

# MONTE CARLO TOOLS AND TECHNIQUES

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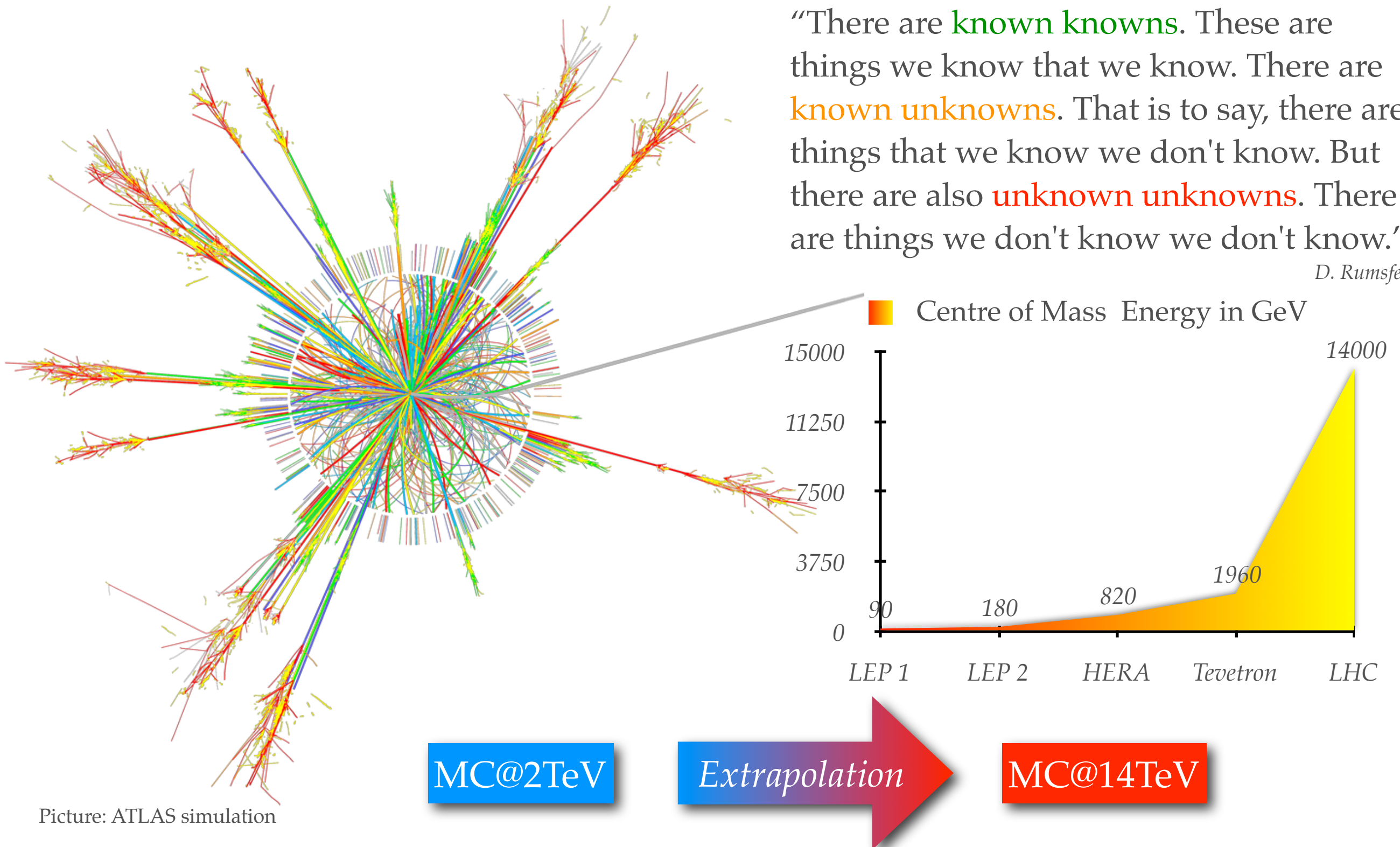
Parton shower, matching at LO and NLO level

# Introduction

The expectations for LHC physics can be sorted into three categories:

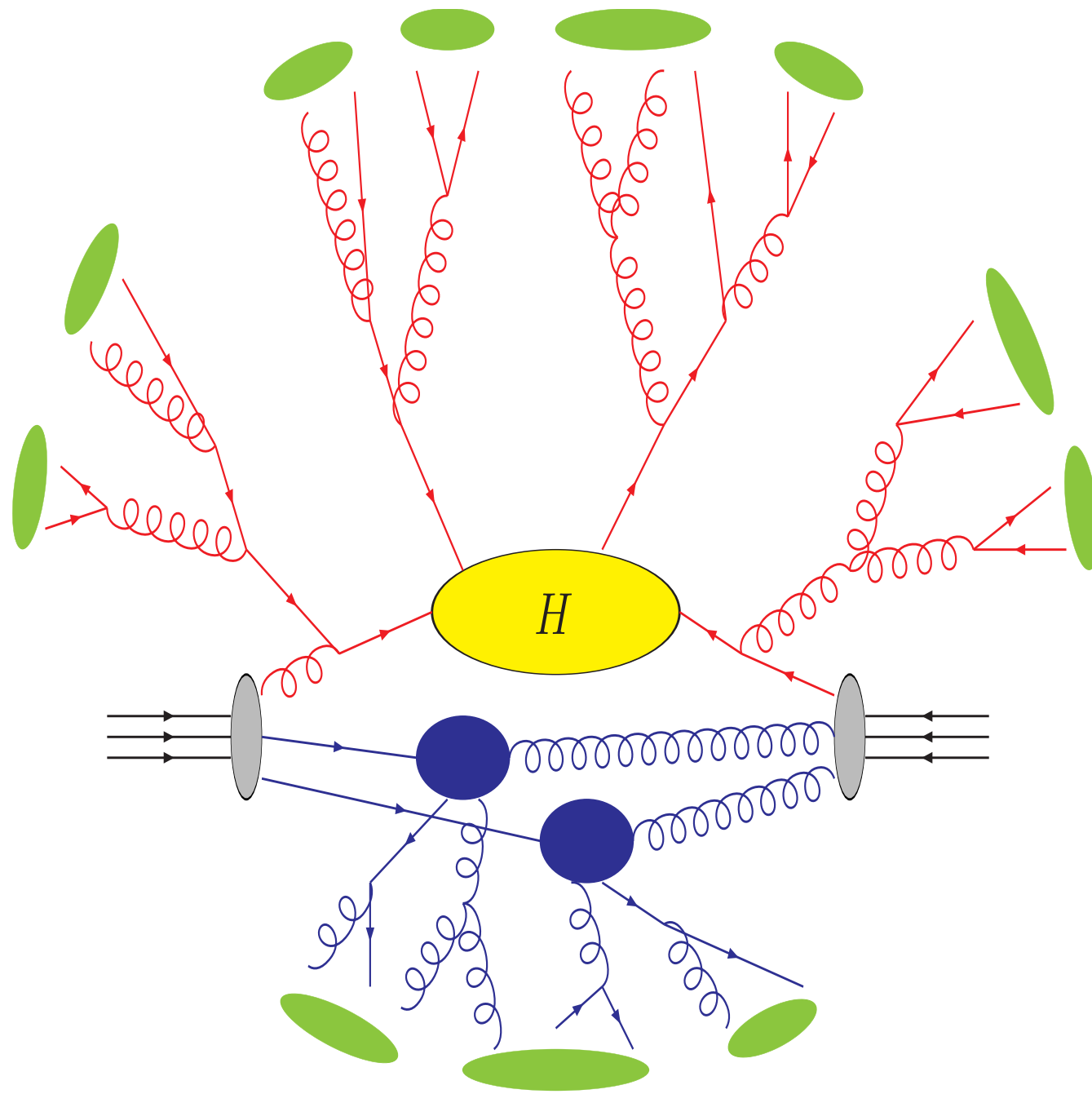
“There are **known knowns**. These are things we know that we know. There are **known unknowns**. That is to say, there are things that we know we don't know. But there are also **unknown unknowns**. There are things we don't know we don't know.”

*D. Rumsfeld*



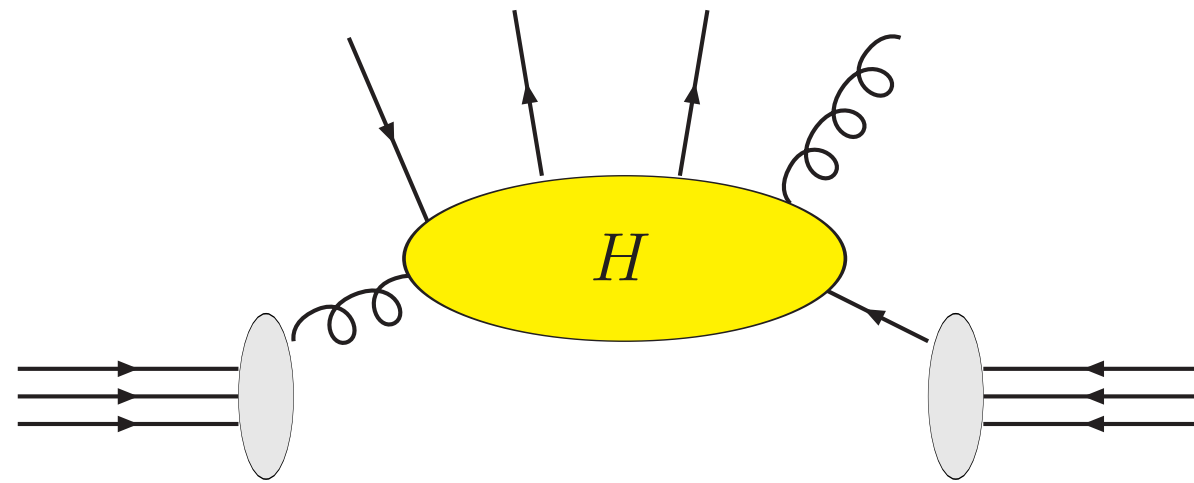
# Introduction

From theory point of view this event looks very complicated



- 1. Incoming hadron** *(gray bubbles)*
  - ⇒ Parton distribution function
- 2. Hard part of the process** *(yellow bubble)*
  - ⇒ Matrix element calculation, cross sections at LO, NLO, NNLO level
- 3. Radiations** *(red graphs)*
  - ⇒ Parton shower calculation
  - ⇒ Matching to the hard part
- 4. Underlying event** *(blue graphs)*
  - ⇒ Models based on multiple interaction
- 5. Hadronization** *(green bubbles)*
  - ⇒ Universal models

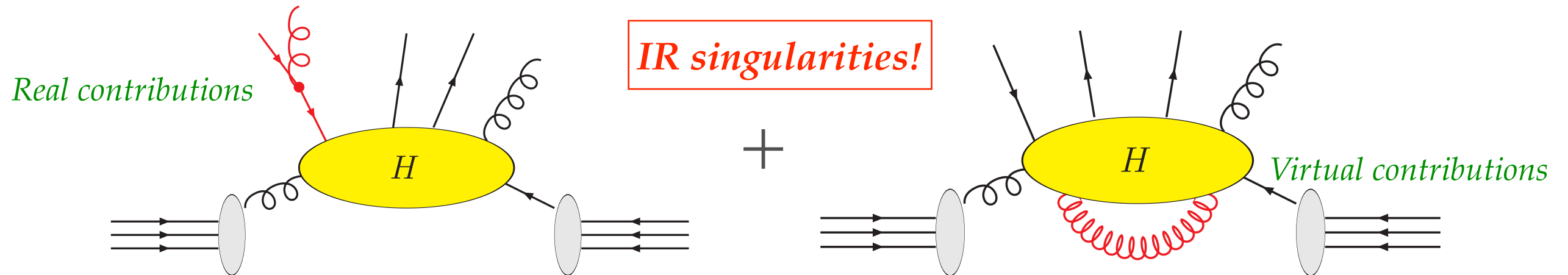
# Born Level Calculation



$$\sigma[F_J] = \int_m d\Gamma^{(m)}(\{p\}_m) |\mathcal{M}(\{p\}_m)|^2 F_J(\{p\}_m)$$

- ✓ Easy to calculate, no IR singularities. Several matrix element generators are available (Alpgen, Helac, MadGraph, Sherpa)
- ✗ Strong dependence on the unphysical scales (renormalization and factorization scales)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ Every jet is represented by a single parton
- ✗ No quantum corrections
- ✗ No hadronization

# NLO Level Calculation



$$\sigma_{\text{NLO}} = \int_N d\sigma^B + \int_{N+1} [d\sigma^R - d\sigma^A]_{\epsilon=0} + \int_N \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

$$d\sigma^A \sim d\Gamma(\{p\}_{N+1}) \underbrace{V \otimes |\mathcal{M}(\{\tilde{p}\}_N)|^2}_{\text{Based on soft and collinear factorization}} F_J(\{\tilde{p}\}_N)$$

Based on soft and collinear factorization

- ✓ Includes quantum corrections, in most of the cases it significantly reduces the unphysical scale dependences
- ✓ One of the jets consists of **two** partons (still very poor)
- ✓ Hard to calculate, the most complicated available processes are  $2 \rightarrow 3$  (NLOJET++<sup>1</sup>, MCFM, PHOX,...)
- ✗ Exclusive quantities suffer on large logarithms
- ✗ No hadronization

<sup>1</sup> <http://cern.ch/nagy/Site/NLOJET++/NLOJET++.html>

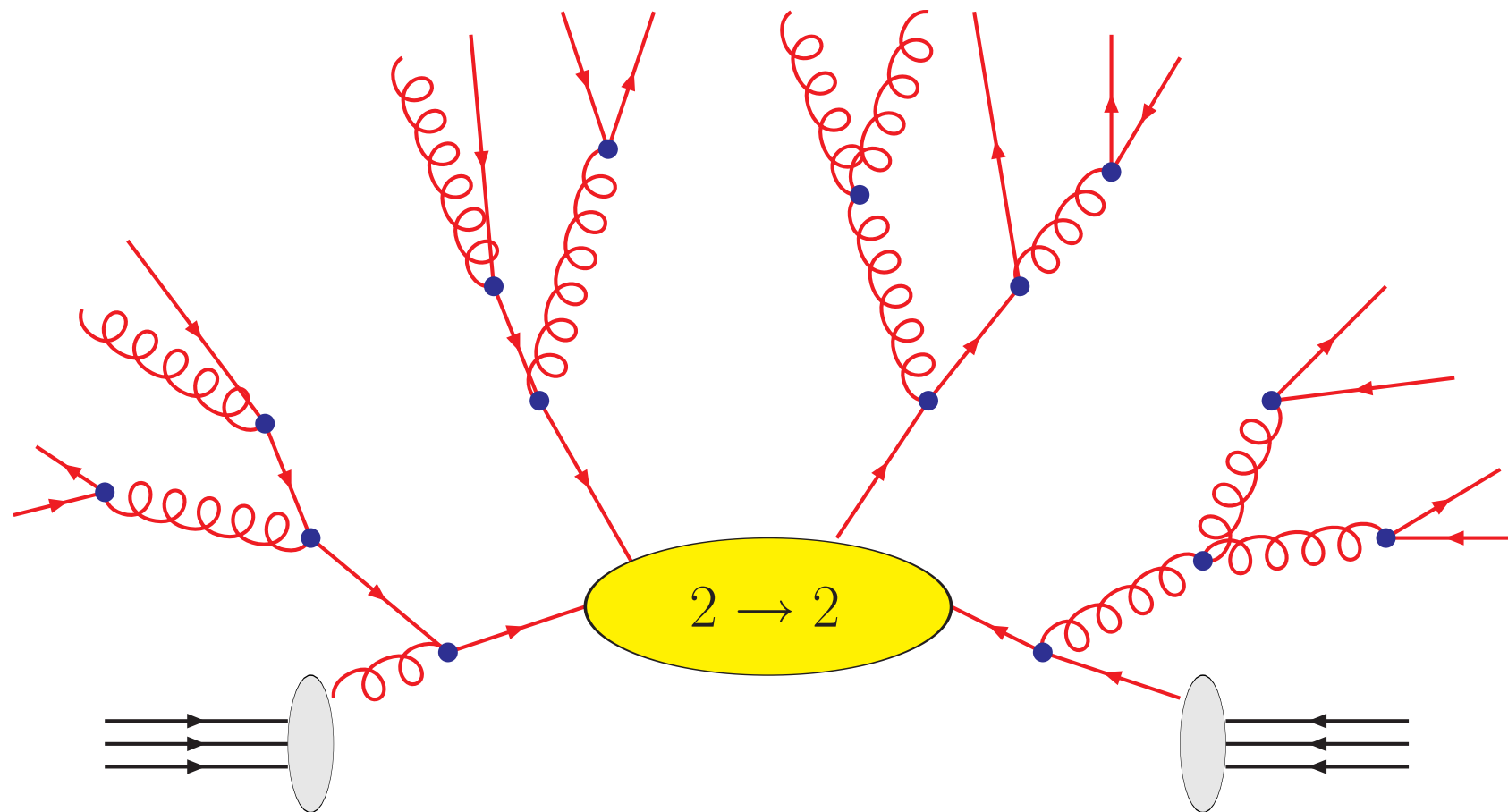
# Experimenter's NLO Wish List

Single boson	Diboson	Triboson	Heavy Flavor
<i>Run II Monte Carlo Workshop, April 2001</i> (Almost 7 years to the day and only two calculation finished!)			
$V + \leq 5\text{jets}$ $V + \textcolor{red}{bb} + \leq 3\text{jets}$ $V + \textcolor{red}{cc} + \leq 3\text{jets}$	$VV + \leq 5\text{jets}$ $VV + \textcolor{red}{bb} + \leq 3\text{jets}$ $VV + \textcolor{red}{cc} + \leq 3\text{jets}$ $WZ + \leq 5\text{jets}$ $WZ + \textcolor{red}{bb} + \leq 3\text{jets}$ $WZ + \textcolor{red}{cc} + \leq 3\text{jets}$ $W\gamma + \leq 3\text{jets}$ $Z\gamma + \leq 3\text{jets}$	$WWW + \leq 3\text{jets}$ $WWW + \textcolor{red}{bb} + \leq 3\text{jets}$ $WWW + \textcolor{red}{cc} + \leq 3\text{jets}$ $Z\gamma\gamma + \leq 3\text{jets}$ $WZZ + \leq 3\text{jets}$ $ZZZ + \leq 3\text{jets}$	$t\bar{t} + \leq 3\text{jets}$ $\textcolor{red}{bb} + \leq 3\text{jets}$ $t\bar{t} + V + \leq 2\text{jets}$ $t\bar{t} + H + \leq 2\text{jets}$ $t\bar{t}b + \leq 2\text{jets}$
<i>Les Houches Workshop 2005</i>			
$V + 3\text{jets}$ $\textcolor{green}{H} + 2\text{jets}$	$VV + \leq 2\text{jets}$ $VV + \textcolor{red}{bb}$	$\textcolor{green}{ZZZ}$	$t\bar{t} + 2\text{jets}$ $t\bar{t} + \textcolor{red}{bb}$
$V \in \{W, Z, \gamma\}$			

*Why are these calculations so hard?*



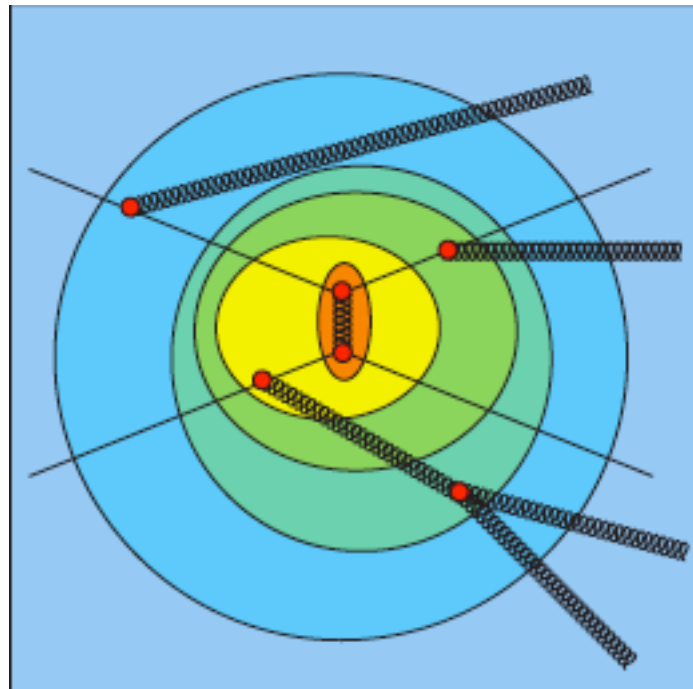
# LO Parton Shower



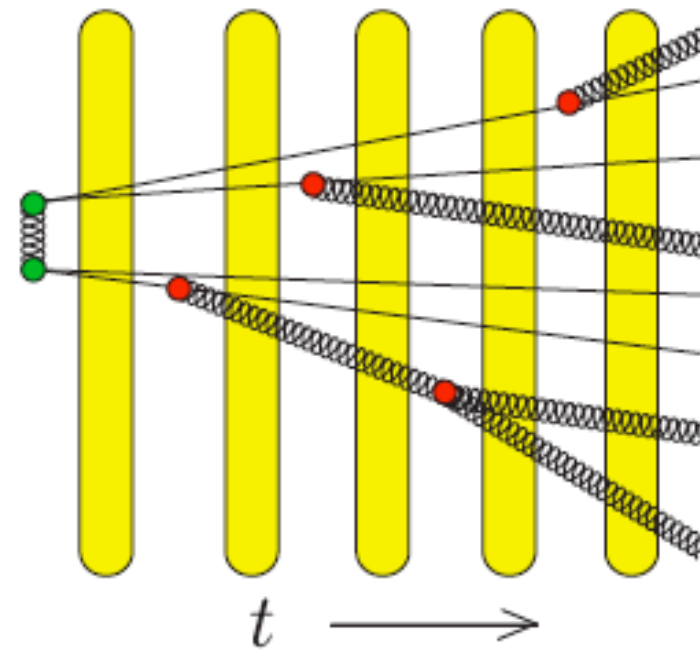
- ✓ It is an iterative algorithm. Arbitrary number of partons.
- ✓ Matched to the hadronization models (which is universal effect).
- ✓ In the best cases it resumes the leading large logarithms properly.
- ✗ Needs more, rather non systematic approximations. *(See next slides!)*
- ✗ Strong dependence on the unphysical scales.
- ✗ The only exact matrix element in the calculations is  $2 \rightarrow 2$  like at Born level.
- ✗ Positive unweighted events. I think it is time to **give up** this concept....

# Shower from Inside Out

Think of shower branching as developing in a “time” that goes from most virtual to least virtual.



Real time picture



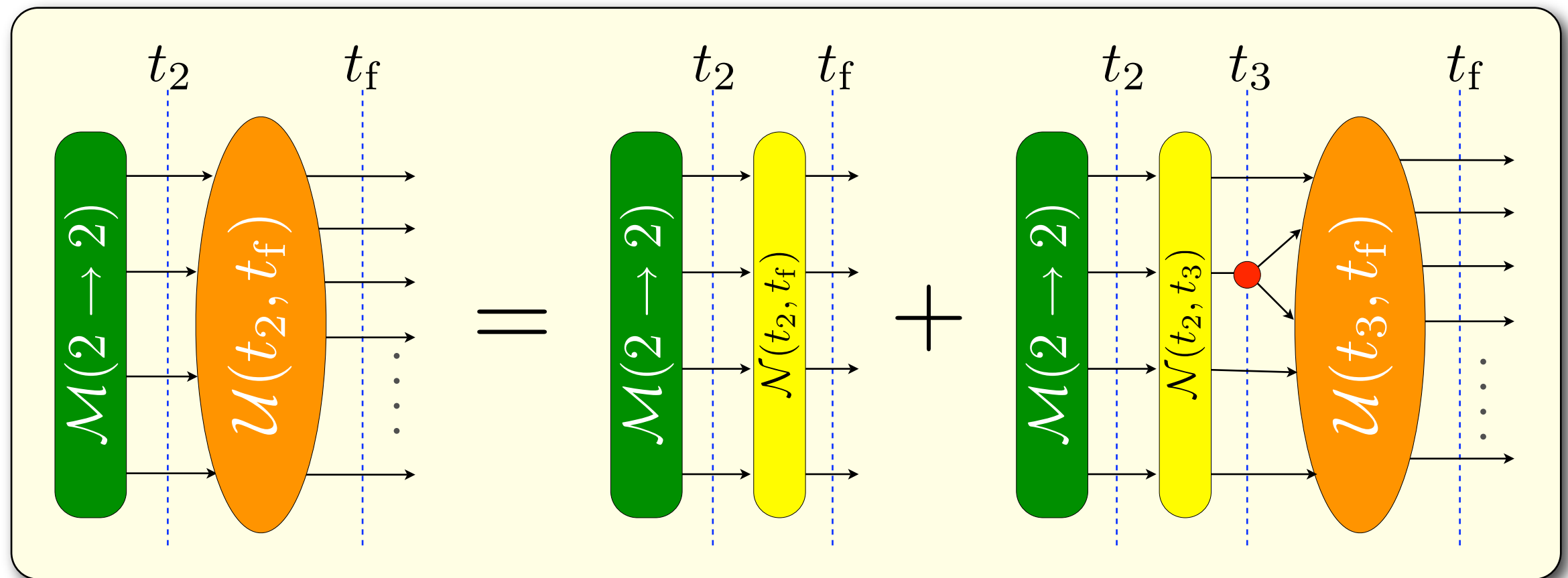
Shower time picture

Thus shower time proceeds backward in physical time for initial state radiation.



# Iterative Algorithm

The parton shower evolution starts from the **simplest hard configuration**, that is usually  $2 \rightarrow 2$  like.

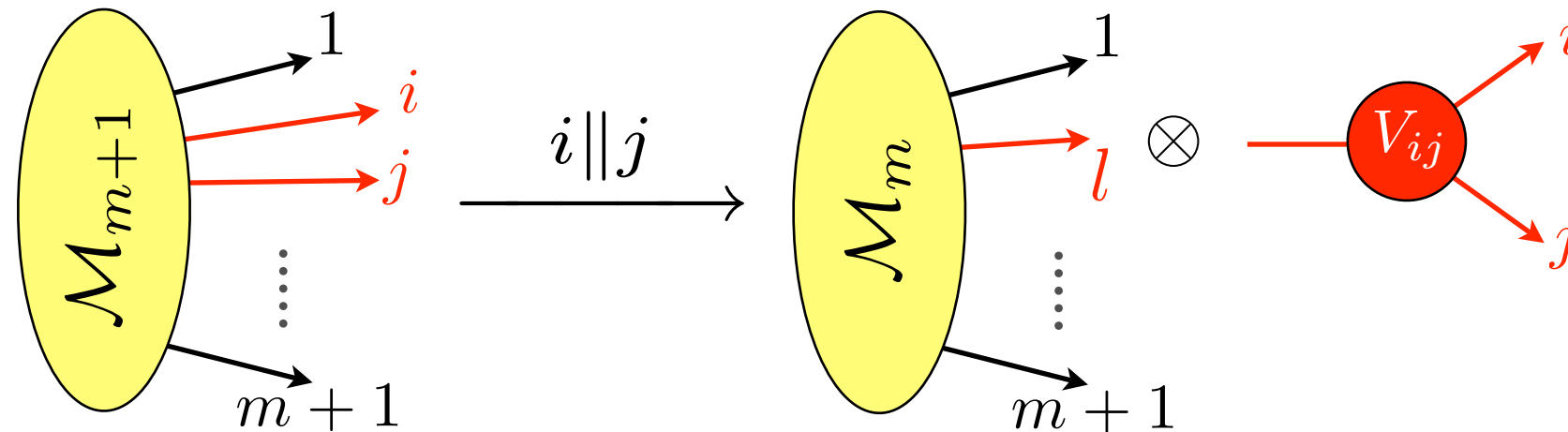


*“Something happens”*

$$\mathcal{U}(t_f, t_2) | \mathcal{M}_2 \rangle = \underbrace{\mathcal{N}(t_f, t_2) | \mathcal{M}_2 \rangle}_{\text{“Nothing happens”}} + \int_{t_2}^{t_f} dt_3 \mathcal{U}(t_f, t_3) \mathcal{H}(t_3) \mathcal{N}(t_3, t_2) | \mathcal{M}_2 \rangle$$

# Collinear Approximation

The QCD matrix elements have universal factorization property when two external partons become collinear

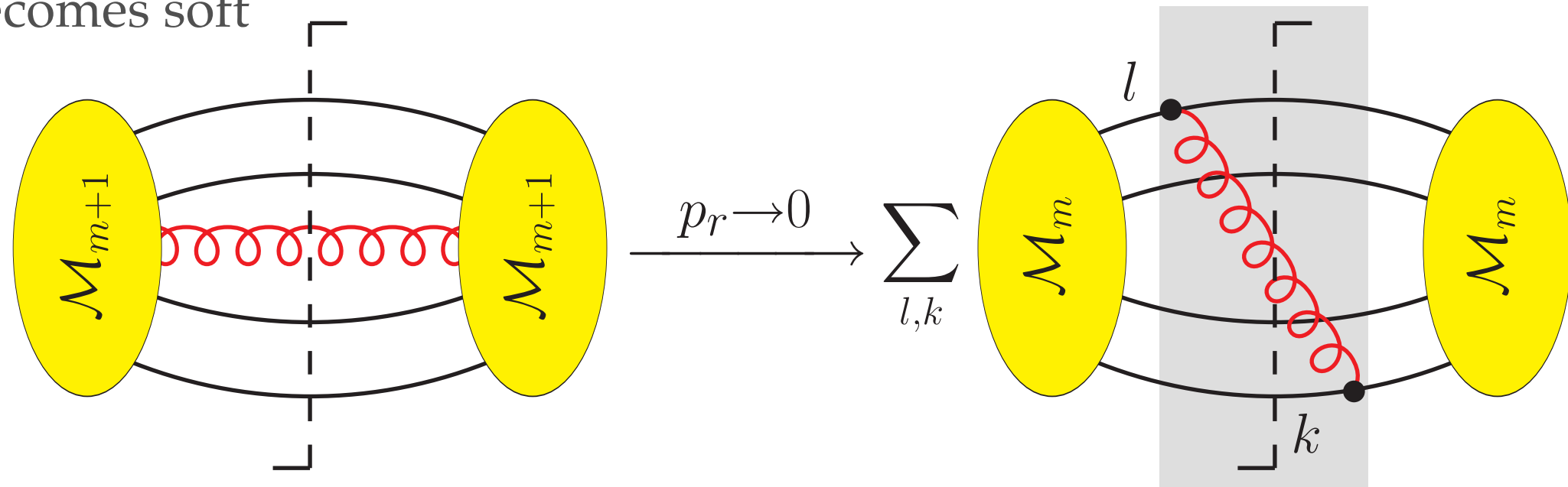


$$\mathcal{H}_C \sim \sum_l t_l \otimes t_l^\dagger V_{ij}(s_i, s_j) \otimes V_{ij}^\dagger(s'_i, s'_j)$$

- Produces leading and next-to-leading logarithms.
- It is diagonal color, no color correlations.
- The gluon splitting is not diagonal in spin.
- The spin correlations are not really complicated but one can use average spin *as extra approximation*.

# Soft Approximation

The QCD matrix elements have universal factorization property when an external gluon becomes soft



$$\mathcal{H}_S \sim - \sum_{\substack{l,k \\ l \neq k}} \frac{\hat{p}_l \cdot \varepsilon(s) \hat{p}_k \cdot \varepsilon(s')}{\hat{p}_l \cdot \hat{p}_{m+1} \hat{p}_k \cdot \hat{p}_{m+1}} t_l \otimes t_k^\dagger$$

- Pure soft contributions produce next-to-leading logarithms.
- Soft gluon connects everywhere and the color structure is not diagonal;  
*quantum interferences in the color space.*
- Does it spoil the independent evolution picture? Yes, it does, but ...

# Color Coherence

There are three way to deal with the soft gluon color interferences:

1. The soft gluon contributions are cancelled in the wide angle region. One can apply **angular ordering** (Herwig / Herwig++) or impose angular ordering by angular veto (old Phytia). This is an extra approximation, especially for massive quarks. In the massive quark case the color coherence breaks down.
2. One can do **leading color approximation**. In the large  $N_c$  limit the soft gluon is radiated from a **color dipole**. The leading color contributions are diagonal in color space, thus no technical complication with colors. (Ariadne, new Phytia, Vincia)
3. **No extra approximation**, treat the soft gluon as it is. Full color correlations.

*ZN and D. Soper: JHEP: 0709 114,2007*

# Color Coherence

There are three way to deal with the soft gluon color interferences:

1. The soft gluon contributions are cancelled in the wide angle region.

Cross sections at  $\sqrt{s} = 1960$  GeV, with structure functions, in nanobarns,  
 $p_T > 10 \text{ GeV}$   $|\eta| < 2.0$ .

- 2.

Process	$\sigma_0$ : Normal	$\sigma_1$ : Large Nc component	$\frac{\sigma_1 - \sigma_0}{\sigma_0}$
ud $\rightarrow$ W+g	0.1029(5)D+01	0.1158(5)D+01	13%
ud $\rightarrow$ W+gg	0.1018(8)D+00	0.1283(10)D+00	26%
ud $\rightarrow$ W+ggg	0.1119(17)D-01	0.1564(22)D-01	40%
ud $\rightarrow$ W+gggg	0.1339(36)D-02	0.2838(71)D-02	120%

*Results were calculated by HELAC*

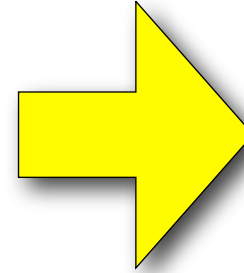
- 3.

correlations.

*ZN and D. Soper: JHEP: 0709 114,2007*

# Classical Parton Shower

- ☀ The parton shower relies on the **universal soft and collinear factorization** of the QCD matrix elements. It is universal property and true at all order. This should be the **only** approximation ...

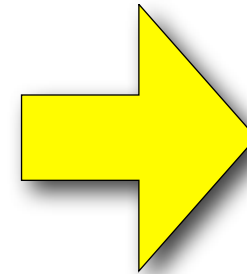


Parton shower as  
Quantum statistical  
mechanics



# Classical Parton Shower

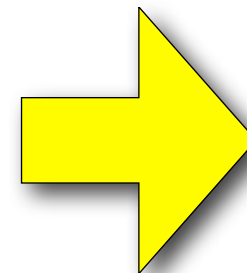
☀ The parton shower relies on the **universal soft and collinear factorization** of the QCD matrix elements. It is universal property and true at all order. This should be the **only** approximation ...



Parton shower as  
Quantum statistical  
mechanics

... but we have some further approximations:

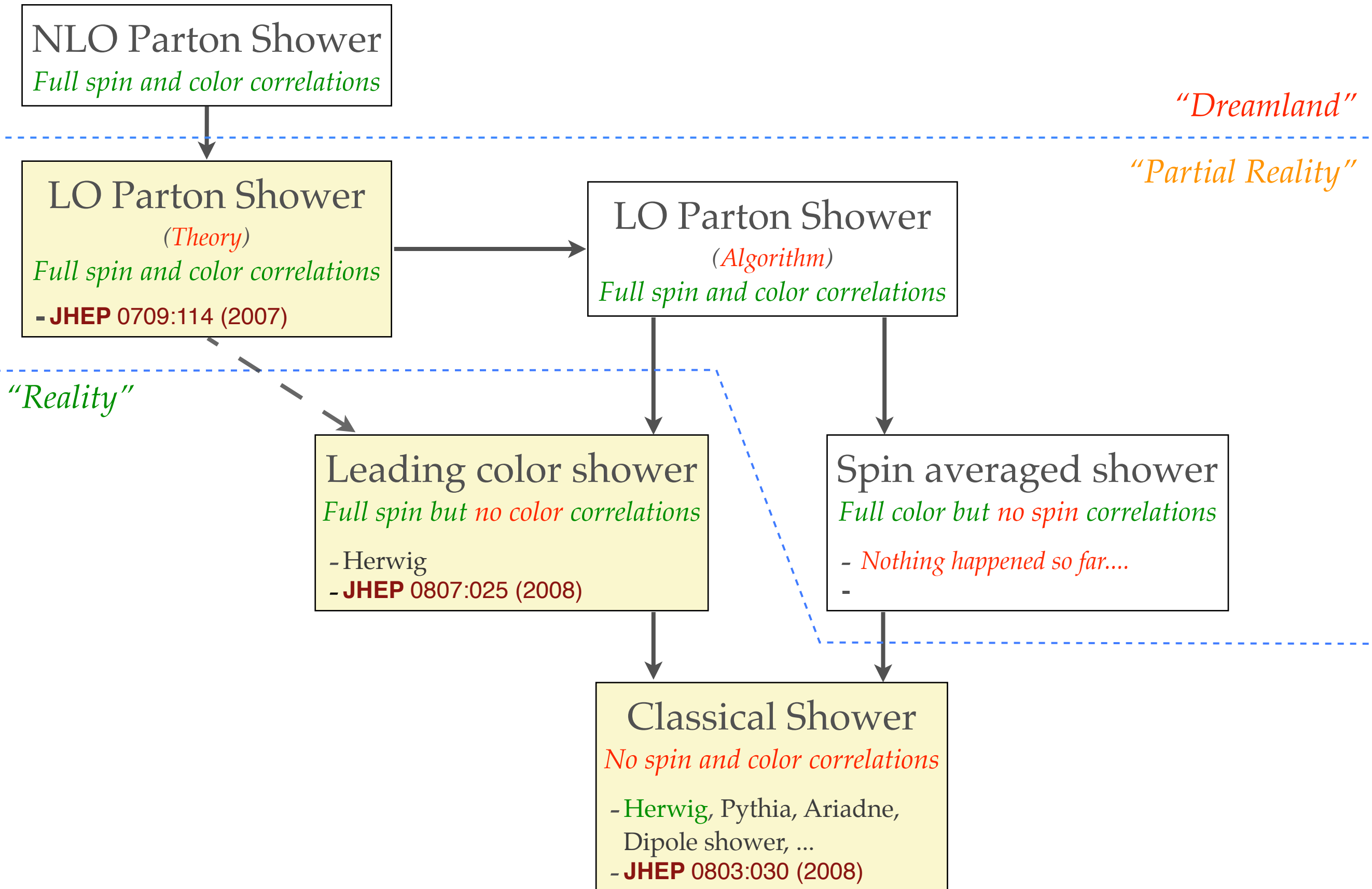
- ✗ Interference diagrams are treated approximately with the angular ordering
- ✗ Color treatment is valid in the  $N_c \rightarrow \infty$  limit
- ✗ Spin treatment is usually approximated.
- ✗ Usually very crude approximation in the phase space



Parton shower as  
classical statistical  
mechanics

... non-systematic approximations lead to systematically **NOT improvable** algorithm.

# Shower Family Tree



# LO Matching Schemes



There are two algorithm available in the literature for LO matching:

☀ **CKKW-L algorithm**: Reweighting Born matrix elements with Sudakov factors

*S. Catani, R. Kuhn, F. Krauss, B. Webber: **JHEP** 0111:063,2001*

*L. Lönnblad: **JHEP** 0205:046,2002*

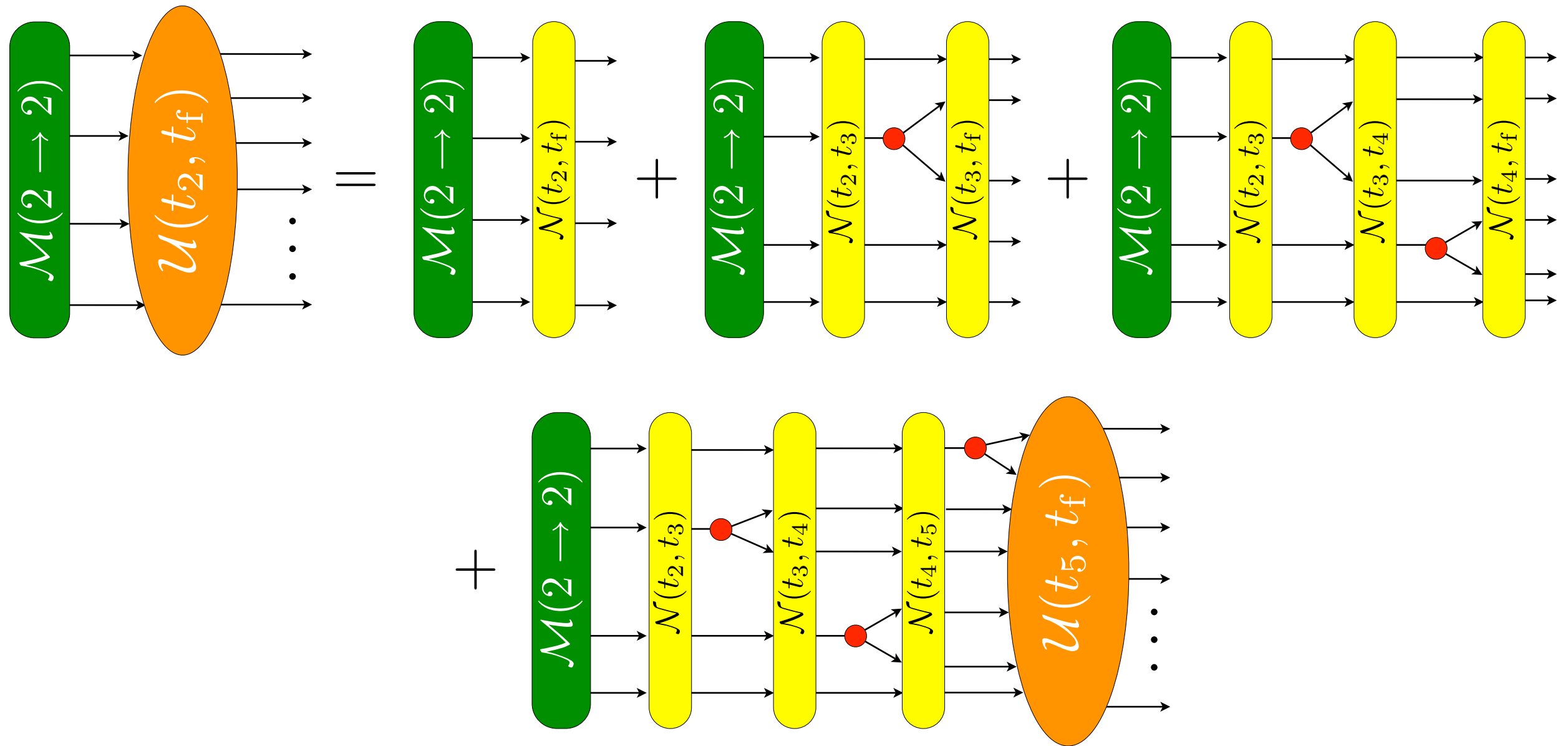
☀ **MLM algorithm**: Reweighting shower contributions with Born level matrix elements

*M. Mangano*

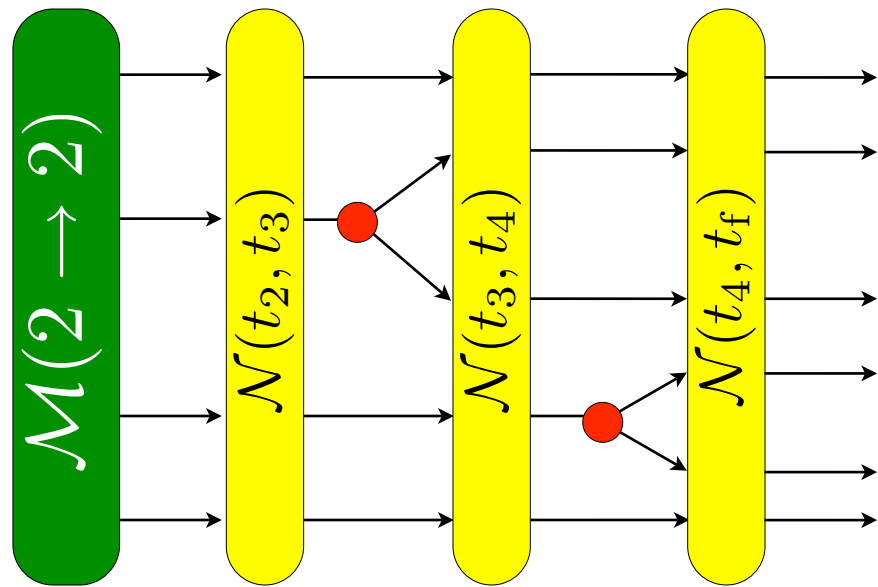
*M. Mangano , M. Moretti, F. Piccinini, M. Treccani: **JHEP** 0701:013,2007*

# Shower Cross Section

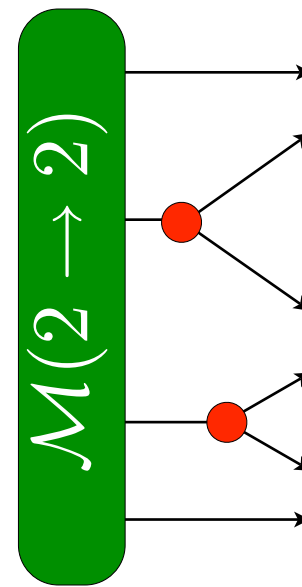
Iterating the evolution twice, then we have



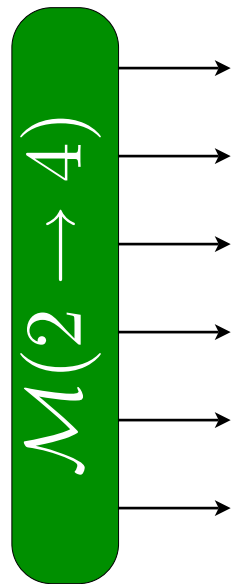
# Deficiency of Shower



*Standard shower contribution*

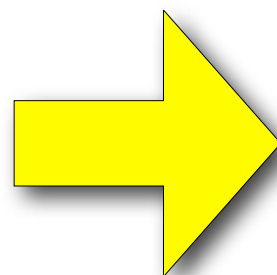


*Small  $p_T$  approximation*



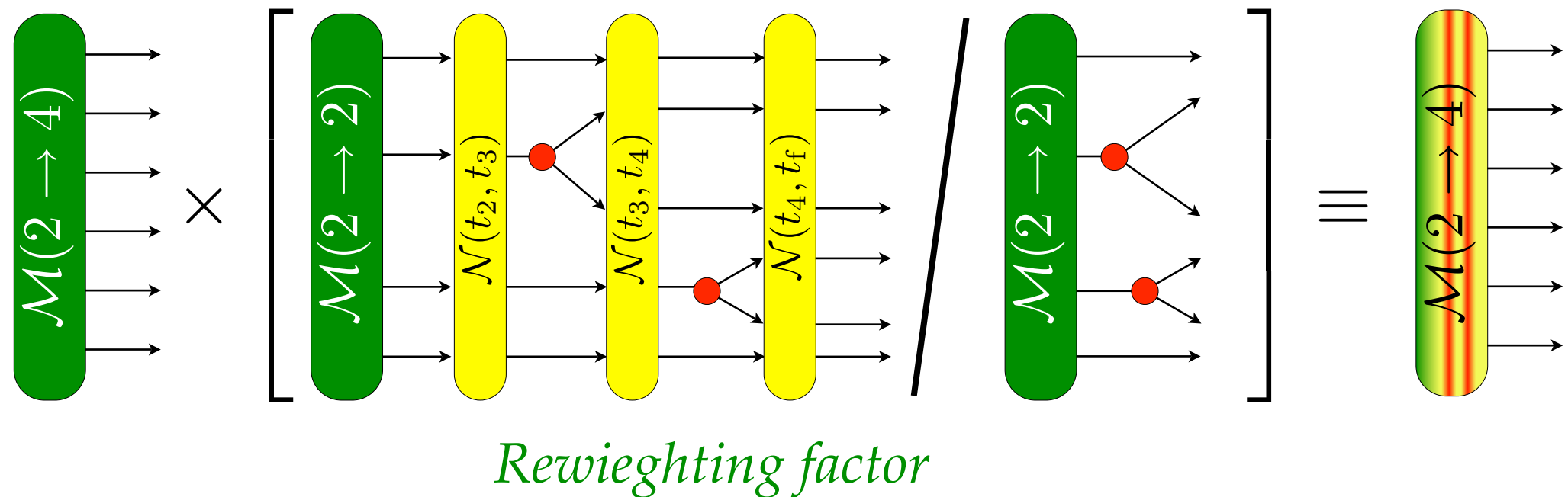
$|\mathcal{M}(2 \rightarrow 4)|^2$

- The shower approximation relies on the small  $p_T$  splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponentials.



*Rewieght the exact matrix elements with Sudakov factors*

# Improved weighting

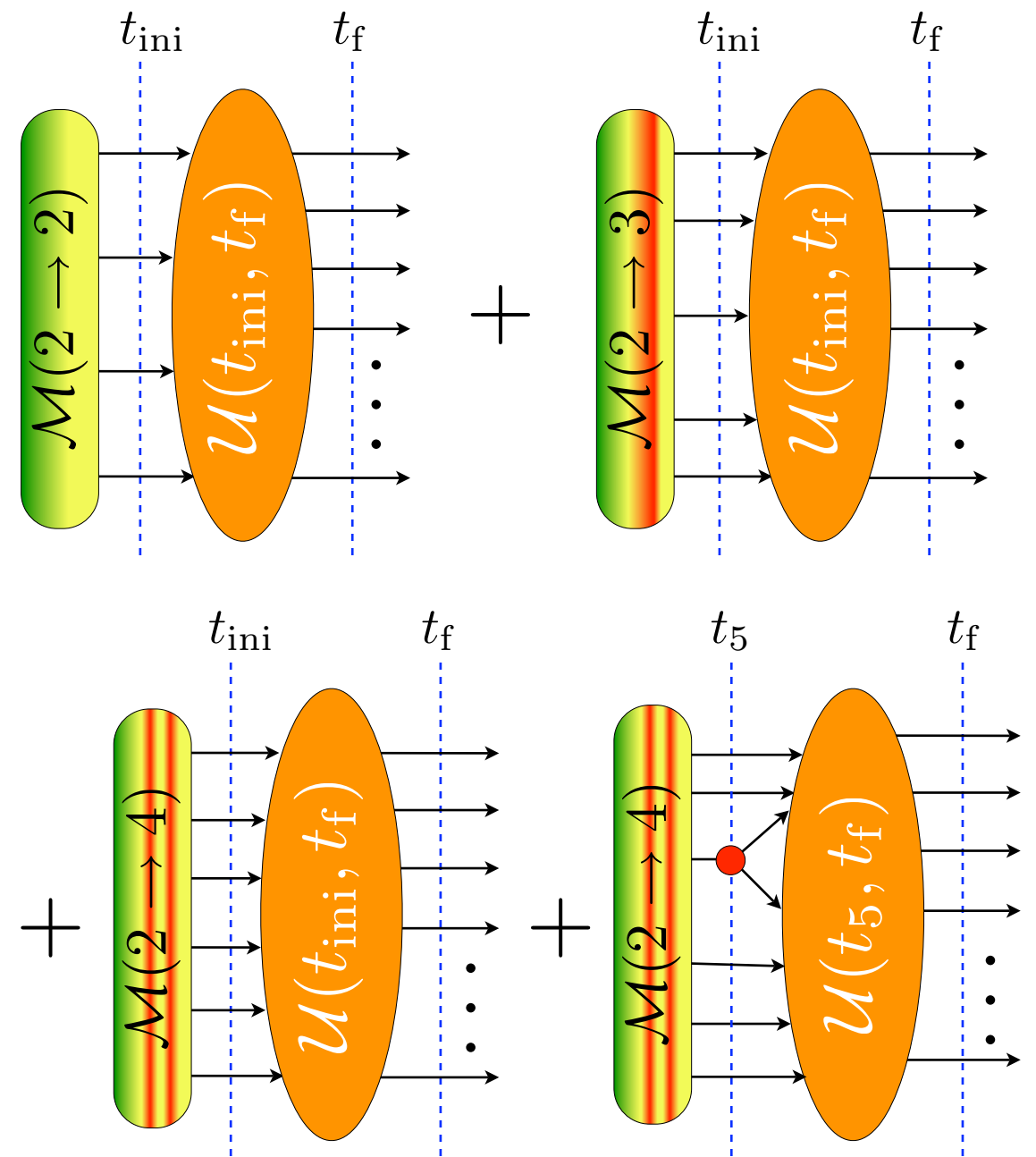
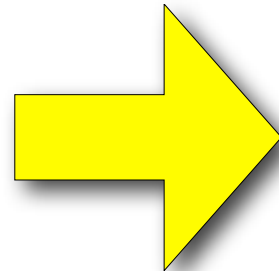
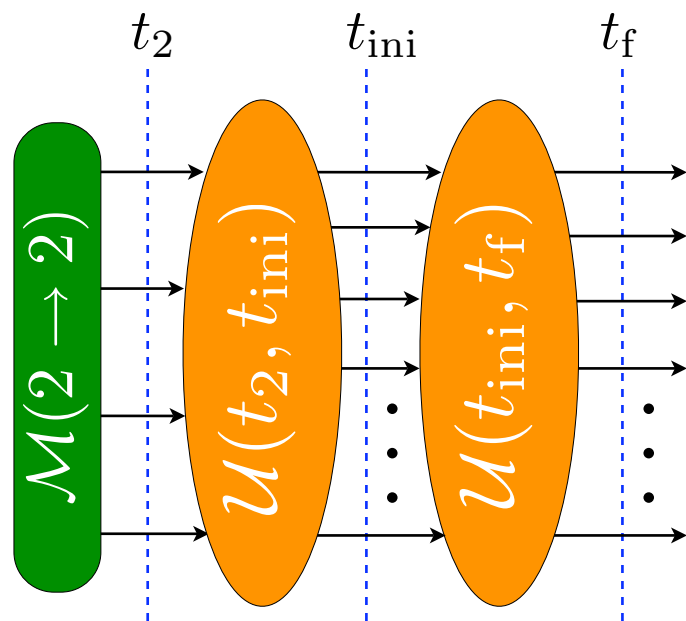


- This is the essential part of the CKKW matching procedure.
- In general there are many ways to get from  $2 \rightarrow 2$  configuration to  $2 \rightarrow m$  configuration.
- CKKW use the kT algorithm to find a unique history to define the Sudakov reweighting.
- The unique history requires to introduce matching scale.



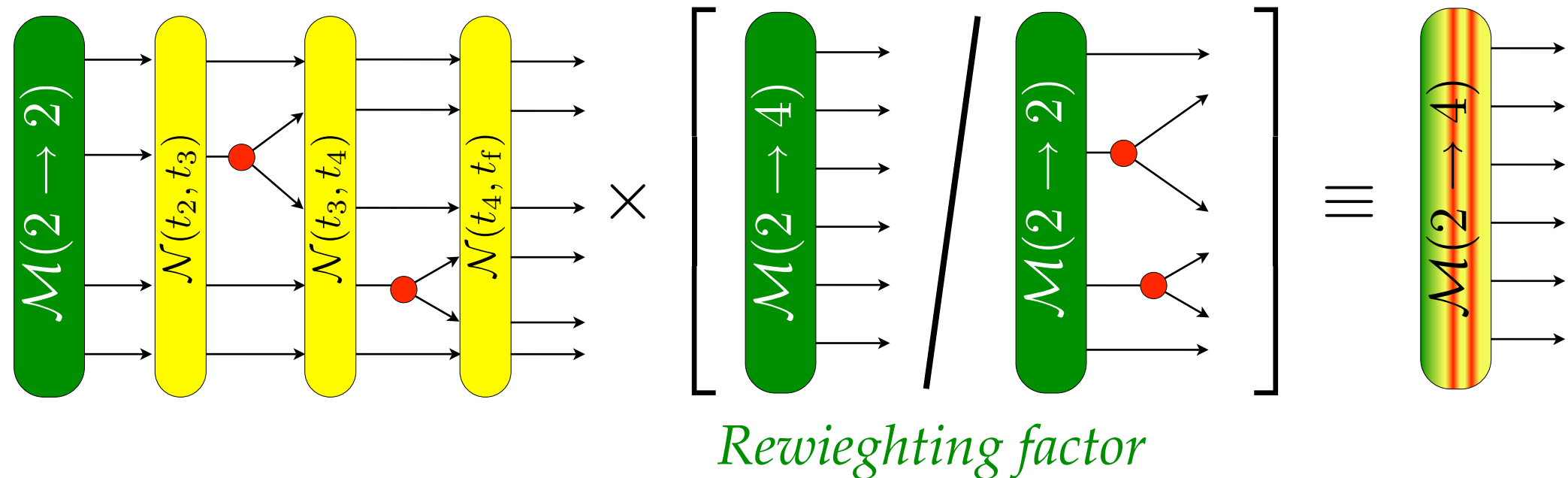
# CKKW Algorithm

CKKW break the evolution  
into  $0 < t < t_{\text{ini}}$  and  $t_{\text{ini}} < t < t_{\text{f}}$



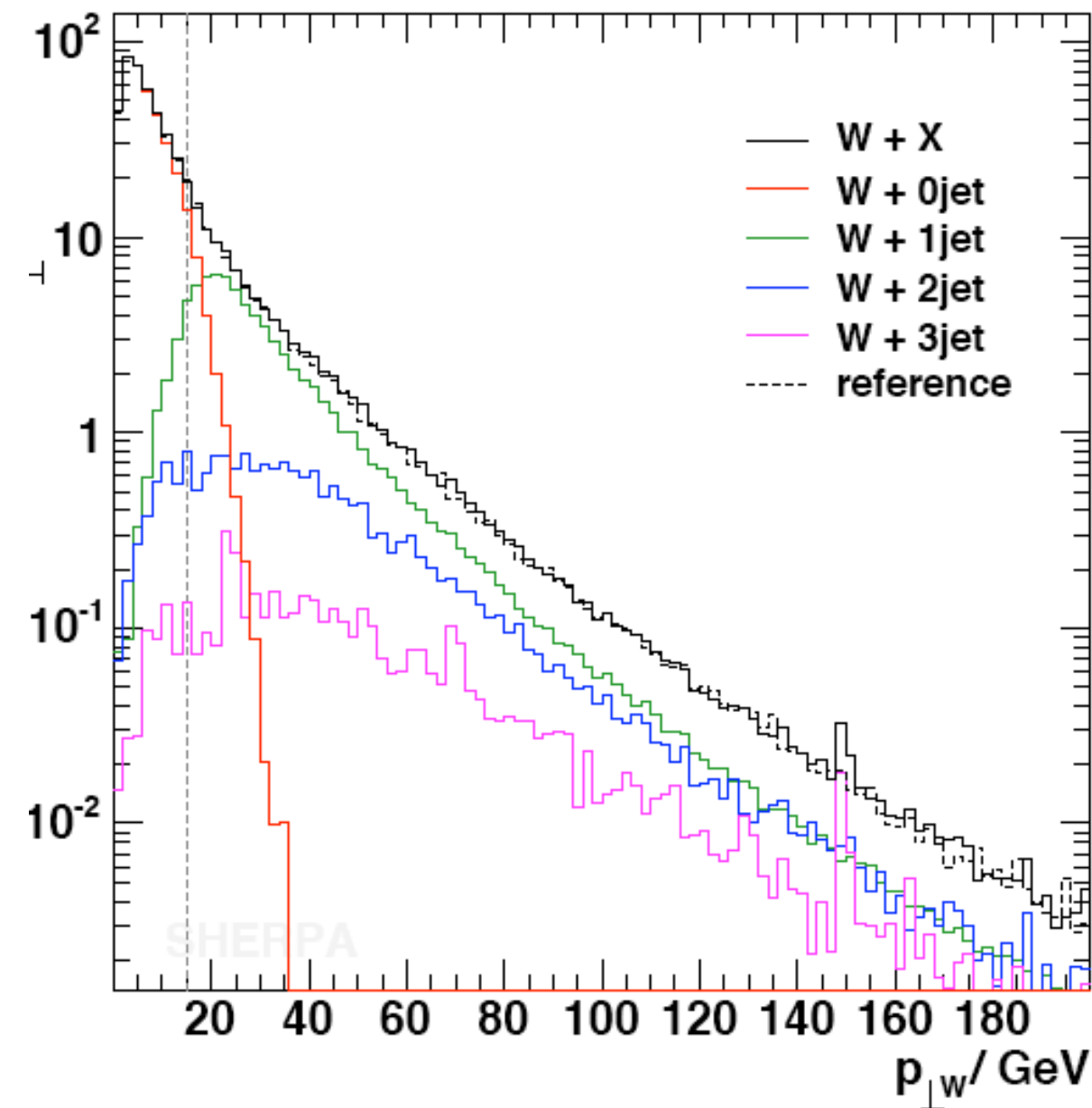
- CKKW use improve weighting for  $0 < t < t_{\text{ini}}$
- For  $t_{\text{ini}} < t < t_{\text{f}}$  they have standard shower (in Herwig and old Phytia case transverse momentum veto is needed)
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

# MLM Algorithm



- This is the essential part of the MLM matching procedure.
- MLM algorithm use the cone jet finding algorithm to define the ratio
- No analytic Sudakov factors, it use the native Sudakov of the underlying parton shower.
- Matching parameters:  $p_{T_{\min}}$ ,  $\eta_{\max}$ ,  $R_{\min}$

# LO Matching Schemes



- ✓ The CKKW-L algorithm is implemented in Sherpa and in Ariadne. MLM in AlpGen and MadGraph
- ✓ It is certainly an improvement.
- ✗ Only normalized cross section can be calculated.
- ✗ The result could strongly depend on the matching scale.
  - ➔ It would be nice **NOT to use** matching scale.
- ✗ Matching scale dependence cancelled at NLL level but only in  $e+e^-$  annihilation.
- ✗ No matching at quantum level.
- ✗ It is still LO order calculation thus the scale dependence is large.
  - ➔ The algorithm can be generalized at NLO level.

# NLO Matching Schemes

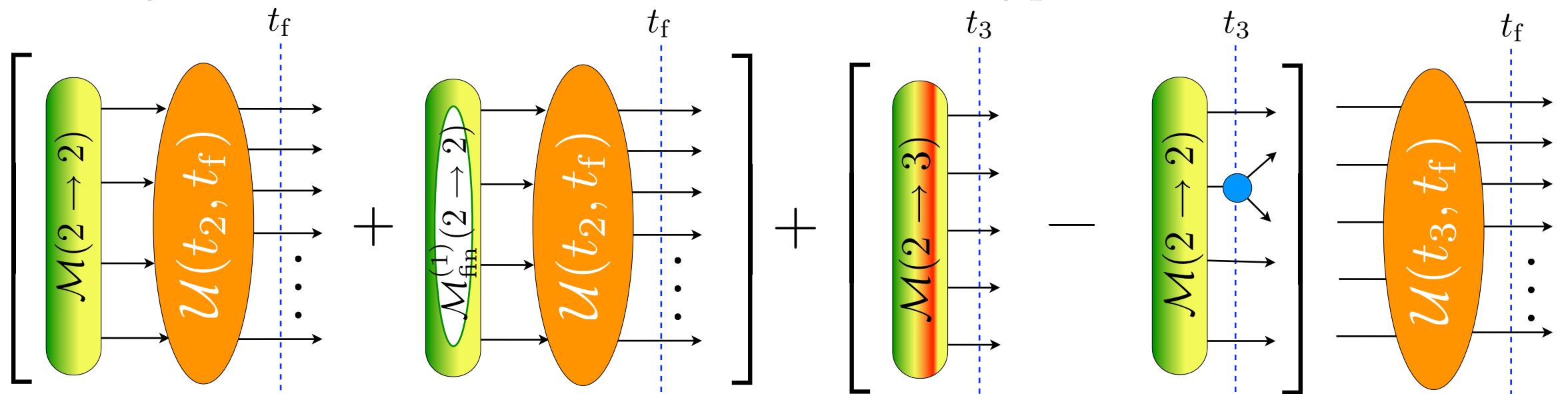


There are several algorithm available in the literature for NLO matching:

- ✱ **MC@NLO**: Avoiding double counting by introducing extra subtraction terms.  
*S. Frixione and B. Webber: JHEP 0206:029,2002*  
*S. Frixione, P. Nason and B. Webber: JHEP 0308:007,2003*
- ☀ **KS approach**: The main idea is to include the first step of the shower in NLO calculation and then start the shower from this configuration.  
*M. Krämer and D. Soper: Phys.Rev. D69:054019,2004*  
*P. Nason: JHEP 0411:040,2004*
- ☀ **“CKKW@NLO”** Combines the KS approach and the CKKW matching procedure.  
*ZN and D. Soper: JHEP 0510:024,2005*  
*Giele, Kosower, Skands: arXiv:0707.3652 [hep-ph]*

# MC@NLO

It might be a good idea to illustrate the MC@NLO matching procedure:

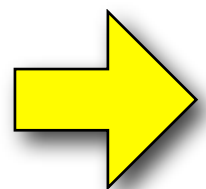


- ✓ Several simple processes are implemented in the MC@NLO framework.
- ✗ The MC@NLO is worked out for HERWIG. If you want to use it with PYTHIA you have to redo the MC subtraction.
- ✗ MC@NLO is defined only for the simplest processes, like  $2 \rightarrow 0, 1, (2)$  processes.
- ✗ No quantum correlations.

$$+ \int_1 [dV_{\text{MC}} - dV] \text{ [Diagram: Green cylinder } M(2 \rightarrow 2) \text{ with 4 arrows pointing to orange oval } U(t_2, t_f) \text{ with 4 arrows pointing to vertical dashed line at } t_f]$$

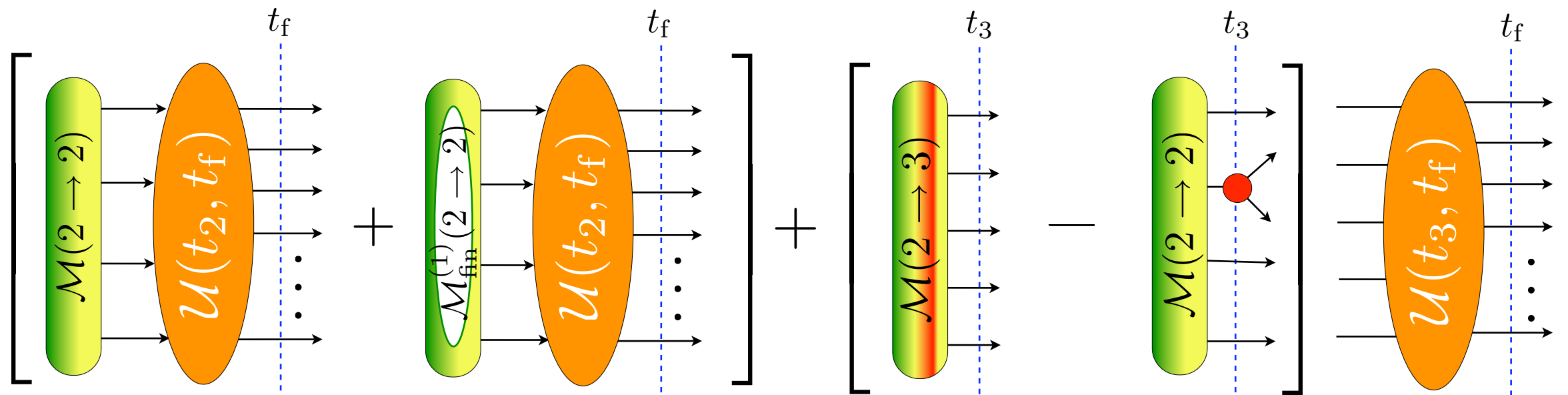
*Obvious step to choose*

$$dV_{\text{MC}} = dV$$



# Other approaches

At least the **first step** of the shower is done with the **NLO** splitting functions.



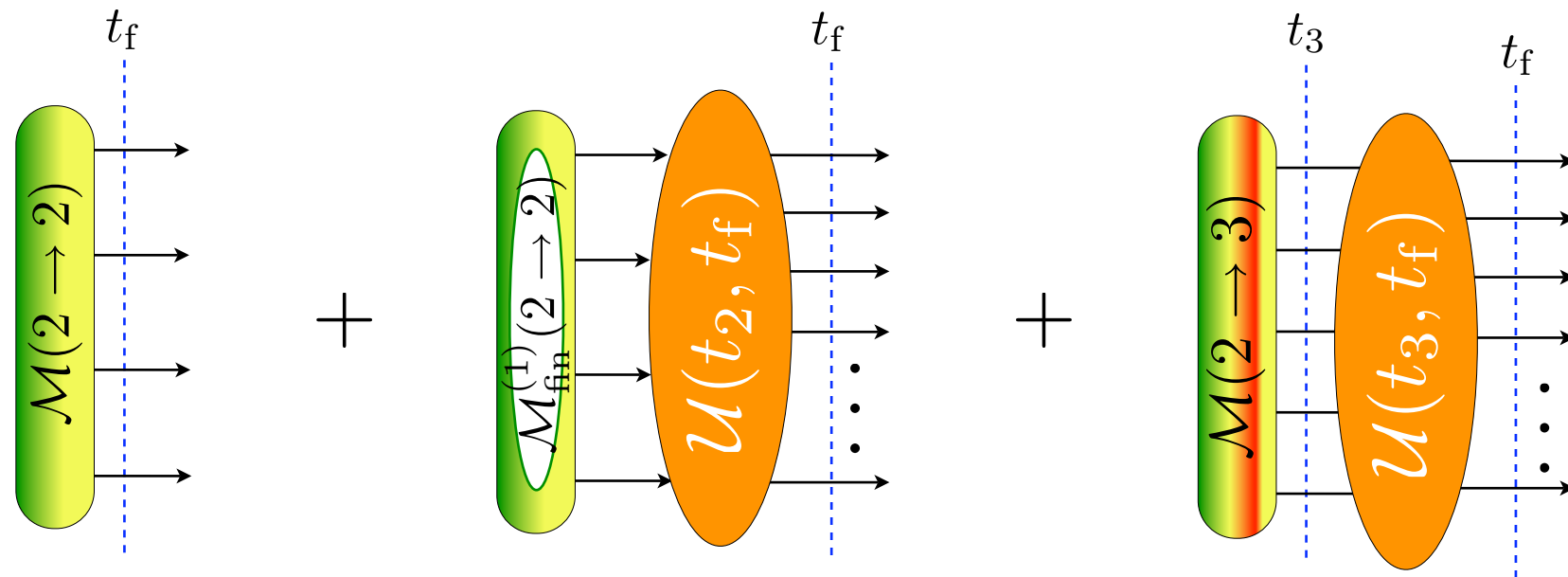
- ✓ This matching works with any shower algorithm.
- ✗ Several proposal but NO implementation in a general purpose program so far.
- ✗ No quantum correlations. Matching only in the momentum and flavor space.
- ✗ It is usually defined only for the simplest processes, like  $2 \rightarrow 0, 1, 2$  processes.
- ✓ To apply for other processes one has to combine the NLO matching with the CKKW algorithm.

*ZN and D. Soper: JHEP 0510:024,2005*



# Quantum level NLO matching

Including the quantum correlations (color and spin) properly the structure of the shower with NLO matching is simpler (no subtraction).



- ✓ This matching requires shower with quantum interference.
- ✓ All the quantum correlations are included.
- ✓ Systematically defined for **any** process.
- ✗ No complete algorithm worked out, No implementation.

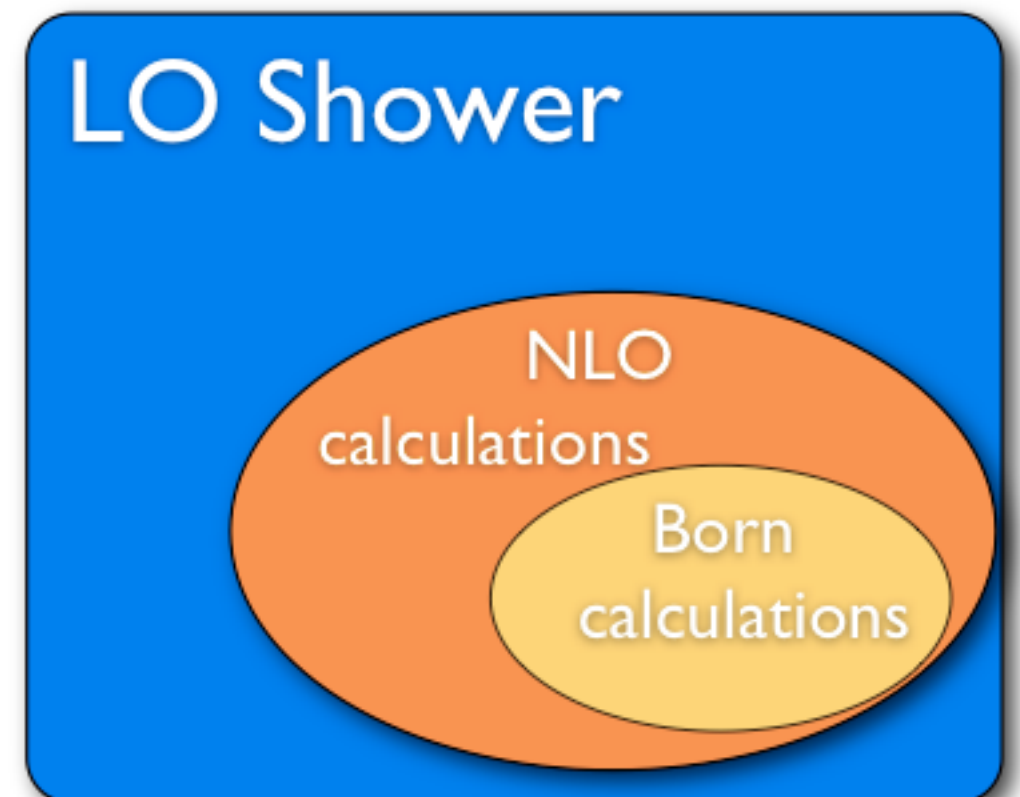
*ZN and D. Soper:* **hep-ph/0601021**  
<http://cern.ch/nagyz>

# Conclusions

Instead of having defined LO, NLO and shower calculation separately and patching the gap between them by matching schemes



we should define a new shower concept that can naturally cooperate with NLO calculations



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