Relativistic viscous hydrodynamics and AdS/CFT correspondence at finite temperature

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Abstract

Second-order viscous hydrodynamics in conformal field theories at high temperature is reviewed and the transport coefficients in strong-coupling are given obtained from gauge-gravity duality. Results for bulk physics are compared with RHIC data.

1 Introduction

I start with a definition: a fluid which has no shear stresses, viscosity or heat conduction is called a PERFECT FLUID, i.e. it looks isotropic in its rest frame, and a quotation: "Top physics story of 2005 is the RHIC discovery of the strongly interacting quark-gluon plasma (called sQGP), which behaves almost like a perfect fluid, with very low viscosity" [1].

Today it is still a crucial story, however, its content needs to be carefully tested!

This talk is based on the work by R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov on "Relativistic viscous hydrodynamics, conformal invariance, and holography" [2], and by M. Luzum and P. Romatschke on "Conformal relativistic viscous hydrodynamics: applications to RHIC results at $\sqrt{s(NN)} = 200$ GeV" [3].

2 Hydrodynamics

Relativistic hydrodynamics [4] is written in terms of the energy momentum tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{perfect} + \Pi^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu} + \Pi^{\mu\nu} \ , \tag{1}$$

where ϵ is the energy density, p the pressure and u^μ the fluid velocity which fulfills $u_\mu u^\mu = -1$. In the following only shear viscosity terms are kept and no net charge in the system is assumed. The symmetric shear tensor $\Pi^{\mu\nu}$ satisfies $u_\mu \Pi^{\mu\nu} = 0, \ \ \Pi^\mu_\mu = 0$. The evolution equations are given by the local conservation law (geometric covariant derivative ∇_μ): $\nabla_\mu T^{\mu\nu} = 0$.

To be noted: in case of interactions present in the system, e.g. in underlying QCD dynamics, a non-vanishing $\Pi^{\mu\nu}$ is present. The main question to be answered is: is the contribution by $\Pi^{\mu\nu}$ large or small?

2.1 Approximation

To first-order in gradients with respect to u^μ, the shear tensor reads

$$\Pi^{\mu\nu} = -2\eta < \nabla^\mu u^\nu > \equiv -\eta \sigma^{\mu\nu} \ , \tag{2}$$

with η the shear viscosity, and

$$\sigma^{\mu\nu} \equiv (\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha \ , \ \ \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \ . \tag{3}$$

The projection $\Delta^\mu_\alpha \nabla_\beta T^{\alpha\beta} = 0$ leads to the relativistic Navier-Stokes equation in first-order theory

$$(\epsilon + p) u^\alpha \nabla_\alpha u^\mu = \nabla^\mu p - \Delta^\mu_\alpha \nabla_\beta \left[-2\eta < \nabla^\alpha u^\beta > \right] , \tag{4}$$

which is a parabolic differential equation: the time derivative is of first order ($u^\alpha \nabla_\alpha \equiv D \rightarrow \partial/\partial t$), while the space derivative is of second order (∇^2): "Relativistic first-order dissipative theory is highly pathological, and therefore should be discarded in favor of the second-order one" [5].

To see this problem differently, apply a small linear perturbation in the first-order theory, e.g. in the transverse mode $\delta u_\perp$, to find a diffusion equation in the shear channel with a Gaussian solution, which propagates outside the light-cone.

A minimal modification beyond the diffusion equation by introducing a relaxation time $\tau_\pi > 0$ leads to a hyperbolic equation,

$$[\tau_\pi \partial_t^2 + \partial_t - \frac{\eta}{(\epsilon + p)} \partial_x^2] \, \delta u_\perp = 0 \ , \tag{5}$$

which becomes second-order in gradients [6].

2.2 Conformal hydrodynamics

Going much beyond the above conjecture, having CFT in mind, a new result has been obtained [2, 7]: *all* second-order terms have been classified by conformal symmetry. Starting from the Weyl transformations: $g^{\mu\nu} \rightarrow e^{2\omega(x)} \, g^{\mu\nu}, \ T^{\mu\nu} \rightarrow e^{6\omega} \, T^{\mu\nu}$, ... the constitutive relation of causal viscous hydrodynamics expressed by the derivative expansion to second-order is derived:

$$\begin{aligned} \Pi^{\mu\nu} = & - \eta\sigma^{\mu\nu} + \eta\tau_\pi \left[^{\langle} D\sigma^{\mu\nu\rangle} + \frac{1}{3}\sigma^{\mu\nu}(\nabla_\alpha u^\alpha) \right] \\ & + \kappa \left[R^{<\mu\nu>} - 2u_\alpha R^{\alpha<\mu\nu>\beta} u_\beta \right] + \lambda_1 \sigma^{<\mu}{}_\lambda \sigma^{\nu>\lambda} + \lambda_2 \sigma^{<\mu}{}_\lambda \Omega^{\nu>\lambda} + \lambda_3 \Omega^{<\mu}{}_\lambda \Omega^{\nu>\lambda} , \end{aligned} \tag{6}$$

where $R^{\alpha\beta\gamma\delta}$ is the Riemann tensor, and $R^{\alpha\beta}$ is the Ricci tensor, present in case of curved spaces. $\Omega^{\alpha\beta}$ denotes the antisymmetric vorticity tensor. An independent elegant derivation of this result introducing a Weyl-covariant formalism can be found in [8].

2.3 Müller-Israel-Stewart theory

Keeping just one term in the derivative expansion at second-order, namely

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta \, \tau_\pi \, ^{\langle} D\sigma^{\mu\nu\rangle} \ \ , \tag{7}$$

defines the Müller-Israel-Stewart theory [11,12]. In [2] it is remarked that it does not match with AdS/CFT $\mathcal{N} = 4$ SYM and that therefore all second-order terms in Eq. (6) consistent with conformal symmetry have to be included into the shear tensor.

3 AdS/CFT correspondence

Following Maldacena [9] a strongly coupled quantized conformal gauge theory in $d = 4$ dimensions ($\mathcal{N} = 4$ SYM with $8N_c$ (1 gauge and 6 scalar) bosons and ($4N_c$) Weyl fermions), which is obviously NOT QCD, is dual to a weakly coupled classical supergravity (type IIB) in $d = 10$ dimensions (on $AdS_5 \times S^5$) via a holographic property based on the near extremal black $D3-$ brane metric with horizon $r = r_0$,

$$ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f(r)}dr^2, \;\; f(r) = 1 - \frac{r_0^4}{r^4} \, , \tag{8}$$

where the radial (bulk) coordinate is bounded by $r_0 \leq r < \infty$, with the gauge theory on the boundary at ∞. $D3-$ branes are dynamical walls on which strings can end: the theory of open strings is living on $D3-$branes $\Longleftrightarrow$ the gravity theory of fields is living in the space curved by the branes. The Hawking temperature is given by $T = \frac{r_0}{\pi R^2}$.

The hydrodynamic transport coefficients are calculated in the limit of large 't Hooft coupling $\lambda = g^2_{YM} N_c$, $N_c \to \infty$, $g^2_{YM} << 1$, i.e. the string coupling $g_s = \frac{g^2_{YM}}{4\pi} << 1$ is small, implying no loops and small curvature $\frac{l_s^4}{R^4} = \frac{1}{\lambda} << 1$. The radius R of curvature is large compared to the string scale l_s, implying classical gravity.

The rather involved AdS/CFT-gravity calculations [2, 7], e.g. from the sound channel dispersion for momentum $\omega, k << T$, leads up to $O(k^3)$, etc.:

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad \tau_\pi = \frac{2 - \ln 2}{2\pi T}, \quad \kappa = \frac{\eta}{\pi T} = 2\lambda_1 \, , \;\; \lambda_2 = -\frac{\ln 2}{2\pi T}\eta \, , \quad \lambda_3 = 0 \, . \tag{9}$$

The essence of the calculation is to consider the quasi-normal modes in order to relate the gravitational perturbations to a black hole/brane to the ones of a hydrodynamic system, e.g. see figures in [10].

4 Heavy-ion collisions

4.1 Ambiguities

Heavy-ion collisions require beyond well-understood hydrodynamics, which consists of a set of differential equations:

- initial conditions, i.e. equilibration time and distribution of energy density [13],
- a QCD equation of state,
- a hadronisation prescription.

4.2 Results

The main results obtained in [3] using the code based on viscous conformal hydrodynamics [2] are:

- viscous hydrodynamics simulation give a good description of RHIC data, including the elliptic flow v_2, with (s.. entropy density)

$$\frac{\eta}{s} = 0.1 \pm 0.1(theory) \pm 0.08(experiment) \, , \tag{10}$$

- the modest estimate is: $\frac{\eta}{s} < 0.5$,
- an early thermalisation time τ_0 is questioned, but $\tau_0 < 2\ fm$,
- weak dependence on the values of the second-order parameters $\tau_\pi, \lambda_1, ...$
  This is a consequence of the interplay between small gradients and the values of the parameters, which are at weak coupling: $\frac{\eta/s}{\tau_\pi T} = 1/6 = 0.167,\ \kappa = \lambda_1 = \lambda_3 = 0,\ \lambda_2 = -2\tau_\pi \eta$, and are not very different from the ones for $\lambda >> 1$: $\frac{\eta/s}{\tau_\pi T} = 0.383\ (1 - 3.52\ \lambda^{-3/2} + ...)$ (including corrections [14]).

These results imply for the viscosity: near equilibrium there is an estimate

$$\frac{\eta}{s} \simeq \hbar\ \frac{mean\ free\ path\ \lambda_f}{deBroglie\ wavelength}\ , \tag{11}$$

which allows to distinguish between

- a dilute system (QFT $->$ kinetic theory $->$ hydro):
  with the scale $\lambda_f \ \rightarrow \frac{\eta}{s} >> \hbar$, e.g. pQCD $(N_f = 0)$ [15]

$$\frac{\eta}{s} \simeq 3.8 \frac{1}{g^4 \ln(2.8/g)} \simeq O(1)\ \ for\ g = 2.5\ , \tag{12}$$

  BUT with $\ln(2.8/g) \simeq O(1)$: $\frac{\eta}{s} \simeq 0.1 \quad \rightarrow$ is sensitive to the constant under the log !
- a strongly coupled system (QFT $->$ hydro):
  the only scale is $1/T\ \rightarrow \frac{\eta}{s} = \frac{\hbar}{4\pi} \simeq 0.08$, which is the KSS bound [16, 17].

The modest estimate given above, however, does not rigorously exclude a perturbative QCD plasma versus a sQGP. A related statement follows from the estimates of the thermalisation time; for pQCD see [18].

5 Conclusions

There is excitement in the heavy-ion community about the beautiful ideas of the gauge/gravity correspondence, which strongly helps to gain intuition into STRONG COUPLING phenomena.

But one may ask for more [19], e.g. ”Is there an experiment whose outcome could cast strong doubts on the relevance of AdS/CFT to understand QCD” ? One answer maybe jet physics [20].

For me one of the most challenging questions of the theory is related to the detailed microscopic mechanism for the rather RAPID EQUILIBRATION of matter in RHIC collisions.

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