Partons and jets at strong coupling from AdS/CFT

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Abstract
Calculations using the AdS/CFT correspondence can be used to unveil the short–distance structure of a strongly coupled plasma, as it would be seen by a 'hard probe'. The results admit a natural physical interpretation in terms of parton evolution in the plasma: via successive branchings, essentially all partons cascade down to very small values of the longitudinal momentum fraction x and to transverse momenta smaller than the saturation momentum $Q_s \sim T/x$. This picture has some striking consequences, like the absence of jets in electron–proton annihilation at strong coupling, of the absence of particle production at forward and backward rapidities in hadron–hadron collisions.

1 Introduction

One of the most interesting suggestions emerging from the experimental results at RHIC is that the deconfined, 'quark–gluon', matter produced in the early stages of an ultrarelativistic nucleus–nucleus collision might be strongly interacting. This observation motivated a multitude of applications of the AdS/CFT correspondence to problems involving a strongly–coupled gauge plasma at finite temperature and/or finite quark density. While early applications have focused on the long–range and large–time properties of the plasma, so like hydrodynamics, more recent studies have been also concerned with the response of the plasma to a 'hard probe' — an energetic 'quark' or 'current' which probes the structure of the plasma on space–time scales much shorter than the characteristic thermal scale $1/T$ (with T being the temperature).

From the experience with QCD one knows that the simplest hard probe is an electromagnetic current. In deep inelastic scattering (DIS), the exchange of a highly virtual space–like photon between a lepton and a hadron acts as a probe of the hadron parton structure on the resolution scales set by the process kinematics: if Q^2 is (minus) the photon virtuality and s is the invariant photon–hadron energy squared, then the photon couples to quark excitations having transverse momenta $k_\perp \lesssim Q$ and a longitudinal momentum fraction $x \sim Q^2/s$. Also, the partonic fluctuation of a space–like current can mimic a quark–antiquark 'meson', which is nearly on–shell in a frame in which the current has a high energy. Furthermore, the decay of the time–like photon produced in electron–positron annihilation is the simplest device to produce and study hadronic jets in QCD. Thus, by studying the propagation of an energetic current through the plasma one has access to quantities like the plasma parton distributions, the meson screening length, or the energy loss and the momentum broadening of a jet.

At strong coupling and large number of colors $N_c \gg 1$, the AdS/CFT correspondence allows one to study the propagation of an Abelian '$\mathcal{R}$–current' through the finite–temperature plasma described by the $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory. (For a recent review and more references see [1].) In this context, DIS has been first addressed for the case of a dilaton target, in Refs. [2,3]. These studies led to an interesting picture for the partonic structure at strong coupling: through successive branchings, all partons end up by 'falling' below the 'saturation line', i.e., they occupy — with occupation numbers of order one — the phase–space at transverse momenta below the saturation scale $Q_s(x)$, which itself rises rapidly with $1/x$. Such a rapid increase, which goes like $Q_s^2(x) \sim 1/x$ and hence is much faster than in perturbative QCD, comes about because the high–energy scattering at strong coupling is governed by a spin $j \simeq 2$ singularity (corresponding to graviton exchange in the dual string theory), rather than the usual $j \simeq 1$ singularity associated with the gluon exchange at weak coupling.

In Refs. [4], this partonic picture has been extended to a finite–temperature SYM plasma in the strong 't Hooft coupling limit $\lambda \equiv g^2 N_c \to \infty$ (meaning $N_c \to \infty$). The results of these analyses will be briefly described in what follows.

2 Deep inelastic scattering at strong coupling from AdS/CFT

The strong coupling limit $\lambda \to \infty$ in the $\mathcal{N} = 4$ SYM gauge theory corresponds to the semiclassical, 'supergravity', approximation in the dual string theory, which lives in a ten–dimensional curved space–time with metric $AdS_5 \times S^5$. The finite–temperature gauge plasma is 'dual' to a black hole in AdS_5 which is homogeneous in the four Minkowski dimensions and whose AdS radius r_0 is proportional to the temperature: $r_0 = \pi R^2 T$, with R the curvature radius of AdS_5. The interaction between the $\mathcal{R}$–current J_μ and the plasma is then described as the propagation of a massless vector field A_μ which obeys Maxwell equations in the AdS_5 Schwarzschild geometry. The fundamental object to be computed is the retarded current–current correlator,

$$\Pi_{\mu\nu}(q) \equiv i \int \mathrm{d}^4 x \, \mathrm{e}^{-iq\cdot x} \, \theta(x_0) \, \langle [J_\mu(x), J_\nu(0)] \rangle_T \,, \tag{1}$$

whose imaginary part determines the cross–section for the current interactions in the plasma, i.e., the plasma structure functions in the *space–like* case $Q^2 \equiv -q^\mu q_\mu > 0$ ('deep inelastic scattering') and the rate for the current decay into 'jets' in the *time–like* case $Q^2 < 0$ ('e^+e^- annihilation'). The imaginary part arises in the supergravity calculation via the condition that the wave A_μ has no reflected component returning from the horizon. Physically, this means that the wave (current) can be absorbed by the black hole (the plasma), but not also regenerated by the latter. The classical solution $A_\mu(r)$ is fully determined by this 'no–reflected–wave' condition near the horizon together with the condition that the fields take some prescribed values at the Minkowsky boundary: $A_\mu(r \to \infty) = A_\mu^{(0)}$. The current–current correlator is then obtained as

$$\Pi_{\mu\nu}(q) = \frac{\partial^2 \mathcal{S}_{\mathrm{cl}}}{\partial A_\mu^{(0)} \partial A_\nu^{(0)}} \,, \tag{2}$$

where $\mathcal{S}_{\mathrm{cl}}$ denotes the classical action density (the Maxwell action evaluated on the classical solution), and is bilinear in the boundary fields $A_\mu^{(0)}$.

In what follows we shall focus on the space–like current, i.e., on the problem of DIS off the plasma [4]. (The corresponding discussion of a time–like current can be found in the second paper in Ref. [4]; see also the related work in Ref. [5].) We choose the current as a plane–wave

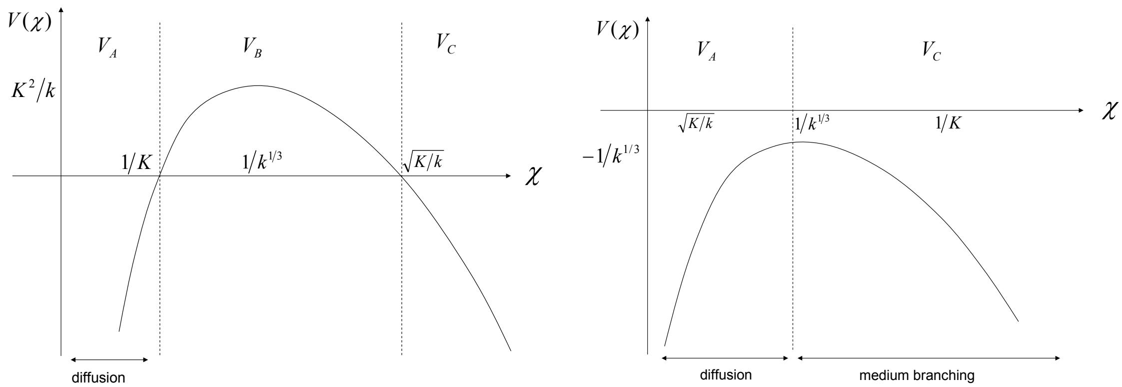

Fig. 1: *The potential in the effective Schrödinger equation describing the propagation of the space–like Maxwell wave in AdS_5–BH. Left: low energy, or large x ($x \gg T/Q$). Right: high energy, or small x ($x \lesssim T/Q$)*

propagating in the z direction in the plasma rest frame: $J_\mu(x) \propto \mathrm{e}^{-i\omega t+iqz}$. Also, we asume the high–energy and large–virtuality kinematics: $\omega \gg Q \gg T$. The physical interpretation of the results can be facilitated by choosing a different definition for the radial coordinate on AdS_5: instead of r, it is preferable to work with the inverse coordinate $\chi \equiv \pi R^2/r$, which via the UV/IR correspondence corresponds (in the sense of being proportional) to the transverse size L of the partonic fluctuation of the current. Then, the AdS_5 boundary lies at $\chi = 0$ and the black–hole horizon at $\chi = 1/T$.

Via a suitable change of function, the Maxwell equations for A_μ can be rewritten as a pair of time–independent Schrödinger equations — one for the longitudinal modes, the other one for the transverse ones. Then, the dynamics can be easily understood by inspection of the respective potential, as illustrated in Fig. 1 for two different regimes of energy. (Note that in plotting the potential in these figures we are using the dimensionless variables $K \equiv Q/T$ and $k \equiv q/T$; also, χ is multiplied by T.) The dynamics depends upon the competition between, on one hand, the virtuality Q^2, which acts as a potential barrier preventing the Maxwell wave A_μ to penetrate deeply inside AdS_5, and, on the other hand, the product ωT^2, which controls the strength of the interactions between this wave and the black hole. (We recall that the gravitational interactions are proportional to the energy density of the two systems in interaction.) The relevant dimensionless parameter is $Q^3/\omega T^2$, which can be also rewritten as xQ/T, where $x \equiv Q^2/2\omega T$ (the Bjorken variable for DIS) has the physical meaning of the longitudinal momentum fraction of the plasma 'parton' struck by the current.

Specifically, in the high–Q^2 regime at $Q^3/\omega T^2 \gg 1$, or $x \gg T/Q$, the interaction with the plasma is relatively weak and the dynamics is almost the same as in the vacuum: the wave penetrates in AdS_5 up to a maximal distance $\chi_0 \sim 1/Q$ where it gets stuck against the potential barrier. Physically, this means that the current fluctuates into a pair of partons (say, a quark–antiquark 'meson') with transverse size $L \sim 1/Q$. At finite temperature, however, the potential barrier has only a finite width — it extends up to a finite distance $\chi_1 \sim (1/T)\sqrt{Q/\omega}$ —, so there is a small, but non–zero, probability for the wave to cross the barrier via tunnel effect. Physically, this means that the plasma structure function at large x is non–vanishing, but extremely small (exponentially suppressed) : $F_2(x, Q^2) \propto xN_c^2Q^2 \exp\{-(x/x_s)^{1/2}\}$ for $x \gg x_s \equiv T/Q$. In other terms, when probing the plasma on a transverse resolution scale Q^2, one finds that there are essentially no partons with momentum fraction x larger than $T/Q \ll 1$.

Where are the partons then ? To answer this question, let us explore smaller values of Bjorken's x, by increasing the energy ω at fixed Q^2 and T. Then the barrier shrinks and eventually disappears; this happens when ω is large enough for $\chi_1 \sim \chi_0$, a condition which can be solved either for x (thus yielding $x \sim x_s = T/Q$), or for Q, in which case it yields the *plasma saturation momentum* : $Q_s^2(x, T) \sim T^2/x^2$. For higher energies, meaning $x < x_s$, the barrier has disappeared and the Maxwell wave can propagate all the way down to the black hole, into which it eventually falls, along a trajectory which coincides with the 'trailing string' of a heavy quark [6]. Physically, this means that the current has completely dissipated into the plasma. We interpret this dissipation as *medium–induced branching* : the current fragments into partons via successive branchings, with a splitting rate proportional to a power of the temperature. This branching continues until the energy and the virtuality of the partons degrade down to values of order T. The lifetime of the current (estimated as the duration of the fall of the Maxwell wave into the black hole) is found as $\Delta t \sim \omega/Q_s^2 \propto \omega^{1/3}$ — a result which agrees with a recent estimate of the 'gluon' lifetime in Ref. [7]. Since the current is tantamount to a 'meson' with size $1/Q$ and rapidity $\gamma = \omega/Q$, our analysis also implies an upper limit on the transverse size of this 'meson' before it melts in the plasma: $L_{\max} \sim 1/Q_s \sim 1/\sqrt{\gamma}T$. This limit is consistent with the meson screening length computed in Refs. [8]. The saturation momentum Q_s turns out to also be the scale which controls the energy loss [4, 6] and the transverse momentum broadening [9, 10] of a parton moving into the plasma. For instance, the rate for the energy loss of a heavy quark reads (in the ultrarelativistic limit $\gamma \gg 1$) [4, 10]

$$-\frac{\mathrm{d}\omega}{\mathrm{d}t} \sim \sqrt{\lambda}\, Q_s^2\,, \tag{3}$$

where one should keep in mind that the saturation scale in the r.h.s. is itself a function of ω, and hence of time: $Q_s^2 \sim (\omega T^2)^{1/3}$. Eq. (3) may be viewed as the time–dependent generalization of the 'drag force' first computed in Refs. [6].

The complete absorbtion of the current by the plasma is tantamount to the 'black disk' limit for DIS: in this high–energy, or small–x, regime the structure function is not only non–zero, but in fact it reaches its maximal possible value allowed by unitarity. This value is found as $F_2(x, Q^2) \sim xN_c^2Q^2$ for $x \sim x_s$, a result with a natural physical interpretation: for a given resolution Q^2, essentially all partons have momentum fractions $x \lesssim T/Q \ll 1$ and occupation numbers $n \sim \mathcal{O}(1)$. This is similar to parton saturation in pQCD, except that, now, the occupation numbers at saturation are of order one, rather than being large ($n \sim 1/g^2N_c$), as it was the case at weak coupling.

This result has interesting consequences for a (hypothetic) high–energy hadron–hadron collision, in which these partons would be liberated: Since there are no partons carrying large longitudinal momenta, there will be no 'forward/backward jets' in the wake of the collision, that is, no hadronic jets following the same directions of motion as the incoming hadrons. Rather, all particles will be produced at central rapidities and will be isotropically distributed in the transverse space. Similar conclusions hold for a *time–like* virtual photon decaying in the vacuum [4], that is, for the analog of electron–positron annihilation at strong coupling (see Fig. 2): unlike

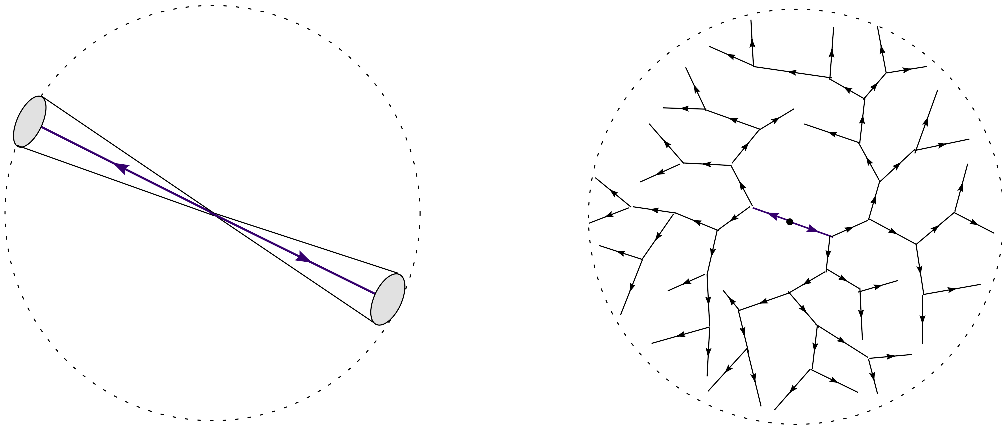

Fig. 2: *Final state produced in* e^+e^- *annihilation: (left) weak coupling; (right) strong coupling.*

at weak coupling, where the typical final state involves a pair of back–to–back hadronic jets, at strong coupling the original pair of partons undergoes a rapid branching process leading to an isotropic distribution of matter in the detector. Similar results have reached in Refs. [11]. This picture for the final state looks quite different from that predicted by perturbative QCD and observed in actual high–energy experiments. Such a discrepancy suggests that much caution should be taken when trying to extrapolate results from AdS/CFT to QCD.

References

[1] D. Son and A. Starinets, `arXiv:0704.0240 [hep-th]`.

[2] J. Polchinski and M. J. Strassler, *JHEP* **05** (2003) 012.

[3] Y. Hatta, E. Iancu, and A. H. Mueller, *JHEP* **01** (2008) 026.

[4] Y. Hatta, E. Iancu, and A. H. Mueller, *JHEP* **01** (2008) 063; *JHEP* **05** (2008) 037.

[5] P.M. Chesler, K. Jensen, and A. Karch, `arXiv:0804.3110 [hep-th]`.

[6] C. P. Herzog et al, *JHEP* **07** (2006) 013; S. S. Gubser, *Phys. Rev.* **D74** (2006) 126005.

[7] S. S. Gubser, D. R. Gulotta, S. S. Pufu, F. D. Rocha, `arXiv:0803.1470 [hep-th]`.

[8] K. Peeters et al, *Phys. Rev.* **D74** (2006) 106008; H. Liu et al *Phys. Rev. Lett.* **98** (2007) 182301; M. Chernicoff et al *JHEP* **09** (2006) 068; E. Caceres et al *JHEP* **10** (2006) 011.

[9] J. Casalderrey-Solana and D. Teaney, *JHEP* **04** (2007) 039; *Phys. Rev.* **D74** (2006) 085012; S. S. Gubser, *Phys. Rev.* **D76** (2007) 126003.

[10] F. Dominguez et al, `arXiv:0803.3234 [nucl-th]`; A.H. Mueller, `0805.3140 [hep-ph]`.

[11] D. M. Hofman and J. Maldacena, *JHEP* **05** (2008) 012.