Monte Carlos — Part III

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GK Mass Spectra Symmetry
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Outline

- Part I — Basics
  - Introduction
  - Monte Carlo techniques
- Part II — Perturbative physics
  - Hard scattering
  - Parton showers
- Part III — Merging/Matching
  - Matrix element corrections
  - Merging multiple tree level MEs with parton showers
  - Matching NLO and parton showers
- Part IV — Non–perturbative physics
  - Hadronization
  - Hadronic decays
  - Comparison to data
- Part V — Multiple Partonic Interactions
  - Minimum Bias/Underlying Event in data
  - Modelling

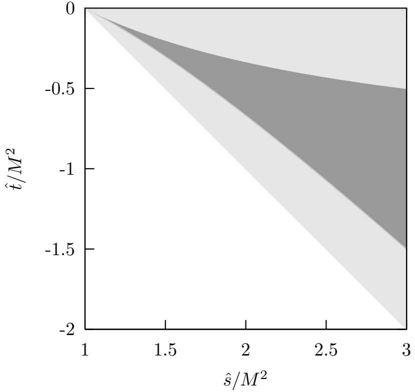
Outline Part III

- Matrix element corrections
- Merging tree level matrix elements
  - Inclusive/exclusive
  - CKKW weights
- Matching NLO and parton showers
  - NLO calculation
  - MC@NLO
  - POWHEG
  - Matching with dipole showers

Matrix element corrections

Hard ME correction (e.g. in DY)

- Light: collinear/soft regions.
- Dark: Dead region, filled with extra hard emissions — not accessable by parton shower.
- To be complemented by soft matrix element corrections.

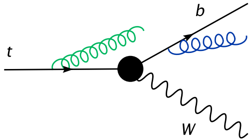


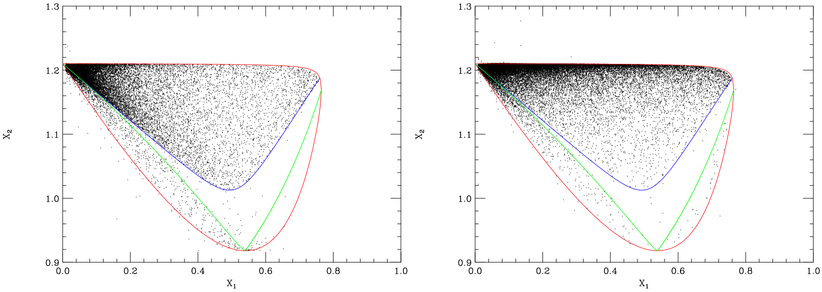
Also for $V^* \to q\bar{q}$, t–decay (2.0) $gg \to h^0$ (2.2),

Simplest matching.

Soft ME Corrections in t Decays

Smooth coverage of soft gluon region from both parton showers.

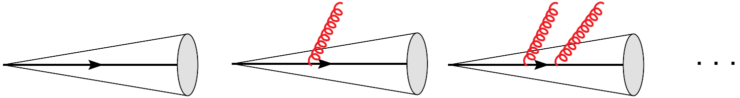




w/o and with soft ME correction

Matching tree level ME and parton showers

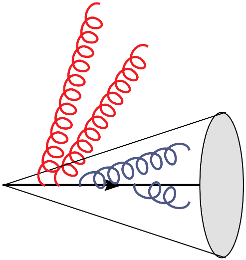
- Problem: have multiple tree level MEs for $X+0,1,\ldots n$ jets.



- Jets well separated and *inclusive*.
- Merge this into one exclusive multijet sample.
- Idea: use Sudakov form factors to disallow "+ anything softer" (which is normally inside an inclusive ME).
- That's done in the CKKW(-L) approach. Catani, Krauss, Kuhn, Webber, JHEP 0111:063,2001, Krauss JHEP 0208:015,2002, L. Lönnblad, JHEP 0205:046,2002, Gleisberg, Höche, Winter, Schälicke, Schumann.
- Alternative: MLM matching. M.L. Mangano
- Systematic study and comparison of implementations. J. Alwall, S. Höche, F. Krauss, N. Lavesson, L. L"onnblad, F. Maltoni, M.L. Mangano, M. Moretti, C.G. Papadopoulos, F. Piccinini, S. Schumann, M. Treccani, J. Winter, M. Worek, EPJC53:473-500,2008.

Matching tree level ME and parton showers

- Separates ME and parton shower at intermediate scale Q_{ini}.
- Parton shower fills region below Q_{ini}.
- All emissions resolvable above Q_0.



Merges ME and parton shower at scale Q_{ini}.

CKKW weights

Starting point: Sudakov form factors

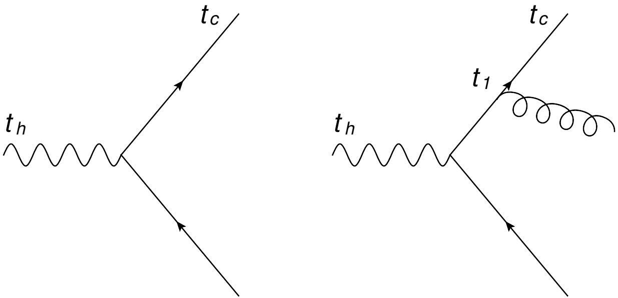
$$\Delta_q(T,t) = \exp\left\{-\int_t^T \frac{\mathrm{d}t'}{t'} \int_{z_-(t')}^{z_+(t')} \frac{\alpha_S(t,z)}{2\pi} P_{qq}(z)\right\}$$
$$= \exp\left\{-\int_t^T \mathrm{d}t\, \Gamma_q(T,t)\right\} ,$$
$$\Delta_g(T,t) = \exp\left\{-\int_t^T \mathrm{d}t\, \Gamma_g(T,t) + \Gamma_f(t)\right\}$$

and integrated splitting functions

$$\Gamma_q(t_h,t) = \frac{C_F}{\pi} \frac{\alpha_S(t)}{t} \left[\frac{1}{2}\ln\frac{t_h}{t} - \frac{3}{4}\right] ,$$
$$\Gamma_g(t_h,t) = \frac{C_A}{\pi} \frac{\alpha_S(t)}{t} \left[\frac{1}{2}\ln\frac{t_h}{t} - \frac{11}{12}\right] ,$$
$$\Gamma_f(t_h,t) = \frac{T_R\, n_F}{3\pi} \frac{\alpha_S(t)}{t} .$$

CKKW weights

Sudakov FF = no branching probability.
Get probability for a jet configuration to survive exclusivly:
e.g.



$$w_2 = \left[\Delta_q(t_h, t_c)\right]^2 \ .$$

CKKW weights

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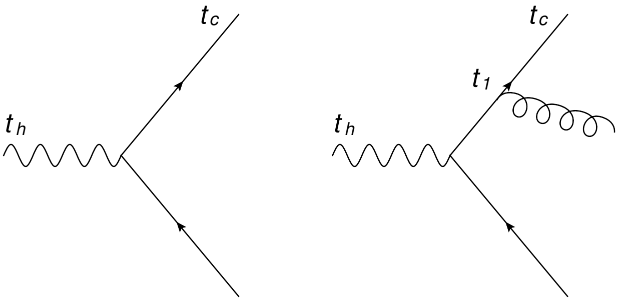
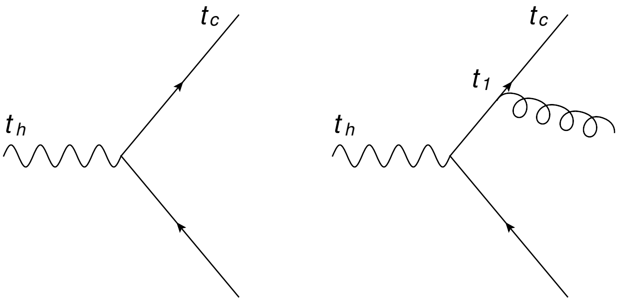
Get probability for a jet configuration to survive exclusivly:
e.g.



$$w_3 = \Delta_q(t_h, t_c)\Delta_q(t_h, t_1)\Gamma_q(t_h, t_1)\Delta_q(t_1, t_c)\Delta_g(t_1', t_c)$$

CKKW weights

Sudakov FF = no branching probability.
Get probability for a jet configuration to survive exclusivly:
e.g.



$$w_3 = \Delta_q(t_h, t_c)\Delta_q(t_h, t_1)\Gamma_q(t_h, t_1)\Delta_q(t_1, t_c)\Delta_g(t_1', t_c)$$
$$= \left[\Delta_q(t_h, t_c)\right]^2 \Gamma_q(t_h, t_1)\Delta_g(t_1', t_c)\ .$$

CKKW algorithm

Results in CKKW algorithm:

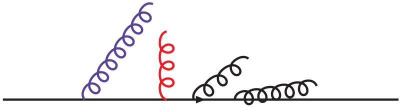
1. Pick multiplicity according to $\sigma_n / \sum_k \sigma_k$.
2. Pick phase space point $\rightarrow w_h$.
3. Jet algorithm $\rightarrow$ shower history $\rightarrow$ nodal scales t_i.
4. Reweighting factor $w_\alpha = \frac{\alpha_S(t_1)}{\alpha_S(t_{\text{ini}})} \cdots \frac{\alpha_S(t_n)}{\alpha_S(t_{\text{ini}})}$.
5. Assign Sudakov weight $\rightarrow w_S$.
6. Final event weight $w = w_h w_S w_\alpha$.

Parton shower $t_c \rightarrow t_0$.

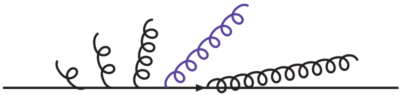
-L variant: Apply trial showers in step 5.

Matching tree level ME and PS — trouble?

Hard emission, to be complemented by parton shower.



$p_\perp$ ordered shower. Angular ordering from additional vetos.



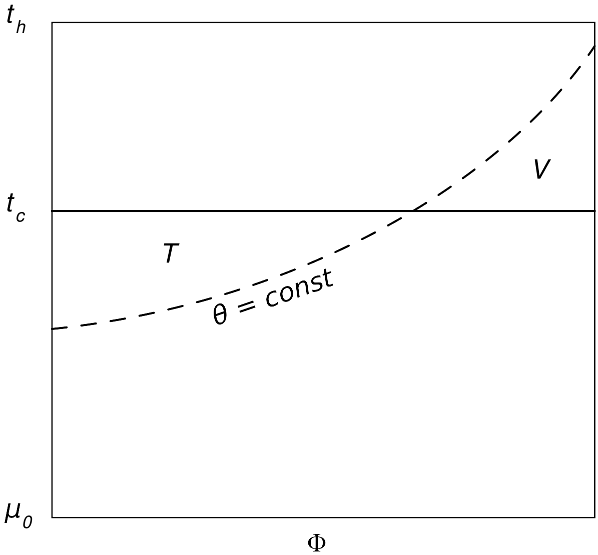
Angular ordered shower. Some softer emissions before hardest one.

Potential holes in phase space $\longrightarrow$ *truncated showers*.

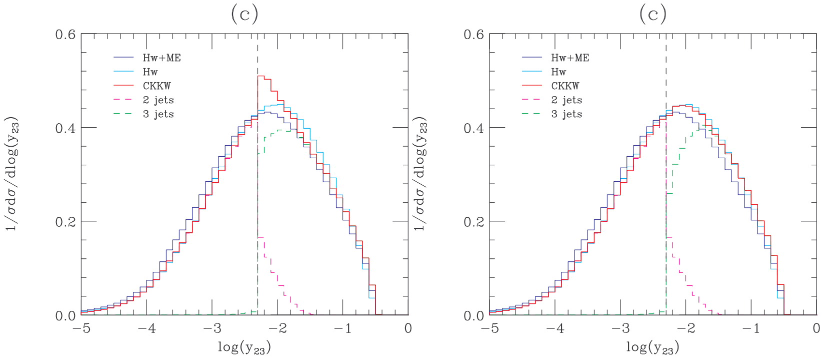
S. Höche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905:053,2009.

K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.

Matching tree level ME and PS — trouble?



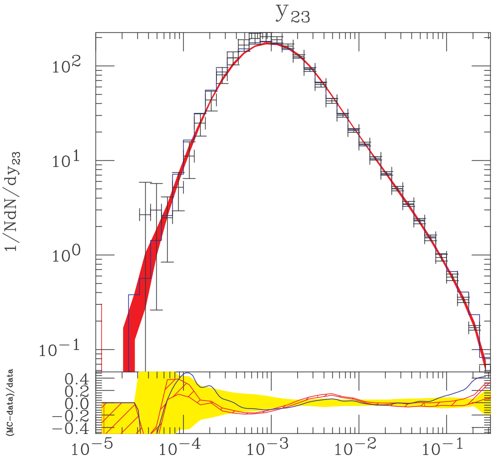
Matching tree level ME and parton showers



K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.

Parton level merging for illustration.
Instabilities at Q_{ini} removed.

Matching tree level ME and parton showers

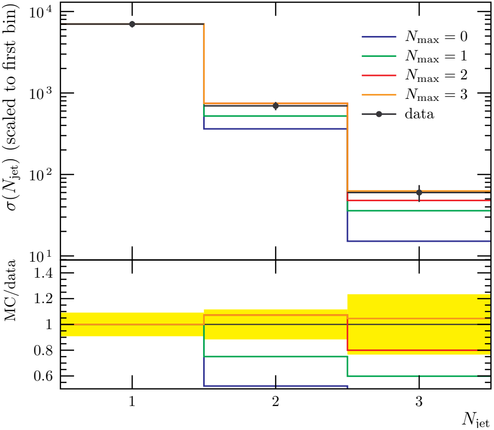


K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.

Hadron level with matching uncertainty band vs OPAL.

Matching tree level ME and parton showers

Sherpa CS shower, matched with $Z^0 + N_{\text{jet}}$ jets vs CDF data.



S. Höche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905:053,2009.

Reached remarkable stability wrt Q_{ini} variation.

Matching NLO computations and parton showers

The problem:
Consider n and $n+1$ body ME

$$|M_n^{(0)}|^2 \qquad 2\mathrm{Re} M_n^{(0)} M_n^{(1)} \qquad |M_{n+1}^{(0)}|^2 \ .$$

- Both present in NLO as Born+Virtual and Real ME.
- Parton shower adds $n+1$ st emission as well (accurate to leading log accuracy).

$\Rightarrow$ Potential double counting!

Matching NLO computations and parton showers

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Two popular approaches:

- MC@NLO
- POWHEG

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
x = real emission phase space, Born, Observable, Real, Virtual.

$$\langle O \rangle_{\rm NLO} = BO(0) + VO(0) + \int_0^1 dx \, \frac{O(x)R(x)}{x},$$

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
x = real emission phase space, Born, Observable, Real, Virtual.

$$\langle O \rangle_{\mathrm{NLO}} = BO(0) + VO(0) + \int_0^1 dx \, \frac{O(x)R(x)}{x} ,$$

Add/subtract soft/collinear piece $A(x)$ ($\lim_{x\to 0} A(x) = R(x)$):

$$\langle O \rangle_{\mathrm{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx \, \frac{O(x)R(x) - O(0)A(x)}{x} ,$$

where

$$\bar{V} = V + \int_0^1 dx \, \frac{A(x)}{x} = \mathrm{IR\ finite} .$$

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \qquad .$$

From Born $\otimes$ zero/one parton shower emission:

$$\langle O \rangle_{\mathrm{PS}} = \int dx\, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu) \right]$$

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \approx 1 - \int dx \, \frac{P(x)}{x} \; .$$

From Born $\otimes$ zero/one parton shower emission:

$$\begin{aligned}\langle O \rangle_{\text{PS}} &= \int dx \, O(x) \left[B \Delta \delta(x) + B \frac{P(x)}{x} \Delta \Theta(x - \mu) \right] \\ &= B O(0) \left[1 - \int_\mu \frac{dx}{x} P(x) \right] + \int_\mu dx \, O(x) B \frac{P(x)}{x} \; .\end{aligned}$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \approx 1 - \int dx \frac{P(x)}{x} \ .$$

From Born $\otimes$ zero/one parton shower emission:

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Terms that contribute at $O(\alpha_S)$/NLO $\Rightarrow$ double counting.

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O \rangle_{\text{NLO}} = B O(0) + \bar{V} O(0) + \int_0^1 dx \, \frac{O(x) R(x) - O(0) A(x)}{x}$$

$$\langle O \rangle_{\text{PS}} = B O(0) \left[1 - \int_\mu \frac{dx}{x} P(x) \right] + \int_\mu dx \, O(x) B \frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\begin{aligned}\langle O\rangle_{\mathrm{NLO}}{}' =& BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x} \\ &+ \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}\end{aligned}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\rm NLO}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x}$$
$$+ \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}$$

Result ("MC@NLO master formula")

$$\langle O\rangle_{\rm MC@NLO} = O(0)\left[B + \bar{V} + \int_0^1 dx\, \frac{BP(x) - A(x)}{x}\right]$$
$$+ \int dx\, O(x) \frac{R(x) - BP(x)}{x}\,.$$

Note: $(O(0)B \otimes$ parton shower) adds back subtracted terms
$\Rightarrow$ NLO result is exactly reproduced after parton shower.

Matching MC and NLO

$$\langle O\rangle_{\text{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-BP(x)}{x}\;.$$

Observations/remarks:

- Events with n and $n+1$ legs are seperately finite. No cancellation of large weights.
- NLO result can be recovered strictly upon expansion in powers of α (with parton shower emission).
- Interface to MC program very well defined.
- Dropping $\mu \to 0$ is only a power correction.

Matching MC and NLO

$$\langle O \rangle_{\text{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{BP(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - BP(x)}{x} \, .$$

Three types of matching

1. MC@NLO (classic, Frixione and Webber).
2. Simpler: parton shower with $P(x) = A(x)/B$.
3. Or, also simpler, $P(x) = R(x)/B$.

Matching MC and NLO

$$\langle O \rangle_{\text{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{BP(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - BP(x)}{x} \, .$$

1. Classic MC@NLO (Frixione and Webber)
   - $A(x) =$ FKS subtraction terms
   - $P(x)$ and phase space specific for HERWIG.
   - Generic, calculate once and for all.
   - New for every process.

Matching MC and NLO

$$\langle O \rangle_{\text{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\,\frac{BP(x) - A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x) - BP(x)}{x}\,.$$

2. 'Custom' parton shower
e.g. with Catani–Seymour subtraction kernels

- CS subtraction already used in many NLO calculations.
- $P(x) = A(x)/B$, so terms vanish.
- $R(x) - A(x)$ already in NLO parton level program.

⇒ (almost) no need to modify NLO calculation!

Matching MC and NLO

$$\langle O\rangle_{\text{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\frac{R(x)-BP(x)}{x}\,.$$

3. Simpler in a different way, $P(x) = R(x)/B$
   - $R(x)-A(x)$ now only needed as integral available in NLO parton level program.
   - No $n+1$ body events.
   - ≥ 1 PS emission from $R(x)/B$ as splitting kernel $\rightarrow$`POWHEG`.
   - Positive weights (terms $\neq 0$ are $\sigma^{\text{incl}}_{\text{NLO}}$).
   - Further emissions from (truncated) standard PS.

MC@NLO

- Introduced 2002 — Frixione, Webber, JHEP 0206:029,2002 [hep-ph/0204244].
- Extended to heavy quarks

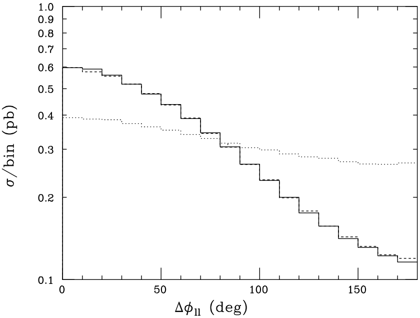
  Frixione, Nason, Webber, JHEP 0308:007,2003 [hep-ph/0305252].
- further extensions to many processes (single top etc.)
- MC@NLO customised to use with HERWIG.
- Some processes in Herwig++ as well
  $e^+e^- \to$ jets, DY, W', h^0 decay

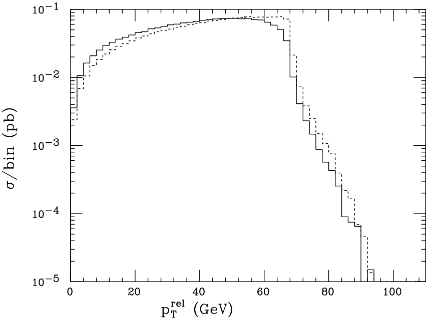
  Latunde–Dada 0708.4390, 0903.4135, Latunde–Dada, Papaefstatiou, 0901.3685.
- MC@NLO package adopted to Herwig++ as well.

  S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

MC@NLO

Examples with Herwig++ (solid) Herwig6 (dash)





$h^0 \to WW \to l\nu l\nu$,
(no spin corr dotted)

$t\bar{t}$, $p_t(b)$ rel to t (right).

S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

POWHEG

- Alternative proposed by P. Nason.
- Modified Sudakov FF for first emission.
- Angular ordered Parton Shower tricky (see below).
- *Truncated Shower* adds in missing radiation afterwards.
- Finally evolution with ‘ordinary’ Parton Shower.

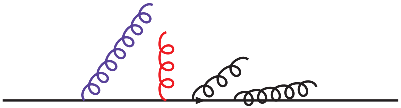
[Nason, hep-ph/0409146; Nason, Ridolfi hep-ph/0606275]

Recently systematically extended.

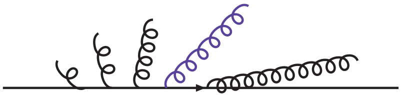
- `POWHEG` formulation independent of the event generator implementation.
- Worked out for different subtraction schemes.

[Frixione, Nason, Ridolfi, 0707.3081, 0707.3088; Frixione, Nason, Oleari, 0709.2092]

Angular ordered showers and POWHEG



$p_\perp$ ordered shower. Angular ordering from additional vetos.



Angular ordered shower. Some softer emissions before hardest one.

Need truncated showers.

POWHEG in Herwig++

- First implementation of method for e^+e^- annihilation

  [O. Latunde–Dada, SG, B. Webber, hep-ph/0612281]

- Many more processes now available with release:
  DY $(\gamma^*/Z^0/W^\pm), h^0, h^0Z^0, h^0W^\pm, W^+W^-, W^\pm Z^0, Z^0Z^0$

  [K. Hamilton, P. Richardson and J. Tully, 0806.0290, 0903.4345, Hamilton, JHEP 1101:009]

- and with contributed code:
  $e^+e^- \to \text{jets}, t\bar{t}, t-\text{decay}, W', h^0-\text{decay}$

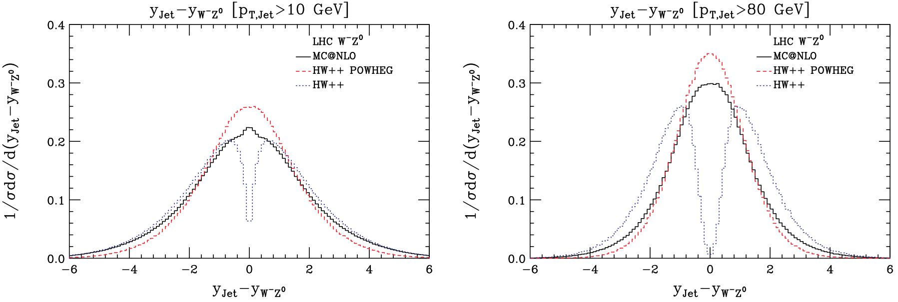
  [O. Latunde-Dada, 0812.3297, Eur. Phys. J. C **58**, 543 (2008)]

  [A. Papaefstathiou and O. Latunde-Dada, JHEP **0907**, 044]

- includes full truncated showers.
- Interface to PowhegBox straightforward.
- More processes underway ($\gamma\gamma$, VBF, SUSY pair prod…).

POWHEG in Herwig++

POWHEG in Herwig++ with full truncated shower.

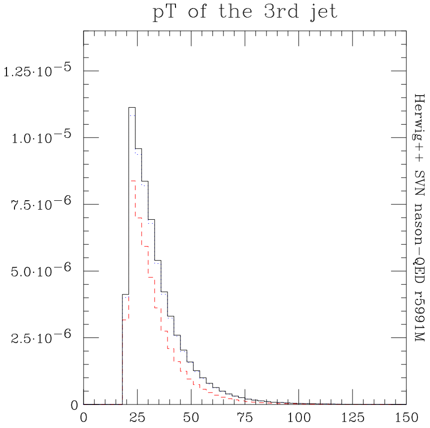


[K. Hamilton, JHEP 1101:009]

VV production. Phase space of radiated gluon properly filled.

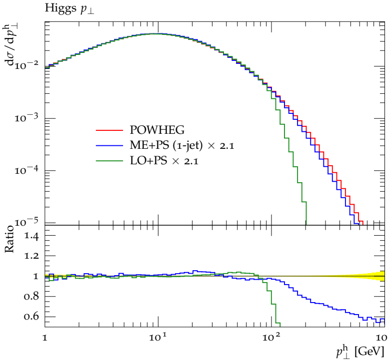
POWHEG in Herwig++

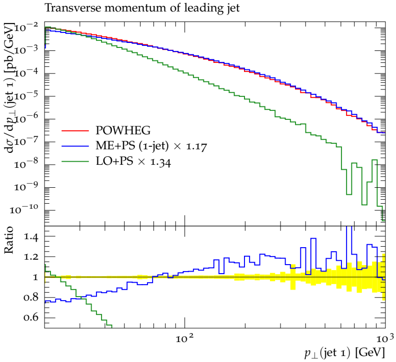
Higgs production in VBF. (POWHEG, MEC, LO+PS)



[L. D'Errico, P. Richardson in preparation]

- Automated POWHEG matching approach.
- Only virtuals needed → Binoth Les Houches Accord.



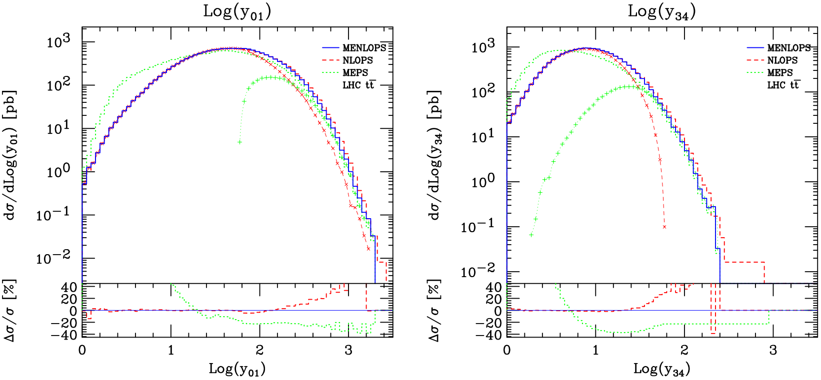


[Hoeche, Krauss, Schönherr, Siegert, JHEP 1104:024]

$gg \to h^0$ (left) WW+jets (right)

MENLOPS

ME+PS merging with lowest multiplicity at NLO.

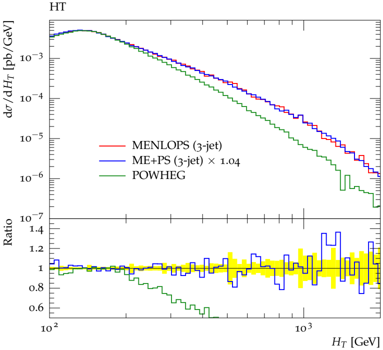


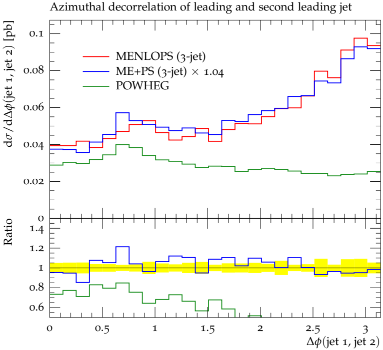
Test generic method with Pythia. y_{nm} in $t\bar{t}$+jets

[Hamilton, Nason, JHEP 1006:039]

MENLOPS

ME+PS merging with lowest multiplicity at NLO.





WW+jets implementation in Sherpa.

[Hoeche, Krauss, Schönherr, Siegert, 1009.1127]

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- Matrix element corrections
- Merging tree level matrix elements
  - Inclusive/exclusive
  - CKKW weights
- Matching NLO and parton showers
  - NLO calculation
  - MC@NLO
  - POWHEG
  - Matching with dipole showers

Outline

- Part I — Basics
  - Introduction
  - Monte Carlo techniques
- Part II — Perturbative physics
  - Hard scattering
  - Parton showers
- Part III — Merging/Matching
  - Matrix element corrections
  - Merging multiple tree level MEs with parton showers
  - Matching NLO and parton showers
- Part IV — Non–perturbative physics
  - Hadronization
  - Hadronic decays
  - Comparison to data
- Part V — Multiple Partonic Interactions
  - Minimum Bias/Underlying Event in data
  - Modelling