How the top-quark mass affects electro-weak vacuum stability

Sven-Olaf Moch

Universität Hamburg & DESY, Zeuthen

Forum on (Meta-)Stability of the electroweak vacuum, Hamburg, June 04, 2013

Fate of the universe

Higgs boson mass Atlas & CMS coll. ‘13

$$m_H = 125.6 \pm 0.3\,\mathrm{GeV}$$

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Bound on from vacuum stability

$$m_H \geq 129.2\,\mathrm{GeV}$$

Fate of the universe

Higgs boson too light ? Are we doomed ?

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WORLD

Scientists studying so-called subatomic 'God particle' say there will be an universe-ending 'catastrophe'

The end of the universe won't come for tens of billions of years, but when it does happen it will destroy everything, according to researchers studying the Higgs boson particle. "If you use all the physics that we know now and you do what you think is a straightforward calculation, it's bad news," theoretical physicist Joseph Lykken said Monday.

Comments (24)

REUTERS

TUESDAY, FEBRUARY 19, 2013, 11:44 AM

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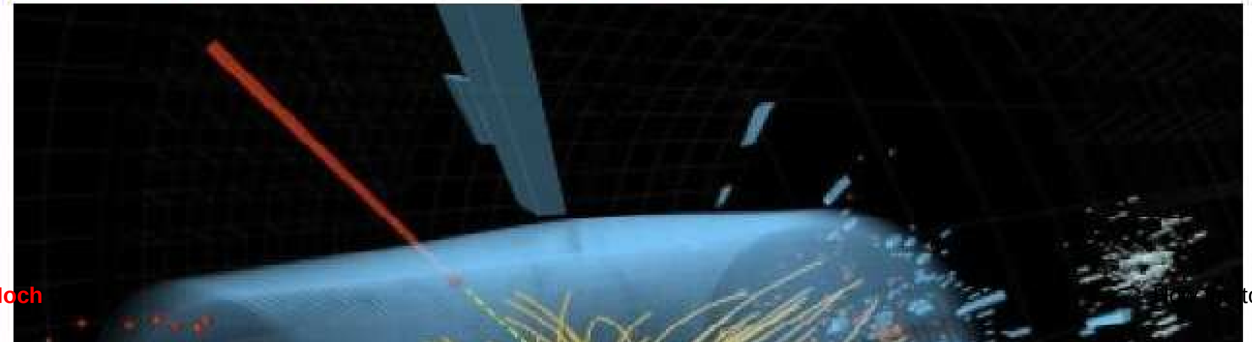

Fate of the universe

Well, ... [check the fine print]

$$m_H \geq 129.2\,\mathrm{GeV} + 1.8 \times \left(\frac{m_t - 173.2\,\mathrm{GeV}}{0.9\,\mathrm{GeV}} \right) + \ldots$$

Top quark mass

Experimental result CDF & D0 coll. 1305.3929

$$m_t = 173.20 \pm 0.87\,\mathrm{GeV}$$

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Which top quark mass has this value ?

$$? = 173.20 \pm 0.87\,\mathrm{GeV}$$

Introduction

Classical mechanics

- Mass is defined as product of density and volume of matter
  - classical concept

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  Newton

PHILOSOPHIÆ NATURALIS

PRINCIPIA MATHEMATICA.

DEFINITIONES.

DEFINITIO I.

Quantitas materiæ est mensura ejusdem orta ex illius densitate et magnitudine conjunctim.

AER densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine corporis vel massæ in sequentibus passim intelligo. Innotescit ea per corporis cujusque pondus: Nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.

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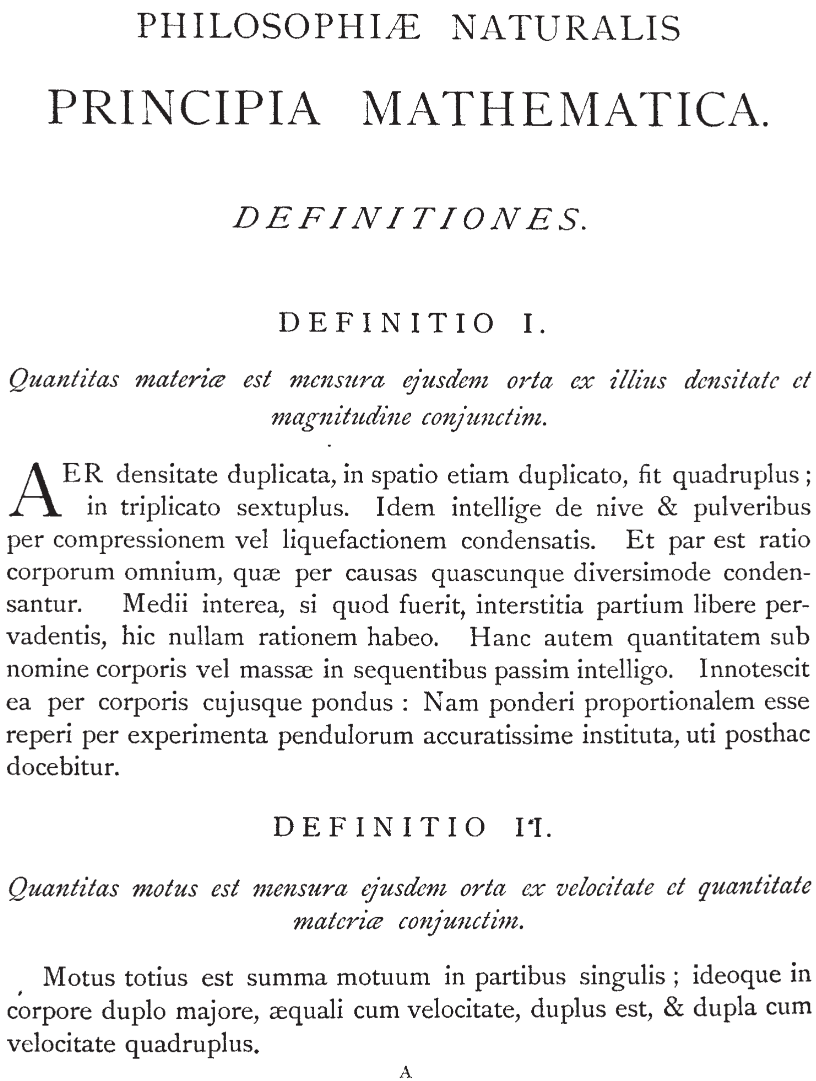

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Atomic theory

- Mass is conserved Lavoisier
- Mass of body is sum of mass of its constituents
  $M(X) = N_A m_a(X)$ Avogadro

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Special relativity

- Equivalence principle
  $E = mc^2$ Einstein

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Heavy-quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a \right)_{ij} A^a_\mu$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

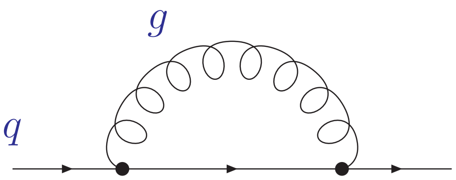
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Heavy-quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV
    Bauer, Bali, Pineda '11

Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ perturbation theory

Running quark mass

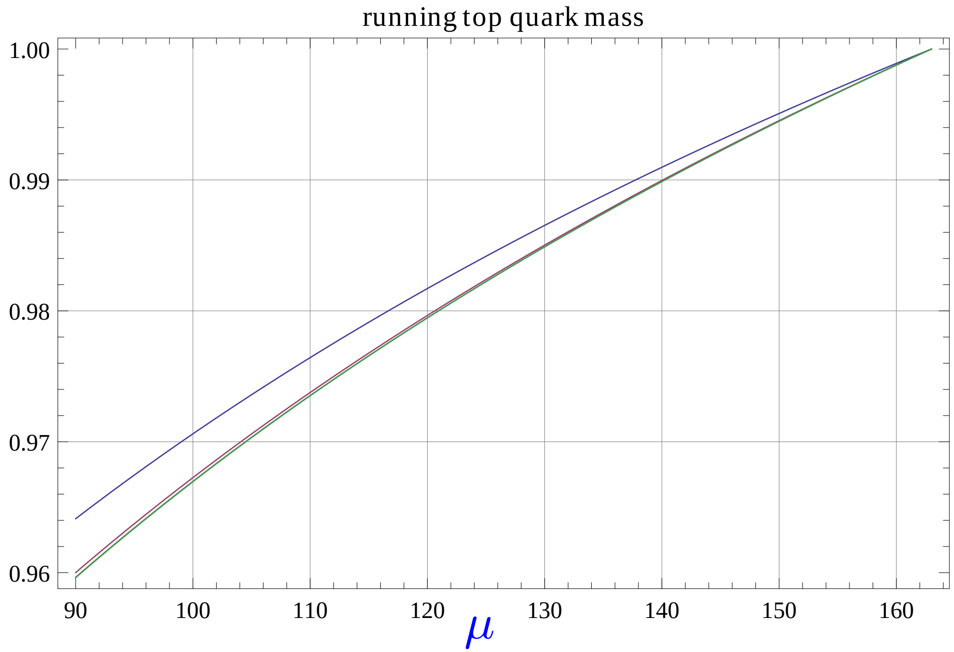
Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops

    Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

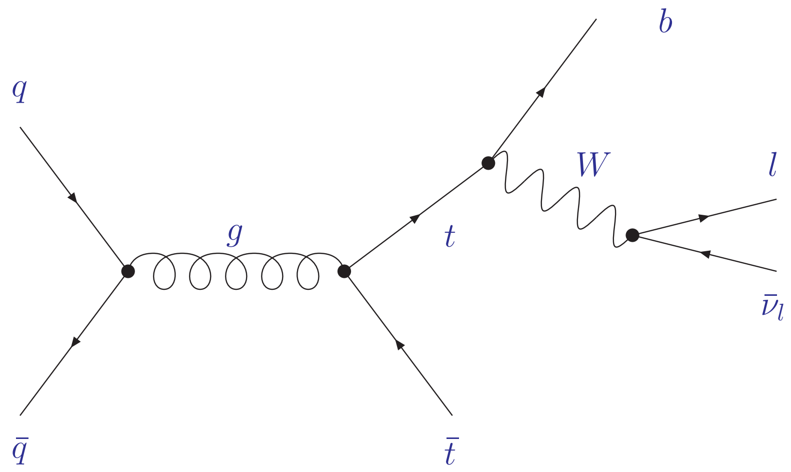
$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) = \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



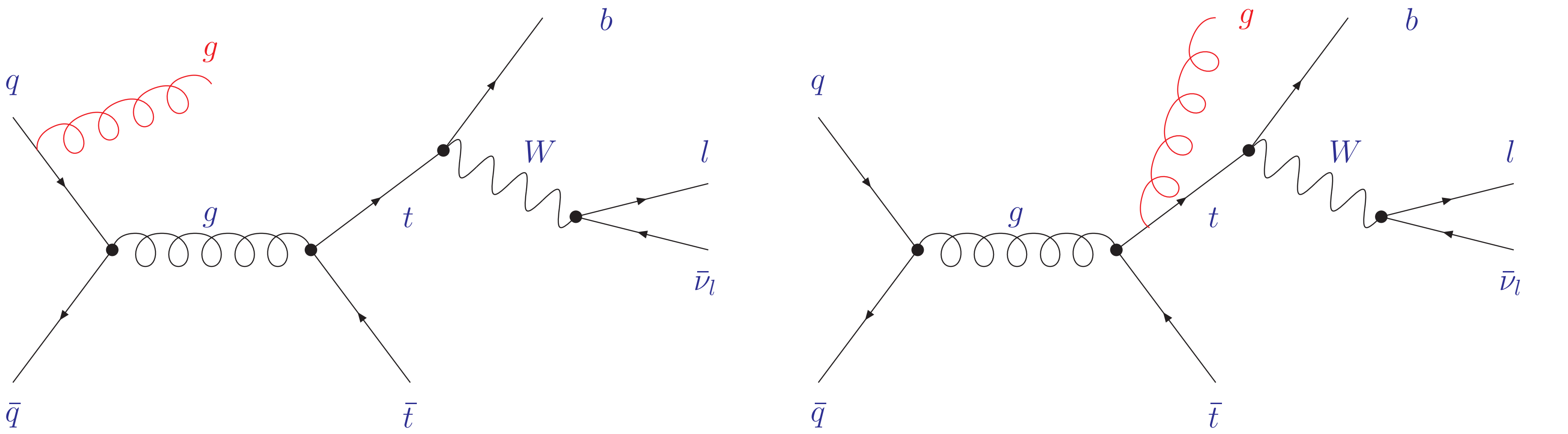
Hard scattering process

- Born process ($q\bar{q}$-channel) with leptonic decay $t \to bl\bar{\nu}_l$



Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration $\rightarrow$ infrared divergences (soft/collinear singularities)

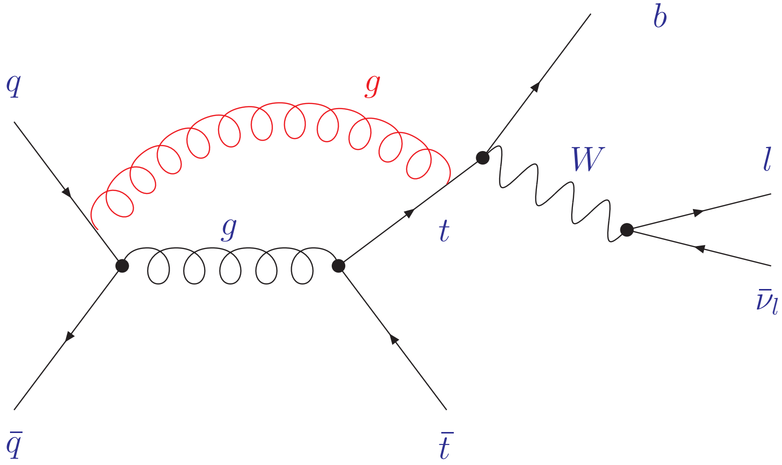


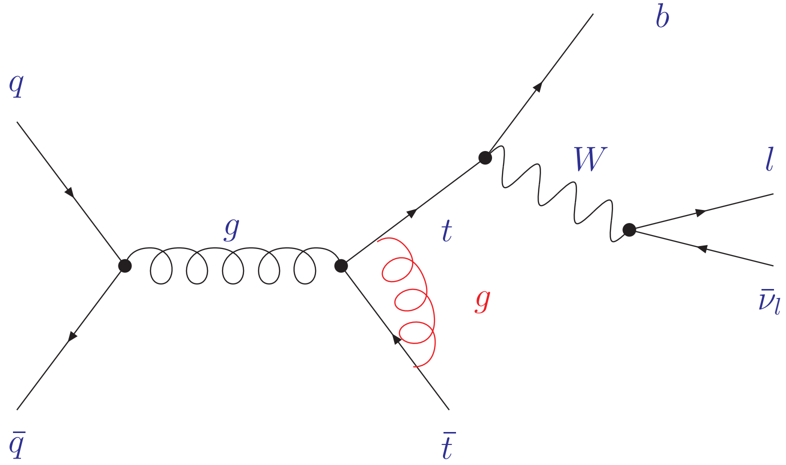
- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off Q_0
  - leading logarithmic accuracy

$$\Delta\left(Q^2, Q_0^2\right) \;=\; \exp\left(-C_F\,\frac{\alpha_s}{2\pi}\,\ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions





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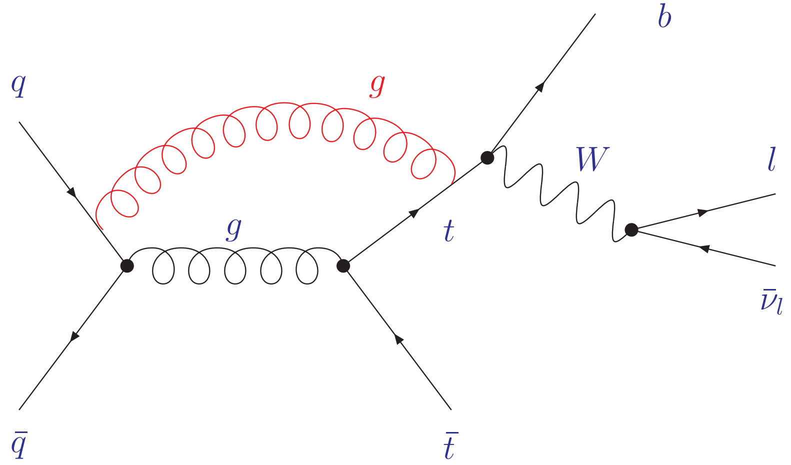
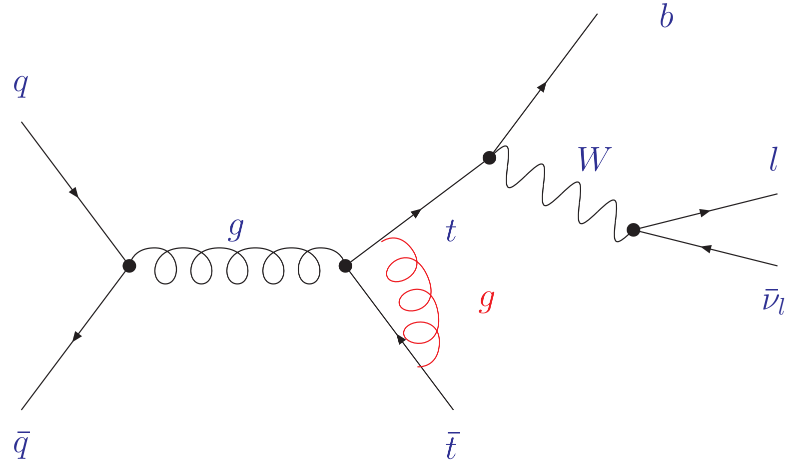
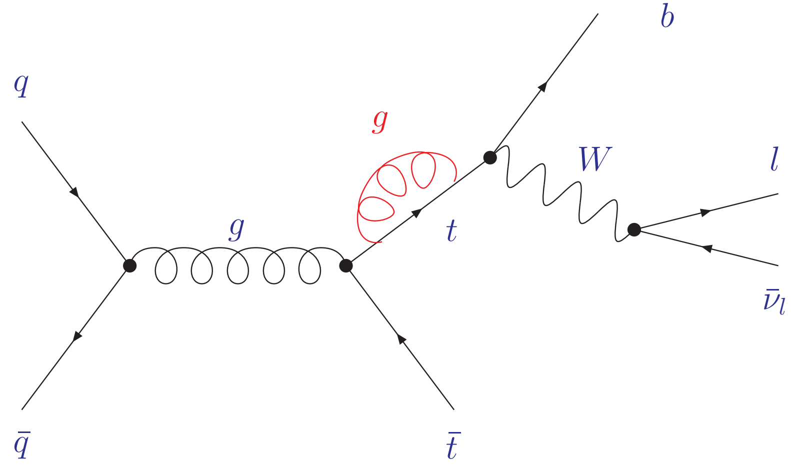


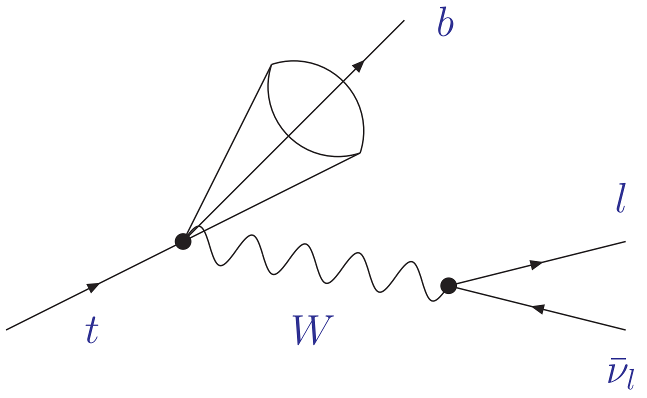


- Mass renormalization from self-energy corrections to top quark



Current methods

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$



Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - no NLO corrections applied

Matrix element method

- Event-by-event likelihood for kinematic configurations arising from events of a given top mass.
  - tree level matrix elements only
  - combinatorics of assignment of jets to top quarks

Tevatron combination

- Error budget in Tevatron determination
  CDF & D0 coll. 1207.1069
  - lepton+jets channel with matrix element method
- *Modeling signal* encompasses all perturbative uncertainties
  - radiative corrections (initial/final)
  - higher order QCD corrections
  - ...
- Uncertainties too optimistic
  $\Delta m_t \simeq 150 \ldots 250$ MeV
- Contradicts lattice bound
  $\Delta m_t \geq 200$ MeV
  (if interpreted as pole mass)

TABLE VIII: Individual components of uncertainty on CDF and D0 m_t measurements in the lepton+jets channel for Run II data [26, 27].

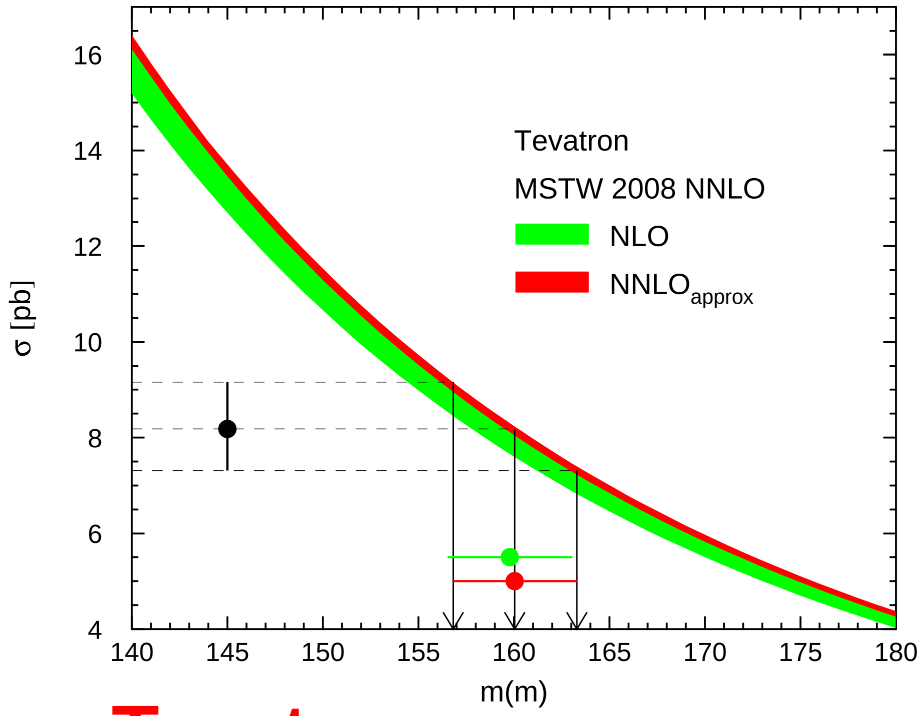
Systematic Source	Uncertainty [GeV] CDF (5.6 fb^{-1}) $m_t = 173.00$ GeV	Uncertainty [GeV] D0 (3.6 fb^{-1}) $m_t = 174.94$ GeV
DETECTOR RESPONSE		
Jet energy scale		
Light-jet response (1)	0.41	n/a
Light-jet response (2)	0.01	0.63
Out-of-cone correction	0.27	n/a
Model for b jets	0.23	0.07
Semileptonic b decay	0.16	0.04
b-jet hadronization	0.16	0.06
Response to $b/q/g$ jets	0.13	0.26
In-situ light-jet calibration	0.58	0.46
Jet modeling	0.00	0.36
Jet energy resolution	0.00	0.24
Jet identification	0.00	0.26
Lepton modeling	0.14	0.18
MODELING SIGNAL		
Signal modeling	0.56	0.77
Parton distribution functions	0.14	0.24
Quark annihilation fraction	0.03	n/a
Initial and final-state radiation	0.15	0.26
Higher-order QCD corrections	n/a	0.25
Jet hadronization and underlying event	0.25	0.58
Color reconnection	0.37	0.28
Multiple interactions model	0.10	0.05
MODELING BACKGROUND		
Background from theory	0.27	0.19
Higher-order correction for heavy flavor	0.03	0.07
Factorization scale for W+jets	0.07	0.16
Normalization to predicted cross sections	0.25	0.07
Distribution for background	0.07	0.03
Background based on data	0.06	0.23
Normalization to data	0.00	0.06
Trigger modeling	0.00	0.06
b-tagging modeling	0.00	0.10
Signal fraction for calibration	n/a	0.10
Impact of multijet background on the calibration	n/a	0.14
METHOD OF MASS EXTRACTION		
Calibration method	0.10	0.16
STATISTICAL UNCERTAINTY	0.65	0.83
UNCERTAINTY ON JET ENERGY SCALE	0.80	0.83
OTHER SYSTEMATIC UNCERTAINTIES	0.67	0.94
TOTAL UNCERTAINTY	1.23	1.50

Alternative methods

- Top mass from leptonic decay: m_{lb} distribution
- Top mass from jet rates
- Top mass from total cross section

Top mass from total cross section

- Total top quark cross section as function of $\overline{MS}$ mass
  Langenfeld, S.M., Uwer '09



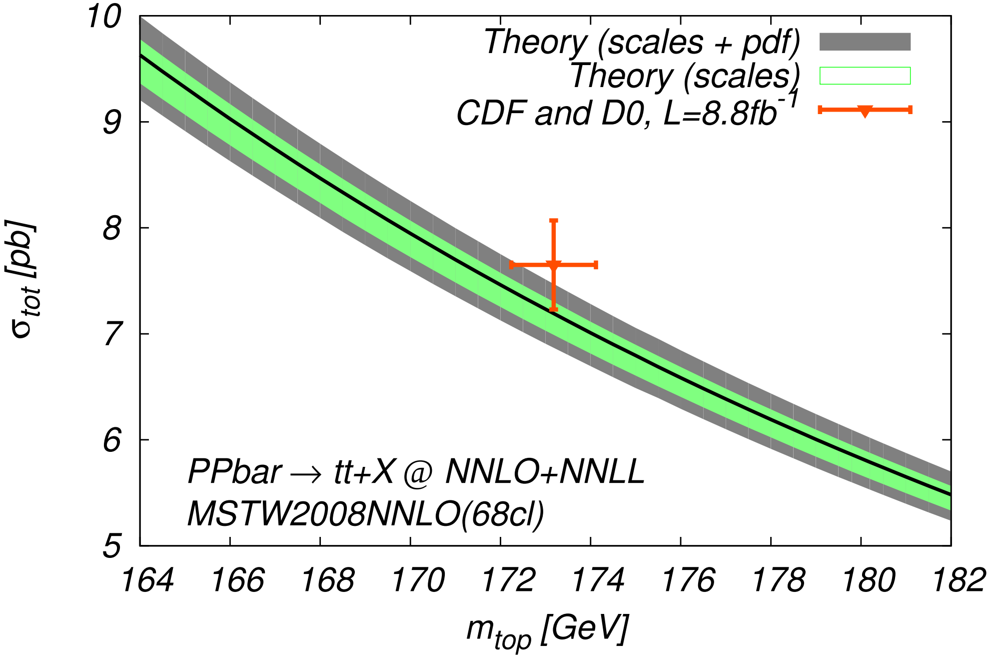
Tevatron

Top-quark pair-production

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13

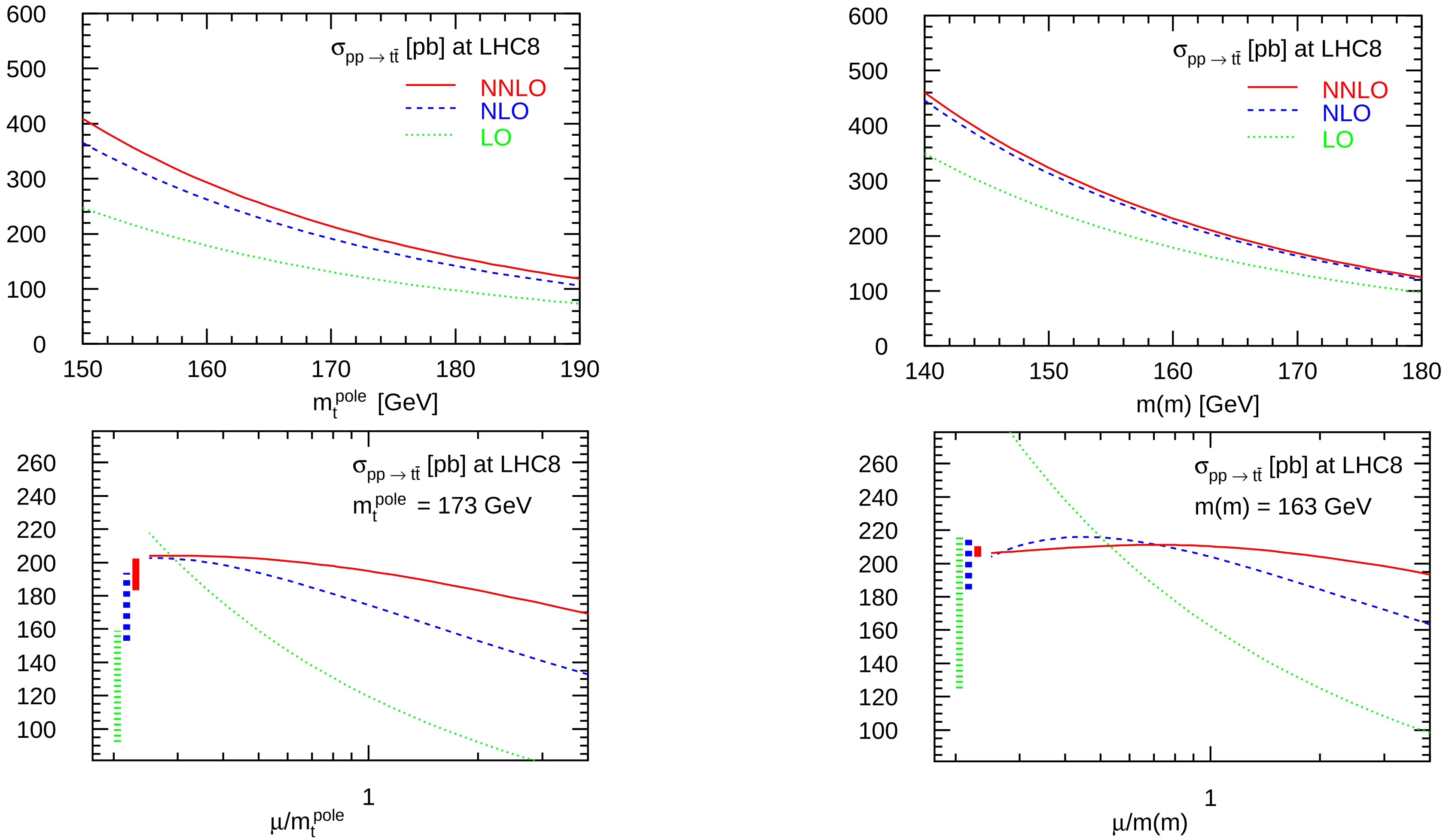
- Illustration of mass dependence for Tevatron



- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with running mass

Comparison pole mass vs. $\overline{MS}$ mass



- good apparent convergence of perturbative expansion
- small theoretical uncertainity form scale variation

Tevatron

- Determine top quark mass from Tevatron cross section data
  - $\sigma_{t\bar{t}} = 7.56^{+\,0.63}_{-\,0.56}$ pb D0 coll. arXiv:1105.5384
  - $\sigma_{t\bar{t}} = 7.50\,^{+0.48}_{-0.48}$ pb CDF coll. CDF-note-9913
- Fit of m_t for individual PDFs
  - parton luminosity at Tevatron driven by $q\bar{q}$
  - $\overline{MS}$-scheme for $m_t^{\overline{\mathrm{MS}}}(m_t)$, then scheme transformation to pole mass m_t^{pole} at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\mathrm{MS}}}(m_t)$	162.0 $^{+2.3}_{-2.3}\,^{+0.7}_{-0.6}$	163.5 $^{+2.2}_{-2.2}\,^{+0.6}_{-0.2}$	163.2 $^{+2.2}_{-2.2}\,^{+0.7}_{-0.8}$	164.4 $^{+2.2}_{-2.2}\,^{+0.8}_{-0.2}$
m_t^{pole}	171.7 $^{+2.4}_{-2.4}\,^{+0.7}_{-0.6}$	173.3 $^{+2.3}_{-2.3}\,^{+0.7}_{-0.2}$	173.4 $^{+2.3}_{-2.3}\,^{+0.8}_{-0.8}$	174.9 $^{+2.3}_{-2.3}\,^{+0.8}_{-0.3}$
(m_t^{pole})	(169.9 $^{+2.4}_{-2.4}\,^{+1.2}_{-1.6}$)	(171.4 $^{+2.3}_{-2.3}\,^{+1.2}_{-1.1}$)	(171.3 $^{+2.3}_{-2.3}\,^{+1.4}_{-1.8}$)	(172.7 $^{+2.3}_{-2.3}\,^{+1.4}_{-1.2}$)

- Good consistency within errors for $m_t^{\mathrm{pole}} = 171.7 \ldots 174.9$ at NNLO

The fine print

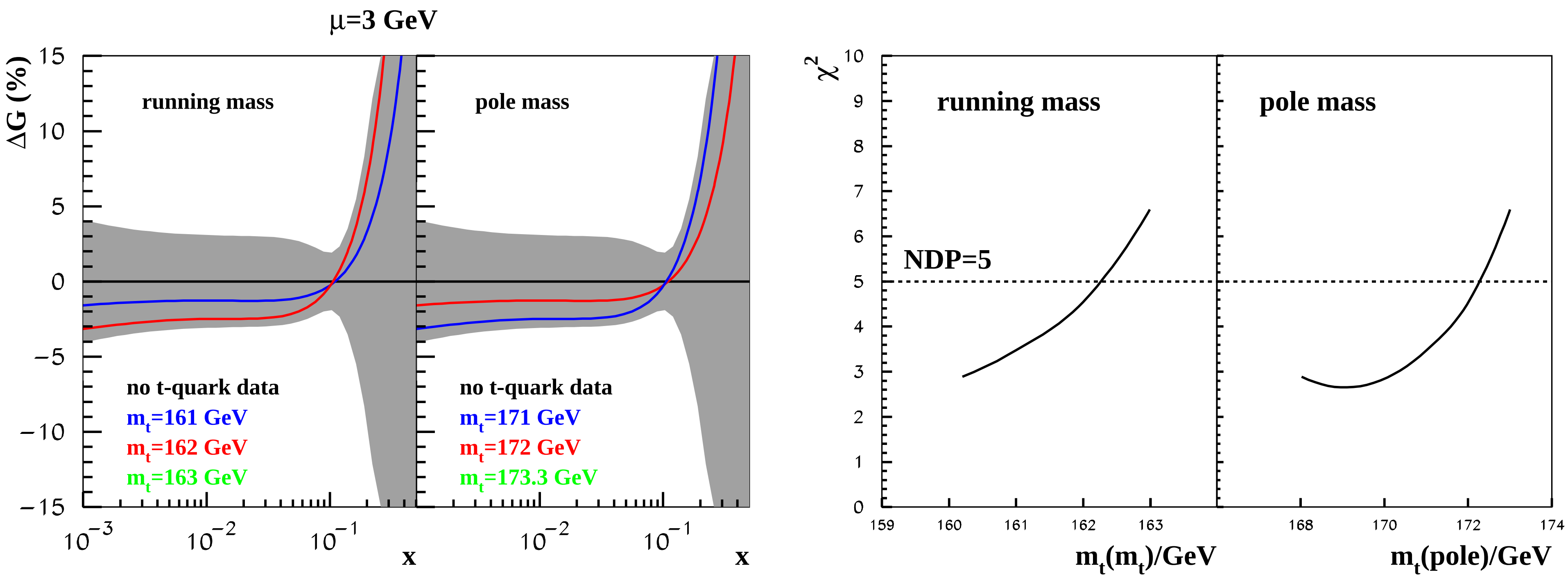
- Intrinsic limitation of sensitivity in total cross section

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$, gluon PDF
  $\sigma_{t\bar{t}} \sim \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \dots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo ‘13

The fine print

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162$ GeV Alekhin, Blümlein, S.M. [in progress]



Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} \;=\; 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

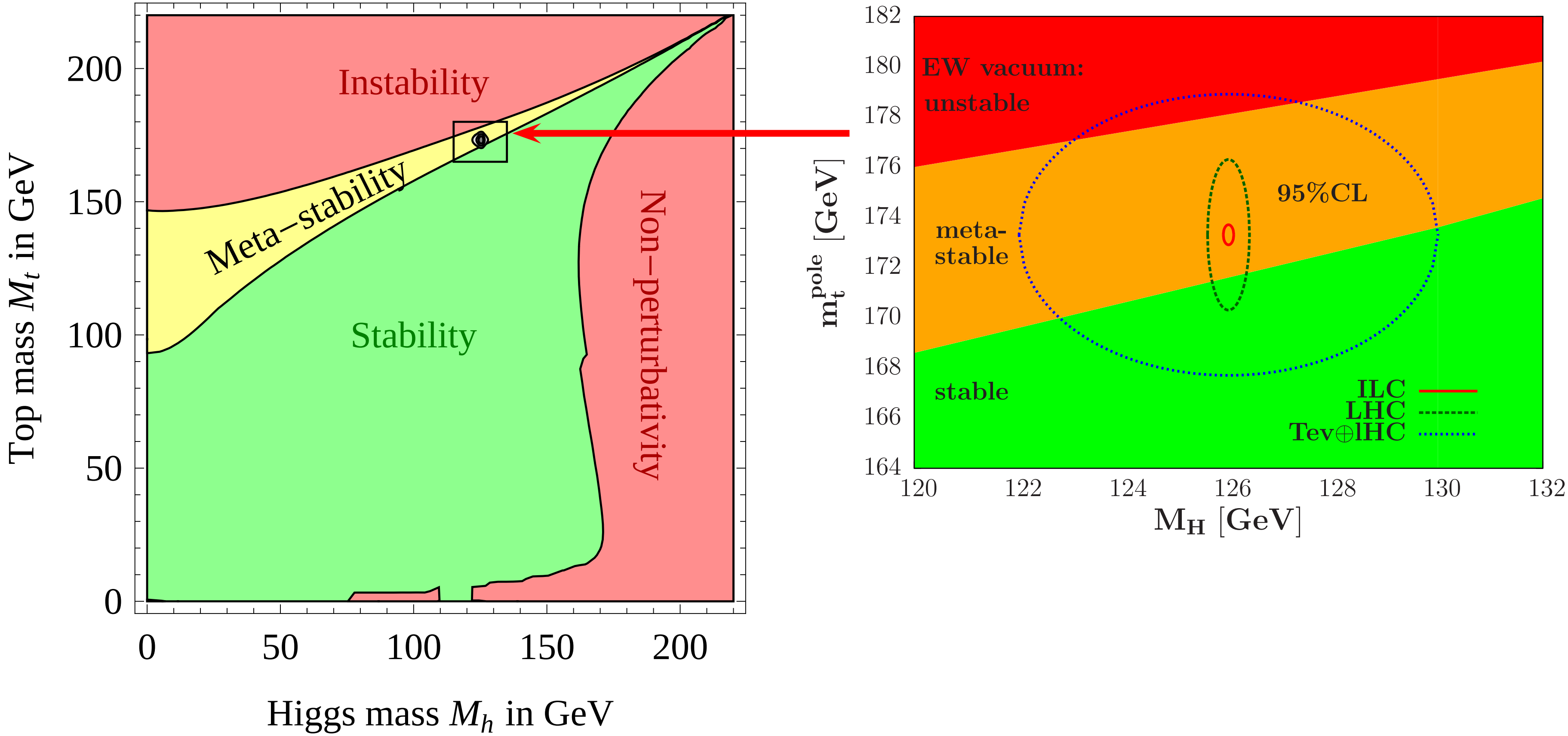
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7$ GeV implies in pole mass scheme $m_t^{\text{pole}} = 173.3 \pm 2.8$ GeV
  - good consistency of mass value between different PDF sets

Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 129.4 \pm 5.6$ GeV
  - "fate of universe" still undecided

Summary

Top quark mass

- On-shell scheme (pole mass) at NNLO in QCD

$$m_t = 173.20 \pm 0.87 \pm \mathcal{O}(\text{few})\,\text{GeV}$$

- Running mass ($\overline{\mathrm{MS}}$ scheme) at NNLO in QCD

$$m_t(m_t) = 163.3 \pm 2.7\text{GeV}$$

Summary

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition

Current measurements

- Kinematic reconstruction
  - very precise value, but only leading order/leading logarithm
  - lacking renormalization scheme definition
- $\overline{\mathrm{MS}}$ mass from total cross section
  - NNLO QCD determination available
  - uncertainty $\mathcal{O}(3)$ GeV from Tevatron analyses
  - LHC analyses affected by uncertainty in parton distributions, $\alpha_S(M_Z)$

Future challenge

- Study of new observables which meet all requirements
- Joint effort theory and experiment