Squeezed correlations among particle-antiparticle pairs

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Motivation

- Usually we think about looking to in-medium modifications of hadronic masses ↔ effects on dilepton yields and spectra

- Hadron mass shifts (interactions in dense medium) → vanish on the freeze-out surface → naively, no effects expected on Correlations

- However, a quantum mechanical correlation can be induced → medium-modified hadrons ↔ two-mode squeezed states of the asymptotic ones, which are the observables

→ How can correlations be used to determine the size of the interaction and phase transitions?

- Late 90's: Back-to-Back Correlations (BBC) among boson-antiboson pairs → shown to exist if the masses of the particles were modified in a hot and dense medium [Asakawa, Csörgo" & Gyulassy, P.R.L. 83 (1999) 4013]

- Shortly after → similar BBC shown to exist among fermion-antifermion pairs with medium modified masses [Panda, Csörgo", Hama, Krein & SSP, P. L. B512 (2001) 49]

Similarities

→ **Properties:**

- Similar positive correlations with **unlimited intensity of** both **fBBC** and **bBBC** Correlations
- Similar formalism for both bosonic (**bBBC**) and fermionic (**fBBC**) Back-to-Back
- Expected to appear for $p_T \leq 1\text{–}2$ GeV/c

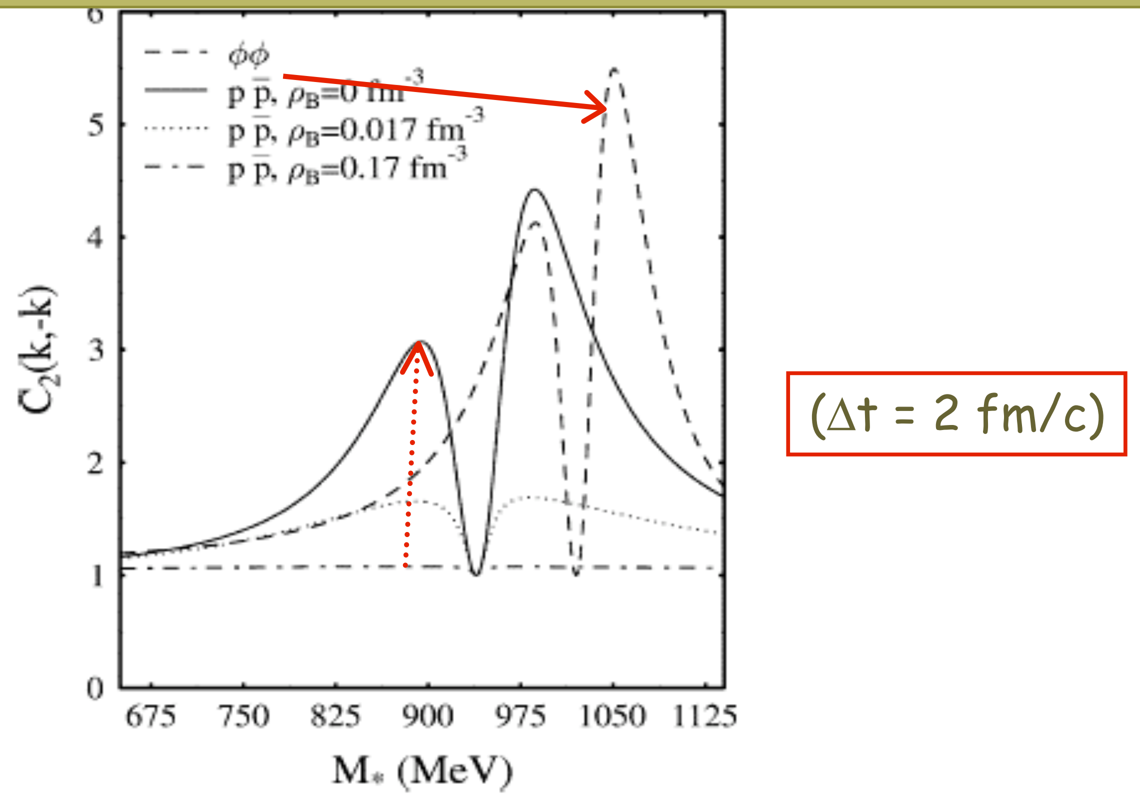


Fig. 1. Back-to-back correlations of proton–anti-proton pairs and ϕ-meson pairs, for T = 140 MeV, Δt = 2 fm/c and $|\mathbf{k}|$ = 800 MeV/c.

Outline

- Brief review of (minimal) formalism and previous results (infinite systems)
- Focus on finite systems expanding with non-relativistic flow → illustration: $\phi\phi$ pairs & K^+K^- pairs
- How to search for squeezed BBC pairs in experiments → suitable variables (relativistic analogue)
- Effects of modified-mass and squeezing on correlations of $\phi\phi$ and K^+K^- pairs
- Summary and conclusions → urge for experimental discovery

Full Correlation Function ($\pi^0\pi^0$ or $\phi\phi$)

$$\boxed{\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \underbrace{\langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle}_{} \pm \underbrace{\langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle}_{} + \underbrace{\langle a^\dagger_{k_1} a^\dagger_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle}_{}}$$

NOTATION

$$N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3N}{d^3k} = G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i,i) = \omega_{k_1} \langle a^\dagger_{k_i} a_{k_i} \rangle \longrightarrow \text{Spectra}$$

$$G_c(\vec{k}_1, \vec{k}_2) \equiv G_c(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a^\dagger_{k_1} a_{k_2} \rangle \longrightarrow \text{Chaotic amplitude}$$

$$G_s(\vec{k}_1, \vec{k}_2) \equiv G_s(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle \longrightarrow \text{Squeezed amplitude}$$

$$\boxed{C_2(\vec{k}_1, \vec{k}_2) = 1 \pm \underbrace{\frac{\left|G_c(1,2)\right|^2}{G_c(1,1)\,G_c(2,2)}}_{\text{HBT}} + \underbrace{\frac{\left|G_s(1,2)\right|^2}{G_c(1,1)\,G_c(2,2)}}_{\text{BBC}}}$$

In-medium & asymptotic operators

– a_k ($a^\dagger{}_k$) → annihilation (creation) operator of the asymptotic quanta with 4-momemtum p^μ;

– b_k ($b^\dagger{}_k$) → in-medium annihilation (creation) operator

(a-quanta → observed; b-quanta → thermalized in medium)

They are related by the Bogoliubov transformation:

$$\begin{cases} a^\dagger{}_k = c_k^* \, b^\dagger{}_k + s_{-k} \, b_{-k} \\ a_k = c_k \, b_k + s^*_{-k} \, b^\dagger{}_{-k} \end{cases} \quad ; \quad \boxed{c_k = \cosh[f_k]} \quad ; \quad \boxed{s_k = \sinh[f_k]}$$

– $\boxed{f_k = \frac{1}{2} \ln(\omega_k \, / \, \Omega_k)}$ → squeezing parameter (Bogoliubov transformation is equivalent to a squeezing operation)

$$\boxed{m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)} \qquad \boxed{\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)}$$

limit of no-squeezing: $\Omega_k \to \omega_k \Rightarrow f_k \to 0 \Rightarrow s_k \to 0 \wedge c_k \to 1$

Finite expanding systems

- Does the BBC survive
  - Finite emission interval? → OK!
  - Finite medium (volume V) ?
  - Flow ?

Answer is YES!

- For a hydrodynamical ensemble → amplitudes can be written as [Makhlin & Sinyukov, N.P. A566 (1994) 598c]

Results for a static infinite medium

$$G_c(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{iq_{1,2}.x}\left[\left|c_{1,2}\right|^2 n_{1,2} + \left|s_{-1,-2}\right|^2\left(n_{-1,-2}+1\right)\right]$$

$$G_s(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{i2K_{1,2}.x}\left[s^*_{-1,2}\, c_{2,-1}\, n_{-1,2} + c_{1,-2},1\, s^*_{-2,1}\left(n_{1,-2}+1\right)\right]$$

$$2 * K^\mu_{i,j} = (k_i + k_j) \qquad q^\mu_{i,j} = (k_i - k_j)$$

Finite system expanding with non-relat. flow

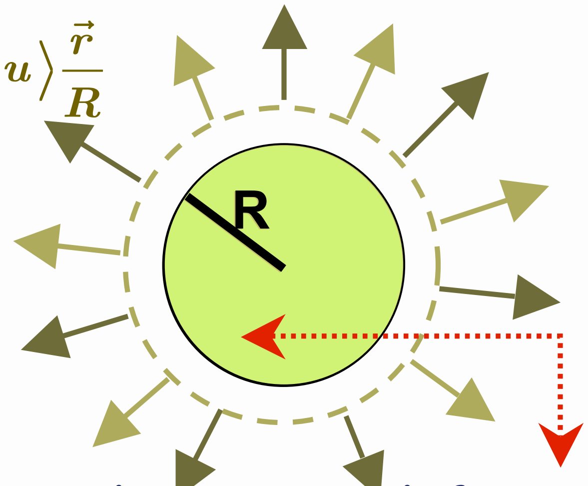
– Neglecting flow effects on squeezing parameter $f_{i,j}$

– Non-relativistic flow

– Simplest finite squeezing Vol. profile → analytical calculations: 3-D Gaussian → circular cross-sectional area of radius R

$$\approx \exp[-\vec{r}^2 \,/\, (2R^2)]$$

$$\vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$



Region where mass-shift is non-vanishing

Freeze-out

– Sudden freeze-out

$$\int dt\, E_{i,j} e^{-2iE_{i,j}.\tau}\, \delta(\tau - \tau_0)\, d\tau_f$$

$$= E_{i,j} e^{-2iE_{i,j}.\tau_0}$$

– Finite emission time interval

$$\int dt\, E_{i,j}\, F(\tau_f)\, e^{-iE_{i,j}(\tau-\tau_0)}\, d\tau_f$$

$$= \frac{E_{i,j}}{[1 + (E_{i,j}\Delta\tau)^2]}$$

$$\boxed{\frac{\theta(\tau-\tau_0)}{\Delta\tau}\, \mathrm{e}^{-(\tau-\tau_0)/\Delta\tau}}$$

$$n_{i,j}(x) \sim \exp\Big[-\Big(K^{\mu}_{i,j} u_{\mu} - \mu(x)\Big) \,/\, T(x)\Big]$$

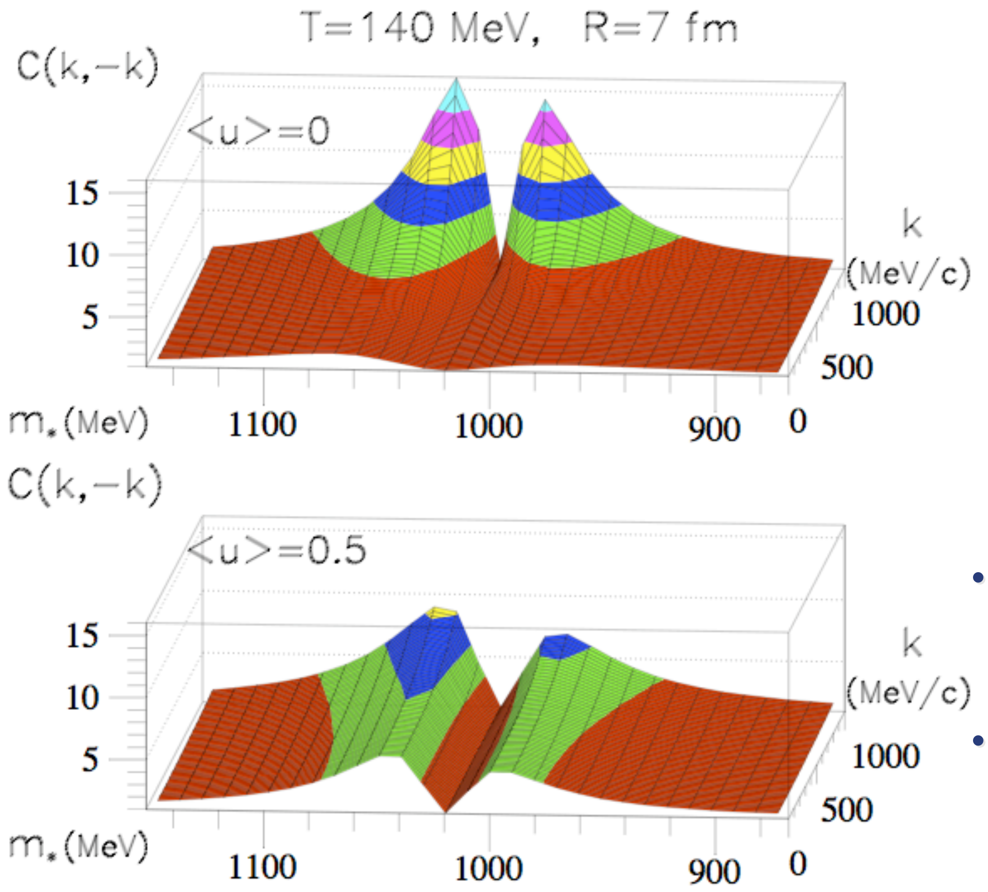
Hydro parameterization → $\dfrac{\mu(x)}{T(x)} = \dfrac{\mu_0}{T(x)} - \dfrac{\vec{r}^2}{2R^2}$

$$u^{\mu} = \gamma(1, \vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

$$\gamma = (1 - \vec{v}^2)^{-1/2} \approx 1 + \tfrac{1}{2}\vec{v}^2 \quad [\mathcal{O}(v^2)]$$

Summary of the previous results

$\phi\phi$ BBC



- Previous results studied before:
  - $C_s(k, -k)$ survives both
    - Finite emission times (Δt=2fm/c) and finite system sizes
    - Moderate flow (could enhance signal at small $\underline{k}$)
  - However, only the behavior of the maximum value (intercept) of $C_s(k,-k)$ vs. m_* vs. k was studied before (not useful for looking for the signal)
- Which would be the basic signal to be searched for?
- → better look for different values of k_1,k_2, i.e.,

$$C_s(k_1,k_2)$$

$\phi\phi$ BBC from simulation (cross-check)

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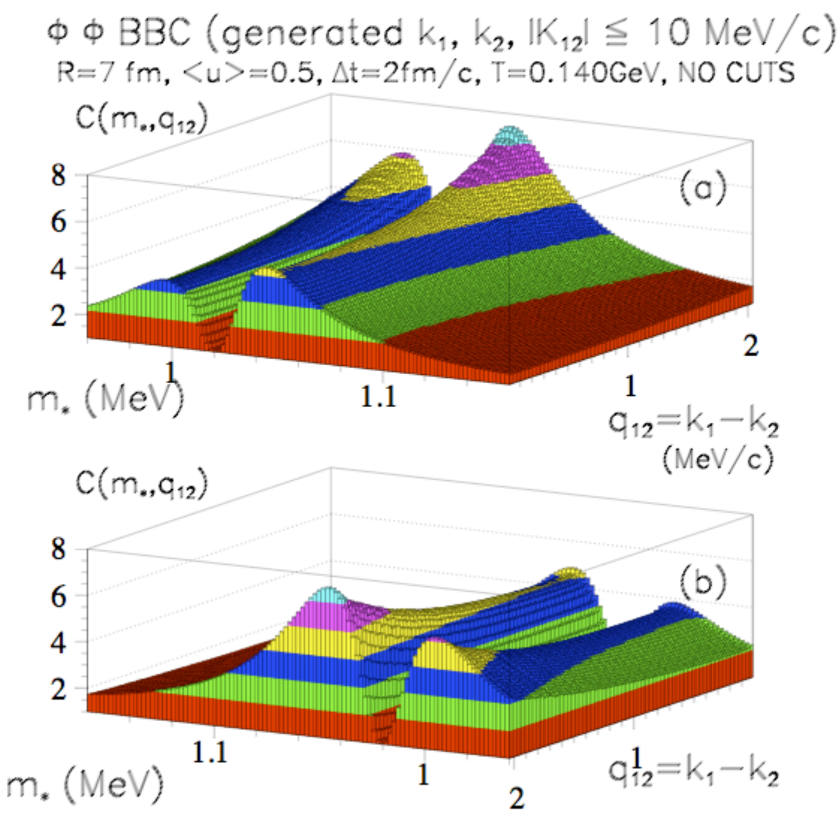
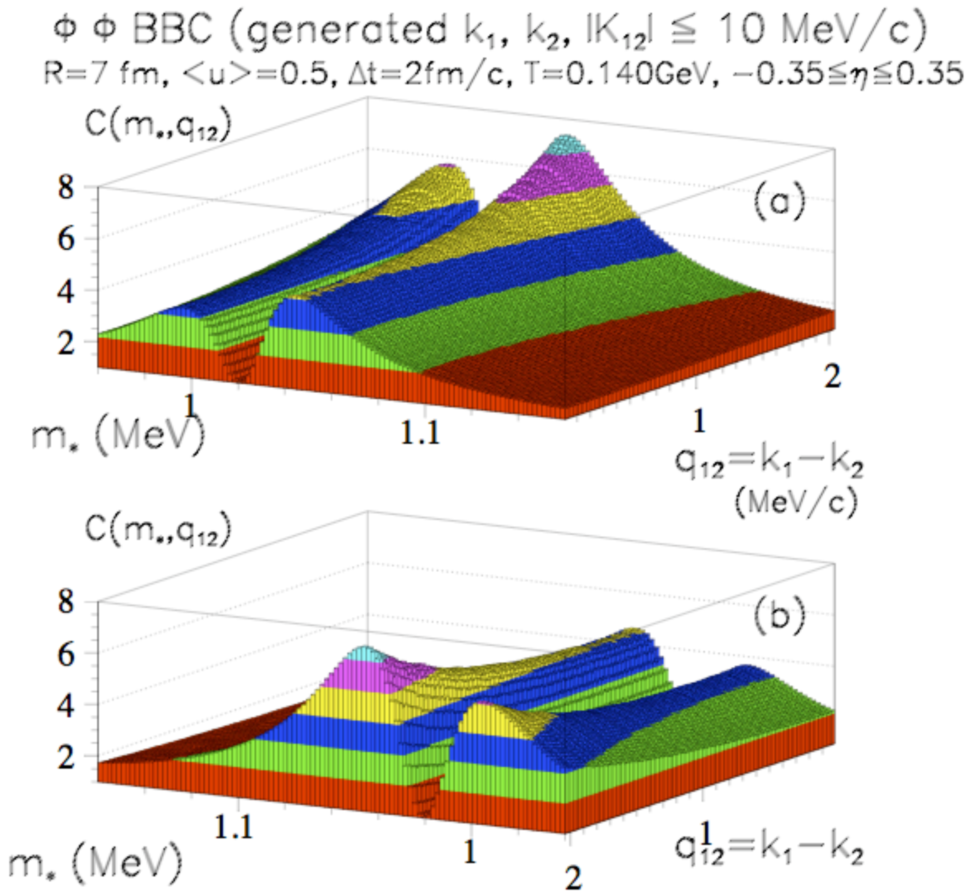
- Redoing older calculation with simulation $\leftrightarrow$ test

Experimental cuts: PHENIX arXiv:nucl-ex/0410012 Phys. Rev. C69, 034909 ('04)





$$\vec{q}_{12} = \vec{k}_1 - \vec{k}_2 \quad \rightarrow \quad \vec{k}_1 = -\vec{k}_2 = \vec{k} \Rightarrow \vec{q}_{12} = 2\vec{k}$$

Suitable variables

Two main possibilities:

1) Combining **particle-antiparticle pairs** → theoretically generating (k_1, k_2) → simulation ↔ Exp: SEv/DEv

2) Rewriting $C_s(k_1, k_2)$ in terms of K and q :

$$2 * \vec{K}_{i,j} = (\vec{k}_i + \vec{k}_j)$$

$$\vec{q}_{i,j} = (\vec{k}_i - \vec{k}_j)$$

The effect is maximum for

$$\boxed{\vec{k}_1 = -\vec{k}_2 = \vec{k}}$$

i.e., for $\vec{K} = 0$ → study for different values of q

Relativistic extension

If we define (suggested by M. Nagy)

$$Q_{back} = (\omega_1 - \omega_2, \vec{k}_1 + \vec{k}_2) = (q_{12}^0, 2\vec{K}_{12})$$

where $q^0 = k_1^0 - k_2^0 = \omega_1 - \omega_2$

However, even better: define a new variable, such as

$$\boxed{Q_{bbc}^2 = -(Q_{back})^2 = 4(\omega_1\omega_2 - K^\mu K_\mu)}$$

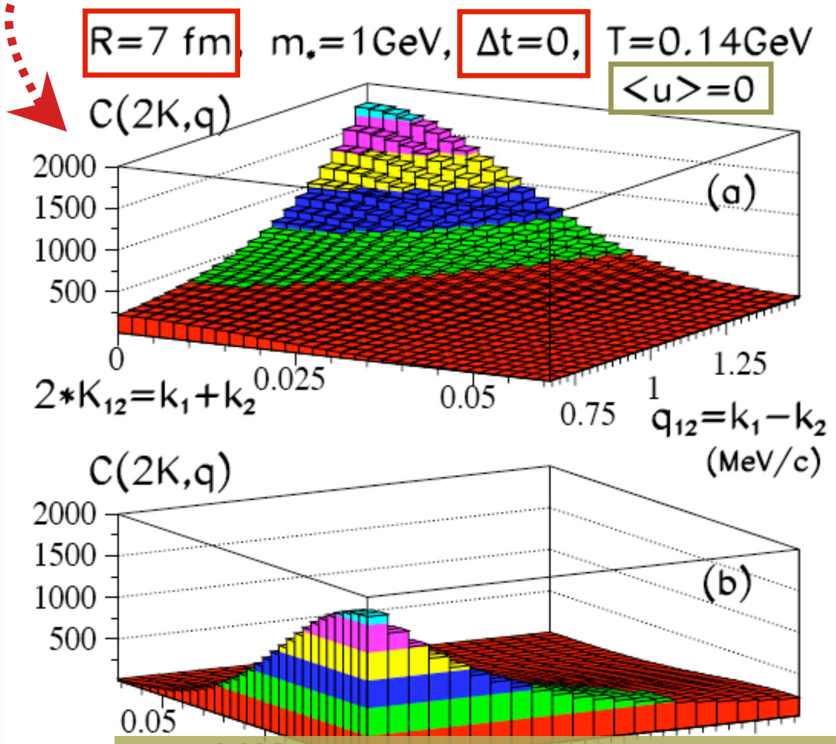
Then, its non-relativ. limit ($\omega_i = \sqrt{m^2 + \vec{k}_i^{\,2}} \approx m + \frac{\vec{k}_i^{\,2}}{2m}$) is

$$\boxed{Q_{bbc}^2 \approx (2\vec{K}_{12})^2}$$

(limit adopted here)

$C_{sq}(K_{12},q_{12})$ vs. K_{12} vs. q_{12} - flow effects

time reduction factor: $\frac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$

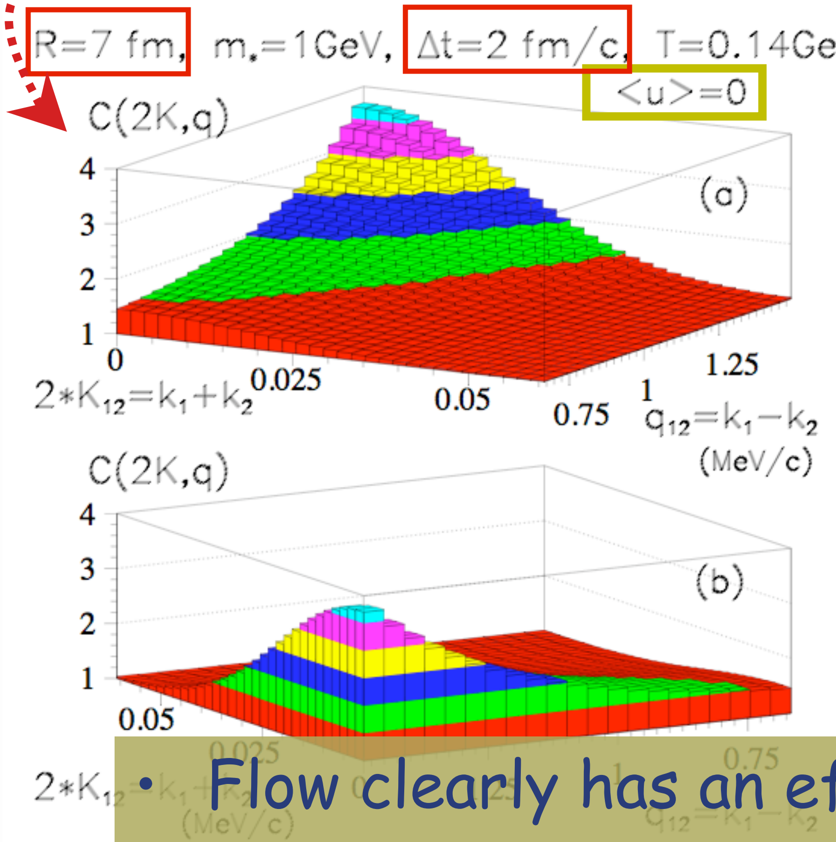


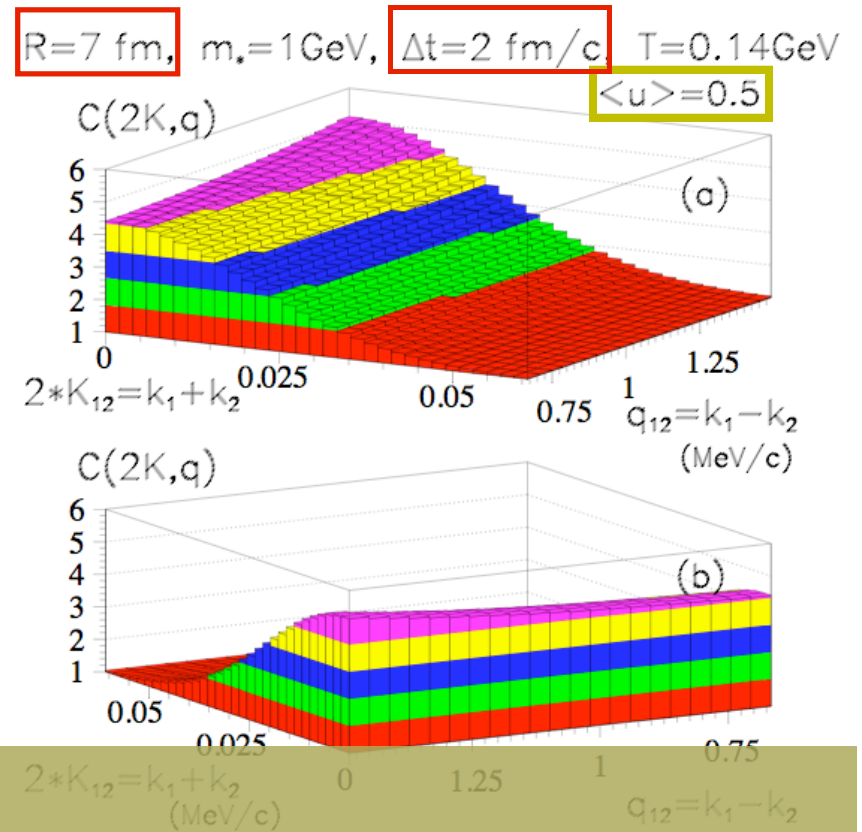
- Flow clearly has an effect:
  - For $<u>=0$ → C_s increases fast for increasing q_{12}
    $<u>=0.5$ → C_s increases more slowly but is significantly higher at lower values of q_{12}
  - Flow **enhances** and **extends** the signal to broader region (K_{12},q_{12})

$C_{sq}(K_{12},q_{12})$ vs. K_{12} vs. q_{12} - flow effects

time reduction factor, $\frac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$

results in:





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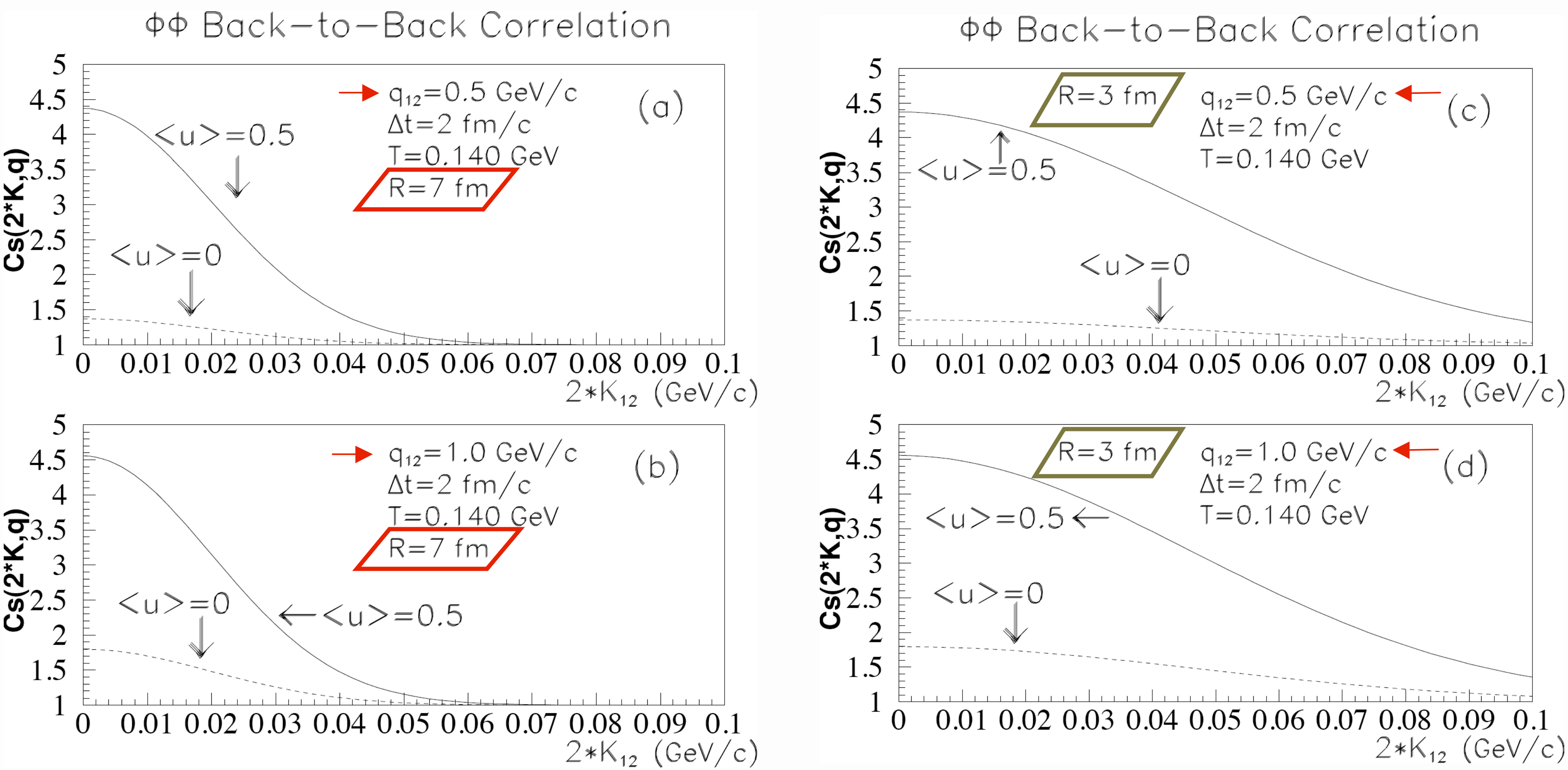
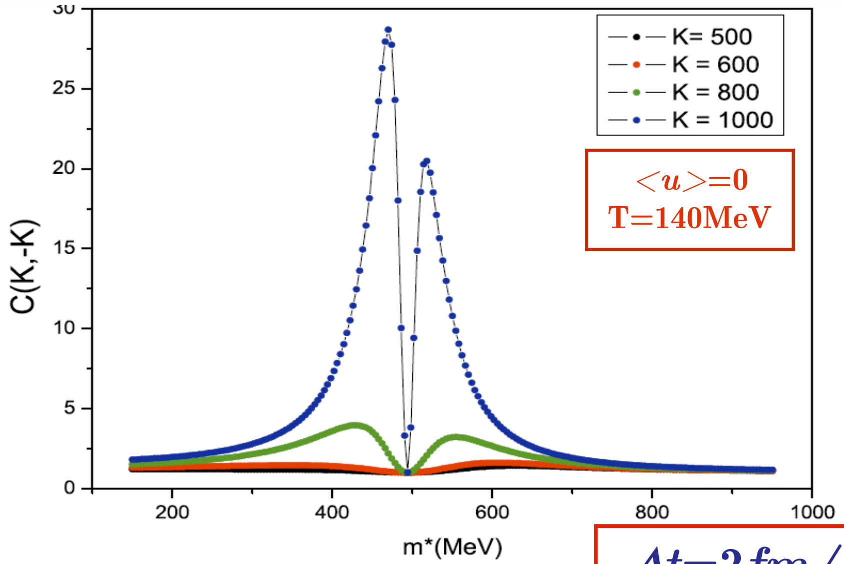
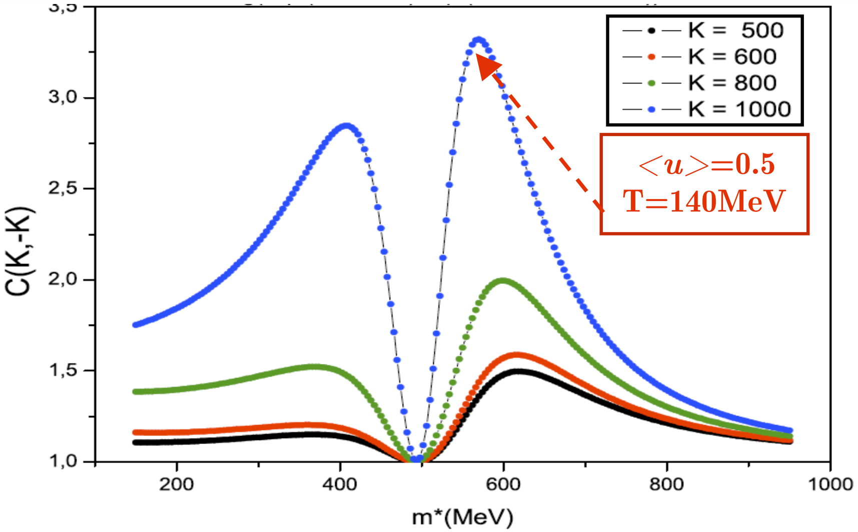
Radial flow and size effects (<*u*> ~ 0.5) on the squeezed correlation function



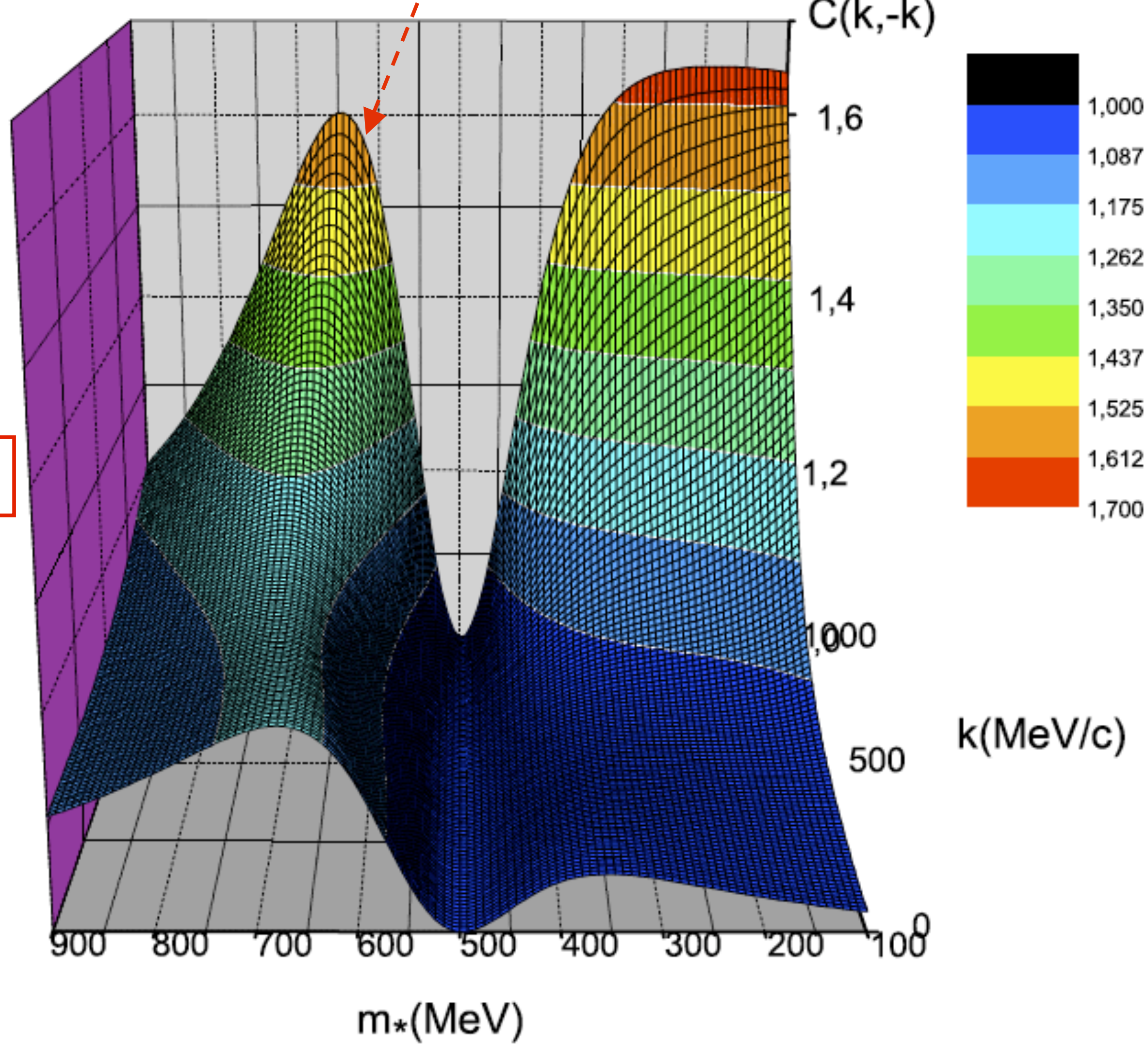
K^+K^- Squeezed Correlation vs modified m_*



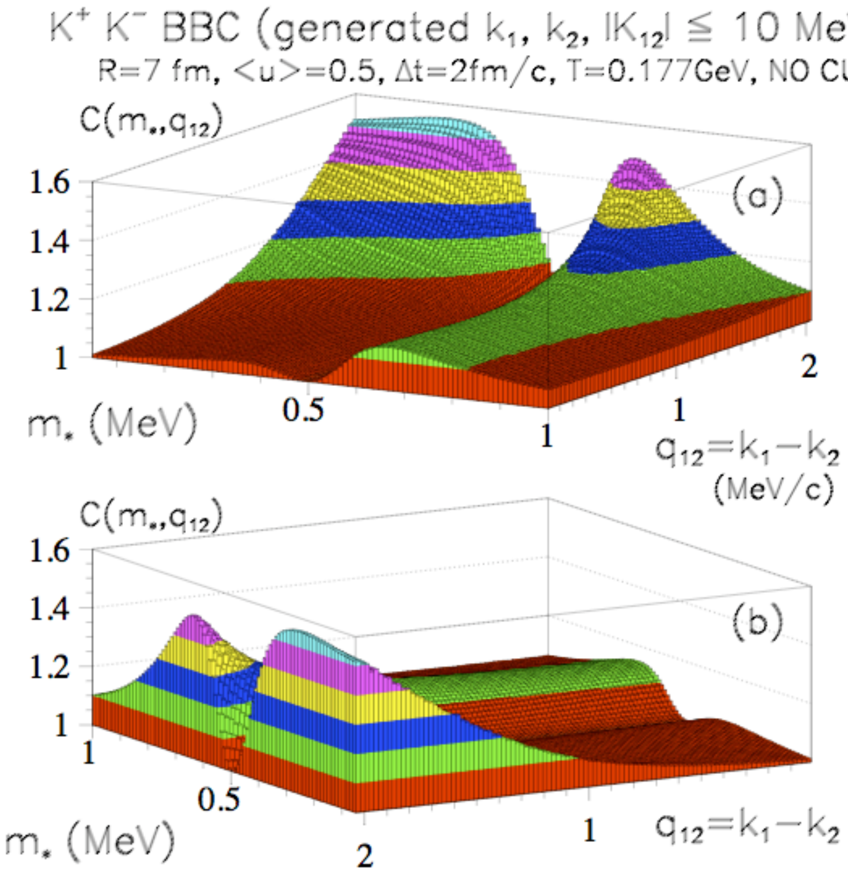


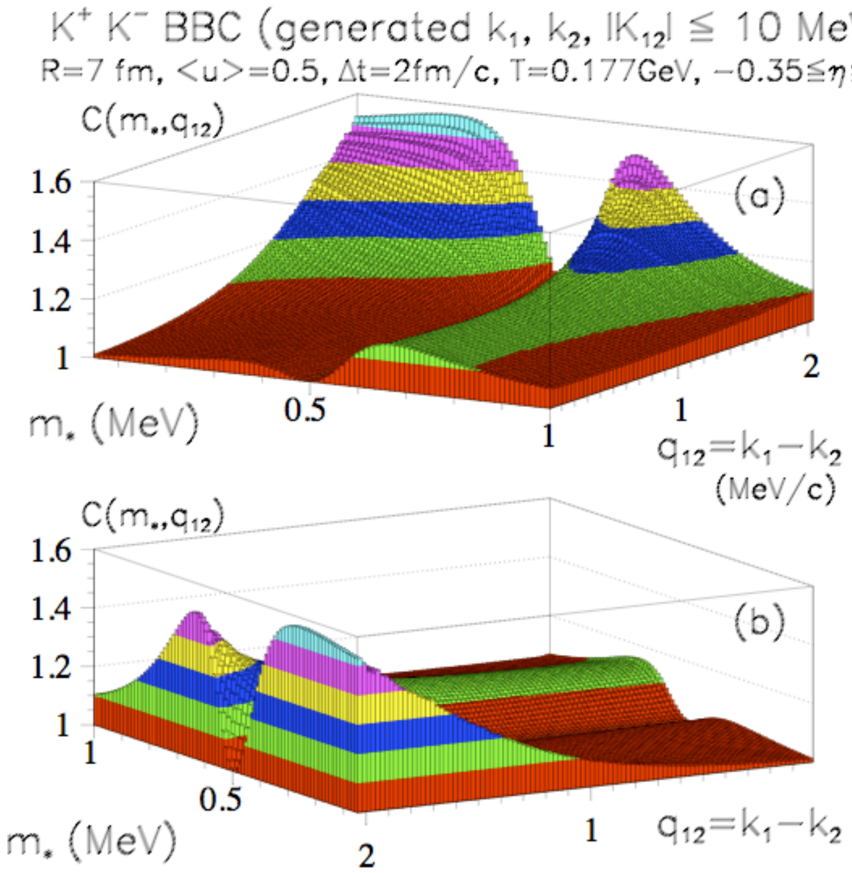
$\Delta t = 2 fm/c$

$R=7\ fm;\ T=177\ MeV;\ <u>=\ 0.5;\ \Delta t=2fm/c$



Kaons - from simulation





Experimental cuts: PHENIX
arXiv:nucl-ex/0410012
Phys. Rev. C69, 034909 ('04)

Summary and Conclusions

- Brief review of squeezed correlations ↔ motivation
- And of the most relevant results of the model (in a non-relativistic treatment of expanding finite systems)
- Suggestion of suitable variables to use in the experimental search of the squeezed states (BBC's): $C_s\ (K_{12},\ q_{12})$ vs. (2^*K_{12}) vs q_{12}, or in invariant terms:

$$\boxed{\phi\phi \ \text{or}\ K^+K^-}$$

$$Q^2_{bbc} = -\,(Q_{back})^2 = 4(\omega_1\omega_2 - K^\mu K_\mu)$$

- Showed some results on the expected behavior of the $C_s(k_1, k_2)$
- **Squeezed Correlations → theoretically well-stablished**
- Essential ingredient missing: **experimental discovery!!**

Urge to start searching NOW !!!

↔ inequivocal signature of hadronic mass shift in hot and dense medium using hadronic probe → Let's go find them!!

Acknowledgments

Thank you for your attention!

I gratefully acknowledge the Organizing Committee of the "ISMD 2008" for their kind support to attend the Symposium

BACK-UPS

Squeezed Correlation vs. k_1 & k_2

$$2 * \vec{K} = \vec{k}_1 + \vec{k}_2 \qquad \vec{q} = \vec{k}_1 - \vec{k}_2$$

$$G_s(k_1,k_2) = \frac{E_{1,2}}{(2\pi)^{3/2}} c_{12} s_{12} \left\{ \begin{array}{l} R^3 \exp\left(-\dfrac{R^2(k_1+k_2)^2}{2}\right) + 2\, n_0^* R_*^3 \exp\left(-\dfrac{(k_1-k_2)^2}{8 m_* T}\right) \times \\ \exp\left[\left(-\dfrac{im\langle u\rangle R}{2m_* T_*} - \dfrac{1}{8 m_* T_*} - \dfrac{R_*^2}{2}\right)(k_1+k_2)^2\right] \end{array} \right\}$$

$$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$$

Remember: $2 * \vec{K} = \vec{k}_1 + \vec{k}_2 \; , \; \vec{q} = \vec{k}_1 - \vec{k}_2$

$$G_c(k_i) = \frac{E_{i,i}}{(2\pi)^{3/2}} \left\{ |s_{ii}|^2 R^3 + n_0^* R_*^3 \left(|c_{ii}|^2 + |s_{ii}|^2\right) \exp\left(-\frac{k_i^2}{2 m_* T_*}\right) \right\}$$

$$R_* = R\sqrt{\frac{T}{T_*}}$$

$$C_s(\vec{k}_1,\vec{k}_2) = 1 + \frac{\left|G_s\left(\vec{k}_1,\vec{k}_2\right)\right|^2}{G_c\left(\vec{k}_1,\vec{k}_1\right) G_c\left(\vec{k}_2,\vec{k}_2\right)}$$

$$T_* = \left(T + \frac{m^2}{m_*}\langle u\rangle^2\right)$$

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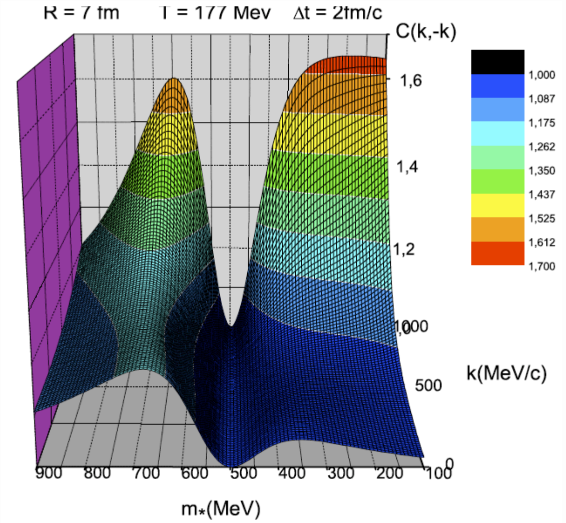
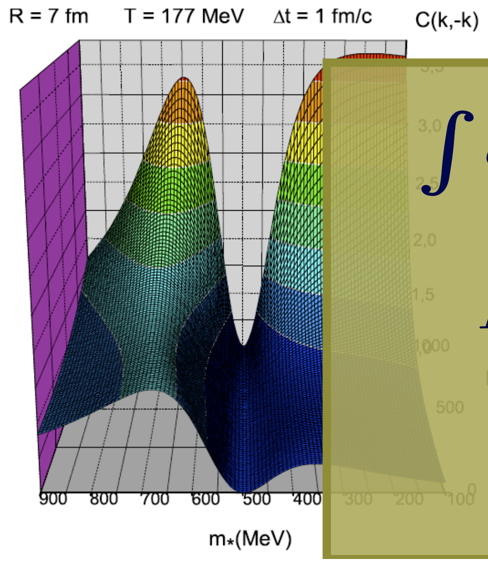
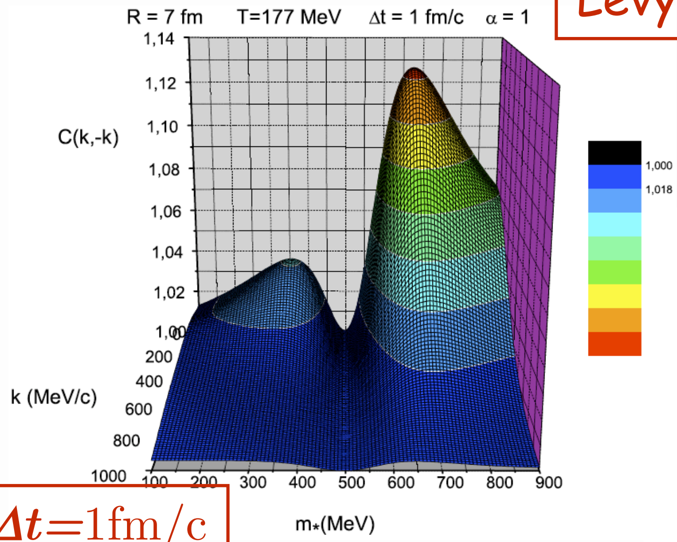
Investigating signal sensitivity to time Lorentzian vs Lévy Time distrib. – K^+K^-

Lorentzian Time Distr.

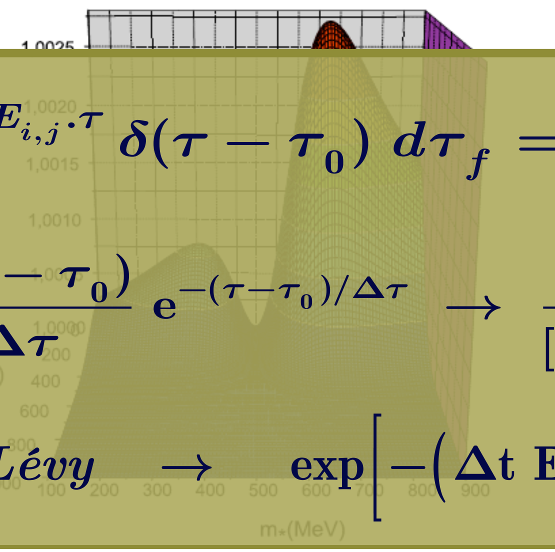




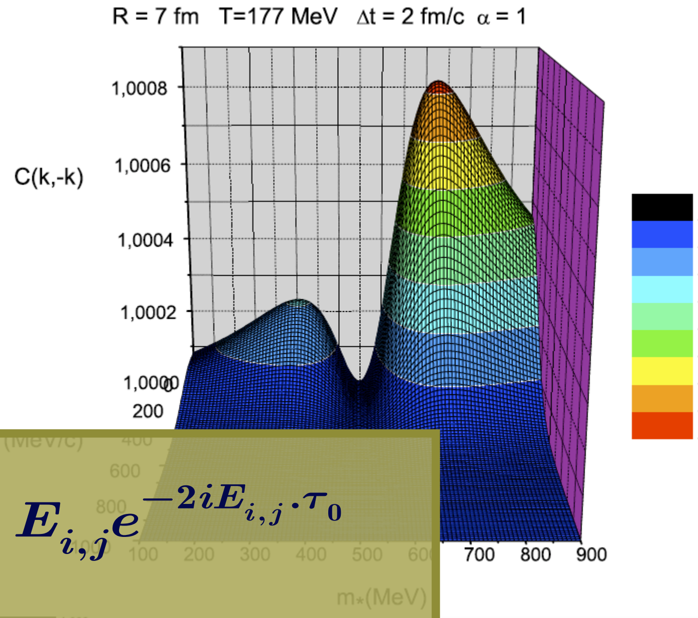
Lévy Time Distribution



$\Delta t = 1\mathrm{fm/c}$



$\Delta t = 2\mathrm{fm/c}$



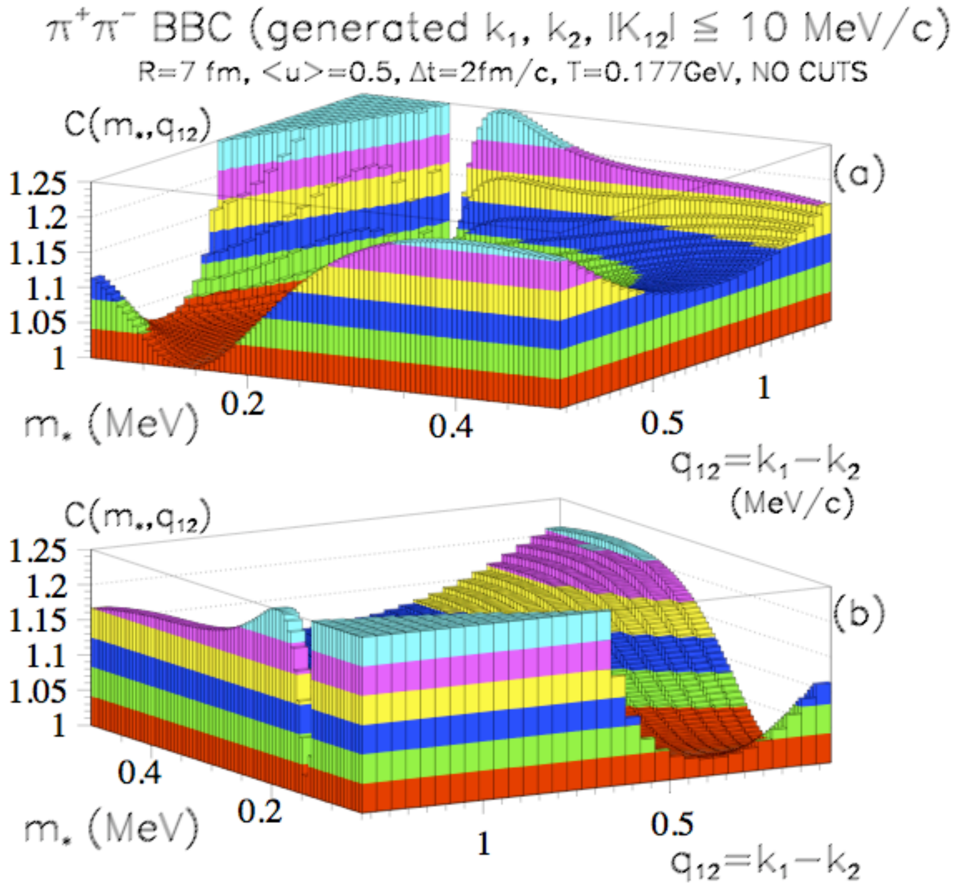
$$\int dt\, E_{i,j} e^{-2iE_{i,j}.\tau}\, \delta(\tau-\tau_0)\, d\tau_f = E_{i,j} e^{-2iE_{i,j}.\tau_0}$$

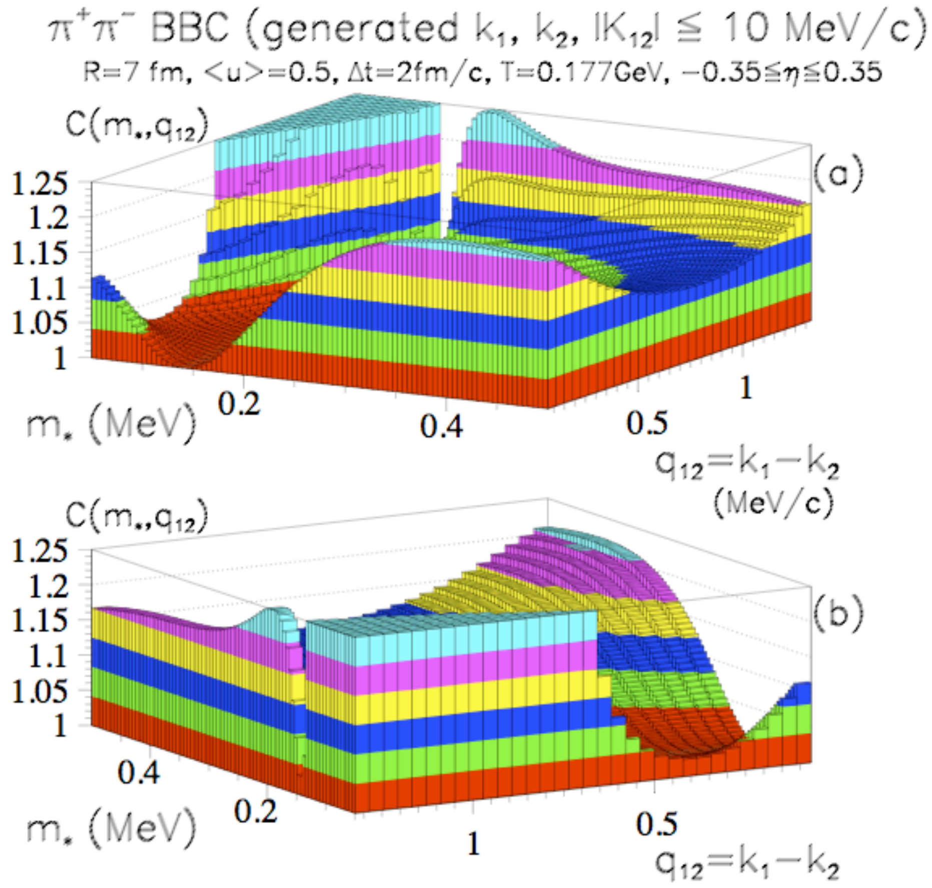
$$F(\tau) = \frac{\theta(\tau-\tau_0)}{\Delta\tau}\, e^{-(\tau-\tau_0)/\Delta\tau} \rightarrow \frac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$$

$$\text{Lévy} \rightarrow \exp\left[-\left(\Delta t\ E_{ij}\right)^{\alpha}\right]$$

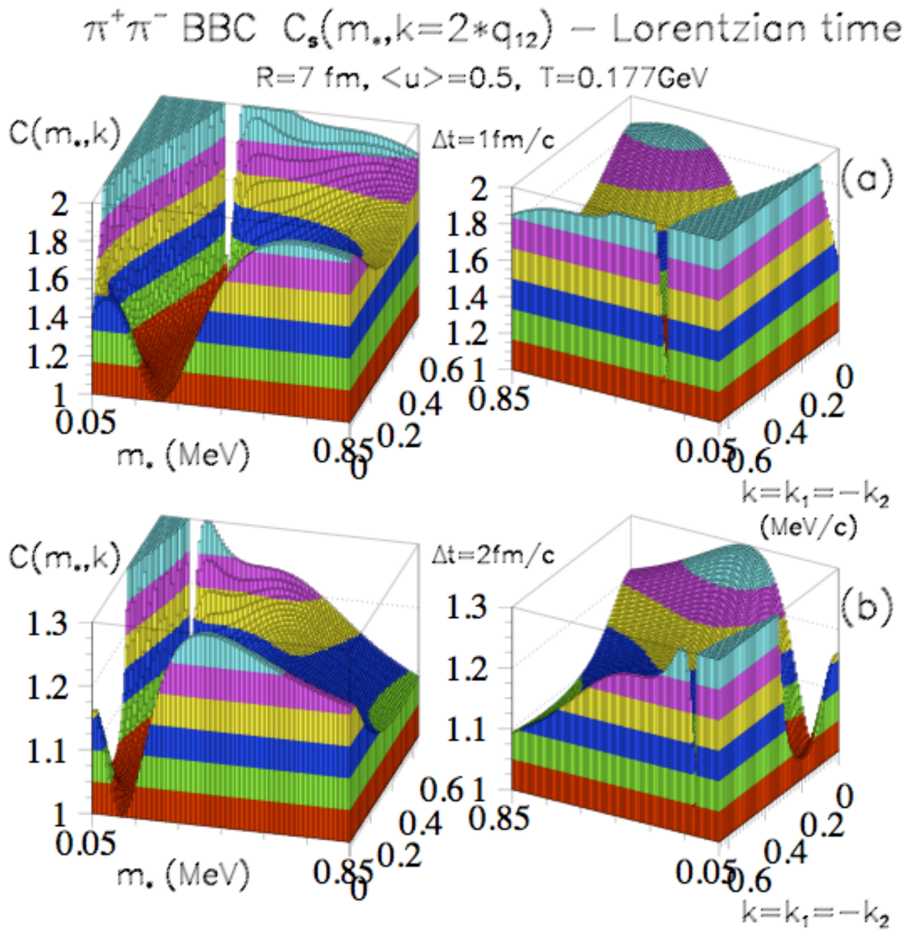
(in collaboration with T. Csörgő and M. Nagy)

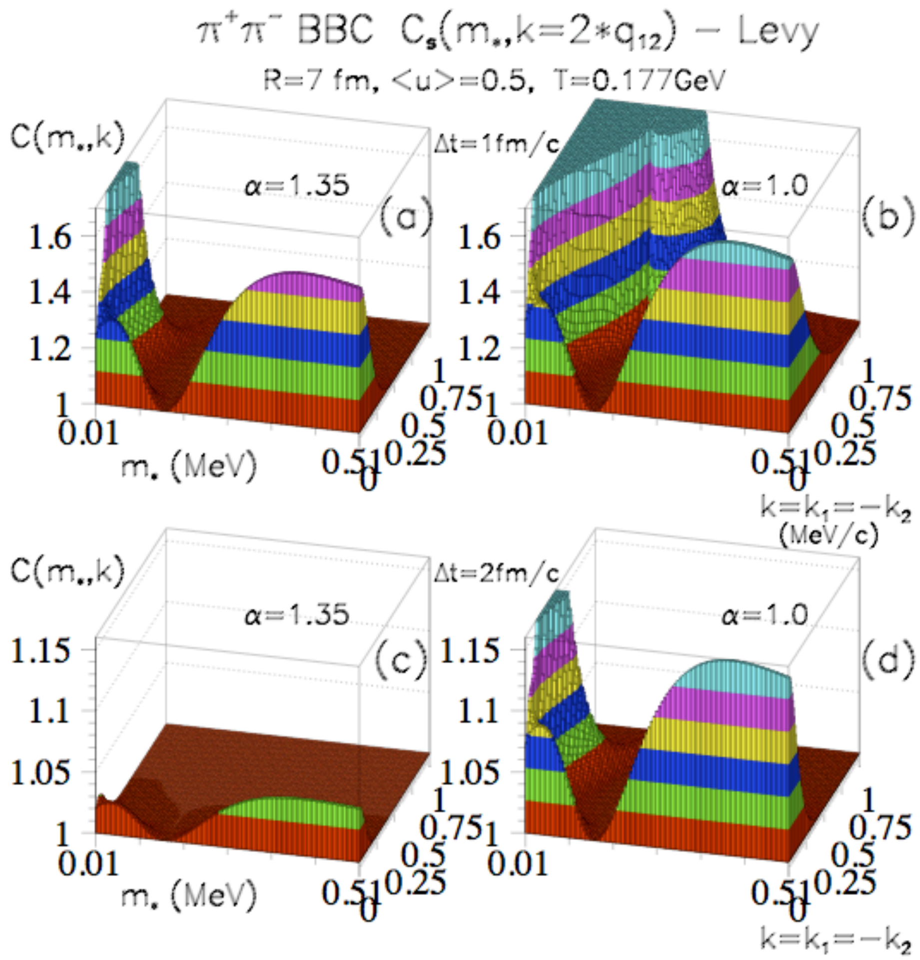
Pions - simulation





Investigating signal sensitivity to time -Lorentzian vs. Lévy distrib.-$\pi^0\pi^0$-$\pi^+\pi^-$





(under investigation, in collaboration with T. *Csörgő* and M. Nagy)

Correlation for strict BBC pairs

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- Momenta of the pair

$$\boxed{k_2 = -k_1 = k}$$

Remember:

$$2 * K^{\mu}_{i,j} = (k_i + k_j) \quad ; \quad q^{\mu}_{i,j} = (k_i - k_j)$$

- Back-to-Back correlation function

$$C_s(k,-k) = 1 + \left\{ |c_0||s_0| \left[R^3 + 2\left(\frac{R^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} \right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + \left(|c_0|^2 + |s_0|^2 \right) \left(\frac{R^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} + \frac{m^2 \langle u \rangle^2 k^2 / m_*^2}{(1 + \frac{m^2 \langle u \rangle^2}{m_* T}) 2T^2} \right) \right\}^{-2}$$

Formalism (bosons)

- Infinite medium

$$H = H_0 - \frac{1}{2}\int d\vec{x}\, d\vec{y}\, \phi(\vec{x})\, \delta M^2(\vec{x}-\vec{y})\phi(\vec{y}) \longrightarrow$$ In-medium Hamiltonian

$$H_0 = \frac{1}{2}\int d\vec{x}\,(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2) \longrightarrow$$ Asymptotic (free) Hamiltonian, in the rest frame of matter

- Scalar field $\phi(x)$ → quasi-particles propagating with momentum-dependent medium-modified effective mass, m_*, related to the vacuum mass, m, by

$$\boxed{m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)}$$

- Consequently:

Ω_k → frequency of the in-medium mode with momentum $\vec{k}$

$$\boxed{\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)}$$

Formalism (fermions)

$$H = H_0 + H_I \quad ; \quad H_0 = \int d\vec{x} : \bar{\psi}(x)(-i\vec{\gamma}.\vec{\nabla} + M)\psi(x):$$

$$\psi(x) = \frac{1}{V} \sum_{\lambda,\lambda',\vec{k}} (u_{\lambda,\vec{k}} a_{\lambda,\vec{k}} + v_{\lambda',-\vec{k}} a^{\dagger}_{\lambda',-\vec{k}}) e^{i\vec{k}.\vec{x}}$$

$$\boxed{\langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^{\dagger}_{k_1} a_{k_1} \rangle \langle a^{\dagger}_{k_2} a_{k_2} \rangle - \langle a^{\dagger}_{k_1} a_{k_2} \rangle \langle a^{\dagger}_{k_2} a_{k_1} \rangle + \langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle}$$

- System described by quasi-particles → medium effects taken into account through self-energy function
- For spin-1/2 particles under mean fields in a many body system:

$$\boxed{\sum{}^{s} + \gamma^0 \sum{}^{0} + \gamma^i \sum{}^{i}}$$ → **to be determined by detailed calculation**

- Σ^s → notation: $\Sigma^s(k) = \Delta M(k)$
- Σ^1 → very small → neglected
- Σ^0 → weakly-dependent on momentum→totally thermalized medium: $\boxed{\mu_* = \mu - \Sigma^0}$ → (results for net barion number)
- Hamiltoniana H_1 → describes a system of quasi-particles with mass- dependent momentum $m_* = m - \Delta M(k)$

bBBC & fBBC - formalism summary

- Bosonic BBC

$$\boxed{c_k = \cosh[f_k]} \; ; \; \boxed{s_k = \sinh[f_k]}$$

$$\begin{cases} a^\dagger{}_k = c_k\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k\, b_k + s^*_{-k}\, b^\dagger{}_{-k_1} \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \dfrac{1}{2}\log\left(\dfrac{\omega_k}{\Omega_k}\right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

- Fermionic BBC

$$\boxed{c_k = \cos[f_k]} \; ; \; \boxed{s_k = \sin[f_k]}$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}^\dagger_{\lambda',-k} \end{pmatrix} = \begin{pmatrix} c_k & \dfrac{f_k}{|f_k|} s_k A \\ -\dfrac{f_k^*}{|f_k|} s_k^* A^\dagger & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}^\dagger_{\lambda',-k} \end{pmatrix}$$

$$A = [\chi^\dagger_\lambda (\sigma.\hat{k}) \tilde{\chi}_{\lambda'}] \; ; \; A^\dagger = [\tilde{\chi}^\dagger_{\lambda'} (\sigma.\hat{k})^\dagger \chi_\lambda]$$

$$\begin{cases} \tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} \; ; \; \hat{k} = \vec{k}/|\vec{k}| \\ \rightarrow \text{ is a Pauli spinor} \\ \tan(2f_k) = -\dfrac{|k|\Delta M(k)}{\omega_k^2 - \Delta M(k) M} \\ m_*(k) = m - \Delta M(k) \\ \omega_k^2 = m^2 + \vec{k}^2 \; ; \; \Omega_k^2 = m_*^2 + \vec{k}^2 \end{cases}$$