Analyticity, Unitarity, and Gauge-String Duality

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High Energy scattering after AdS/CFT,
Conformal Invariance, Confinement,
Saturation and Froissart bound.

References:

- R. Brower, J. Polchinski, M. Strassler, and C-I Tan, “The Pomeron and Gauge/String Duality”, hep-th/0603115;
- R. Brower, M. Strassler, and C-I Tan, hep-th/0707.2408, hep-th/0710.4378;
- R. Brower, H. Nastase, H. Schinitzer, and C-I Tan, arXiv:0809.1632 (hep-th). Also: arXiv:0801.3891 (hep-th).

Outline

- QCD Pomeron as "metric fluctuations" in AdS space
  - Graviton in AdS becomes a fixed Regge Cut: (Conformal Invariance)
  - Pomeron as a Reggeized Massive Graviton: (Confinement)
- Aspects of Analyticity, Unitarity and Confinement
- Conformal Invariance and Transverse Space,
- Phase of Eikonal, Saturation, Confinement.
- Analyticity and Unitarity Constraints on Multi-gluon amplitudes

Issues:

Eikonal Sum in AdS3:

$$\boxed{\begin{aligned} A_{2\to 2}(s,t) \;\simeq\; & -2is\int d^2b\, e^{-ib^{\perp}q_{\perp}} \\ & \times \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^{\perp},z,z')} - 1\right] \end{aligned}}$$

Eikonal, $\chi(s,b^{\perp},z,z')$, given by Pomeron Exchange in AdS.

Saturation:

$$\boxed{\chi(s, x^{\perp} - x'^{\perp}, z, z') \;=\; O(1)}$$

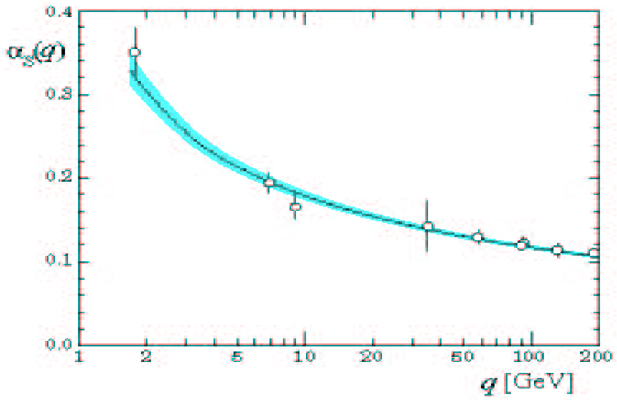
Questions:

Constraints due to Comformal Inv., Analyticity, Unitarity, Confinement, etc.

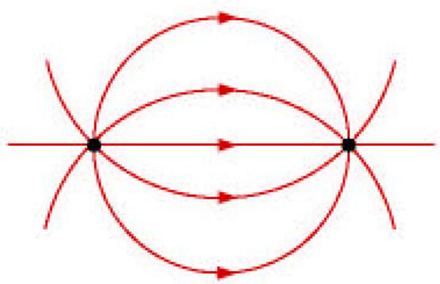
I. Scale Dependence of QCD and History of Hadron Scattering at High Energies

Asymptotic Freedom

perturbative



$$\alpha_s(q) \equiv \frac{\bar{g}(q)^2}{4\pi} = \frac{c}{\ln(q/\Lambda)} + \ldots$$



r* < 0.1 *fm

Confinement

non-perturbative

r* >> 1 *fm

Force at Long Distance--Constant Tension/Linear Potential, Coupling increasing, Quarks and Gluons strongly bound <==> "Stringy Behavior"

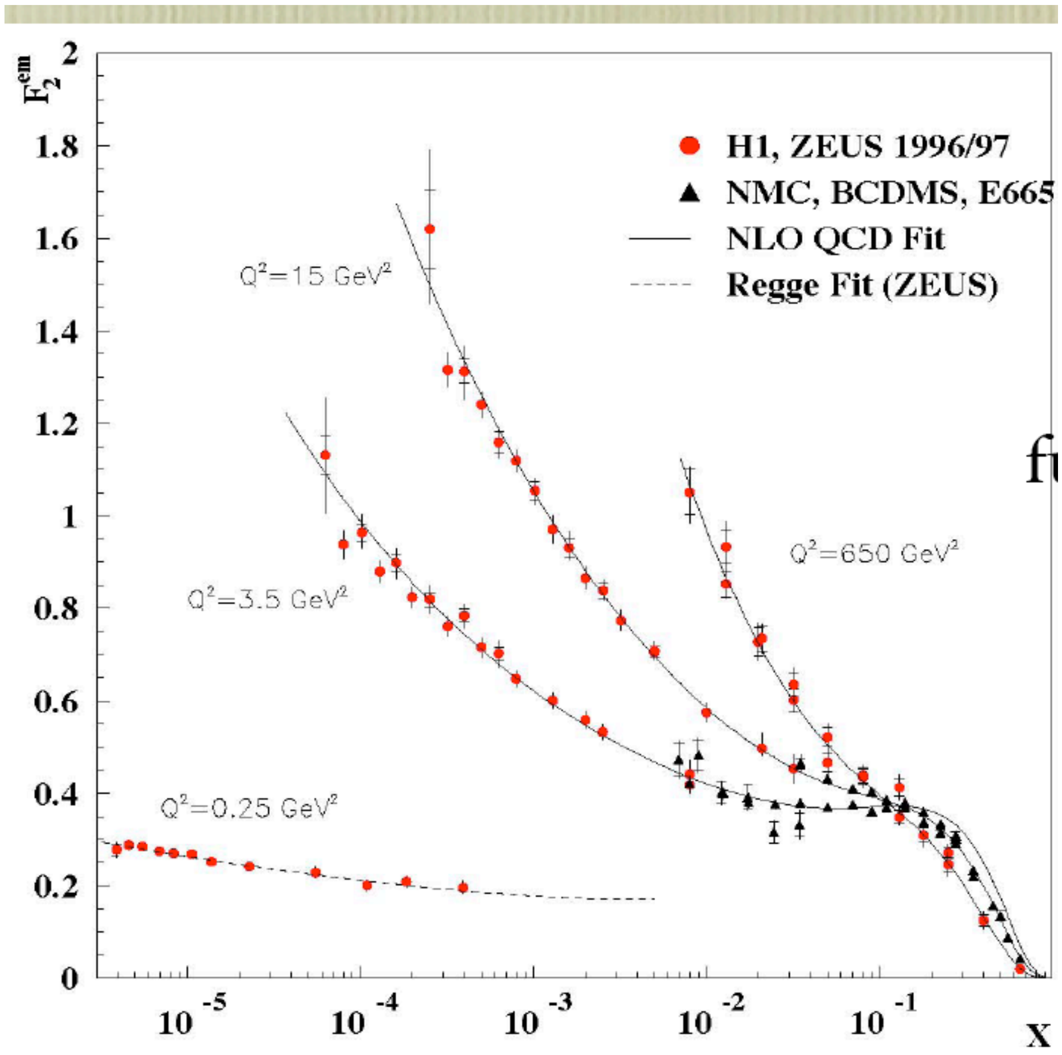
Test of Perturbative QCD-- Deep Inelastic Scattering (DIS)

Anomalous Dimension of

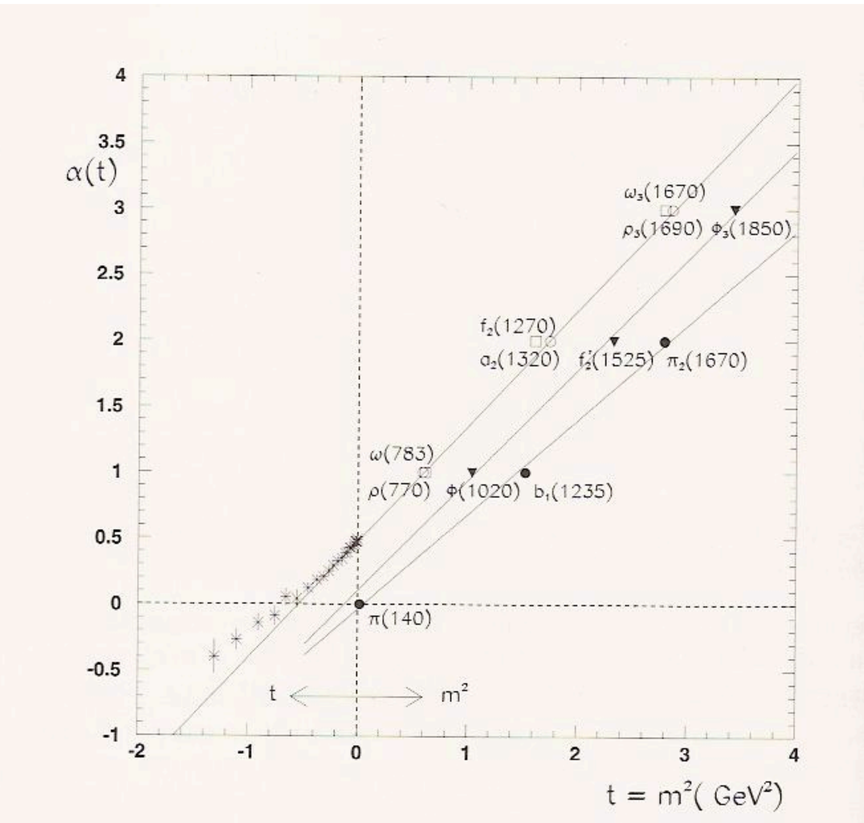
Leading twist operator

DGLAP evolution

$$\boxed{\mathrm{tr}\left(F_{+\mu} D_+^{j-2} F_+{}^{\mu}\right)}$$

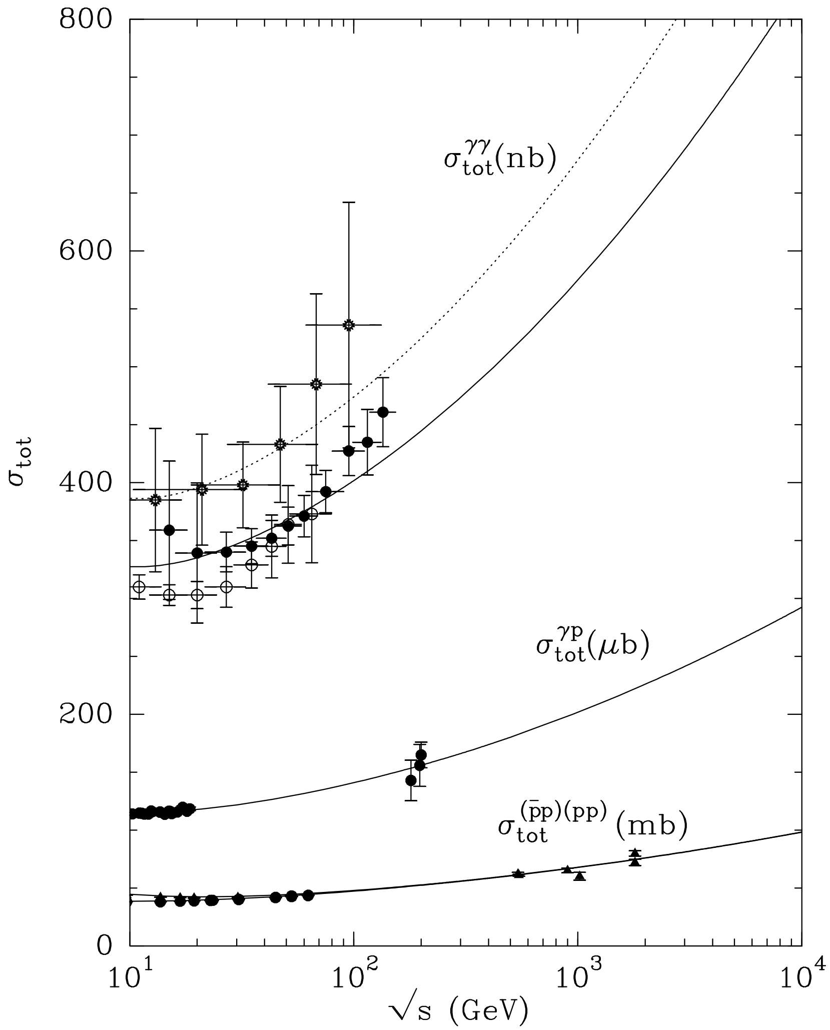


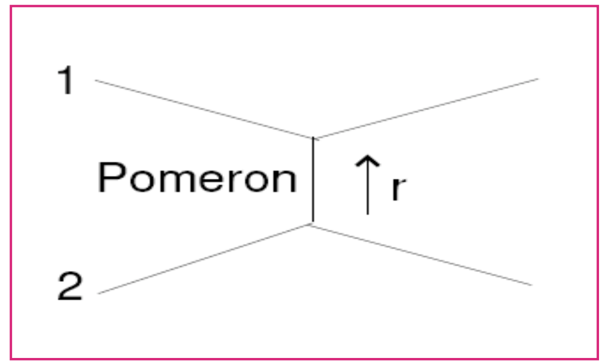
Regge Behavior and Regge Trajectory



$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

Total Cross Sections





$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

$$\sigma_{total} \sim \mathcal{A}(s,0)/s \sim S^{J(0)-1} \sim s^{\alpha(0)-1}$$

$$\alpha(0) > 1$$

(IR) Pomeron as Closed String??

BFKL vs Soft Pomeron

- Perturbative QCD
- Short-Distance
- $\alpha_{BFKL}(0) \sim 1.4$
- Increasing Virtuality
- No Shrinkage of elastic peak
- Fixed-cut in t
- Diffusion in Virtuality
-

- Non-Perturbative
- Long-distance: Confinement
- $\alpha_P(0) \sim 1.08$
- Fixed trans. Momenta
- Shrinkage of Elastic Peak: $<|t|> \sim 1/\log s$
- $\alpha'(0) \sim 0.3$ Gev^{-2}
- Diffusion in impact space

UV Pomeron (BFKL): Scale Invariance

IR Pomeron (Soft Pomeron): Confinement

The QCD Pomeron

We show that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

Pomeron can be associated with a Reggeized Massive Graviton.

Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

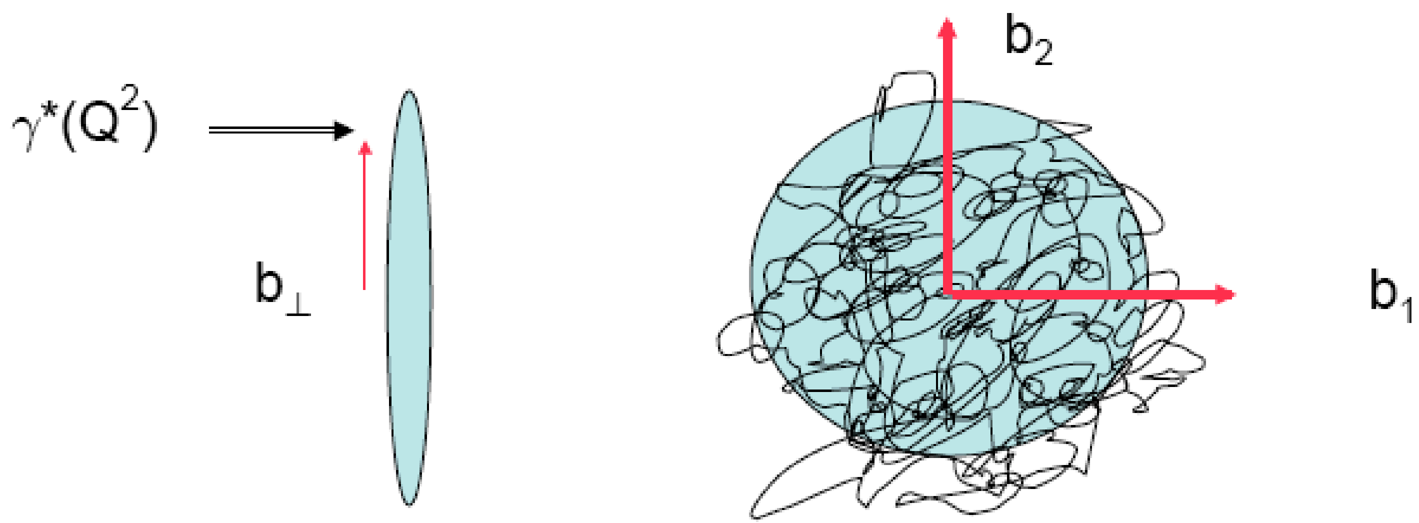
Unification

- *Soft Pomeron*: Diffusion in Impact space, *Hard Pomeron*: Diffusion in Virtuality,
- *Heterotic Pomeron* -- G. M. Levin and CIT (ISMD--1993)
- After nearly15 years, *Unification* through AdS/CFT Correspondence via AdS_5
- Pomeron is the Graviton in Curved Space (AdS)

Emergence of 5-dim AdS-Space

Let $z=1/r$, $0 < z < z_0$, where $z0 \sim 1/\Lambda_{qcd}$

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

2-d Longitudinal $p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$

2-d Transverse space: $x'_\perp - x_\perp = b_\perp$

1-d Resolution: $z = 1/Q$ (or $z' = 1/Q'$)

II: Gauge/String Duality

QCD Pomeron as "metric fluctuations" in AdS

- Strong <==> Weak duality
- Scale Invariance:
- Confinement:
- Pomeron as Reggeized Massive Graviton

II-a. Gauge/String Duality

Degrees of freedom: metric tensor, Kolb-Ramond anti-sym. tensor, etc.

Scale Invariance and AdS

What is the curved space?

Maldacena: UV (large r) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + ds_X^2$$

Captures QCD's approximate UV conformal invariance

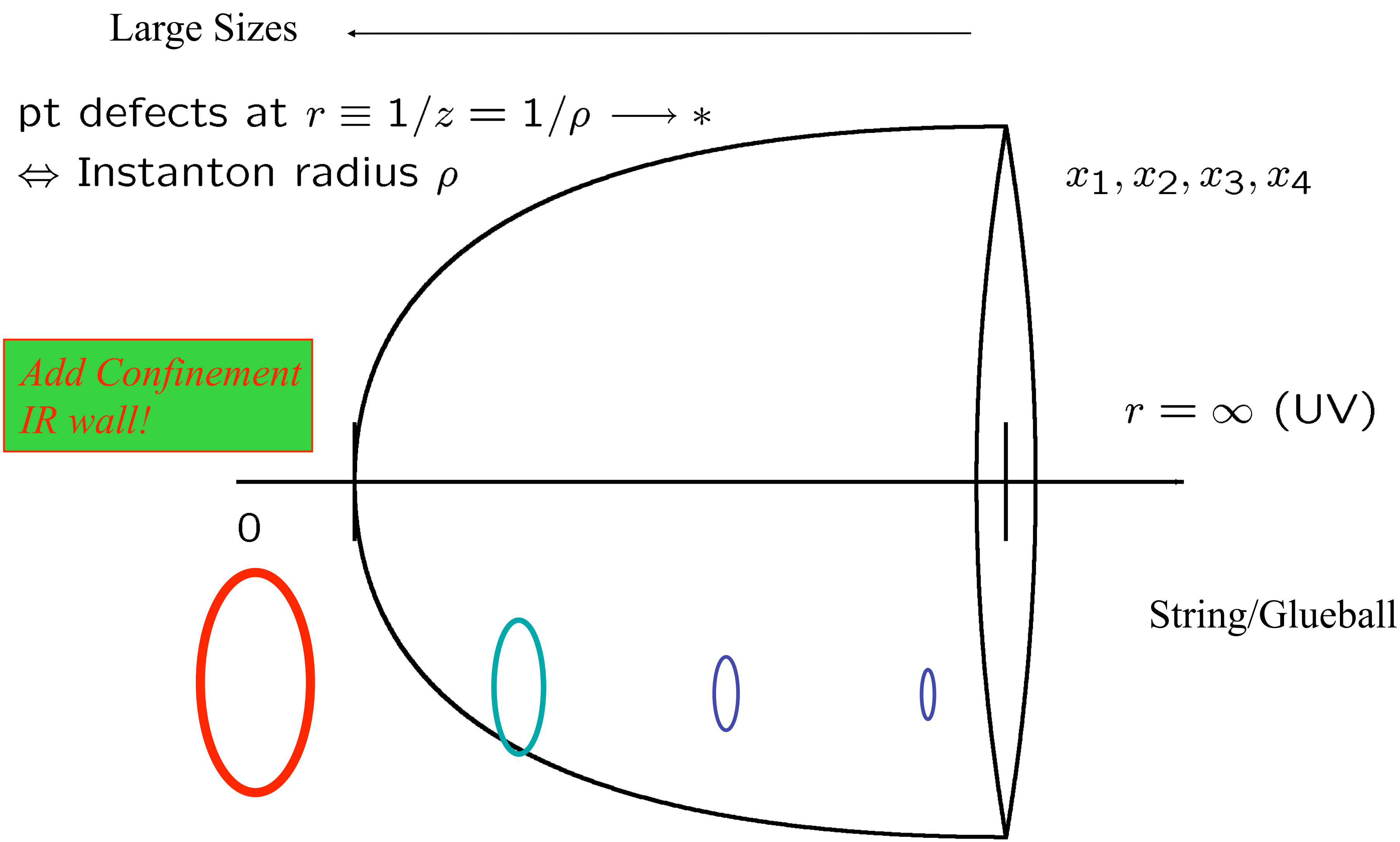
$$x \rightarrow \zeta x \ , \ r \rightarrow \frac{r}{\zeta} \qquad \text{(recall } r \sim \mu\text{)}$$

Confinement: IR (small r) is cut off in some way

$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

For Pomeron: *string theory* on cut-off AdS_5 (X plays no role)

Cutoff AdS_5



Large Sizes
pt defects at $r \equiv 1/z = 1/\rho \longrightarrow *$
$\Leftrightarrow$ Instanton radius ρ
x_1, x_2, x_3, x_4
Add Confinement IR wall!
$r = \infty$ (UV)
0
String/Glueball

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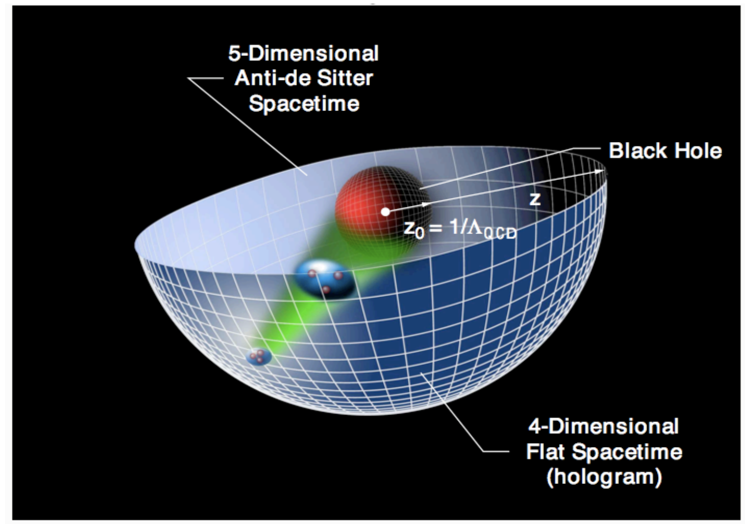
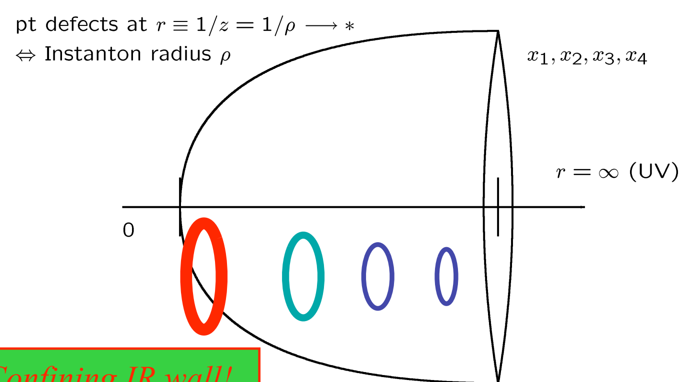


Cutoff AdS_5

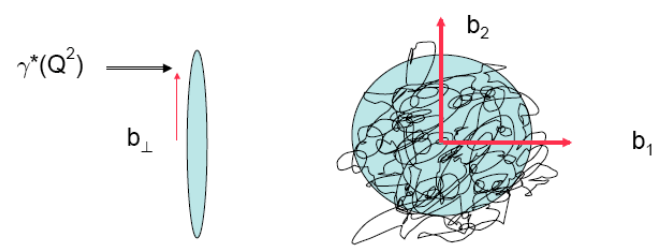
Large Sizes



Add Confining IR wall!

z=1/r,

"Fifth" co-ordinate is size z / z' of proj/target



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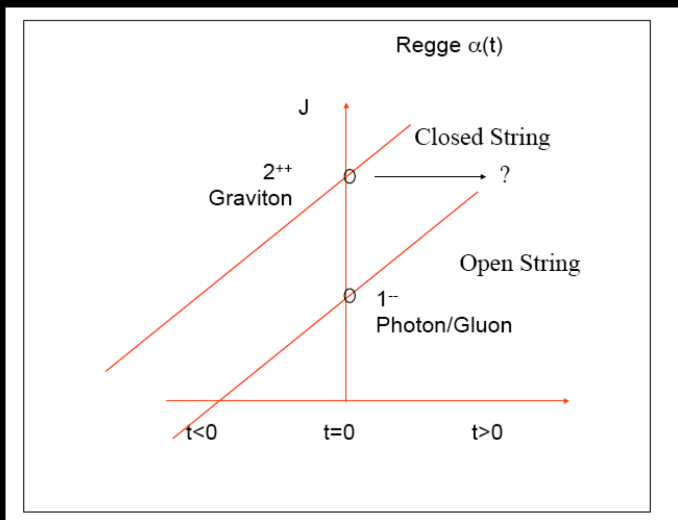
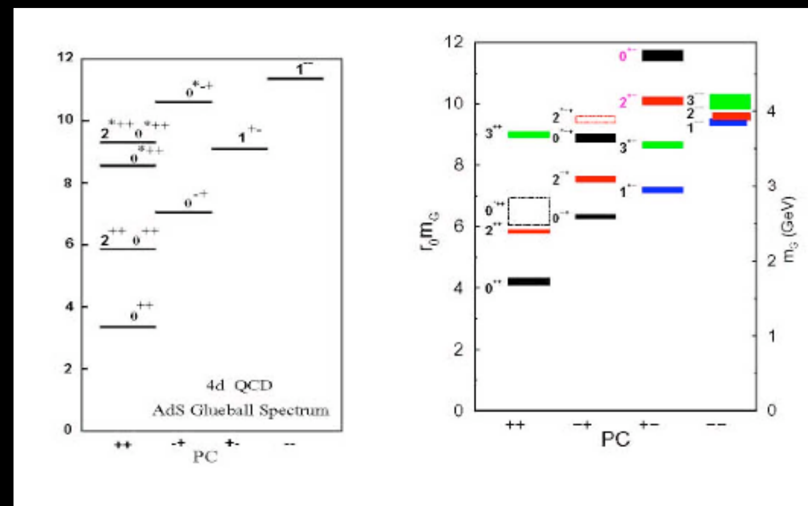
QCD Pomeron <====> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

Fixed cut in J-plane:

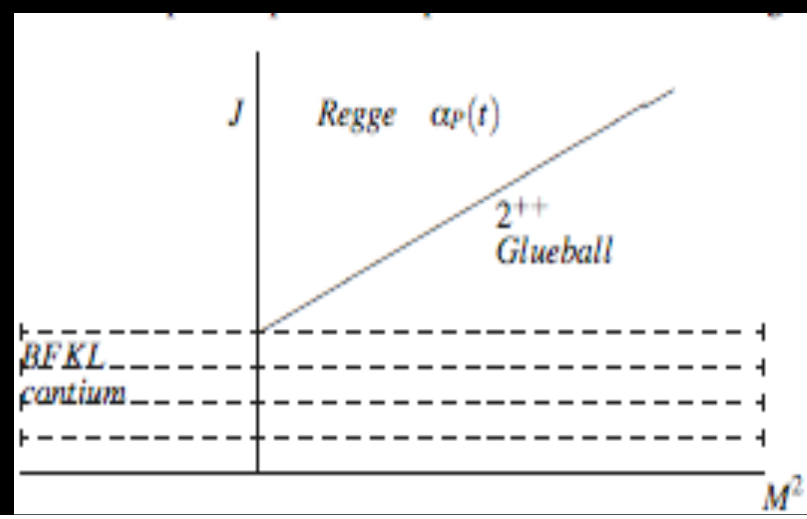
Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

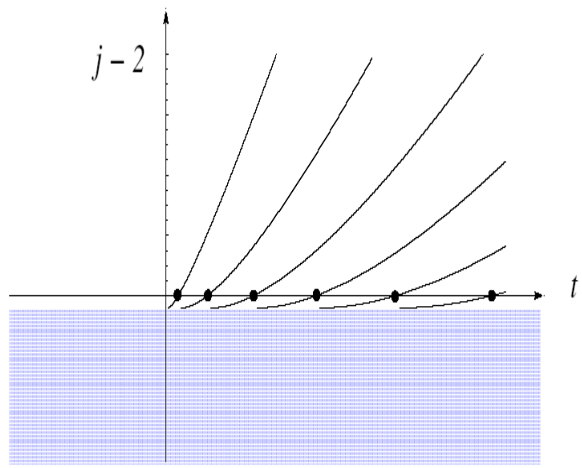
Strong coupling:

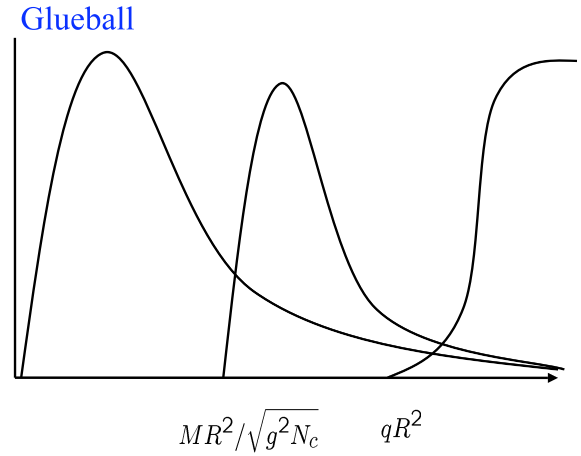
$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Pomeron in AdS Geometry

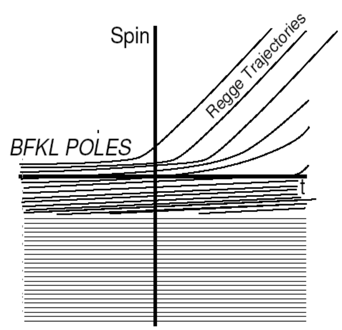


Pomeron in QCD





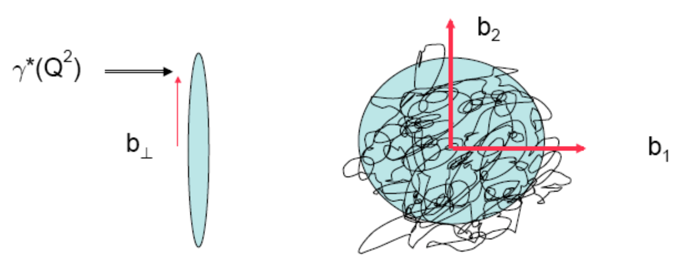
Running UV, Confining IR (large N)



The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

"Fifth" co-ordinate is size z / z' of proj/target



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1-d Resolution: $z = 1/Q$ (or $z' = 1/Q'$)

II-6. Spectrum at strong coupling

4-Dim Massive Graviton

5-Dim Massless Mode:

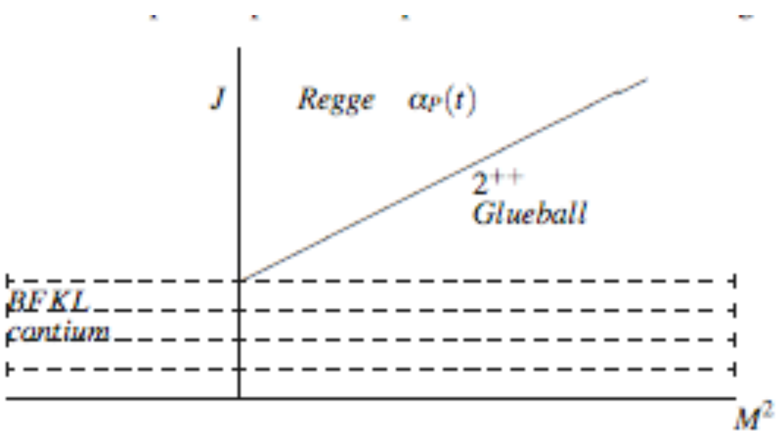
$$\boxed{0 = E^2 - (p_1{}^2 + p_2{}^2 + p_3{}^2 + p_r{}^2)}$$

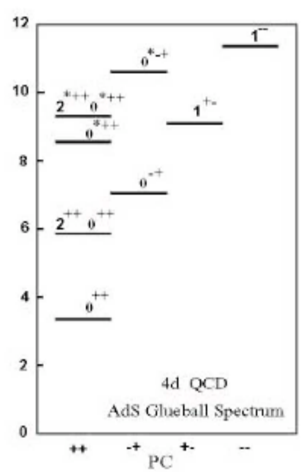
If, due to Curvature in fifth-dim, $p_r{}^2 \neq 0$,

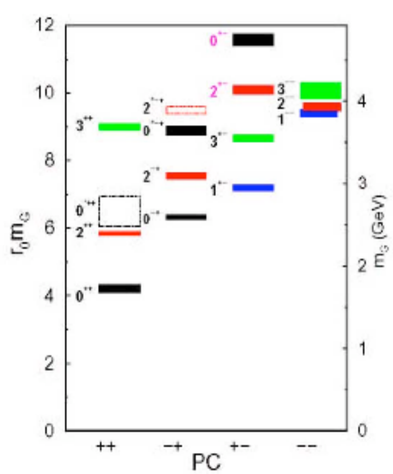
Four-Dimensional Mass:

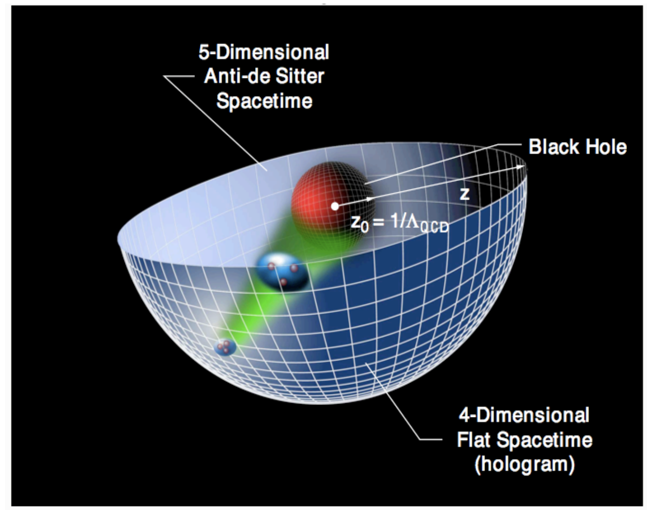
$$\boxed{E^2 = (p_1{}^2 + p_2{}^2 + p_3{}^2) + M^2}$$

QCD Pomeron <====> Graviton (metric) in AdS

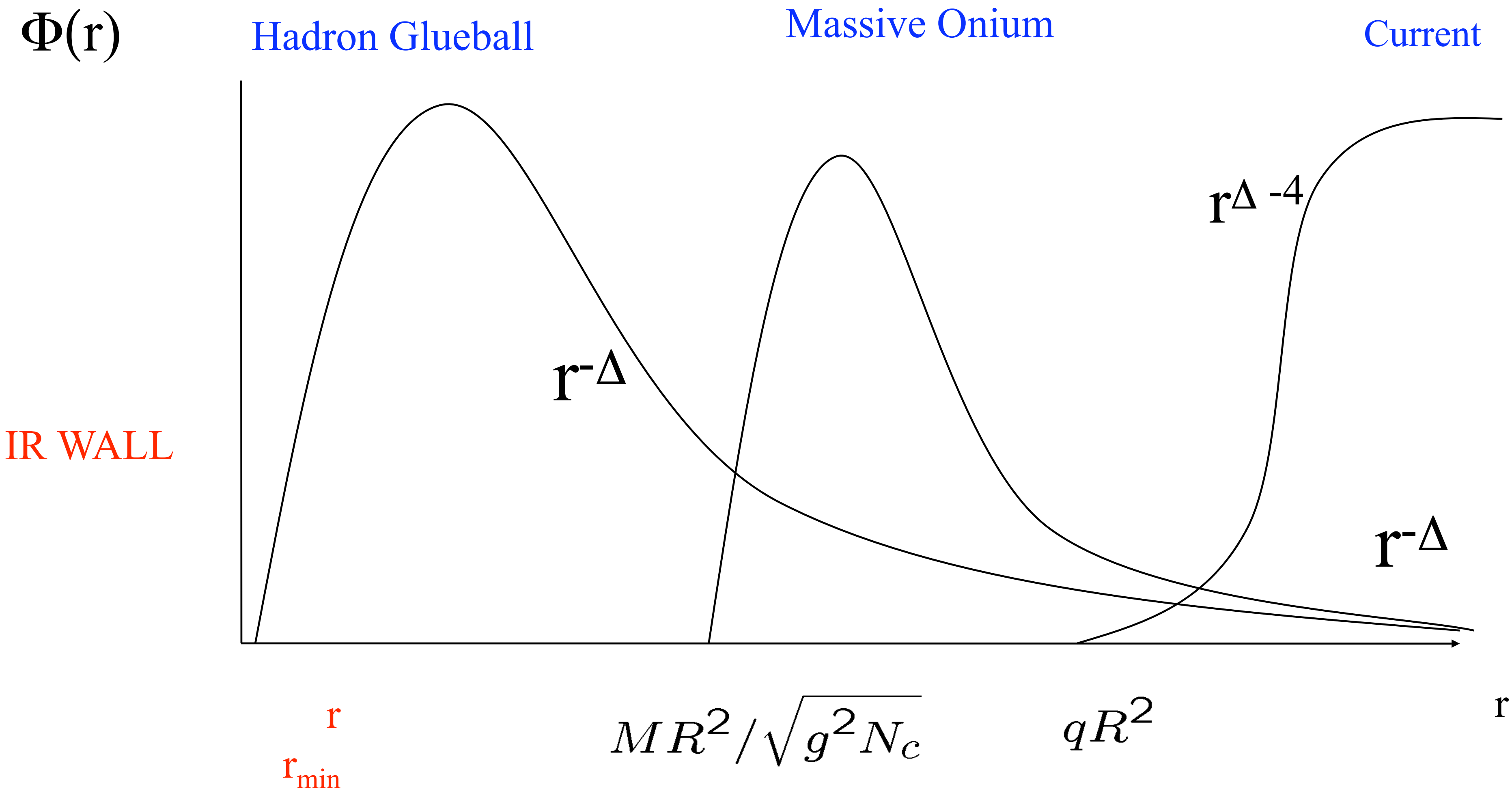








Approx. Scale Invariance and the 5th dimension

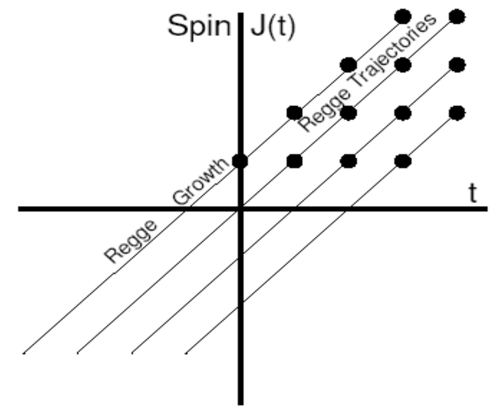


==> Hard Scattering (Polchinski-Strassler)

IIc: Pomeron as Diffusion in AdS

Flat Space String Scattering -- Regge Behavior

$$\mathrm{Im} A \sim \sum_i s^{J_i(t)}$$



$$J(t) = \alpha(t) = \alpha_0 + \alpha' t$$

$$t \leftrightarrow \nabla_b^2$$

$$G(s; \vec{b}, \vec{b}') \longleftrightarrow \langle\, \vec{b}\, |\, s^{2+\alpha' \nabla_b^2/2}\, |\, \vec{b}'\, \rangle$$

$$\sim s^{\alpha_0} \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

$\Leftarrow$ Diffusion in Impact Space

Regge in AdS$_5$

$$\mathrm{Im}\mathcal{A} \sim s^{J(t)} = s^{2+\alpha'\nabla_b^2/2} \quad \text{(flat space)}$$

$$G(s;\vec{b},\vec{b}') \longleftrightarrow \langle\, \vec{b} \mid s^{2+\alpha'\nabla_b^2/2} \mid \vec{b}' \,\rangle$$

$$\mathrm{Im}\ \mathcal{A} \to s^{2+\alpha'\nabla^2/2} \quad \text{(curved space)}$$

$$G(s;\vec{b}_2,z_2,\vec{b}_1,z_1) \longleftrightarrow \langle\, \vec{b}_2, z_2 \mid s^{2+\alpha'\nabla^2/2} \mid \vec{b}_1, z_1 \,\rangle$$

$$\longleftrightarrow \langle\, \vec{b}_2, z_2 \mid e^{-\mathcal{H}\,\tau} \mid \vec{b}_1, z_1 \,\rangle$$

$$\mathcal{H} \longleftrightarrow -2-\alpha'\nabla^2/2 \qquad \tau \longleftrightarrow \log s \qquad u = \log r$$

$$-\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla_{\mathbf{r}}^{\mathbf{2}} + 0 = -\partial_u^2 + (4 - e^{-2u}t/t_0)$$

Diffusion in u=log r: (Effective Hamiltonian at t=0)

where $\tau \propto \ln s$ is again a diffusion time, and for $t = 0$,

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla^2_{\mathbf{r}} + 0 = -\partial_u^2 + 4$$

where $u = \ln r$

A Schrödinger operator with potential $V(u;t) = 4$

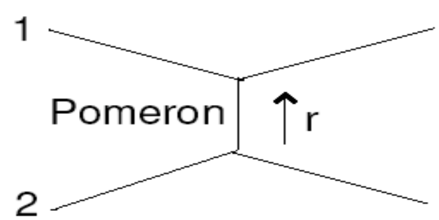
$$\mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

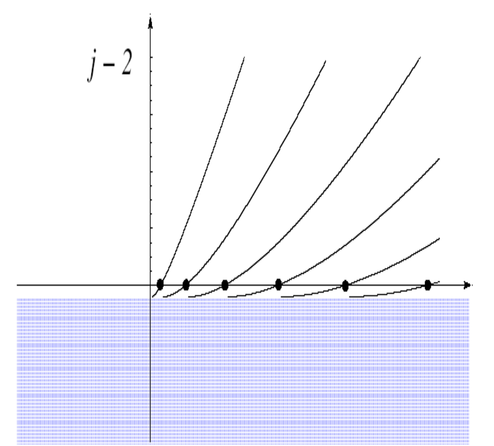
Fixed cut in J-plane:

Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

Strong coupling: $j_0 = 2 - \dfrac{2}{\sqrt{\lambda}}$





Comparison of Diffusion in AdS and BFKL

BFKL:

$$\mathcal{A} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) \quad s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \quad \Phi_2(k'_\perp)$$

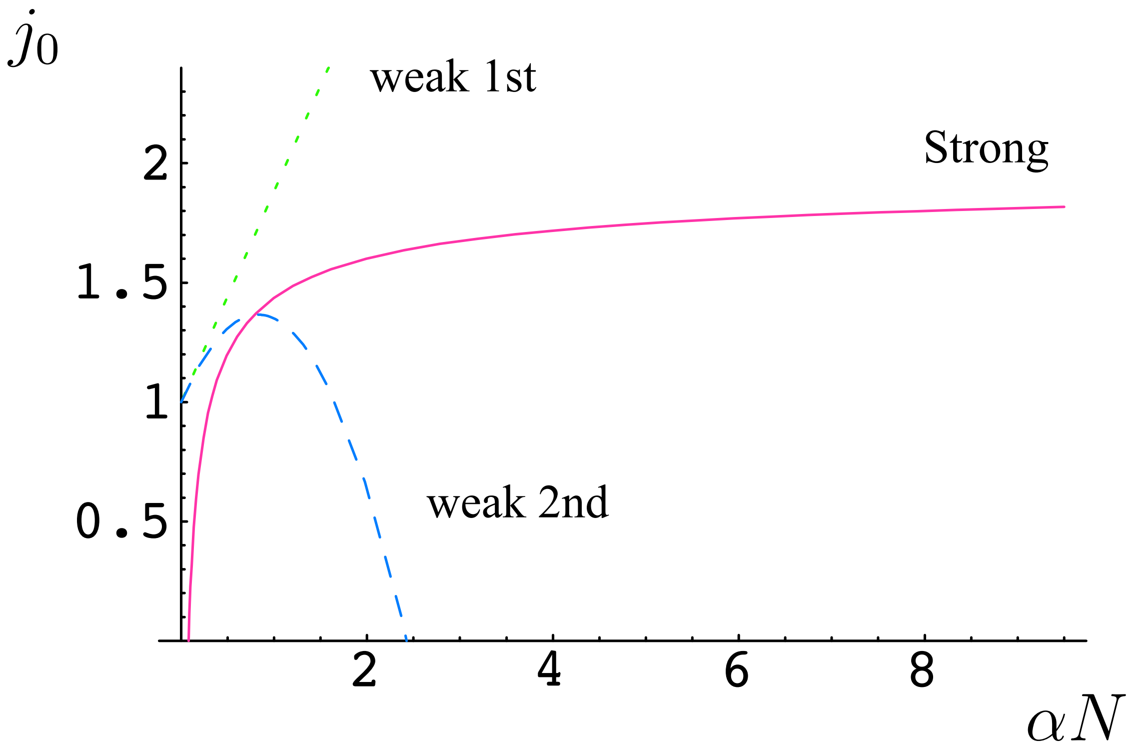
$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \;, \quad \mathcal{D} = \frac{7\zeta(3)}{\pi}\alpha N \;.$$

Pomeron in AdS:

$$\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \; \Phi_1(r) \quad s^{j_0} \frac{e^{-\left[(\ln[r'/r])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \quad \Phi_2(u')$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} \;, \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

$\mathcal{N} = 4$ Strong vs Weak BFKL



Main Lesson from AdS/CFT dual description of Diffraction

Here $\lambda \equiv R^4/\alpha'^2 = g^2_{YM}N = 4\pi\alpha N$ in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory — the numerical coefficient can differ in other theories but the proportionality always holds — so large λ is large 't Hooft coupling.

The identification of r and $k_\perp$ has its source in the UV/IR correspondence and has been suggested in numerous contexts, but here appears as a nontrivial and precise match. The effective diffusion time, $\ln s$, holds for both the BFKL and the Regge diffusions, at both large and small λ.

General form depends on Conformal Symmetry.

The QCD Pomeron and AdS/CFT

- Have shown that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

- Pomeron can be identified as Reggeized Massive Graviton.

- Both the IR Pomeron and the UV Pomeron are dealt in a unified single step.

- Both conceptual and practical advantages.

III. Conformal Invariance at H/E and Graviton

* Reduction to AdS_3
* Conformal Invariance

  - Conformal limit:

  - Confinement:

full $O(4,2)$ conformal group as isometries of AdS_5

15 generators: $P_\mu, M_{\mu\nu}, D, K_\mu$

collinear group $SL_L(2,R) \times SL_R(2,R)$ used in DGLAP

generators: $D \pm M_{+-}\ ,\ P_\pm\ ,\ K_\mp$

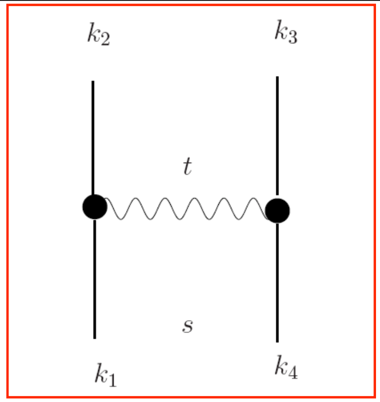
Möbius invariance $SL(2,C)$

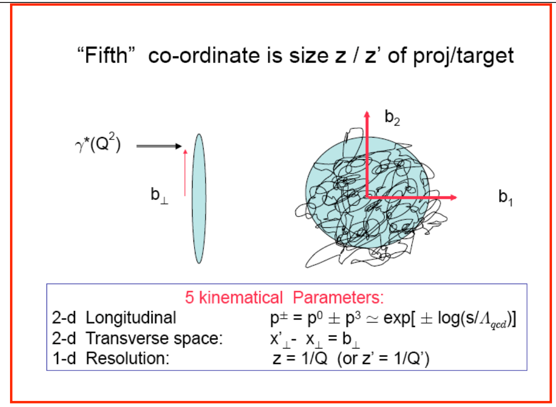
generators: $iD \pm M_{12}\ ,\ P_1 \pm iP_2\ ,\ K_1 \mp iK_2$

isometries of the Euclidean (transverse) AdS_3 subspace of AdS_5

$$\text{Lorentz boost}, \exp[-yM_{+-}]$$

$$ds^2 = R^2[dz^2 + dwd\bar{w}]/z^2$$





AdS_3 is the hyperbolic space H_3. Indeed $SL(2,C)$ is the subgroup generated by all elements of the conformal group that commute with the boost operator, M_{+-} and as such plays the same role as the little group which commutes with the energy operator P_0.

$$\begin{aligned}
J_0 &= w\partial_w + \frac{1}{2}z\partial_z \quad , \quad J_- = -\partial_w \quad , \quad J_+ = w^2\partial_w + wz\partial_z - z^2\partial_{\bar{w}} \\
\bar{J}_0 &= \bar{w}\partial_{\bar{w}} + \frac{1}{2}z\partial_z \quad , \quad \bar{J}_- = -\partial_{\bar{w}} \quad , \quad \bar{J}_+ = \bar{w}^2\partial_{\bar{w}} + \bar{w}z\partial_z - z^2\partial_w \, .
\end{aligned}$$

$$M_{+-} = 2 - H_{+-}/(2\sqrt{\lambda}) + O(1/\lambda)$$

$$H_{+-} = -z^3\partial_z z^{-1}\partial_z - z^2\nabla^2_{x_\perp} + 3 \, .$$

$$[H_{+-} + 2\sqrt{\lambda}(j-2)]G_3(j,v) = z^3\delta(z-z')\delta^2(x_\perp - x'_\perp)$$

$$v = \frac{(x_\perp - x'_\perp)^2 + (z-z')^2}{2zz'}$$

Emergence of 5-dim AdS-Space

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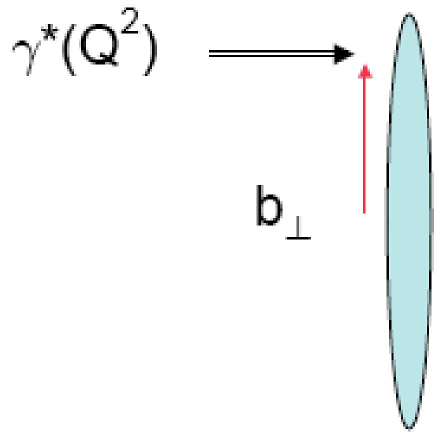

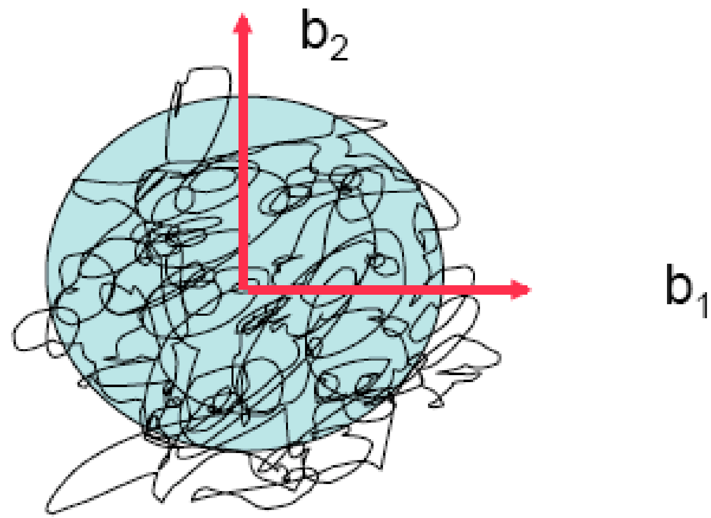

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Remarks on AdS3 Propagator:

$$G_3(j; x^\perp - x'^\perp, z, z') \sim \langle x^\perp, z \,|\frac{1}{2\sqrt{\lambda}(j-2) + H_{+,-}}|\, x'^\perp, z'\rangle$$

- Conformal Invariance, a function of a single AdS3 invariant.

$$v = \frac{(x_\perp - x'_\perp)^2 + (z - z')^2}{2zz'}$$

- Large λ ⇒ j ~ 2.
- λ infinite, s large and fixed ⇒ j=2, and Graviton exchange
- λ and s infinite, $\log s = O(\sqrt{\lambda})$ ⇒ Pomeron exchange, in order to resolve "fine structure", with

$$j \simeq j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Strong Coupling Pomeron Propagator-- Conformal Limit

- AdS-3 propagator:

$$\boxed{\mathcal{K}(j, x_\perp - x'_\perp, z, z') = \frac{1}{4\pi z z'} \frac{\left[y + \sqrt{y^2 - 1}\right]^{(2 - \Delta_+(j))}}{\sqrt{y^2 - 1}} \,,}$$

$$y \pm 1 = \frac{(z \mp z')^2 + (x_\perp - x'_\perp)^2}{2 z z'}$$

- BFKL kernel:

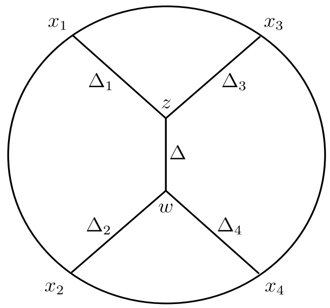
$$\boxed{\Phi_{n,\nu}(b_1 - b_0, b_2 - b_0) = \left[\frac{b_1 - b_2}{(b_1 - b_0)(b_2 - b_0)}\right]^{i\nu + (1+n)/2} \left[\frac{\bar{b}_1 - \bar{b}_2}{(\bar{b}_1 - \bar{b}_0)(\bar{b}_2 - \bar{b}_0)}\right]^{i\nu + (1-n)/2}}$$

One Graviton in Momentum Representation at High Energy

$$J = 2, \quad \Delta = 4$$

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$$p_1 + p_2 \to p_3 + p_4$$



$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

Pomeron Propagator--Conformal Limit

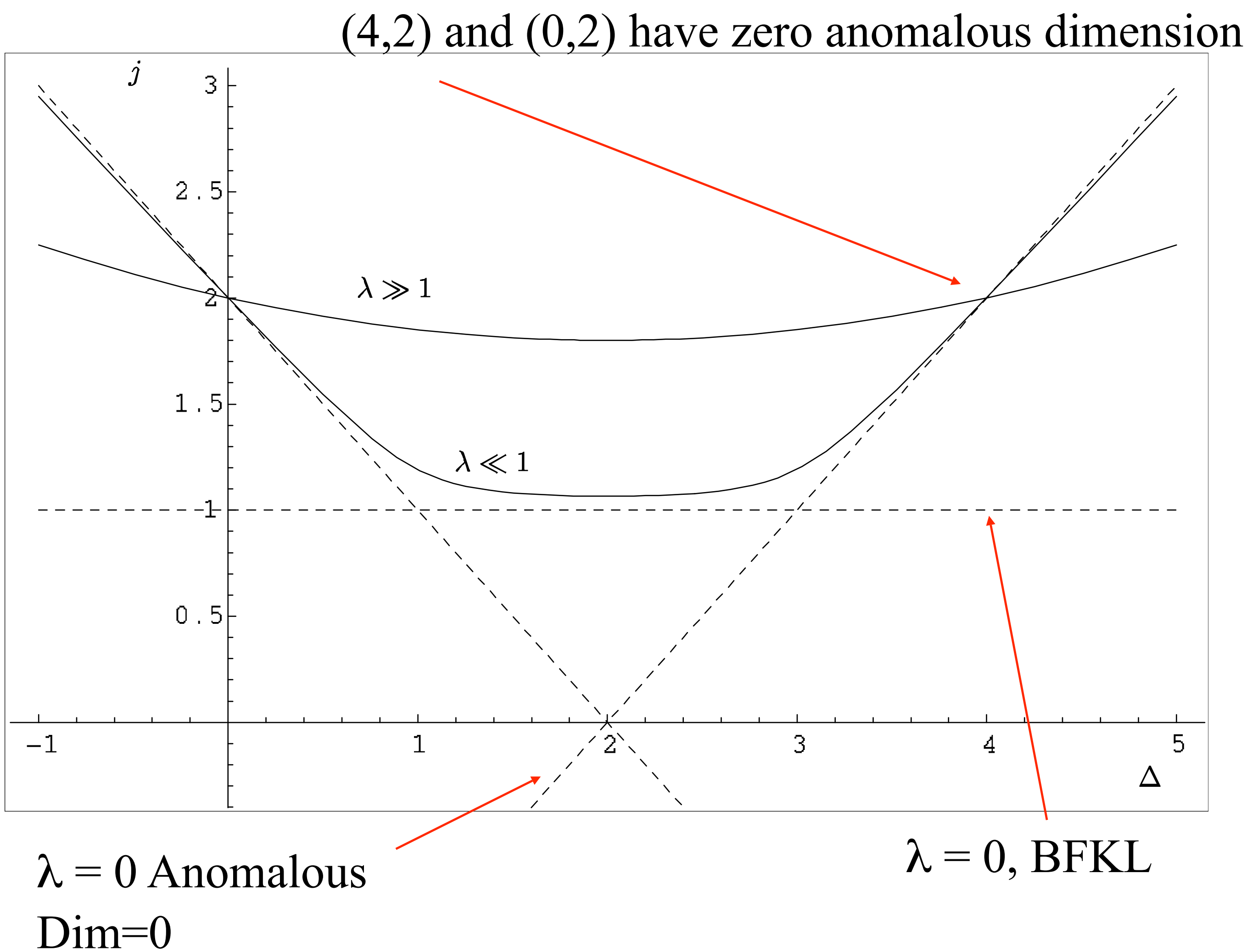
- Spin 2 -------> J by Using Complex angular momentum representation
- Reduction to AdS-3
- Use J-dependent Dimension

$$\Delta: \quad 4 \to \Delta(J) = 2 + [2\sqrt{\lambda}(J - J_0)]^{1/2} = 2 + \sqrt{\bar{j}}$$

- BFKL-cut:

$$J_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

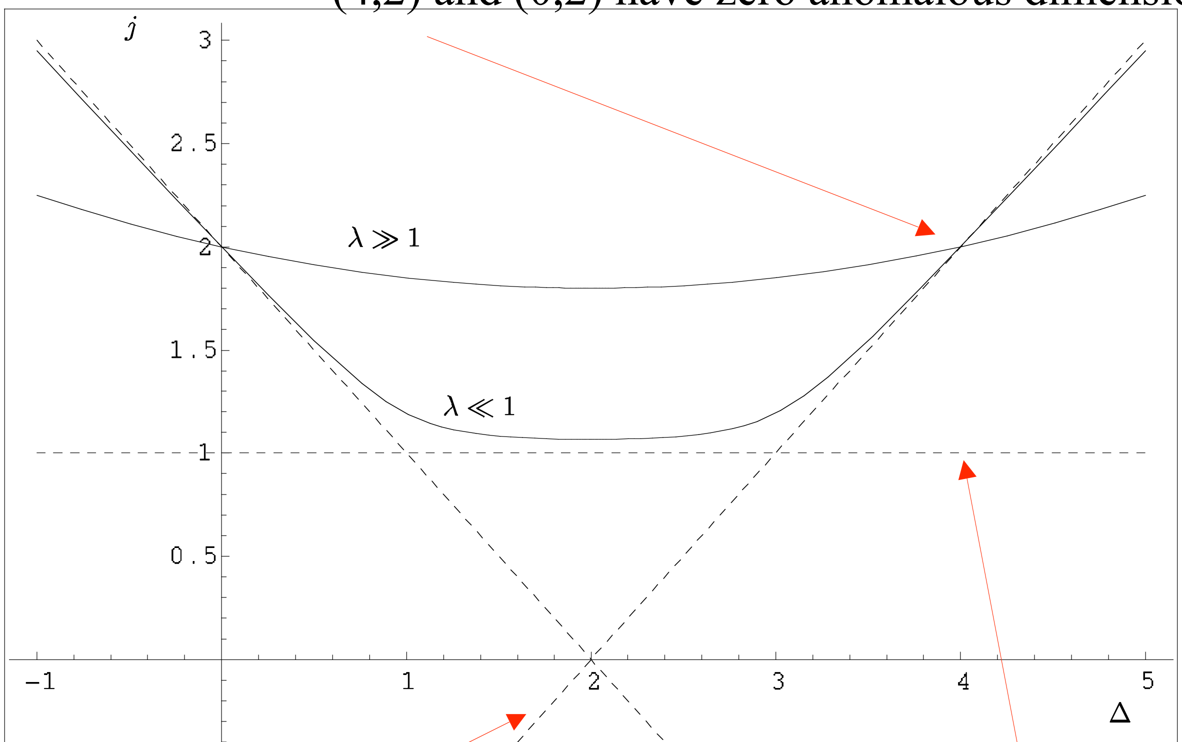
Spin-Dimension Curve



inversion symmetry: $\Delta \rightarrow 4 - \Delta$

All coupling form: $\Delta(j)$ in DGLAP vs BFKL

(4,2) and (0,2) have zero anomalous dimension



$\lambda = 0$ Anomalous Dim=0

$\lambda = 0$, BFKL

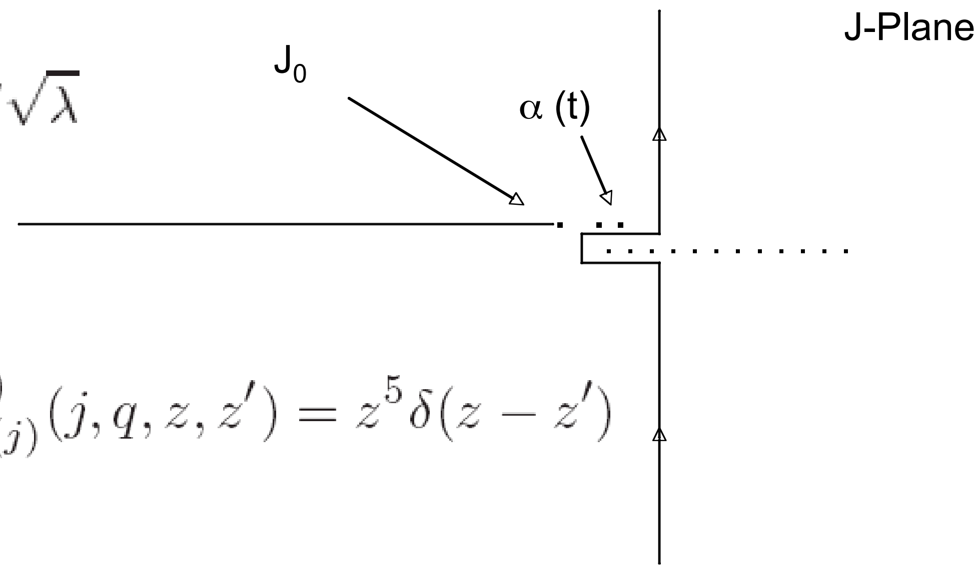
inversion symmetry: $\Delta \rightarrow 4 - \Delta$

Complex j-Plane:

$$\mathcal{T}^{(1)}(p_i, z, z') = \int \frac{dj}{2\pi i} \, \frac{(1 + e^{-i\pi j})}{\sin \pi j} \, (\tilde{s})^j \, G^{(5)}(j, q, z, z')$$

Integration Contour for Mellin Transform

$$j_0 = 2 - 2/\sqrt{\lambda}$$



$$\{2\sqrt{\lambda}(j-2) - z^5 \partial_z z^{-3} \partial_z - z^2 t\} G^{(5)}_{\Delta(j)}(j, q, z, z') = z^5 \delta(z - z')$$

Reduction to AdS-3:

$$G^{(5)}_{\Delta}(j, q^{\pm} = 0, q^{\perp}, z, z') \to (zz') G^{(3)}_{(\Delta-1)}(j, q_{\perp}, z, z')$$

IV. Beyond Pomeron:

- Eikonal Summation:
  - Summing "Reggeized Witten Diagrams"
  - Black Disk Picture
- Froissart Bound
  - Only follows from confinement

IV. Beyond Pomeron: Saturation, etc.

- Sum over Pomeron Exchanges (string perturbative)
- Eikonal Sum in AdS3: (derived both via Cheng-Wu and by Shock-wave method)

$$\boxed{A_{2\to 2}(s,t) \simeq -2is\int d^2b\, e^{-ib^\perp q_\perp}\int dz dz' P_{13}(z)P_{24}(z')\left[e^{i\chi(s,b^\perp,z,z')}-1\right]}$$

$$P_{13}(z) = (z/R)^2\sqrt{g(z)}\Phi_1(z)\Phi_3(z) \qquad P_{24}(z) = (z'/R)^2\sqrt{g(z')}\Phi_2(z')\Phi_4(z')$$

$$\boxed{\chi(s,x^\perp - x'^\perp, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s}\mathcal{K}(s,x^\perp - x'^\perp, z, z')}$$

- Condition for Saturation:

$$\boxed{\chi(s,x^\perp - x'^\perp, z, z') \; = \; O(1)}$$

Unitarity:

- Local Scattering in AdS3 of "String Bits" or "Partons"

$$A_{2\to 2}(s,t) \simeq \int d^2b\, e^{-ib^\perp q_\perp} \int dz dz' P_{13}(z) P_{24}(z') \widetilde{A}(s,b^\perp,z,z')$$

$$\widetilde{A}(s,b^\perp,z,z') = -2is\left[e^{i\chi(s,b^\perp,z,z')} - 1\right]$$

$$\text{Im}\,\widetilde{A}(s,b^\perp,z,z') \geq (1/4s)|\widetilde{A}(s,b^\perp,z,z')|^2 \,.$$

- With J ~ 2, eikonal predominantly real:

$$\begin{aligned} |\,\text{Re}[\chi]\,| \quad &\leq \quad |\,\text{Im}[\chi]\,|\,, \quad 1 \leq J_0 \leq 1.5 \\ |\,\text{Re}[\chi]\,| \quad &\geq \quad |\,\text{Im}[\chi]\,|\,, \quad 1.5 \leq J_0 \leq 2 \end{aligned}$$

- “Parton-Hadron Duality”: Local parton scattering in AdS3 is equiv to Multi-Channel eikonal for hadrons in 2-dim Impact Space

$$A_{n_4,n_3\leftarrow n_2,n_1}(s,t) = -2is\int d^2b e^{-ibq_\perp}\ \left[e^{i\widehat{\chi}(s,b)} - 1\right]_{n_4,n_3;n_2,n_1}$$

$$\chi_{n_4 n_3;n_2 n_1}(s,b) = \int\ dz\ dz'\ P_{n_3 n_1}(z) P_{n_4 n_2}(z')\chi(s,b,z,z')$$

- For real eikonal, quasi-elastic scattering only, and no scattering into “long-string” states.

$$\mathrm{Im}\ A_{n_4 n_3;n_2 n_1}(s,b^\perp) = (1/4s)\sum_{n,m} A^\dagger(s,b^\perp)_{n_4 n_3;nm} A(s,b^\perp)_{nm;n_2 n_1}$$

- Inelastic Production

- Generalized Cutting Rules

$$\cos(j_0\pi)|\chi|^2 = \left[1 - 2\sin^2(j_0\pi/2) - 2\sin^2(j_0\pi/2) + 2\sin^2(j_0\pi/2)\right]|\chi|^2$$

$$\begin{array}{lccccccccc}
j_0 = 1.0: & -1 & = & 1 & - & 2 & - & 2 & + & 2 \\
j_0 = 1.5: & 0 & = & 1 & - & 1 & - & 1 & + & 1 \\
j_0 = 2.0: & 1 & = & 1 & - & 0 & - & 0 & + & 0
\end{array}$$

- Real World: $\underline{j_0 \sim 1.5}$ and $\underline{\lambda \sim O(1)}$

Analyticity:

- Amplitude is crossing even.

$$\mathcal{K}(s,b^{\perp},z,z') = -(zz'/R^4)G_3(j_0,v) \times \hat{s}^{j_0}\int_{-\infty}^{j_0}\frac{dj}{\pi}\,\frac{(1+e^{-i\pi j})}{\sin\pi j}\,\hat{s}^{(j-j_0)}\,\sin\left[\xi(v)\sqrt{2\sqrt{\lambda}(j_0-j)}\,\right]$$

$$\cosh\xi = v+1 \qquad e^{\xi} = 1+v+\sqrt{v(2+v)}$$

- With λ large, the Amplitude has a Large Real Part. Purely real at $\lambda \to \infty$.

- Need to know both Re [K] and Im [K] for all s>0.

- Im [K] can be found more easily. Re [K] can be found by Derivative Dispersion Relation.

- Im [K] can be evaluated analytically, exhibiting Diffusion in AdS3, with diffusion time, τ ~ log s .

$$\mathrm{Im}[\mathcal{K}] \;=\; (zz'/R^4)G_3(j_0,v)(\sqrt{\lambda}/2\pi)^{1/2}\xi \; e^{j_0\tau}\,\frac{e^{-\sqrt{\lambda}\xi^2/2\tau}}{\tau^{3/2}}$$

- With λ large, derivative dispersion relation simplifies,

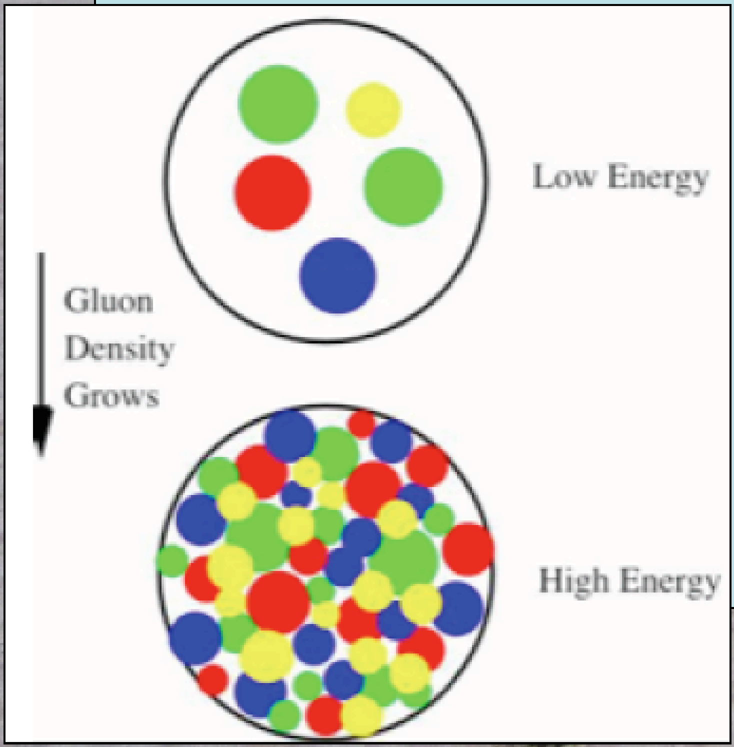
$$\partial_\tau[e^{-2\tau}\mathrm{Re}[\mathcal{K}]] = -(2/\pi)e^{-2\tau}\mathrm{Im}[\mathcal{K}]$$

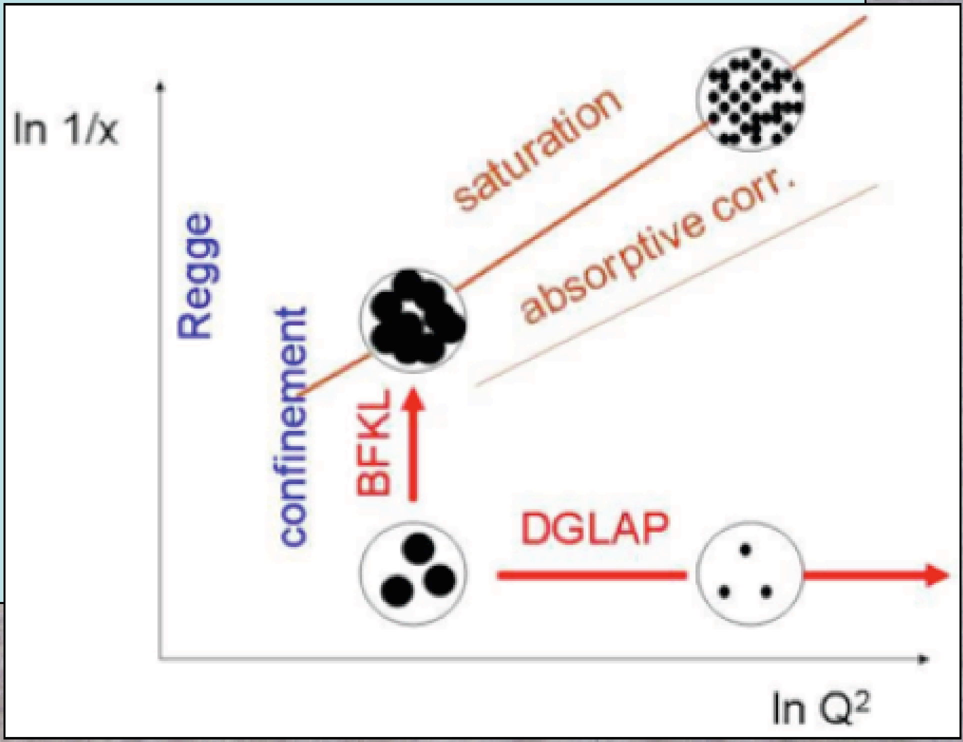
- Re [K] can again be expressed simply as

$$\begin{aligned} \mathrm{Re}[\mathcal{K}] \;&\to\; (\sqrt{\lambda}/\pi)\mathrm{Im}[\mathcal{K}] \sim e^{j_0\tau}\,\frac{e^{-\sqrt{\lambda}\xi^2/2\tau}}{\tau^{3/2}}\,, && \text{if} \quad \log\widetilde{s} > (\sqrt{\lambda}/2)\,\xi \\ &\to\; \frac{2}{\pi}\widehat{s}^2\left(\frac{zz'}{R^4}\right) G_3(2,v) + O(e^{j_0\tau})\,, && \text{if} \quad \log\widetilde{s} < (\sqrt{\lambda}/2)\,\xi \end{aligned}$$

Absorption & Saturation?

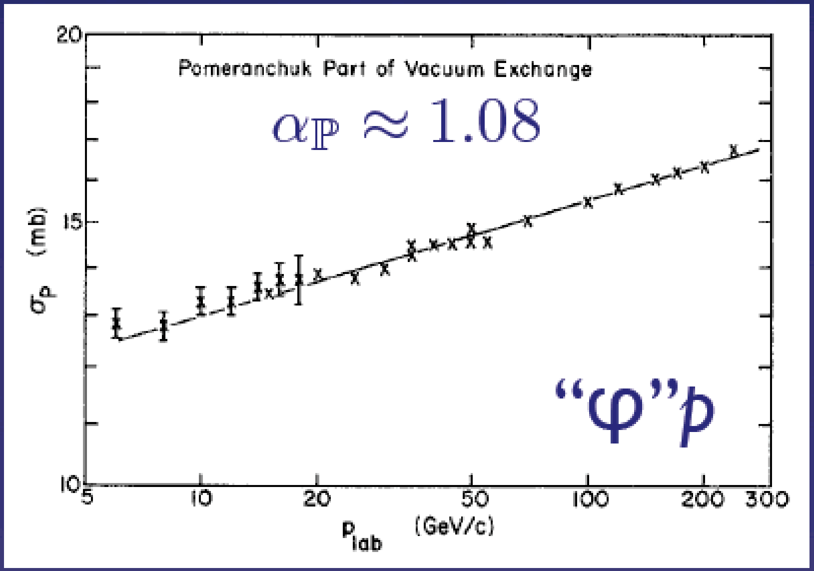
Expected at low x and high Q^2, as number of partons grows, and they overlap





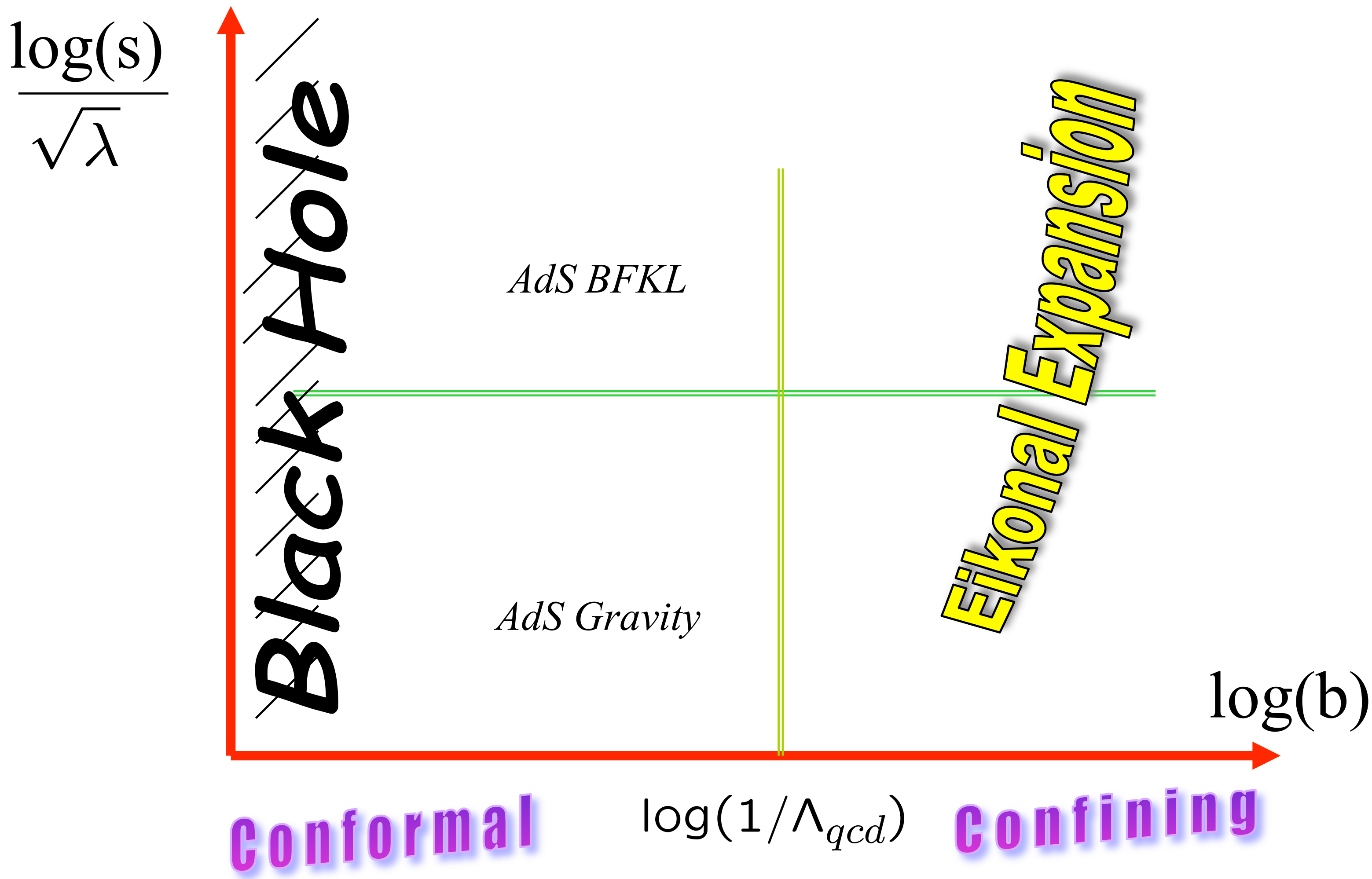
Pomeron > Pomeranchukon > Pomeranchuk singularity

I. Ya. Pomeranchuk



$$\sigma_{\text{tot}} \leq C \cdot \ln^2 s$$

Theory Parameters: N_c & $g^2 N_c$



Unitarity, Confinement and Froissart Bound

Use the condition: $\chi(s, x^{\perp} - x'^{\perp}, z, z') \;=\; O(1)$

Scattering in Conformal Limit:

No Froissart

Elastic Ring: $b_{\rm diff} \sim \sqrt{zz'}\ (zz's/N^2)^{1/6}$

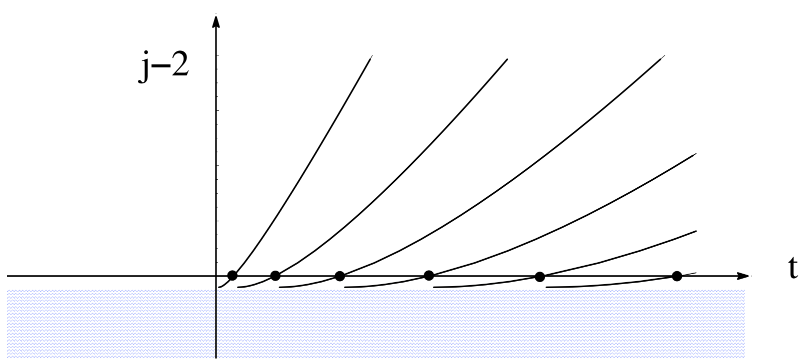
Inner Absorptive Disc:

$$b_{\rm black} \sim \sqrt{zz'}\ \ \frac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N} \qquad b_{\rm black} \sim \sqrt{zz'}\ \left(\frac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2\sqrt{\lambda}}(j_0}$$

Inner Core: “black hole” production ?

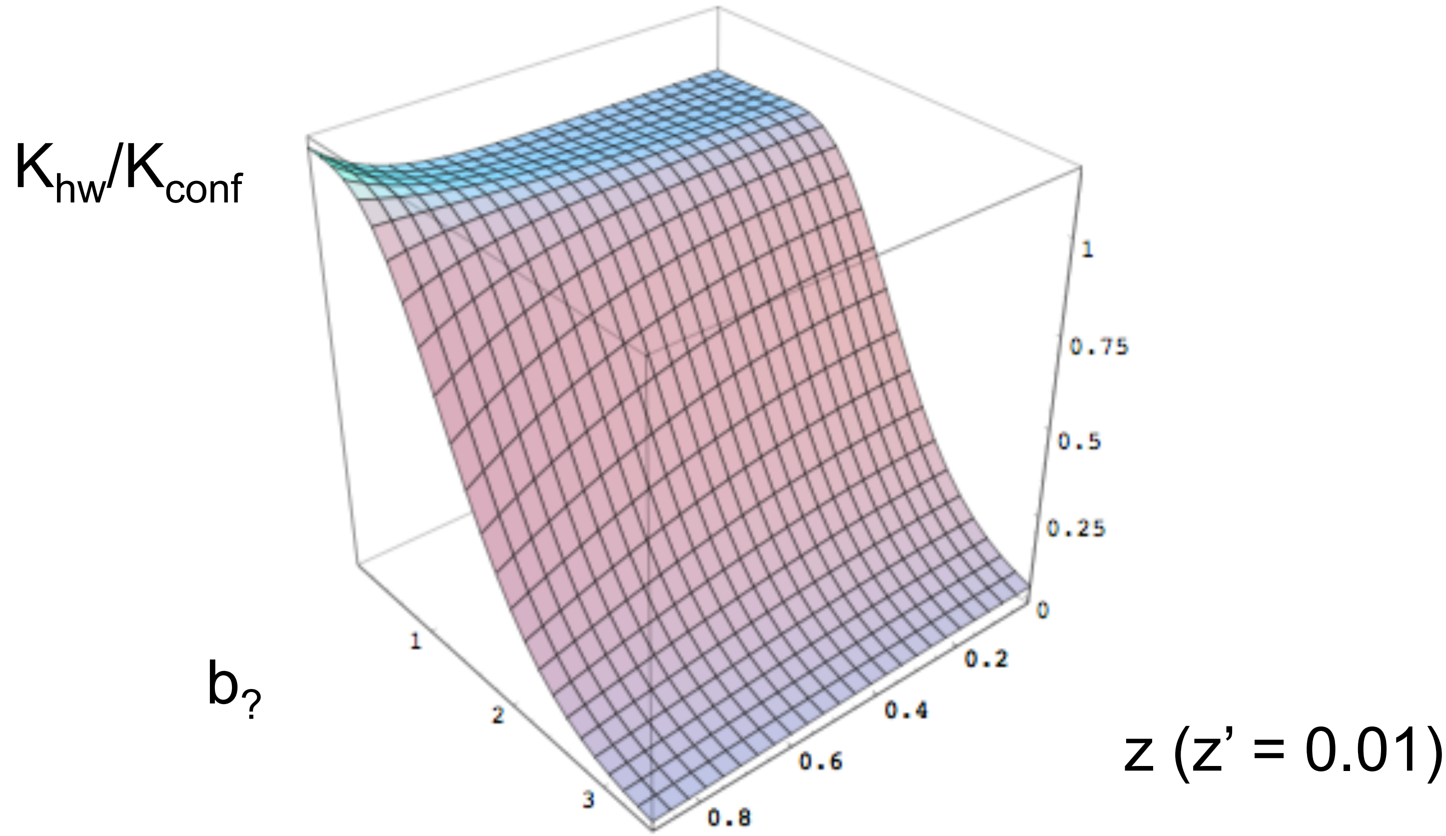
With Confinement

- discrete spectrum



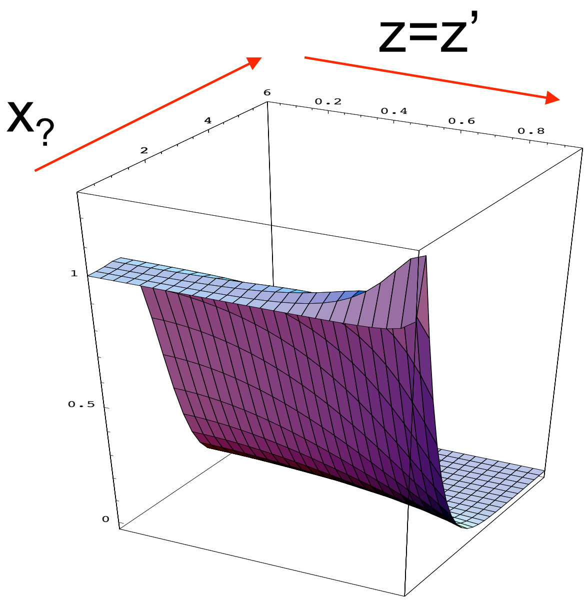
Kernel for hardwall at z =1

$$K_{hw}(x_\perp, zz') \sim \frac{k_5^2 s^2}{zz'} \sum_n \frac{2}{J_2^2(m_n)} J_2(m_n z) K_0(m_n |x_\perp|) J_2(m_n z')$$

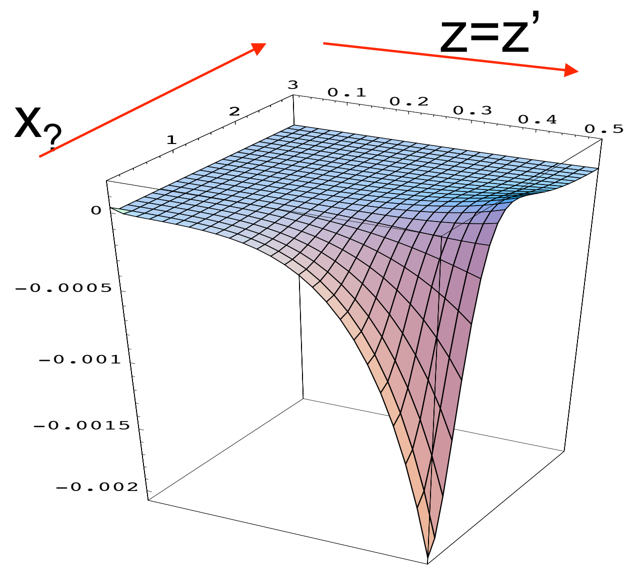


$$lim_{\Lambda\to 0} K_{hw}(x_\perp/\Lambda, z/\Lambda, z'/\Lambda) \sim \frac{\kappa_5^2 s^2}{zz'} \sum_n \frac{2}{y + \sqrt{y^2-1}} 4\pi \sqrt{y^2-1}$$

Born Term for Hard Wall model



$K_{hw}(z,z,x_?)/K_{conf}(z,z,x_?)$



$K_{conf}(z,z,x_?) - K_{hw}(z,z,x_?)$

$$K_{Hardwall}(z,w,x_\perp) = \sum_{n=1}^{\infty} \frac{2}{J_2^2(m_n)} J_2(m_n z) K_0(m_n |x_\perp|) J_2(m_n w)$$

B.C. $\quad \dfrac{d}{dz}[z^2 J_2(z)] = 0 \text{ at } z = 1$

Confinement and Froissart Bound

Mass of the lightest Glueball provides scale

$$e^{-m_0 b}/\sqrt{m_0 b}$$

Elastic Ring: $$b_{\rm diff} \simeq \frac{1}{m_0}\log(s/N^2\Lambda^2) + \dots$$

Absorptive Disc:

Inner Core:

Saturation of Froissart Bound

- The hardwall gives a cut-off so that exponential fall off for $b > \log(s/s_0)$

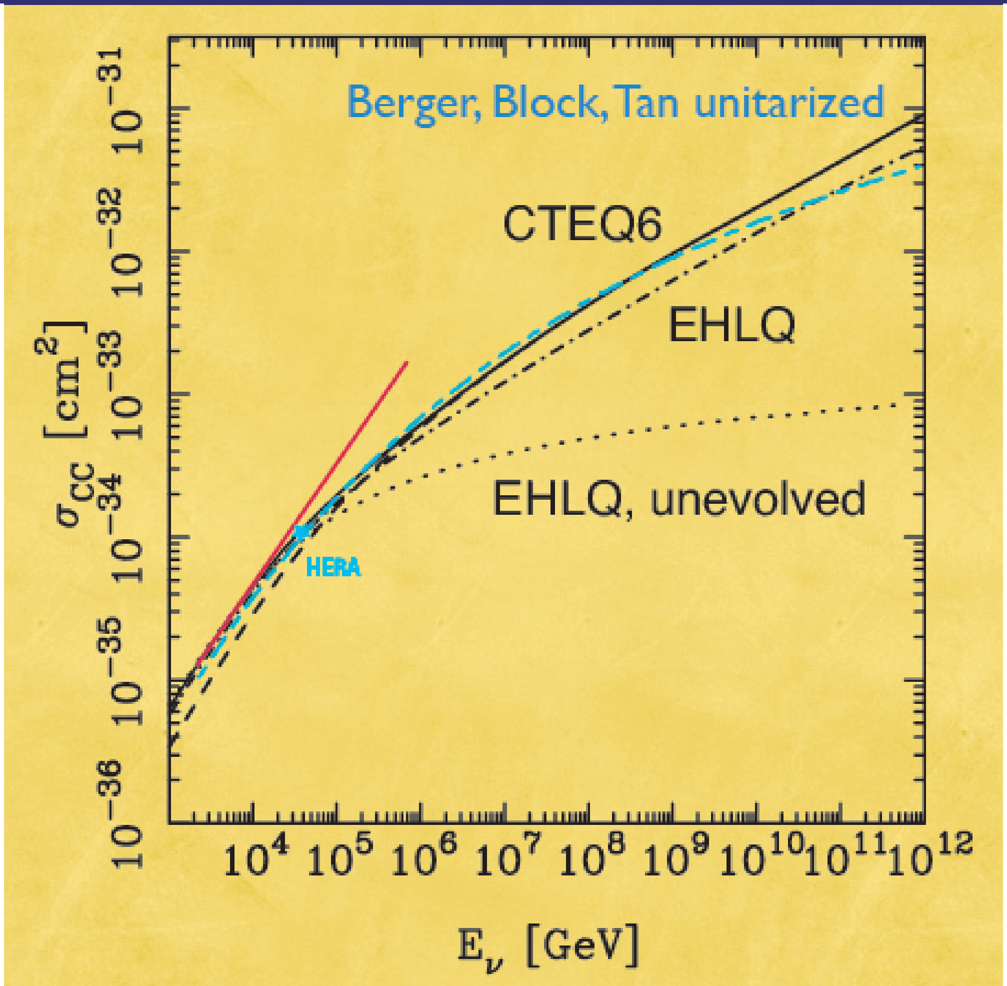
- But there is shell of width ¢ b of $O(\log(s/s_0))$ that is nearly conformal.

- Therefore Froissart is respected and saturated.

Applications beyond the LHC

QCD influence on UHE ν detection

Importance of wee-x parton distributions



V. Summary and Outlook

- Provide meaning for Pomeron Pole non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences.

Further Restrictions:

- Nonlinear effects: e.g., fan diagrams,
- Loops: e.g., AdS-3 Pomeron-Field Theory,
- etc.