Vacuum stability in the Standard Model at two loops

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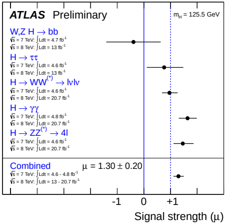
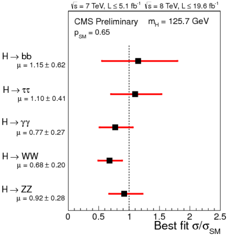
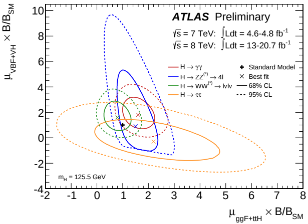
Outline

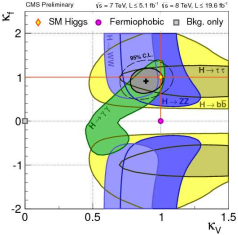
Introduction: experimental status quo

ATLAS	CMS
$(125.5 \pm 0.2\,^{+0.5}_{-0.6})$ GeV	$(125.7 \pm 0.3 \pm 0.3)$ GeV



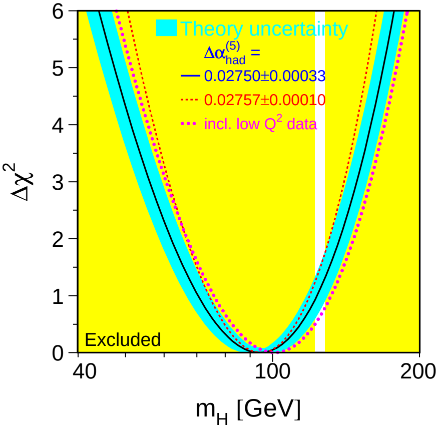


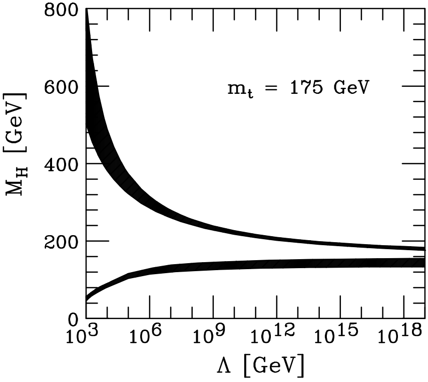




Higgs boson of mass $m_H = (125.6 \pm 0.3)$ GeV w/ SM properties

EW precision tests, triviality, vacuum stability





- $m_H = 125.6\ \mathrm{GeV}$ agrees w/ EW precision data.
- Triviality bound satisfied.
- How about vacuum stability bound?

Higgs potential: spontaneous symmetry breaking

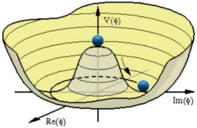
SM Higgs sector: complex scalar doublet Φ

$\mathscr{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(|\Phi|^2), \qquad V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$

Unbroken phase: $\mu^2 > 0$

Broken phase: $\mu^2 < 0$

After SSB and Higgs mechanism:



$$\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$\mathscr{L} = \frac{1}{2} \partial_\mu H \partial^\mu H - V(H), \qquad V = -\frac{\lambda v^4}{4} + \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

- $\left.\frac{\partial V}{\partial H}\right|_{H=0} = 0 \quad \rightsquigarrow \quad -\mu^2 = \lambda v^2 \equiv \frac{m_H^2}{2}$
- $\frac{gv}{2} \equiv m_W \quad \rightsquigarrow \quad v = 2^{-1/4} G_F^{-1/2} = 246.220\ \mathrm{GeV}$
- m_H is free parameter.

So far, bare fields and parameters.

Renormalizaton: RG evolution

Cosmological applications require reliable predictions over very large range of scales: $v \lesssim \mu \lesssim M_P$

Use $\overline{\mathrm{MS}}$ renromalization scheme: running couplings $\lambda(\mu^2), y_t(\mu^2), g_s(\mu^2), \ldots$

Two-step procedure: 1. RG evolution:

$$\mu^2 \frac{d\lambda(\mu^2)}{d\mu^2} = \beta_\lambda = \frac{1}{16\pi^2}(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4) + \cdots$$

$$\mu^2 \frac{dy_t(\mu^2)}{d\mu^2} = \beta_{y_t} = \frac{1}{16\pi^2} y_t \left(\frac{9}{4} y_t^2 - 4g_s^2\right) + \cdots$$

$$\mu^2 \frac{dg_s(\mu^2)}{d\mu^2} = \beta_{g_s} = \frac{1}{16\pi^2} g_s^3 \left(-\frac{11}{2} + \frac{n_f}{3}\right) + \cdots$$

$\beta_\lambda^{(3)}, \beta_{y_t}^{(3)}$	Chetyrkin, Zoller, JHEP06(2012)033; 04(2013)091
	Bednyakov *et al.*, PLB722(2013)336; arXiv:1303.4364
$\beta_{\ldots}^{(3)}, \beta_{g_s,y_t}^{(3)}$	Mihaila *et al.*, PRL108(2012)151602; PRD86(2012)096008
$\beta_{g_s}^{(3)}$	Tarasov *et al.*, PLB93(1980)429

Threshold corrections

2. Matching at $\mu_0 = \mathcal{O}(v)$:

$$\lambda(\mu_0^2) = 2^{-1/2} G_F m_H^2 [1 + \delta_H^{(1)}(\mu_0^2) + \cdots]$$

$$\delta_H^{(1)}(\mu_0^2) = \frac{G_F m_H^2}{8\pi^2\sqrt{2}} \left[6 \ln \frac{\mu_0^2}{m_H^2} + \frac{25}{2} - \frac{3}{2}\pi\sqrt{3} + \mathcal{O}\left(\frac{m_Z^2}{m_H^2} \ln \frac{m_H^2}{m_Z^2}\right)\right]$$

Sirlin, Zucchini, NPB266(1986)389

$$y_t(\mu_0^2) = 2^{3/4} G_F^{1/2} m_t [1 + \delta_t^{(1)}(\mu_0^2) + \cdots]$$

$$\delta_t^{(1)}(\mu_0^2) = \frac{Q_t^2 \alpha + C_F \alpha_s(\mu_0^2)}{4\pi} \left(-3 \ln \frac{\mu_0^2}{m_t^2} - 4\right) + \frac{G_F m_t^2}{8\pi^2\sqrt{2}} \left[\frac{9}{2} \ln \frac{\mu_0^2}{m_t^2} + \frac{11}{2} - 2\pi \frac{m_H}{m_t} + \mathcal{O}\left(\frac{m_H^2}{m_t^2} \ln \frac{m_t^2}{m_H^2}\right)\right]$$

Hempfling, BK, PRD51(1995)1386

$\delta_H^{(\alpha\alpha_s)}, \delta_t^{(\alpha\alpha_s)}$ Bezrukov, Kalmykov, BK, Shaposhnikov, JHEP10(2012)140

$\delta_H^{(y_t^4)}, \delta_t^{(y_t^4)}$ Degrassi *et al.*, JHEP08(2012)098

Triviality and vaccum stability in a nutshell

Recall

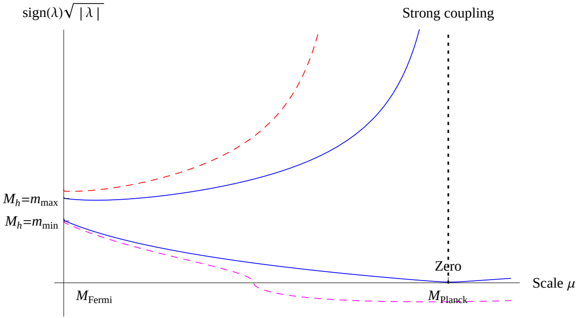
$$\mu^2 \frac{d\lambda(\mu^2)}{d\mu^2} = \frac{1}{16\pi^2}(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4) + \cdots$$

$$\mu^2 \frac{dy_t(\mu^2)}{d\mu^2} = \frac{1}{16\pi^2} y_t \left(\frac{9}{4} y_t^2 - 4g_s^2\right) + \cdots$$

$$\lambda(m_H^2) = 0.130 \times \left(\frac{m_H}{125.6\ \mathrm{GeV}}\right)^2, \quad y_t(m_t^2) = 0.993 \times \frac{m_t}{172.9\ \mathrm{GeV}}, \quad g_s(m_Z^2) = 1.220$$

- Triviality bound: Maiani *et al.*, NPB136(1979)115
  If $m_H > M_{\mathrm{max}}$, then $\lambda(\mu^2) \to \infty$ for $\mu \to \mu_{\mathrm{Landau}}$.
  (For $m_t \ll m_H$, $\mu_{\mathrm{Landau}} \approx m_H \exp \frac{2\pi^2}{3\lambda(m_H^2)} = 1.2 \times 10^{24}\ \mathrm{GeV}$)
  $\rightsquigarrow \lambda(\mu_0^2) = 0$ *trivial*
- Vacuum stability bound: Lindner, ZPC31(1986)295
  If $m_H < M_{\mathrm{min}}$, then $\lambda(\mu^2) < 0$ for $\mu > \mu_{\mathrm{stab}}$.
  $\rightsquigarrow$ *Decay of universe*

Vaccum stability condition



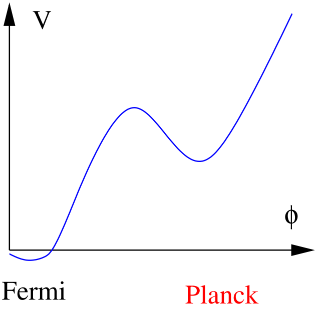
Determine M_{min} so that for $m_H = M_{\text{min}}$

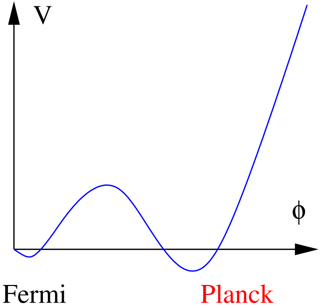
$$\lambda(\mu_{\text{stab}}) = 0 = \beta_\lambda(\lambda(\mu_{\text{stab}}))$$

at some given $\mu_{\text{stab}} \gg v$, *e.g.* $\mu_{\text{stab}} = M_P$.

Caveat: M_{min} is (slightly) scheme dependent. $\rightsquigarrow$ theoretical uncertainty

Effective potential





Determine $\widetilde{M}_{\text{min}}$ so that for $m_H = \widetilde{M}_{\text{min}}$

$$V(\Phi_{\text{SM}}) = V(\Phi_1), \qquad V'(\Phi_{\text{SM}}) = V'(\Phi_1)$$

at some given $\Phi_1 \gg \Phi_{\text{SM}}$, *e.g.* $\Phi_1 = \Phi_P$.

NB: Numerically, $\widetilde{M}_{\text{min}} - M_{\text{min}} = \mathcal{O}(0.1\ \text{GeV})$, *i.e.* well within theoretical uncertainty.

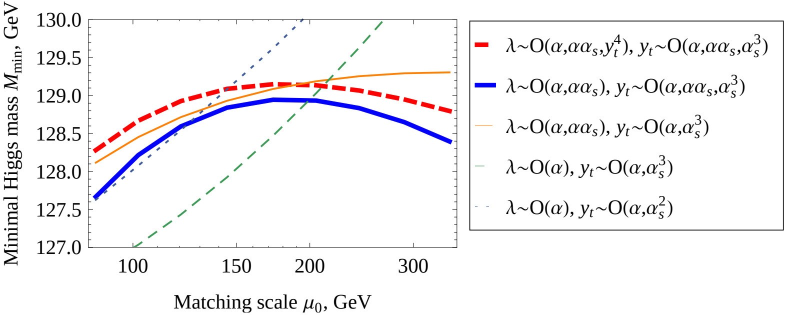
Numerical analysis Bezrukov, Kalmykov, BK, Shaposhnikov, JHEP10(2012)140

3-loop evolution / 2-loop matching yields:

$$M_{\rm min} = \left[128.95 + \frac{M_t - 172.9\ \mathrm{GeV}}{1.1\ \mathrm{GeV}} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56\right]\ \mathrm{GeV}$$

Source of uncertainty	Nature of estimate	$\Delta_{\rm theor} M_{\rm min}$ [GeV]
3-loop matching λ	sensitivity to μ_0	1.0
3-loop matching y_t	sensitivity to μ_0	0.2
4-loop α_s to y_t	educated guess [Kataev, Kim]	0.4
confinement, y_t	educated guess $\sim \Lambda_{\rm QCD}$	0.5
4-loop running $M_W \to M_P$	educated guess	< 0.2
total uncertainty	sum of squares	1.2
total uncertainty	linear sum	2.3

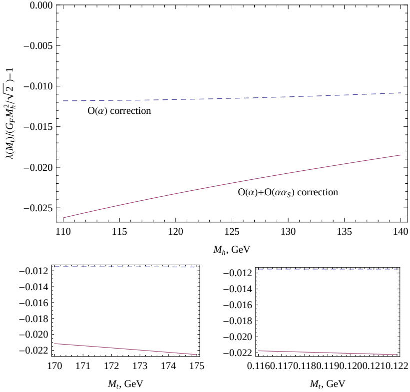
Anatomy



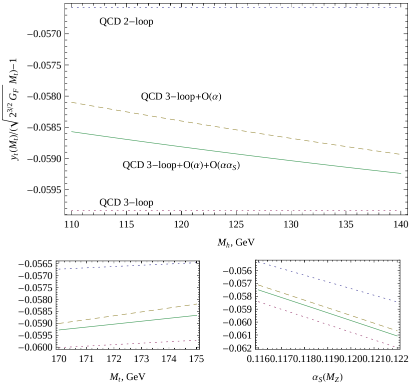
Contribution	$\Delta M_{\rm min}$ [GeV]
3-loop beta functions	-0.23
$\delta y_t \propto O(\alpha_s^3)$	-1.15
$\delta y_t \propto O(\alpha\alpha_s)$	-0.13
$\delta\lambda \propto O(\alpha\alpha_s)$	0.62
$\delta y_t, \delta\lambda \propto O(y_t^4)$	0.2

$\mathcal{O}(\alpha\alpha_s)$ threshold corrections

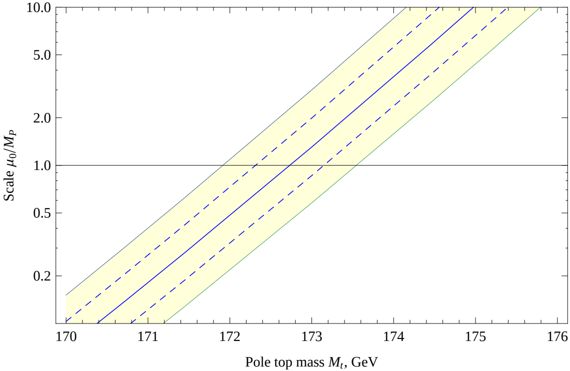
$\delta_H(\mu_0^2)$



$\delta_t(\mu_0^2)$

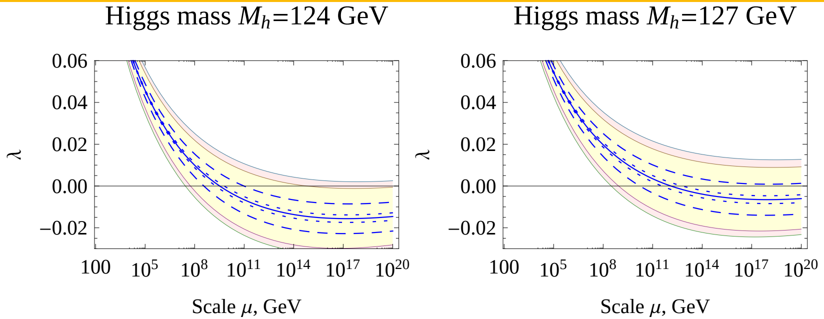


Reduction of fundamental scales

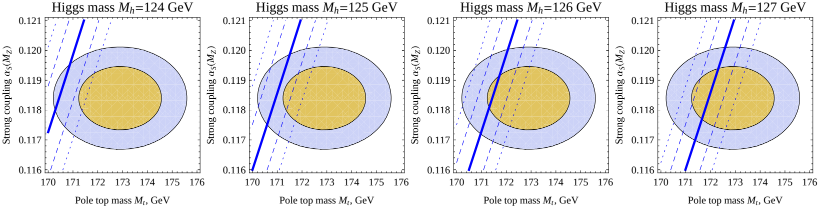


- $\mu_{\rm stab} = 2.9 \times 10^{18}$ GeV stable w.r.t. variations of $m_t = (172.9 \pm 1.1)$ GeV, $\alpha_s^{(5)}(m_Z^2) = 0.1184 \pm 0.0007$ (dashed), and $m_Z < \mu_0 < m_t$ (yellow).
- $\mu_{\rm stab} \approx M_P = 2.44 \times 10^{18}$ GeV
- Electroweak scale is determined by Planck scale physics!

SM stable way up to M_P?

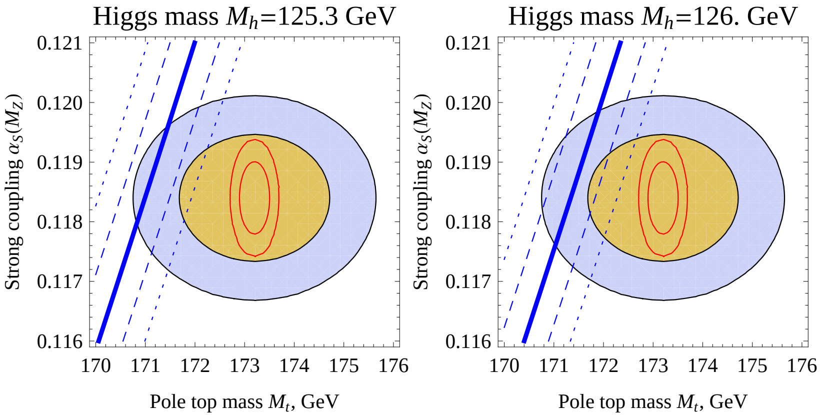


δm_t (dashed); $\delta\alpha_s^{(5)}(m_Z^2)$ (dotted); $+\delta_{\rm theor}$ linear (pink), quadratical (yellow)



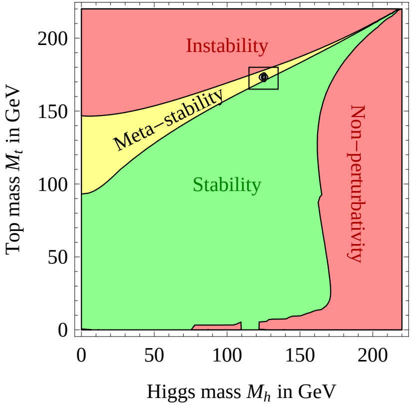
$1\sigma_{\rm exp}$ and $2\sigma_{\rm exp}$ contours; $\delta_{\rm theor}$ linear (dotted), quadratical (dashed)

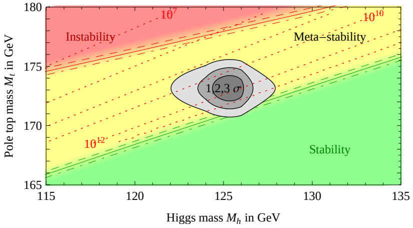
LC as top and Higgs factory



Anticipated $1\sigma_{\rm exp}$ and $2\sigma_{\rm exp}$ contours (red)

Vacuum metastability



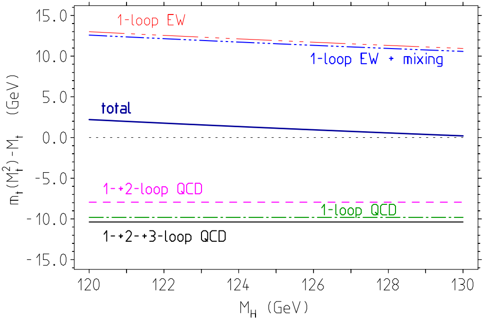


Degrassi *et al.*, JHEP08(2012)098

EW vacuum is metastable / unstable, if its lifetime overshoots / undershoots that of the universe.

Outlook: pole mass m_t

- PDG value $M_X(t \to X) = (172.9 \pm 1.1)$ GeV is not pole mass m_t, but just parameter in MC programs w/o RC to partonic cross sections.
- Rigorous determination of $\overline{\text{MS}}$ mass $\overline{m}_t(\mu^2)$ from $\sigma_{\text{tot}}(p\bar{p}, pp \to t\bar{t} + X)$ ⇝ Moch's talk
- $\overline{m}_t(\mu^2) - m_t$ receives large EW RC from tadpole contributions.



Jegerlehner, Kalmykov, BK, PLB722(2013)123

BSM physics

- Depending on future precision measurements of m_H, m_t, α_s and higher-loop RC calculations, SM may be stable way up to M_P.
- Reduction of m_t by 1.6 GeV [*cf.* $m_t - m_{\bar{t}} = (-1.4 \pm 2.0)$ GeV, $\Gamma_t = (2.0^{+0.7}_{-0.6})$ GeV] $\rightsquigarrow$ $M_{\min} = m_H = 125.6$ GeV
- BSM physics still necessary to solve open problems, *e.g.* smallness of neutrino masses, strong CP problem, dark matter, baryon asymmetry of universe, *etc.* $\rightsquigarrow$ Westphal's talk