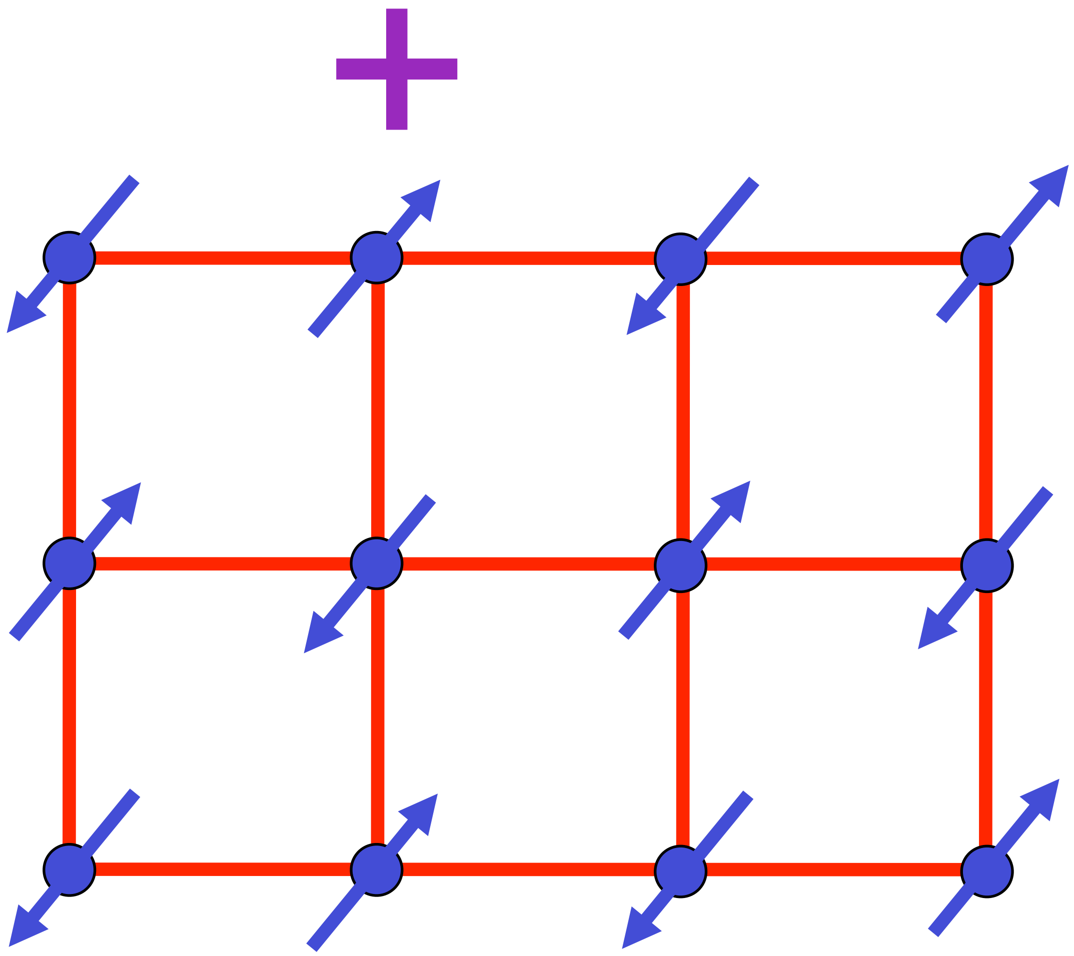
Smaller hole Fermi-pockets

Large hole Fermi surface

Fermi surface+antiferromagnetism

Metal with "large" Fermi surface



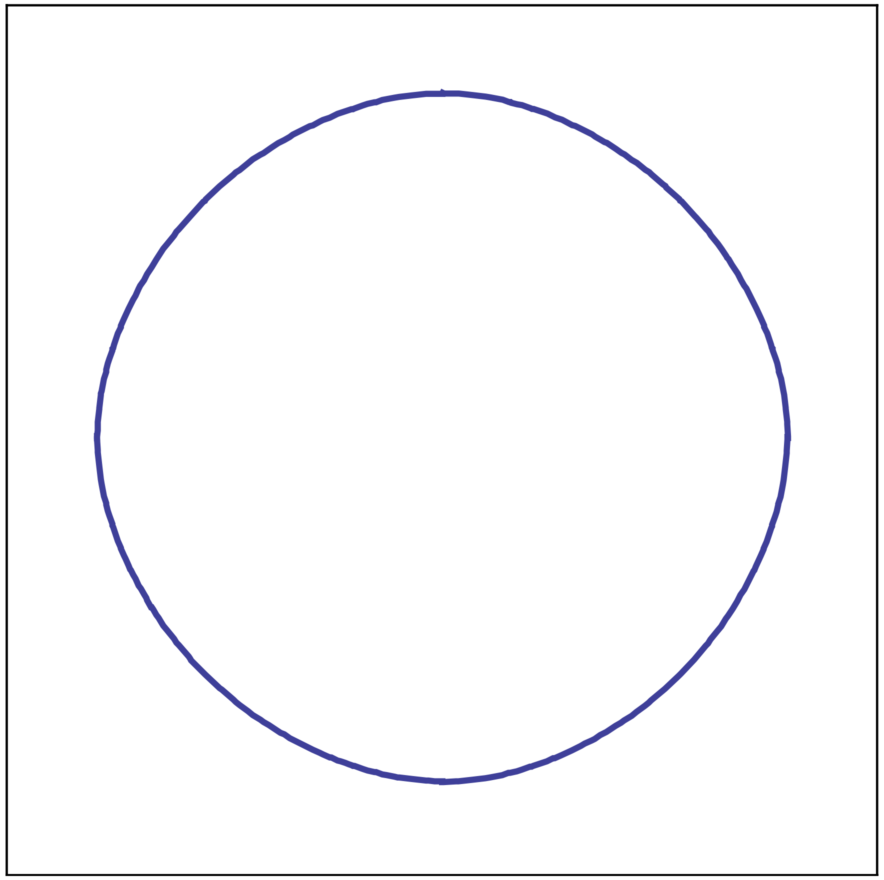


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

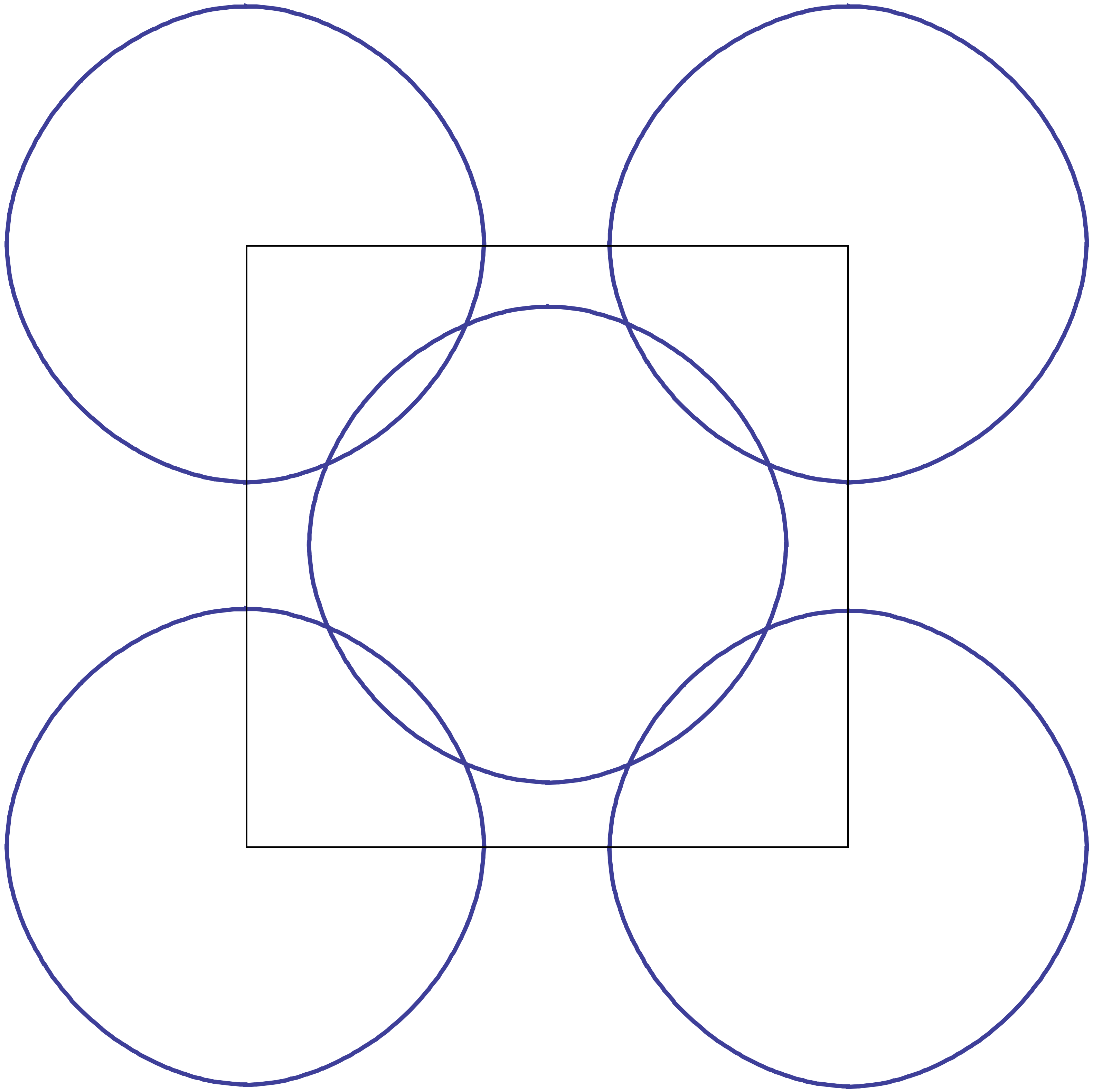
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



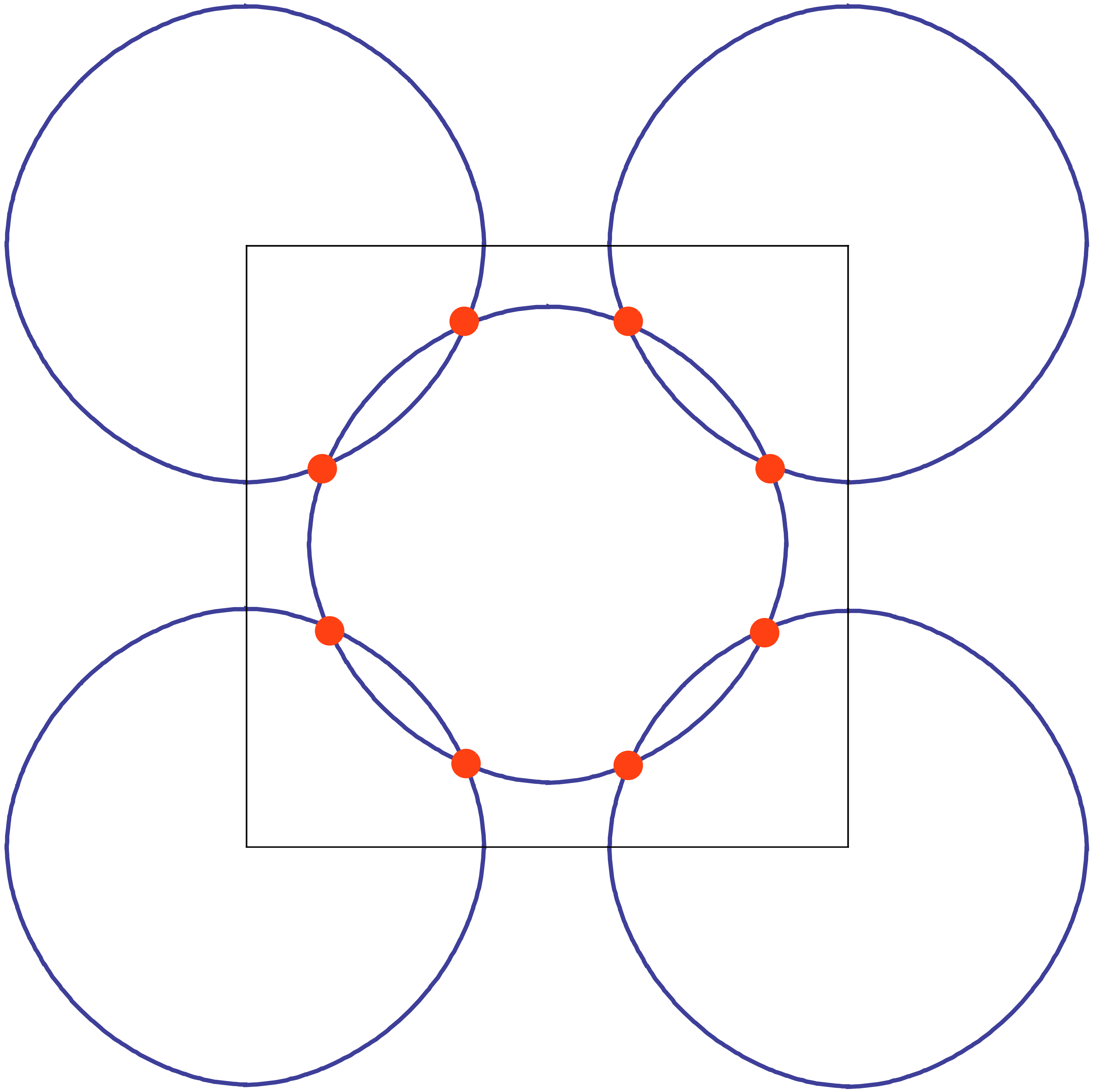
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



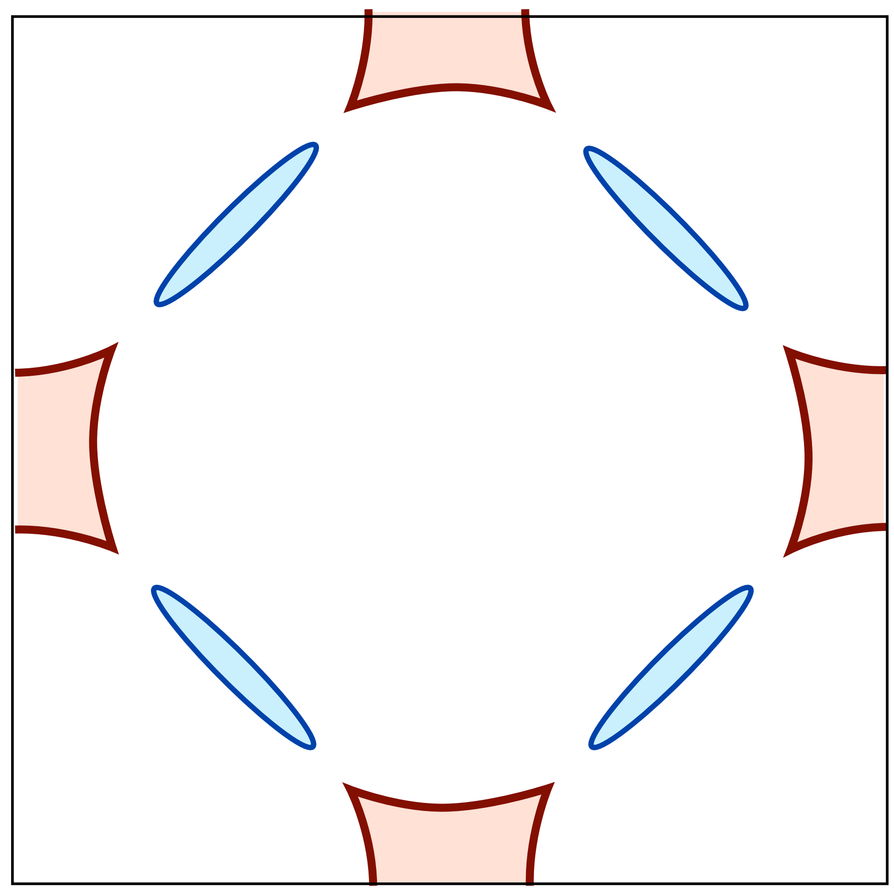
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



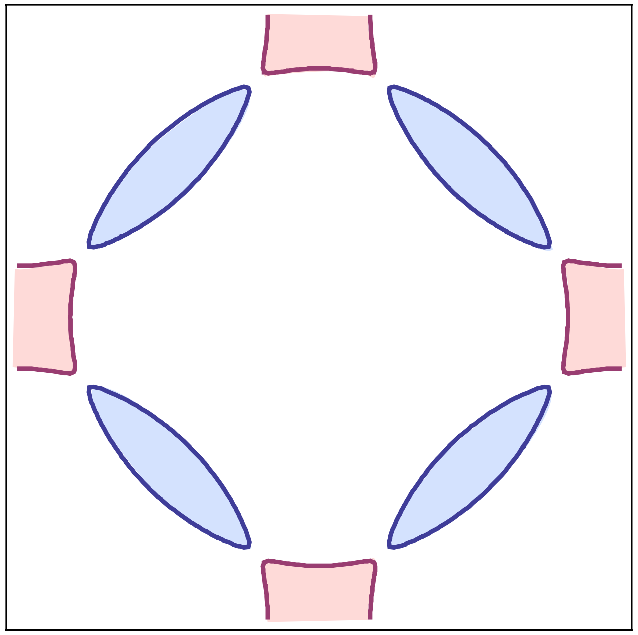
"Hot" spots

Fermi surface+antiferromagnetism



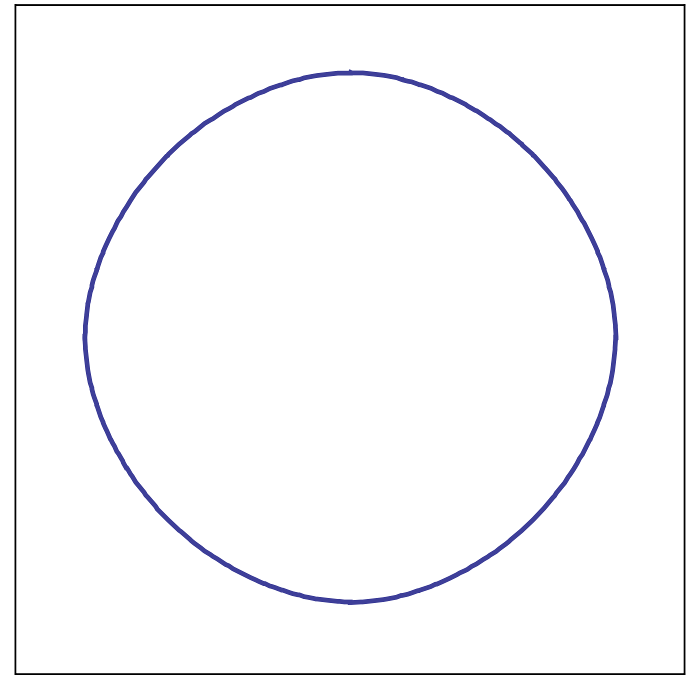
Electron and hole pockets in
antiferromagnetic phase
with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

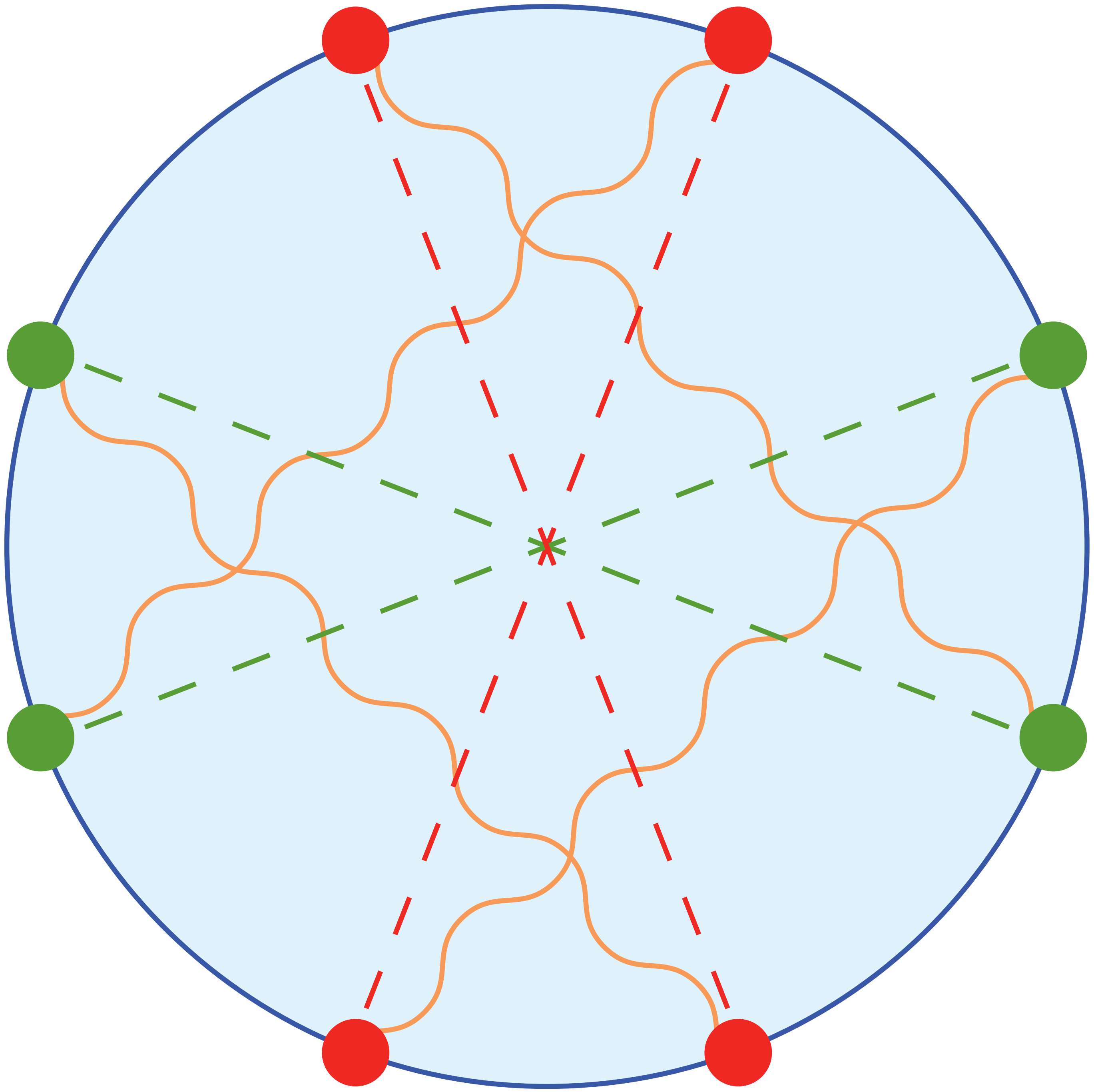


$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

r

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at *and near* hot spots

$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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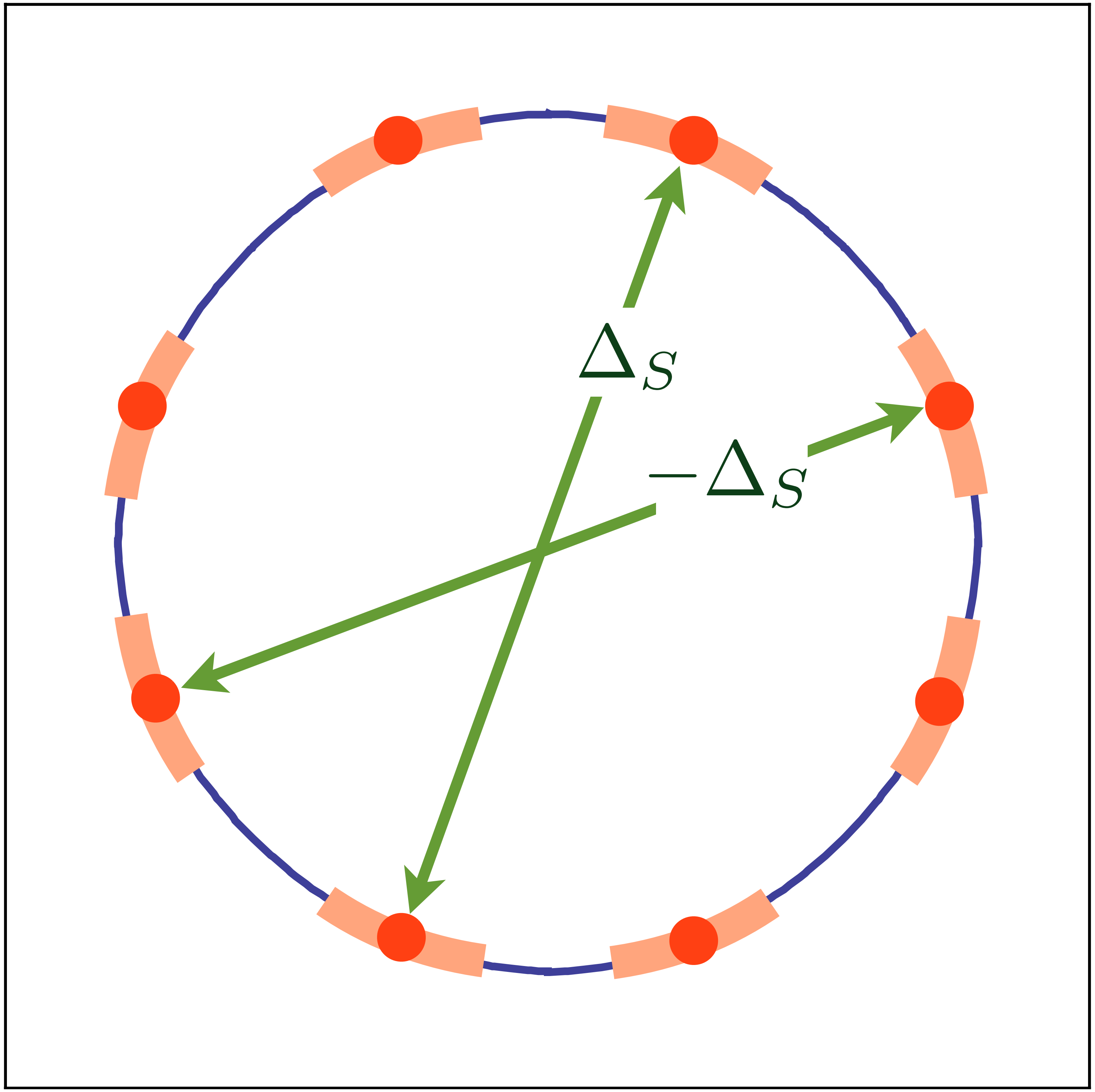
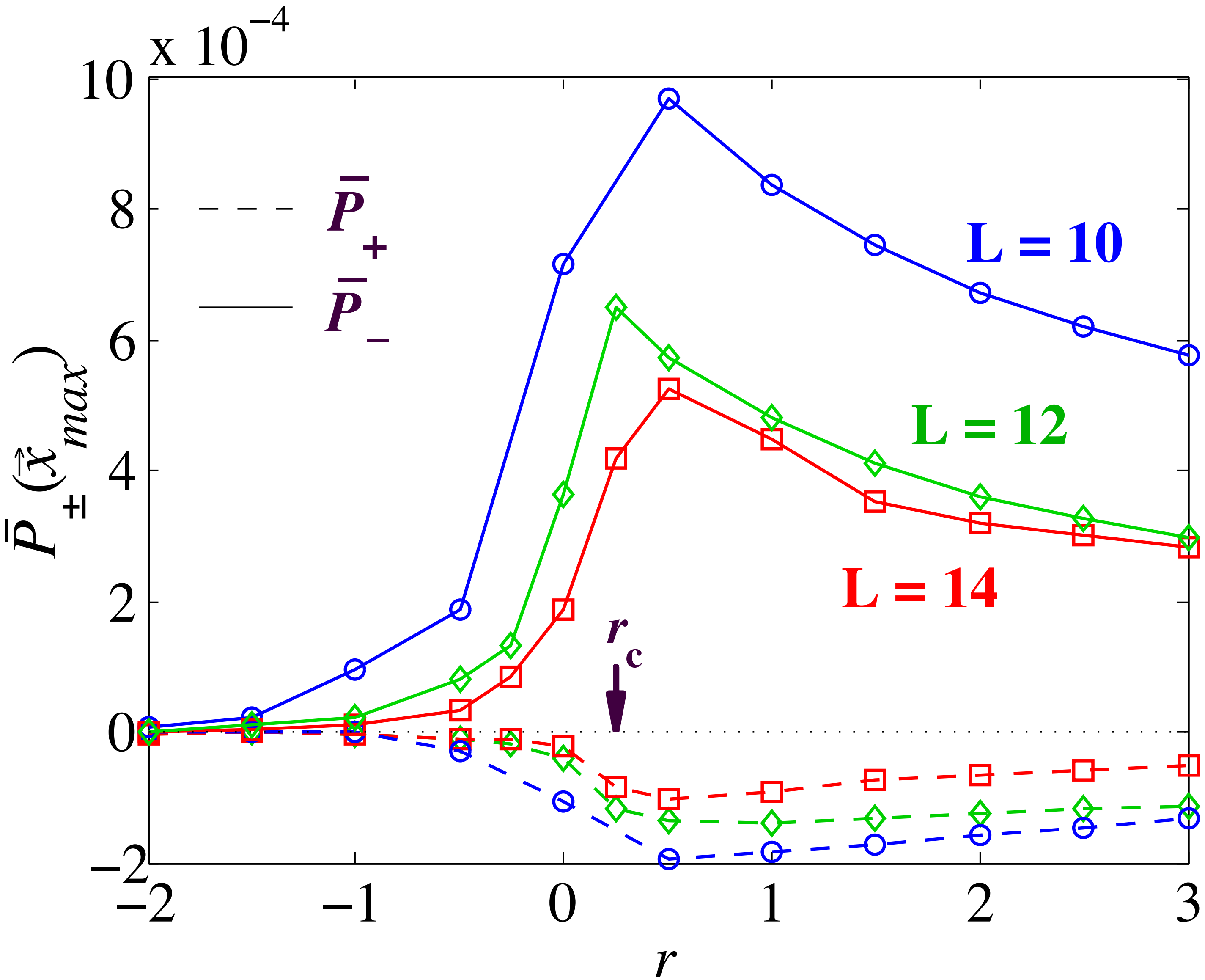


Unconventional pairing at *and near* hot spots

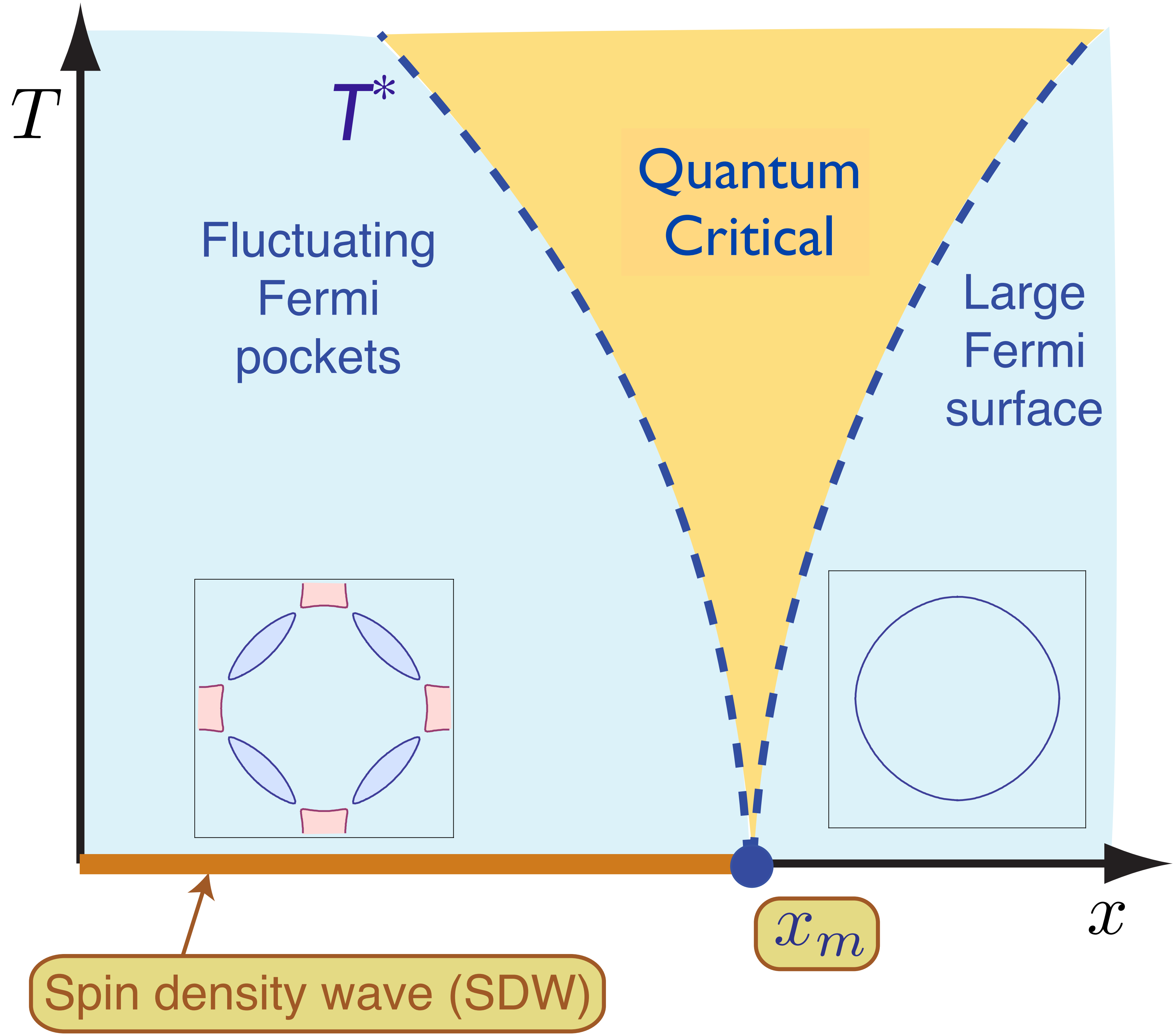
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

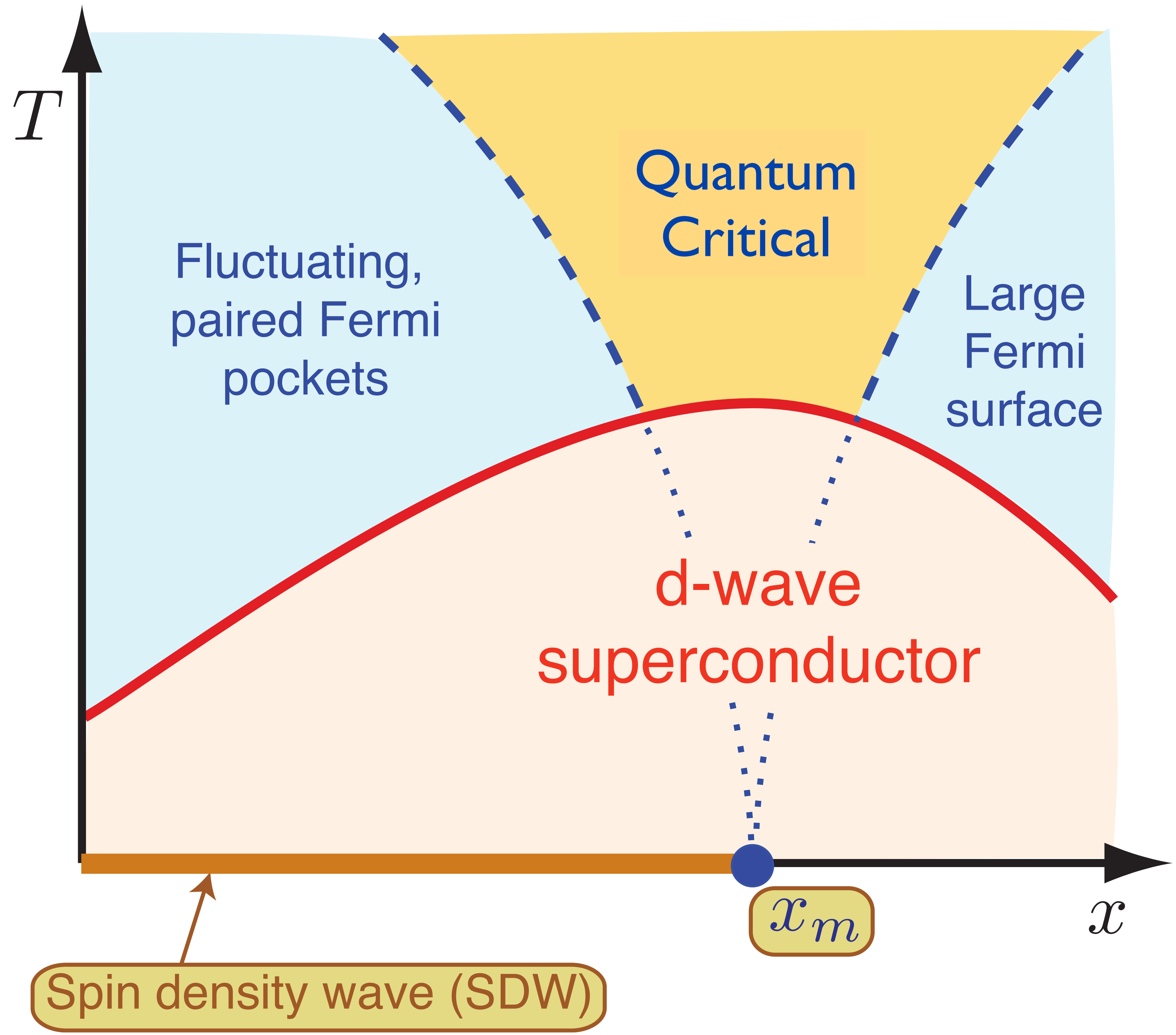
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism

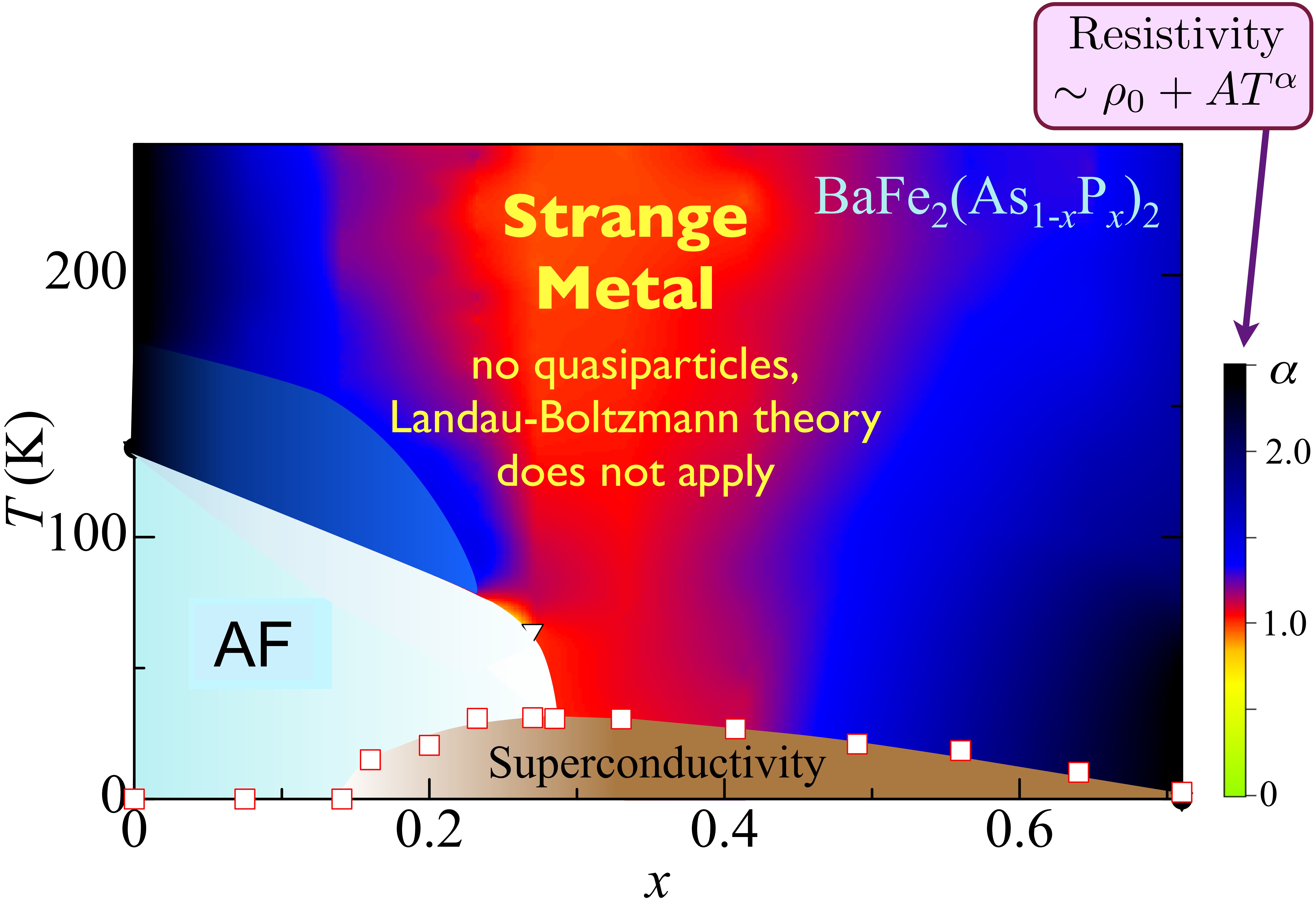


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism

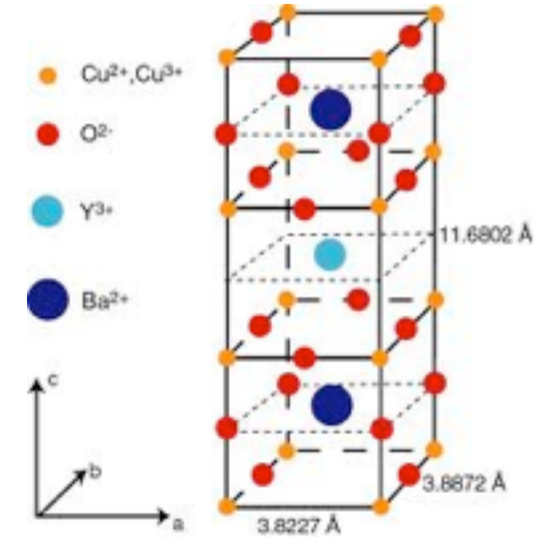


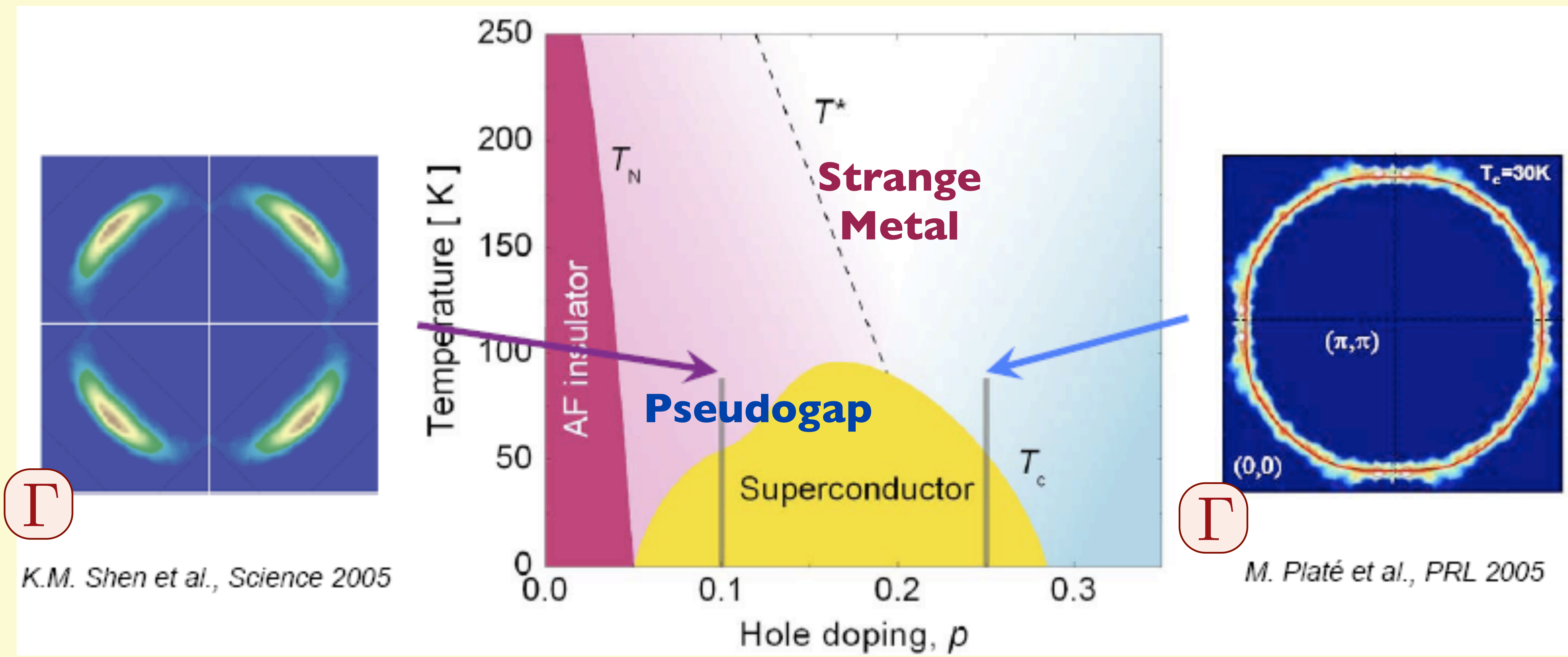
QCP for the onset of SDW order is actually within a superconductor



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

What about the pseudogap ?





Smaller hole Fermi-pockets

Large hole Fermi surface

- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

- These orders form a pseudospin doublet, which is responsible for the "pseudogap" phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)
K. B. Efetov, H. Meier, and C. Pépin, arXiv:1210.3276.
S. Sachdev and R. La Placa, arXiv:1303.2114

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi^\dagger_{j\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

which is invariant under the SU(2) pseudospin transformations

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

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$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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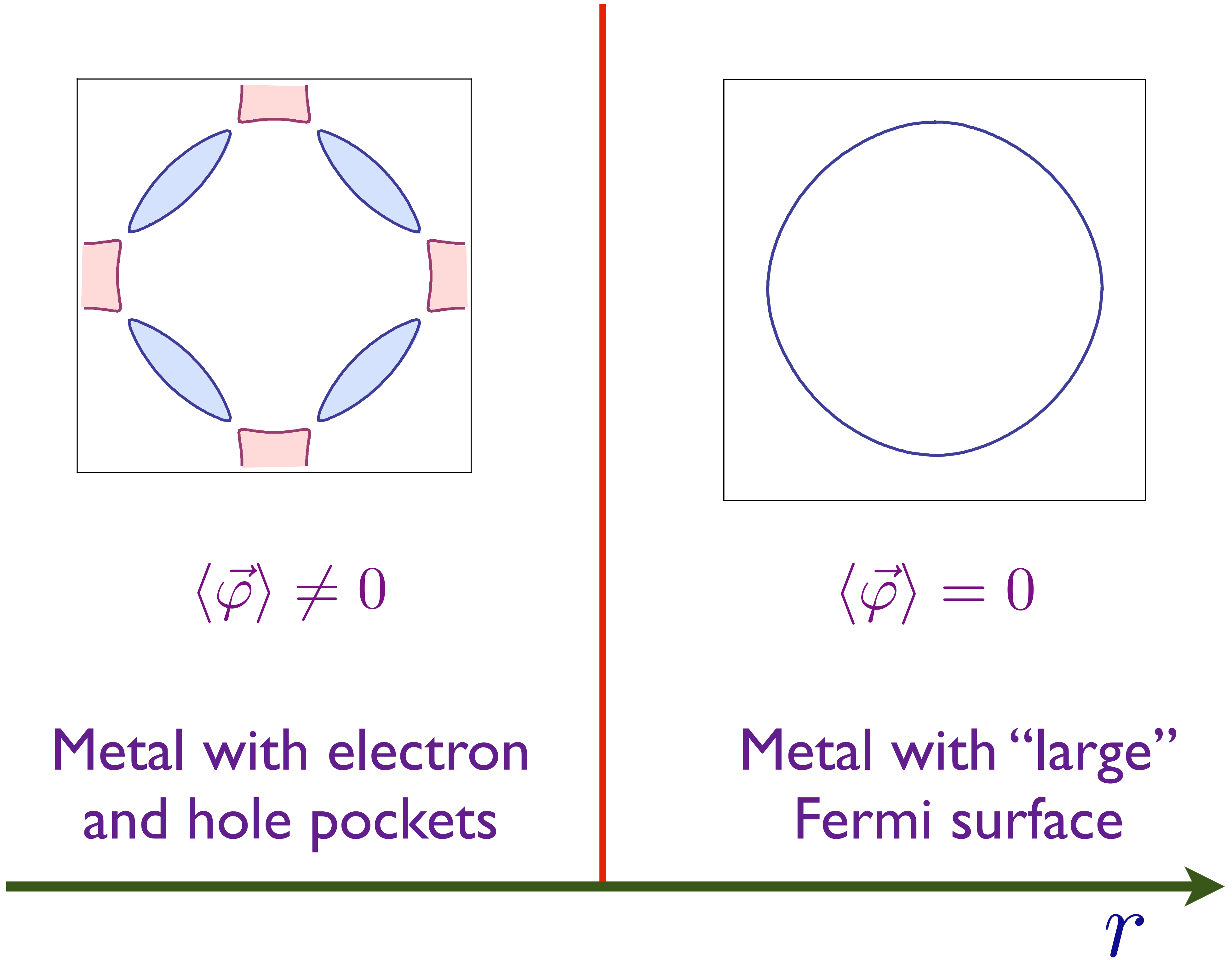
We will start with the Néel state, and find important consequences of the pseudospin symmetry in metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Fermi surface+antiferromagnetism



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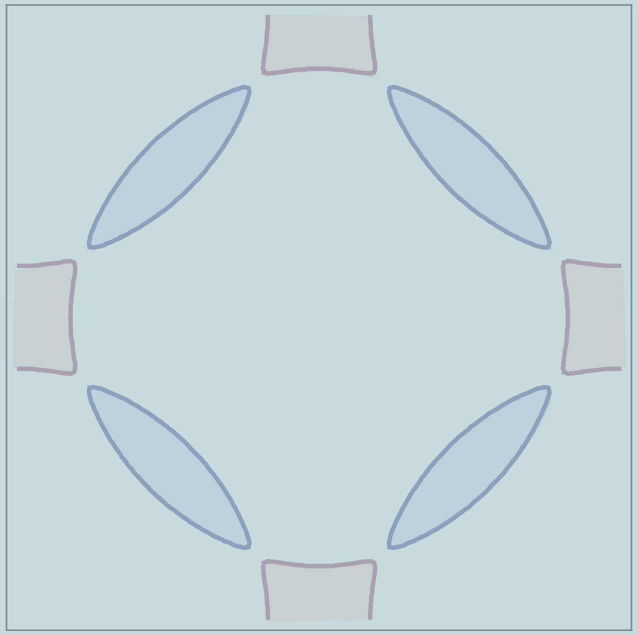
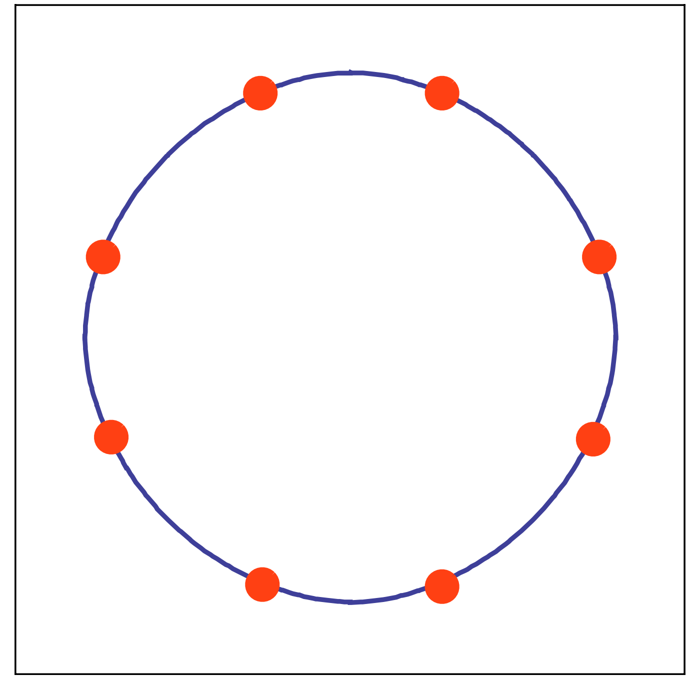




Focus on this region

$\langle \vec{\varphi} \rangle \neq 0$

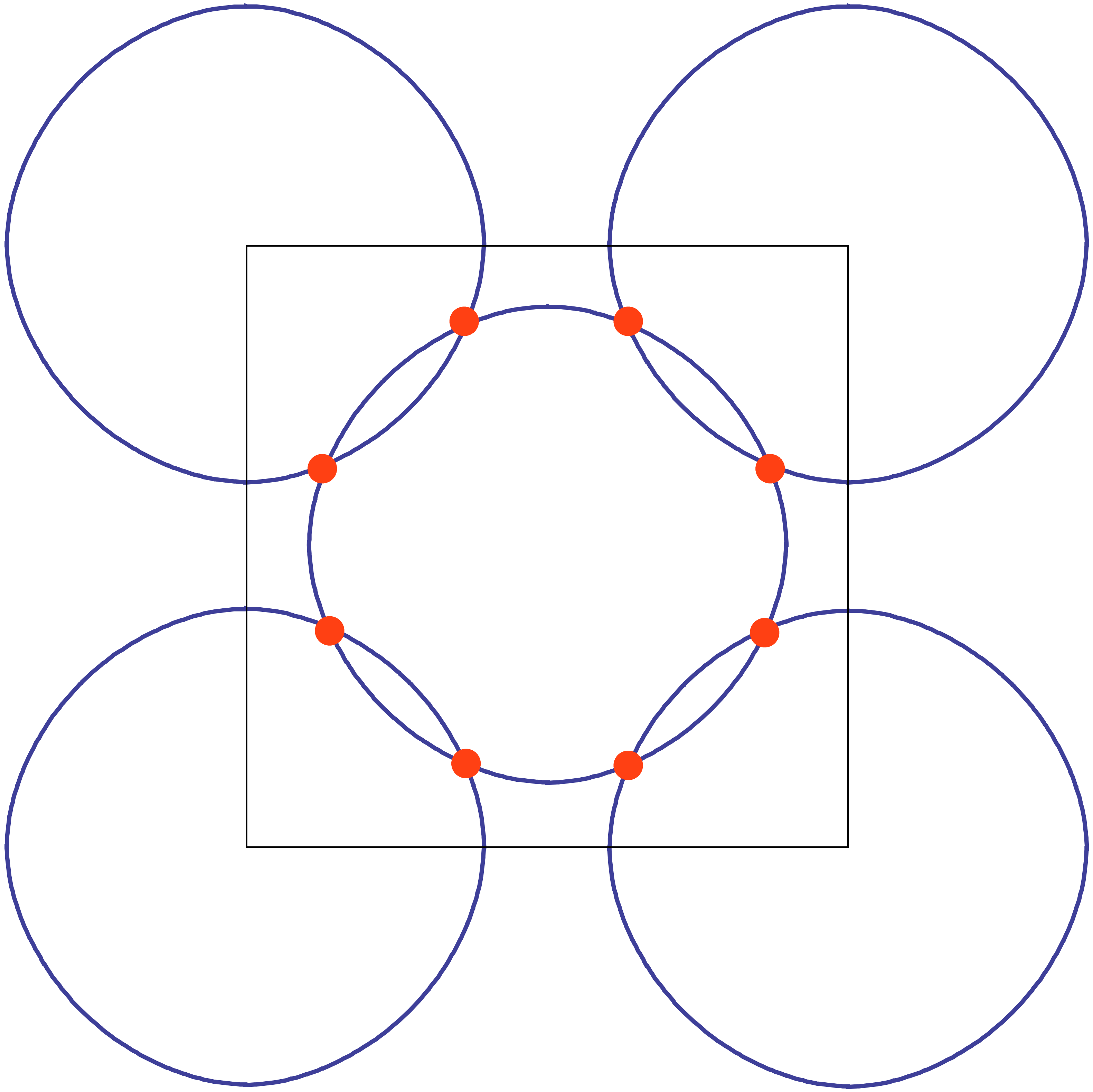
$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

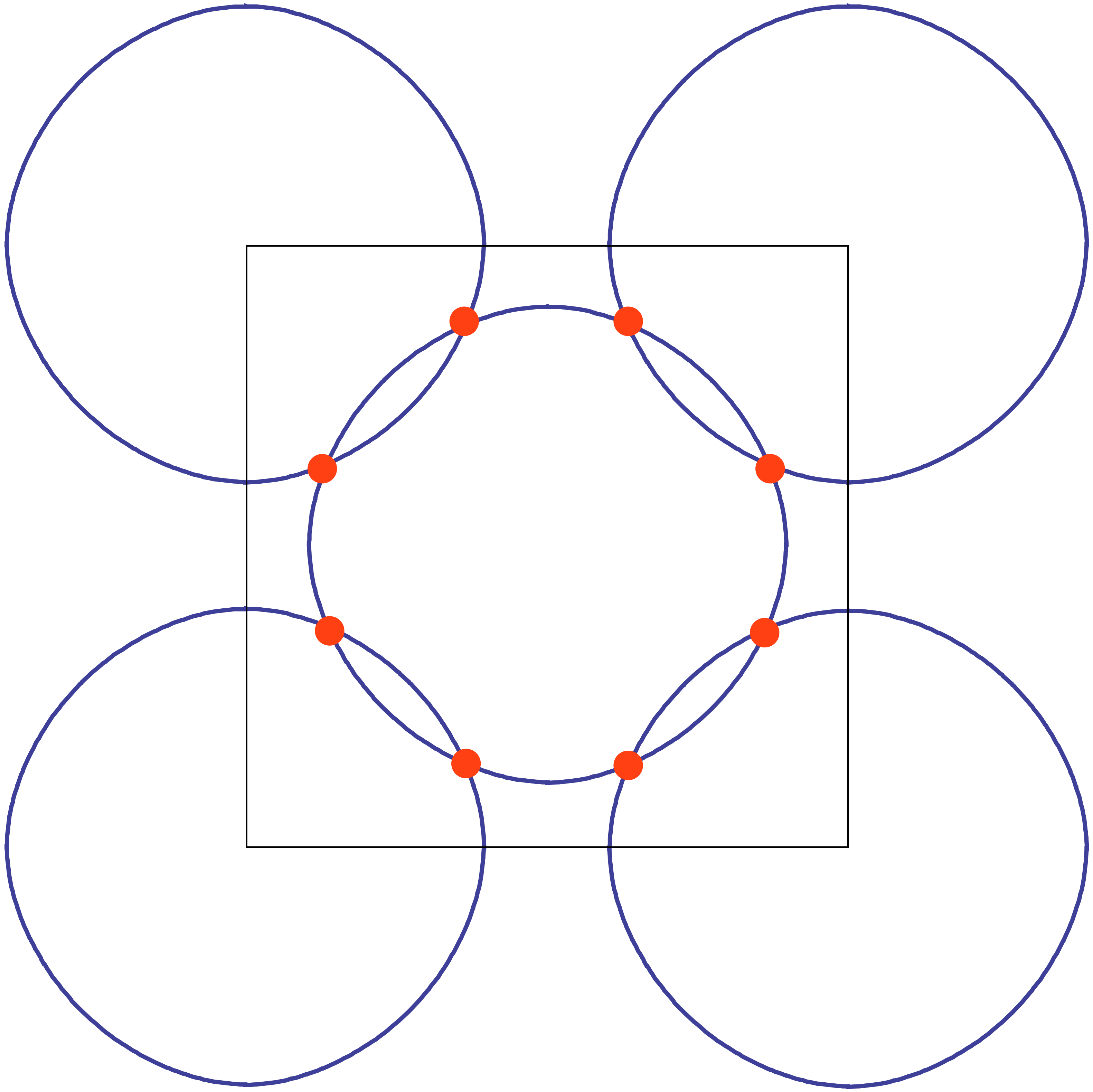
r

Fermi surface+antiferromagnetism



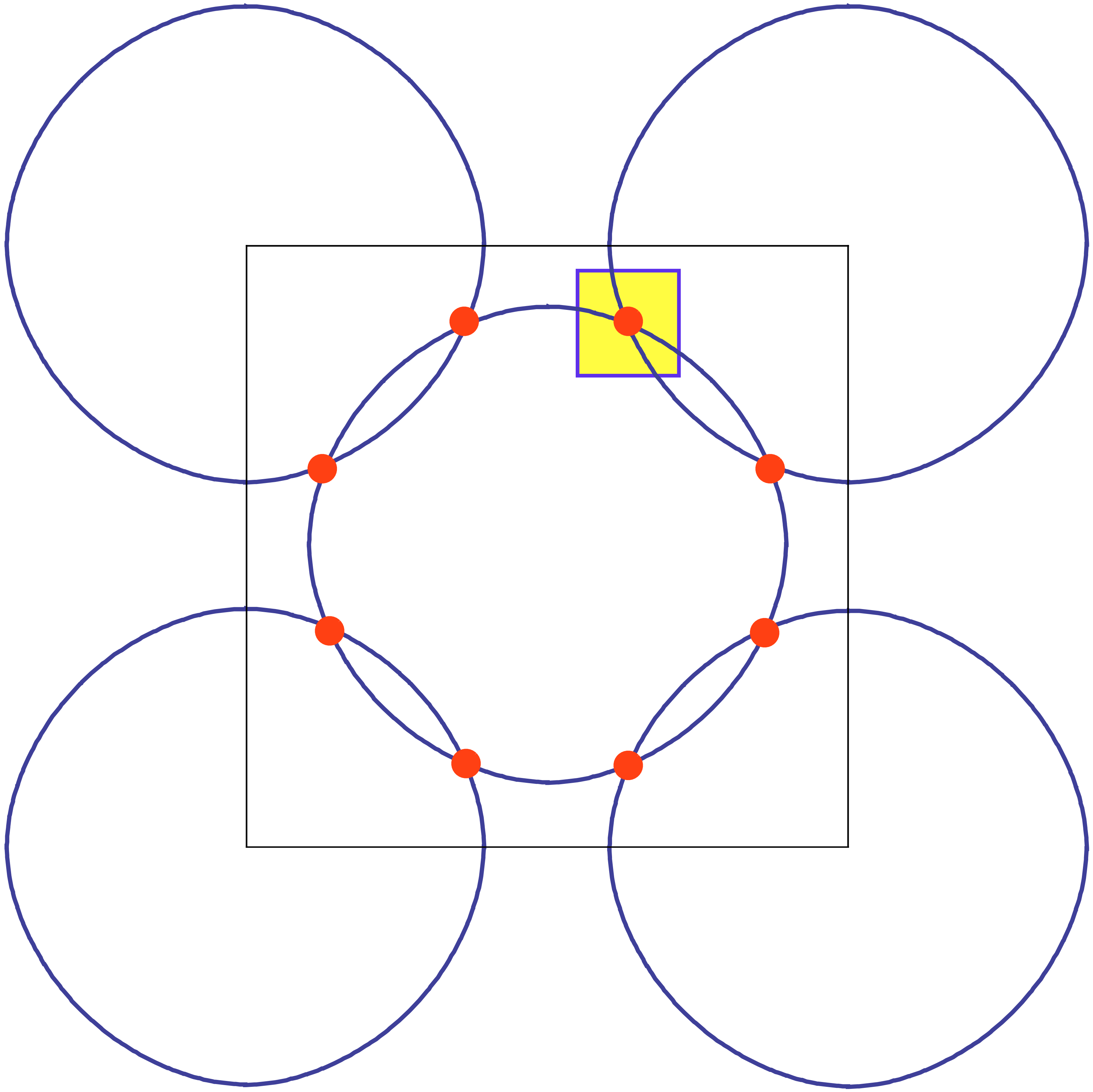
"Hot" spots

Fermi surface+antiferromagnetism



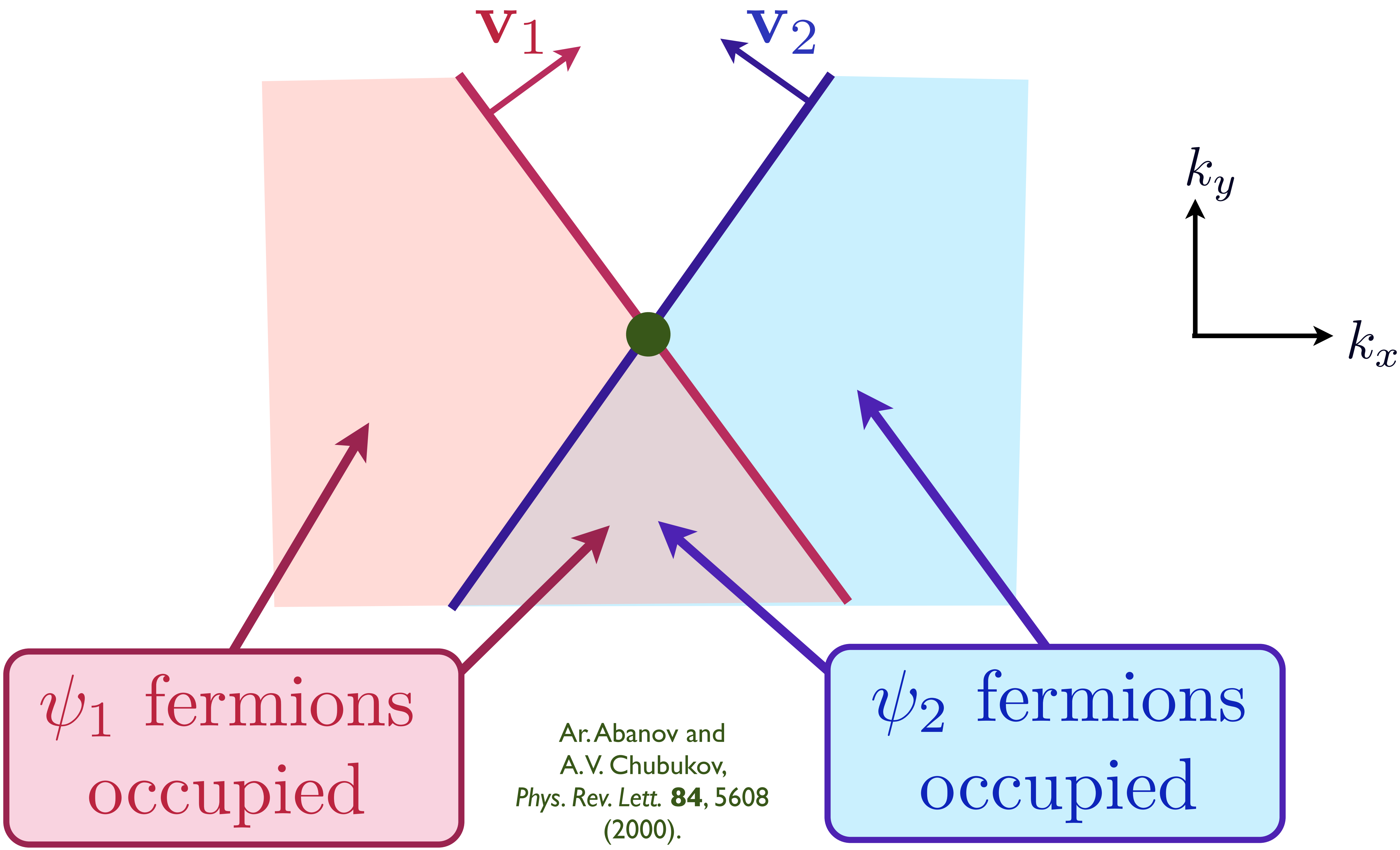
Low energy theory for critical point near hot spots

Fermi surface+antiferromagnetism



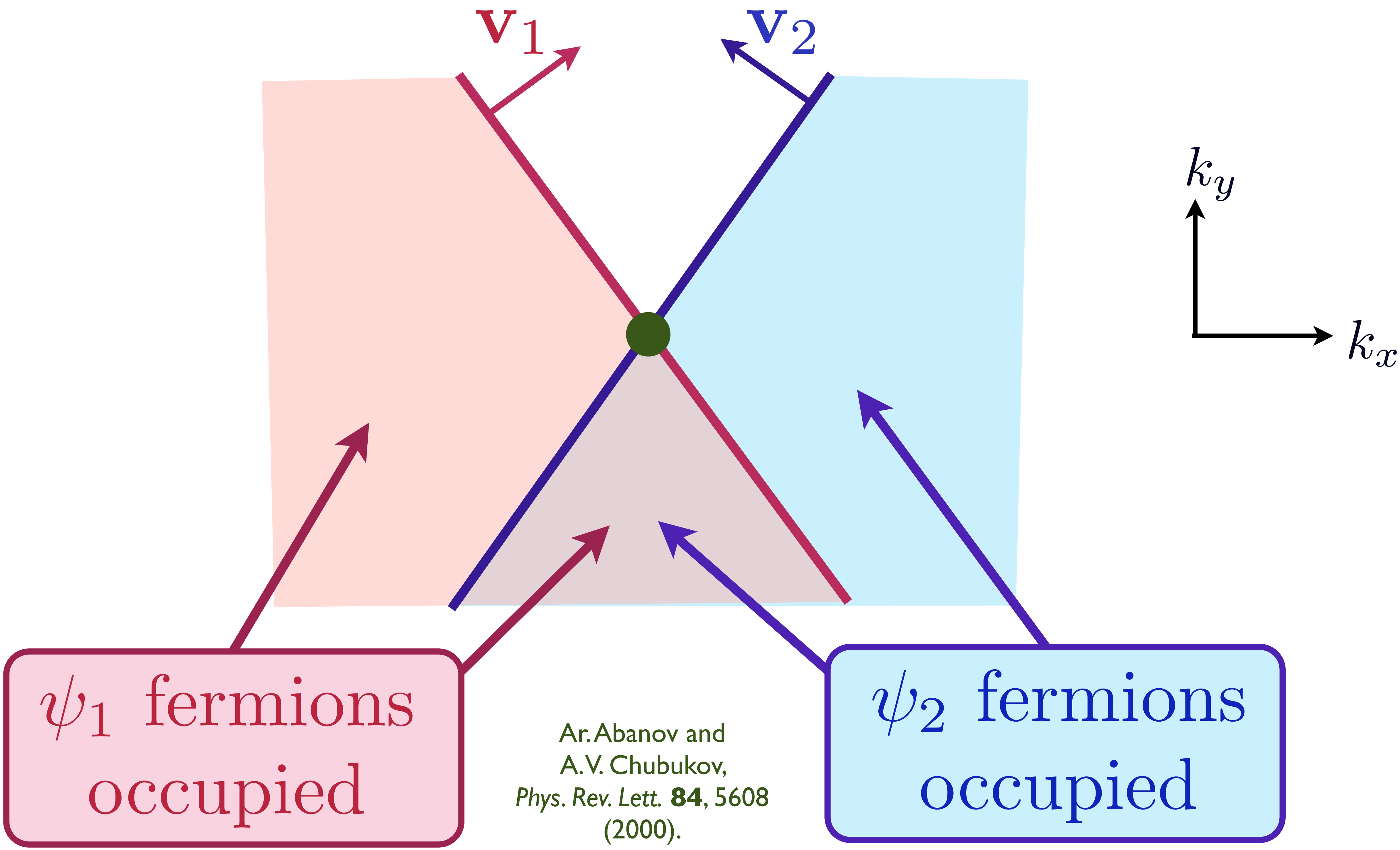
Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



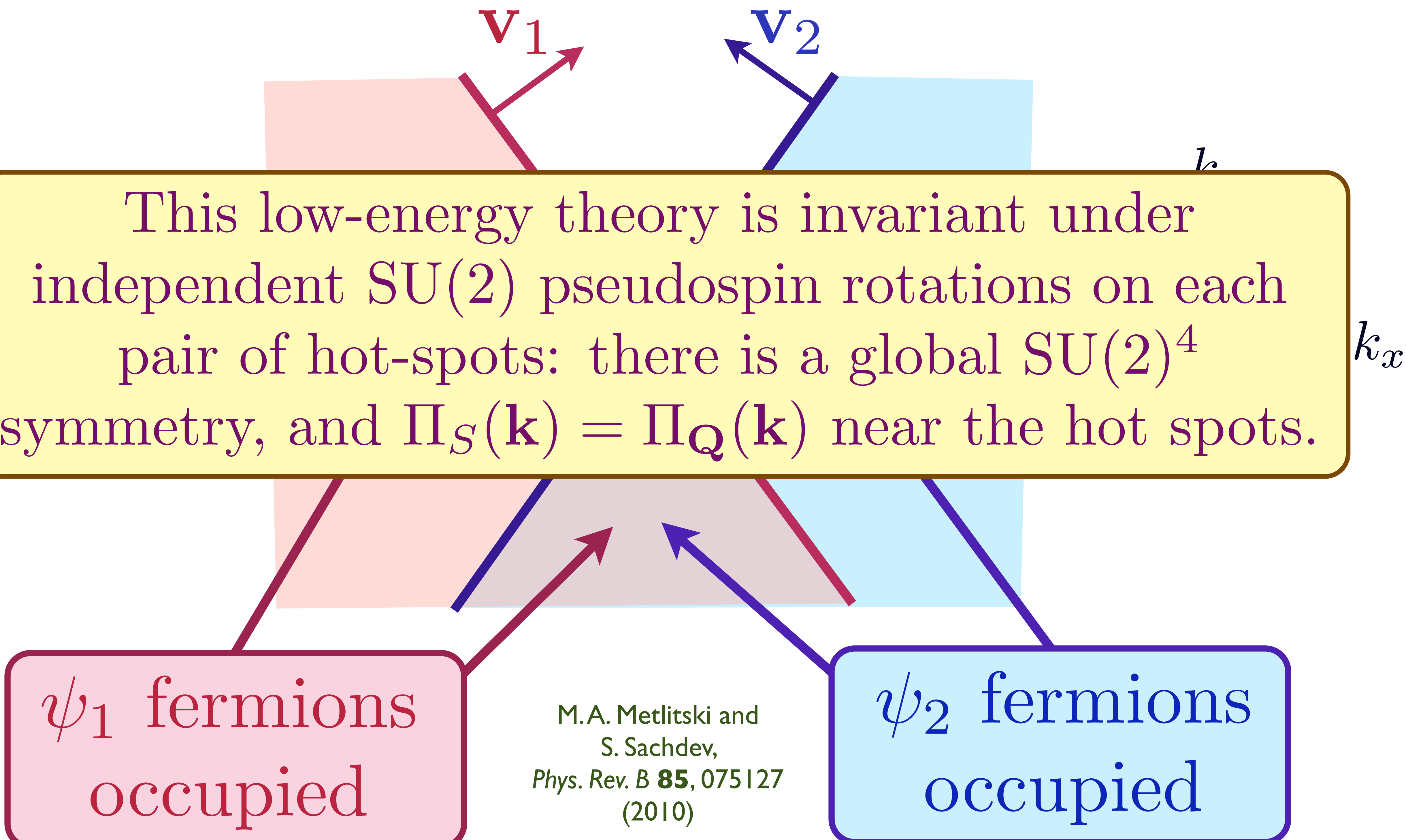
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

$$\mathcal{S} = \int d^2r d\tau \Big[\psi^\dagger_{1\alpha} \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi^\dagger_{2\alpha} \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$
$$+ \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi^\dagger_{2\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \Big]$$



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

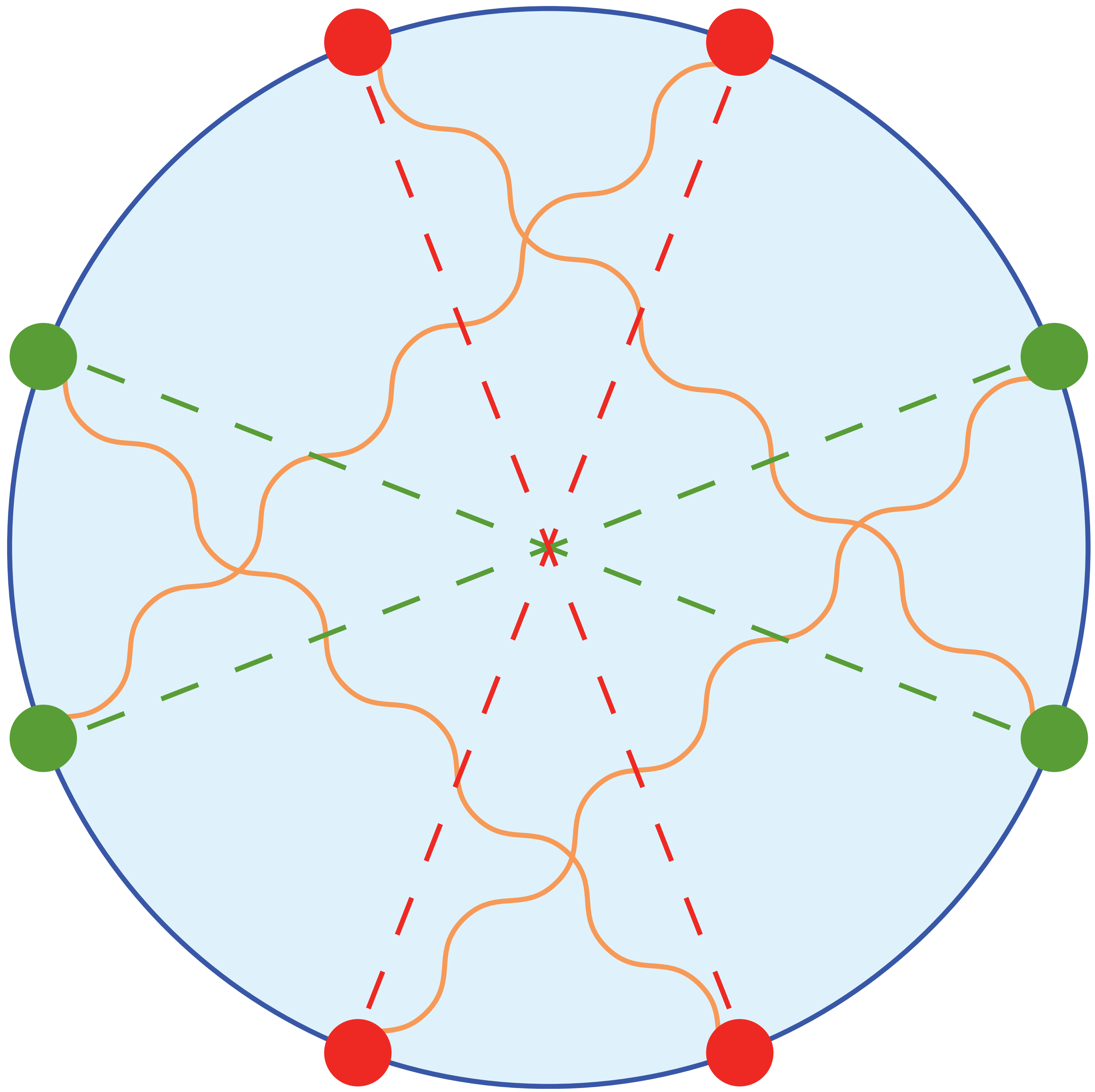
$$\mathcal{S} = \int d^2r d\tau \Big[\psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$
$$+ \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \Big]$$



This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots: there is a global $SU(2)^4$ symmetry, and $\Pi_S(\mathbf{k}) = \Pi_{\mathbf{Q}}(\mathbf{k})$ near the hot spots.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at *and near* hot spots

$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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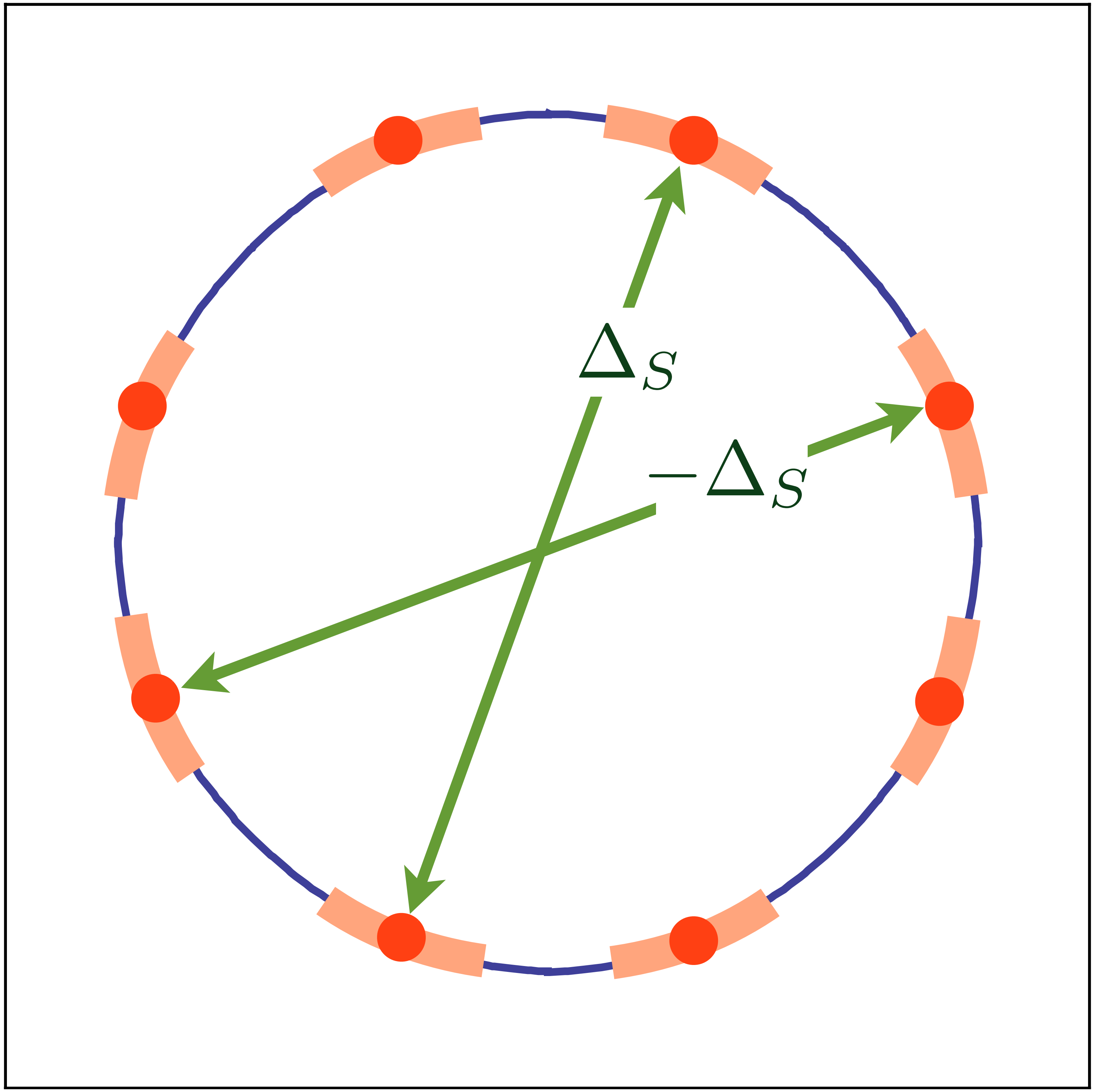
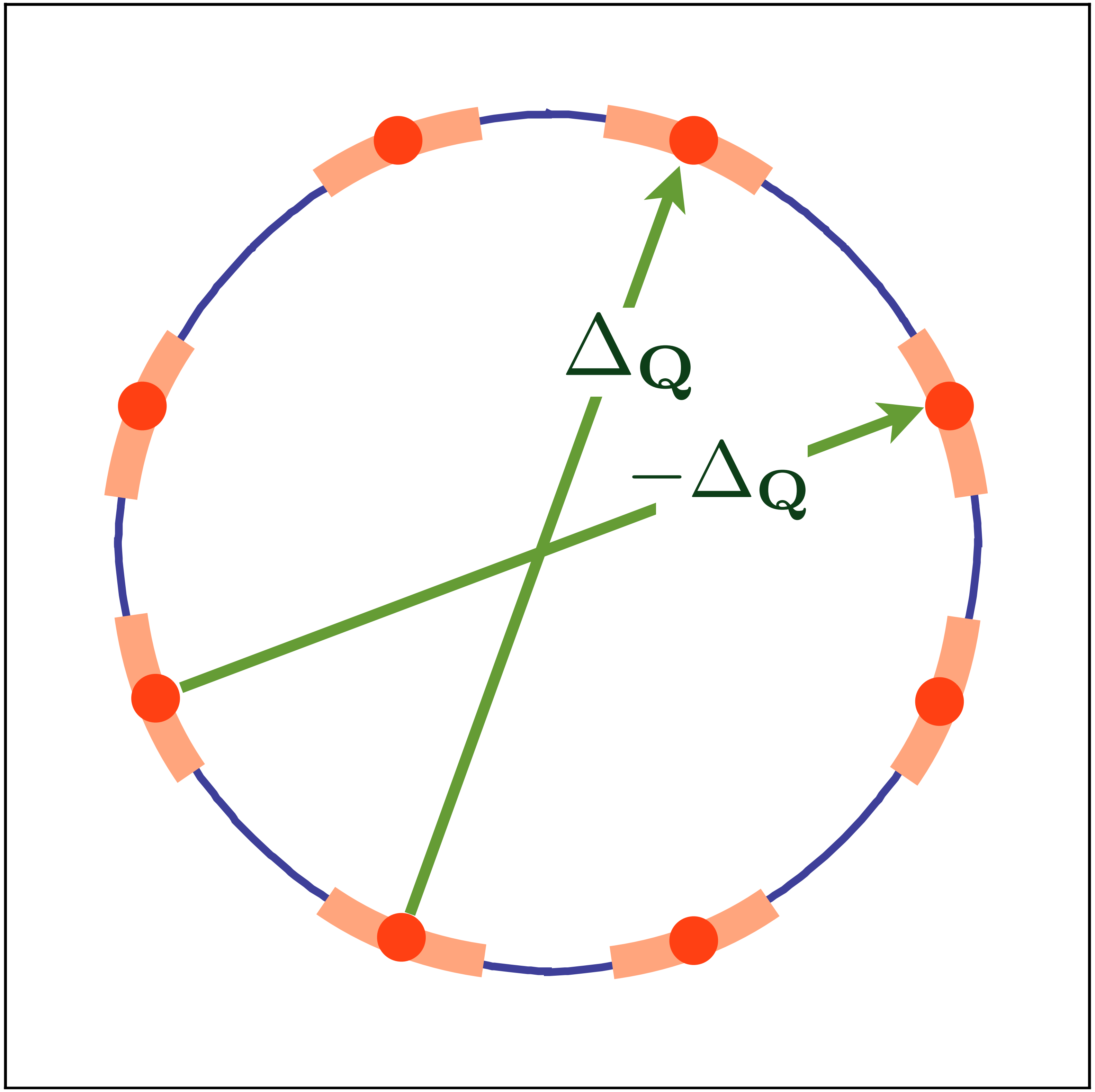


Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

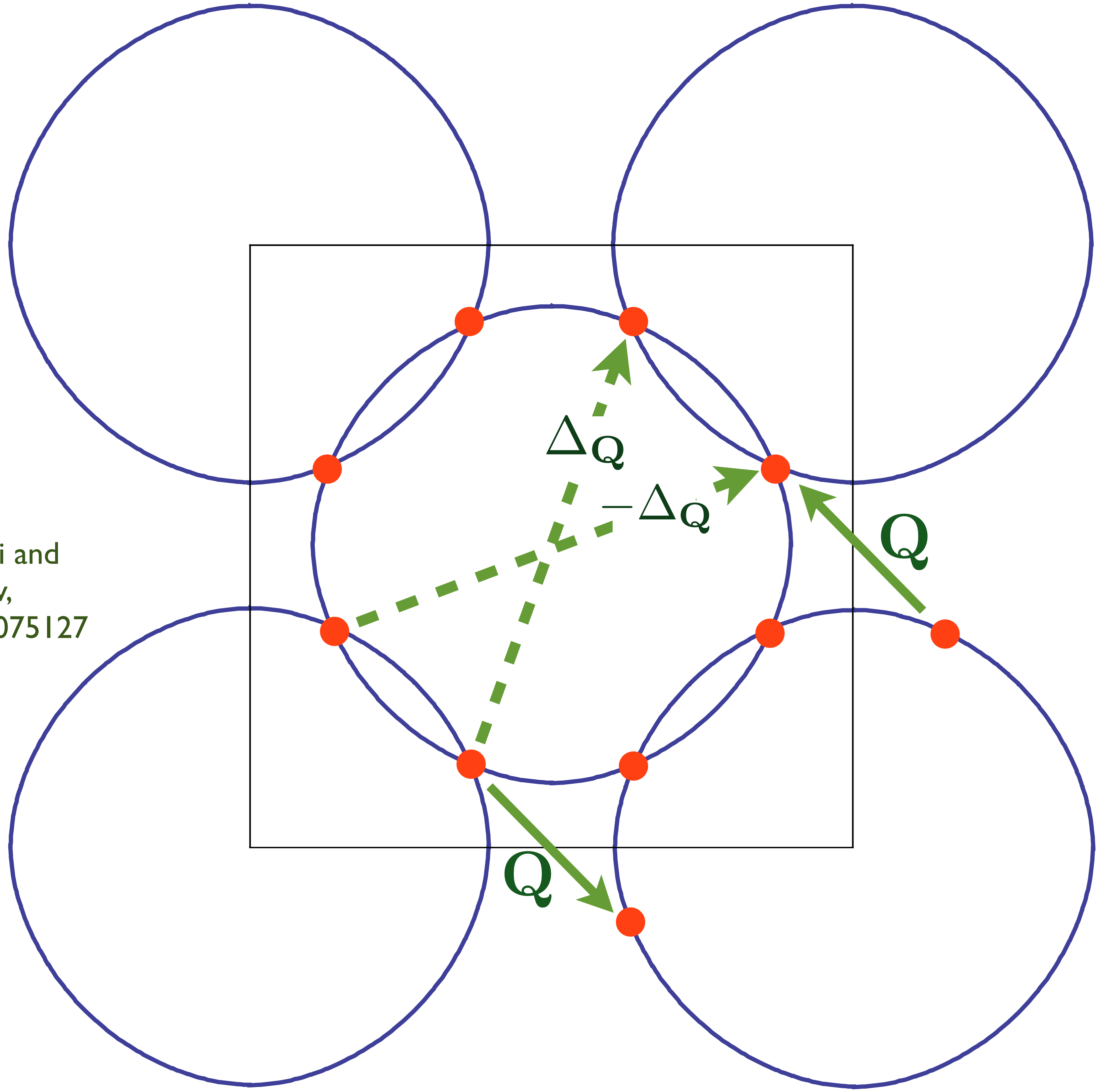


$\mathbf{Q}$ is '$2k_F$' wavevector

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

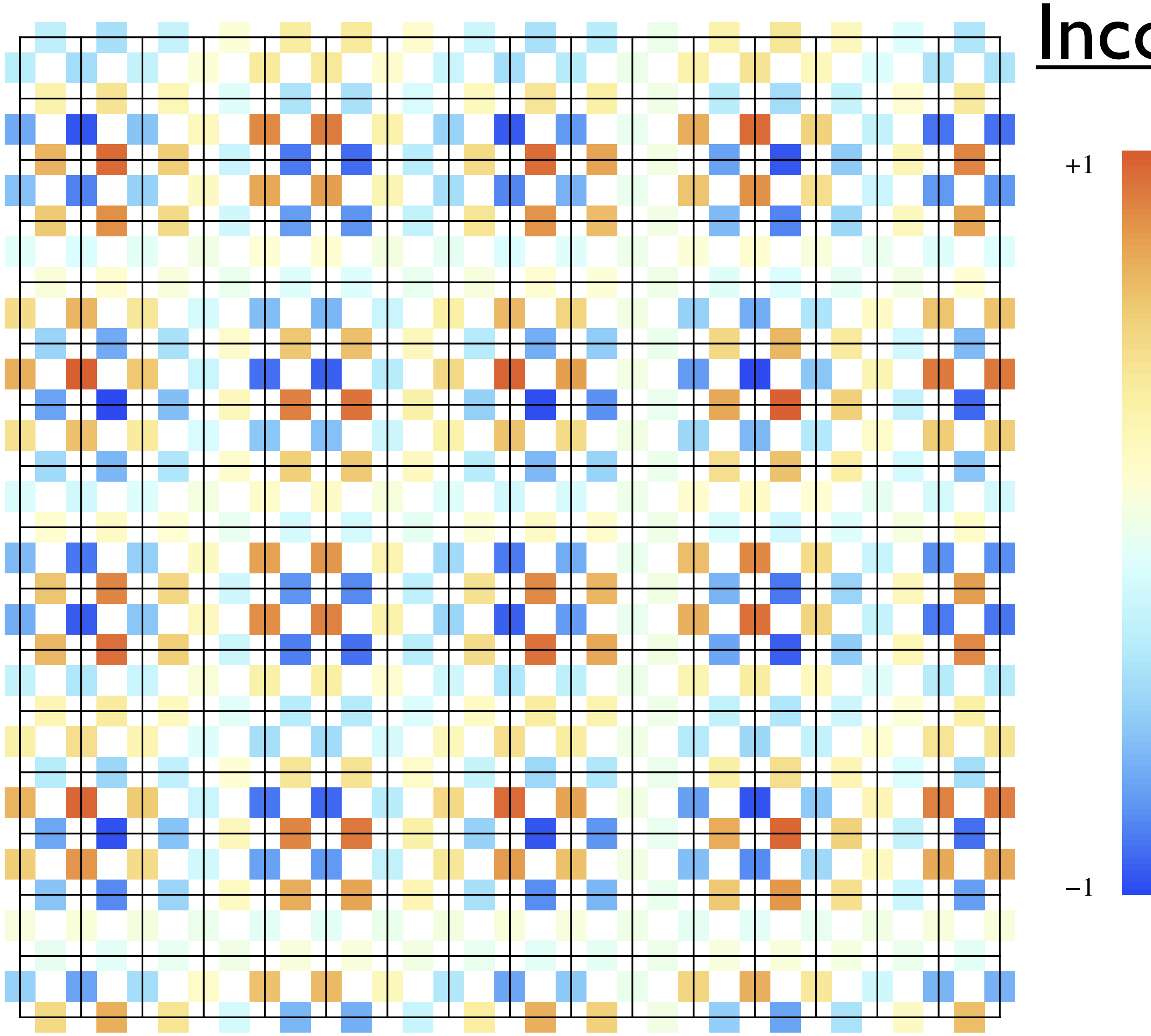
Unconventional particle-hole pairing at *and near* hot spots

Incommensurate *d*-wave bond order



M. A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$



Incommensurate d-wave bond order

"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \right\rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle$$

where $\mathbf{Q}$ extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

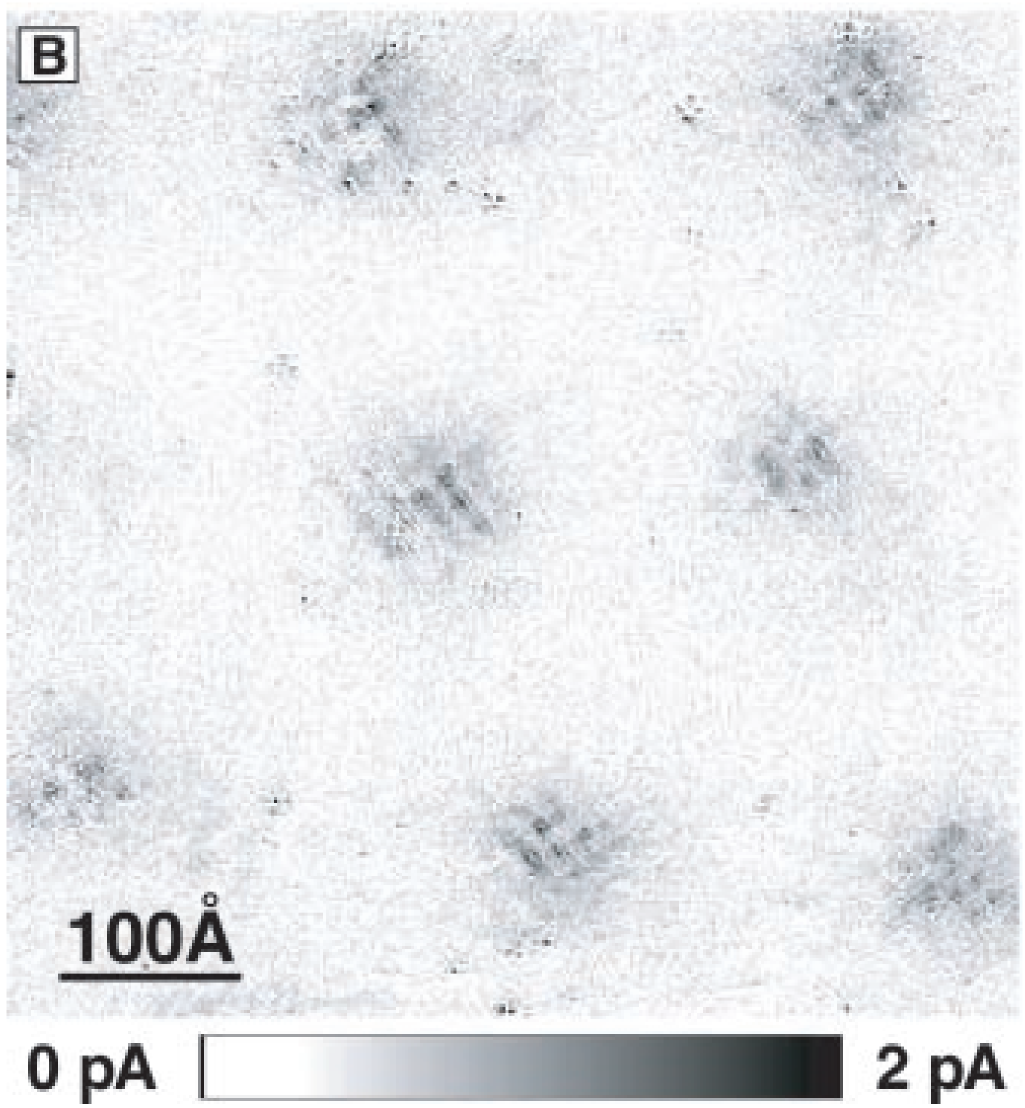
$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\left\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \right\rangle$ is non-zero *only* when $\mathbf{r}, \mathbf{s}$ are nearest neighbors.

A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman,[1] E. W. Hudson,[1,2*] K. M. Lang,[1] V. Madhavan,[1]
H. Eisaki,[3†] S. Uchida,[3] J. C. Davis[1,2‡]

SCIENCE VOL 295 18 JANUARY 2002



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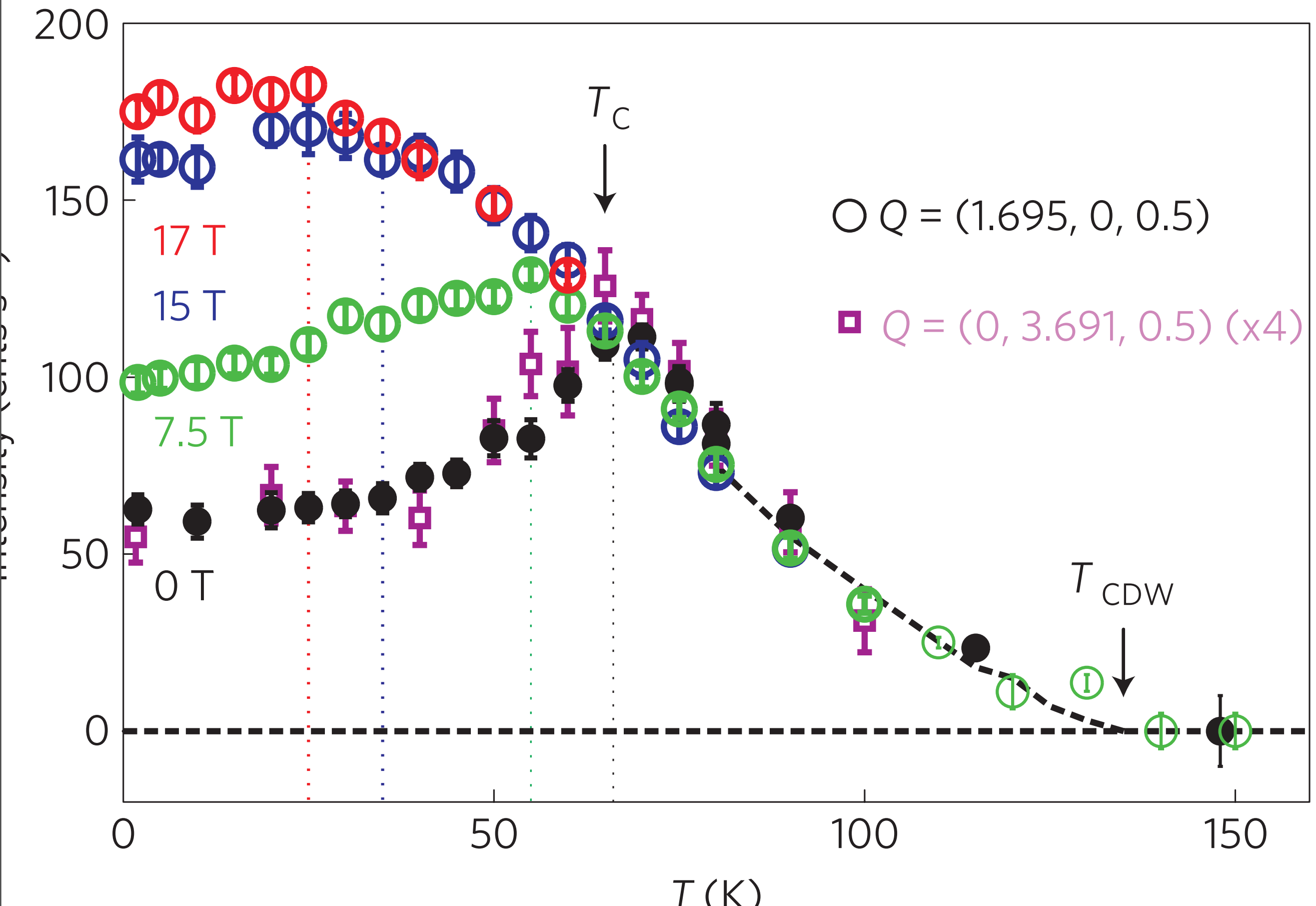
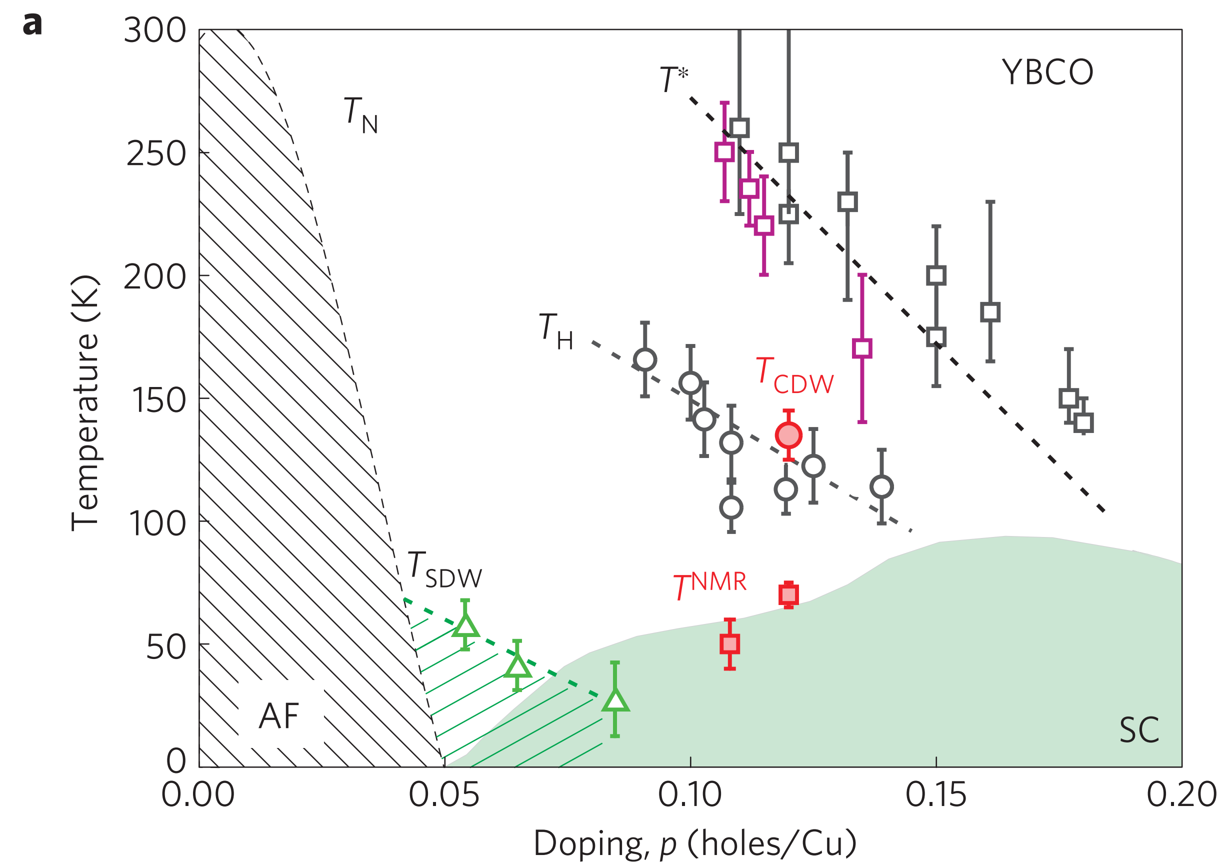
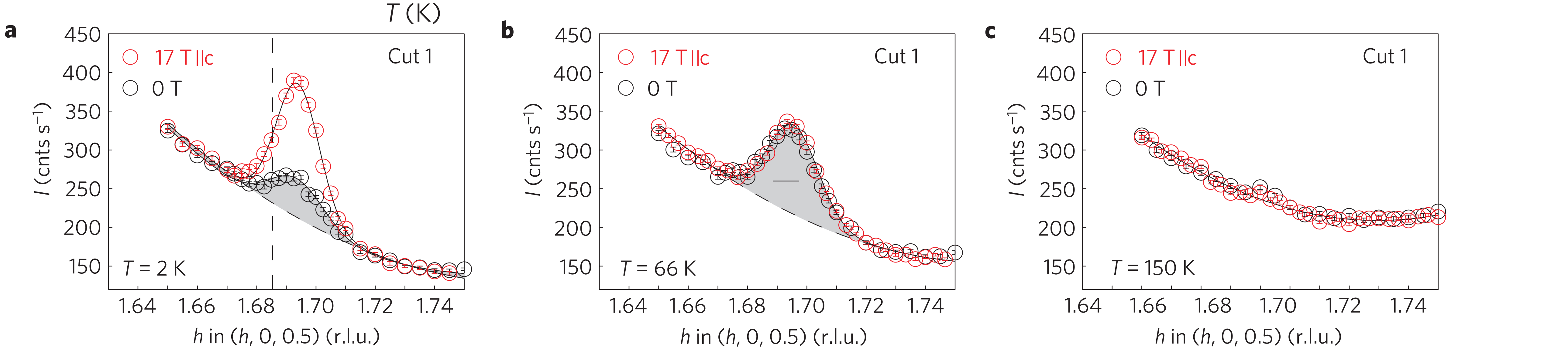
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]







Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

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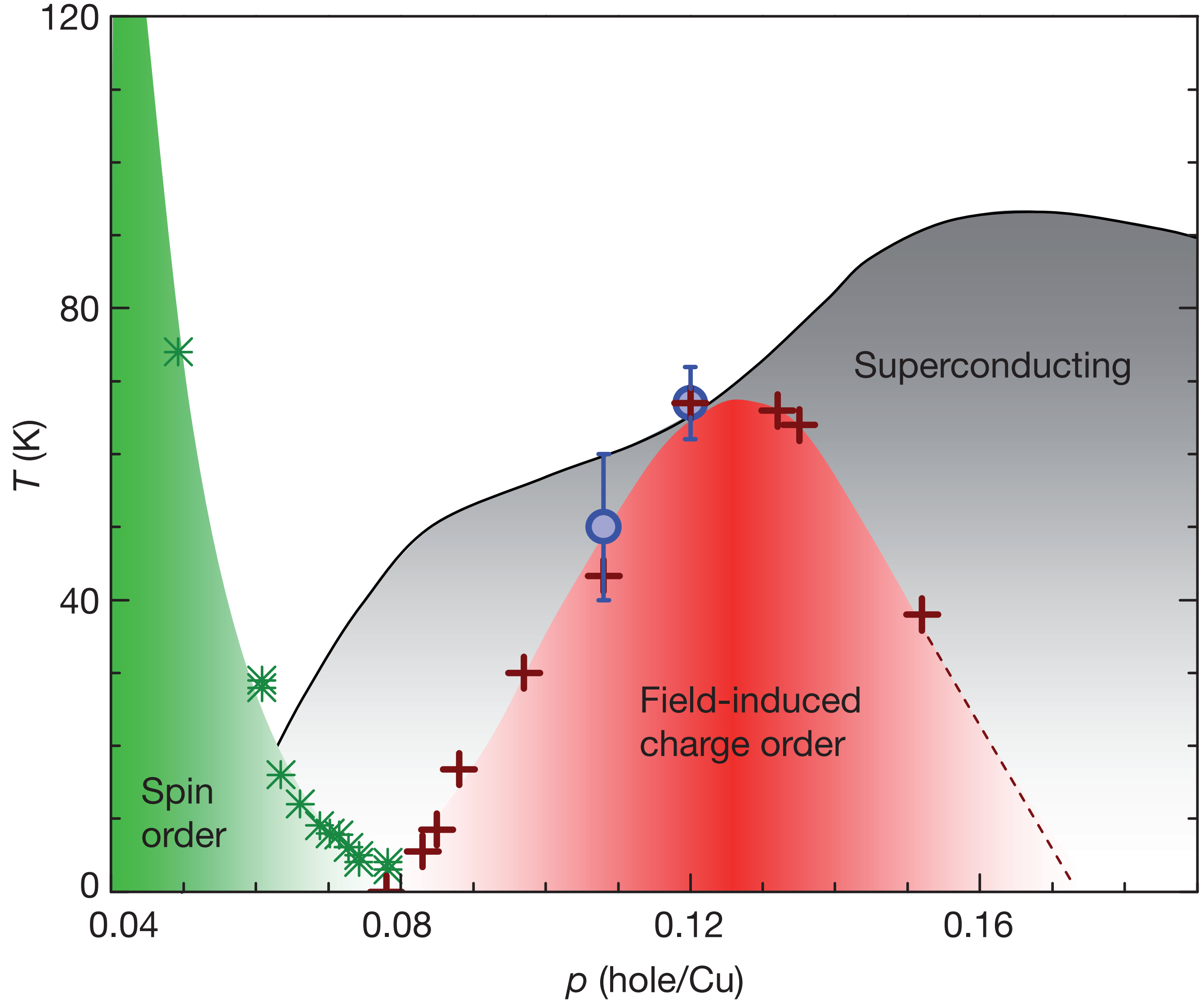

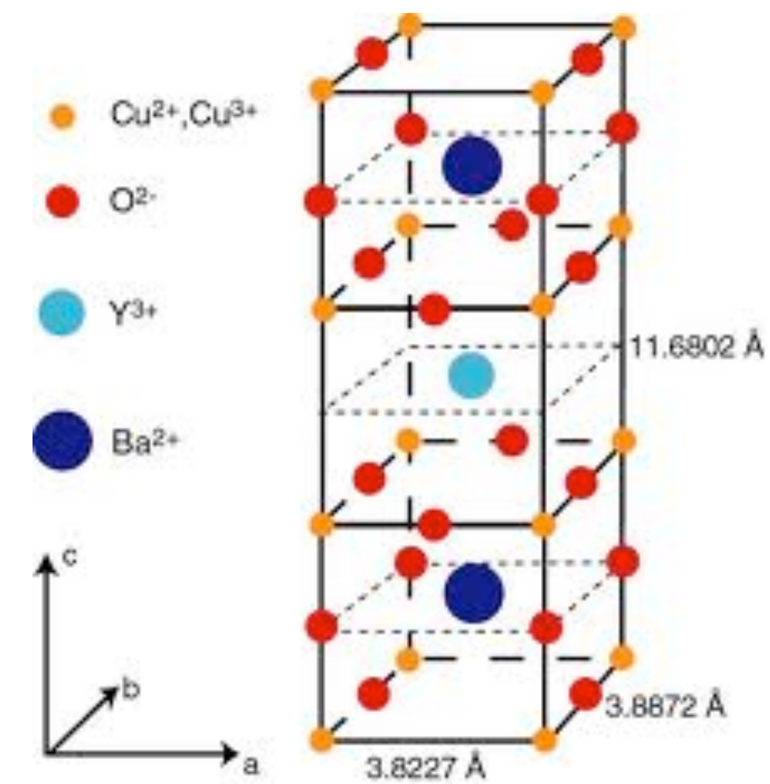

Summary

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points using the methods of gauge-gravity duality.

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Summary

Antiferromagnetism in metals and the high temperature superconductors

- Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)

- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to *d*-wave superconductivity, and to a charge density wave with a *d*-wave form factor. This is a promising explanation of the pseudogap regime.