Calculating amplitudes in the multi-Regge regime of strongly coupled $\mathcal{N} = 4$ SYM

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based on 1207.4204 and 1310.xxxx with J. Bartels and V. Schomerus

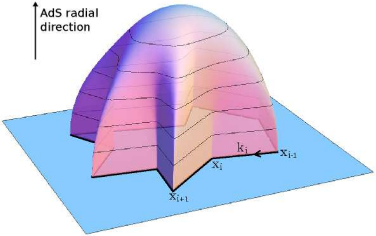
Particles, Strings, and the Early Universe
Collaborative Research Center SFB 676

Motivation

- planar $\mathcal{N} = 4$ integrable $\leftrightarrow$ can compute observables for any coupling
- scattering amplitudes particularly interesting
  $\rightarrow$ techniques for less symmetric theories
- enormous progress on weak coupling side
- how do amplitudes behave at strong coupling?
  - interpolation to intermediate coupling
  - regime inaccessible (analytically) for QCD

Scattering Amplitudes via AdS/CFT

[Alday/Maldacena, A/M/Gaiotto, A/M/Sever/Vieira]



[Figure from 1002.2459]

- $A \sim e^{-\frac{\sqrt{\lambda}}{2\pi}\mathrm{Area}}$
- $k_i = x_{i-1} - x_i$
  $\rightarrow$ polygon depends only on $u = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$
- $Y_{a,s}(\theta)$ generalised cross ratios
  $\rightarrow$ for n gluons: $3n - 15$ Y-functions

$$\log Y_{a,s}(\theta) = - \ m_s \ \cosh\theta \pm \ C_s + \sum_{a',s'} \int d\theta' \ \mathcal{K}_{s,s'}^{a,a'} \ \left(\theta - \theta' + \ i\varphi_s - i\varphi_{s'}\right) \log\left(1 + Y_{a',s'}(\theta')\right)$$

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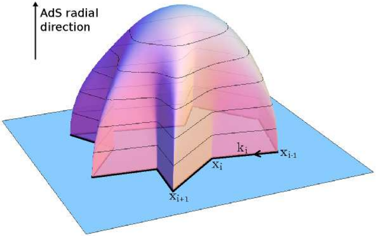

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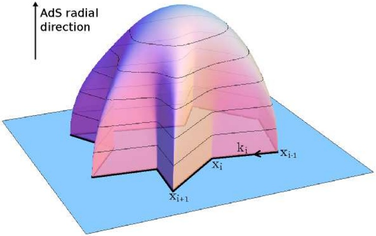

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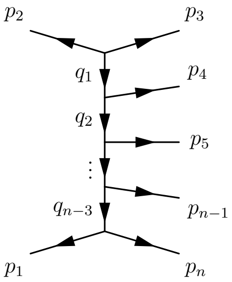
Y-System

$$\text{Area} = \text{A}_{\text{div}}(x_i) + \text{A}_{\text{periods}}(m_s, \varphi_s) + \Delta(u_i) + \sum_s \int \frac{d\theta}{2\pi} |m_s| \cosh\theta \log\left[(1+Y_{1,s})(1+Y_{3,s})(1+Y_{2,s})^{\sqrt{2}}\right](\theta)$$

- non-divergent piece: remainder function $e^{-\frac{\sqrt{\lambda}}{2\pi}R}$
- Y-system easy to solve numerically
- BUT: not solvable analytically for arbitrary kinematics!

Multi-Regge Limit



- for $2 \to n-2$ scattering: $3n-10$ Mandelstam invariants
- Multi-Regge limit: rapidities of produced particles strongly ordered
  $\to$ Regge limit in all subchannels $s \gg s_i \gg -t_i$

- $\mathcal{N}=4$ dual conformal $\to$ choose $3n-15$ cross ratios u_{as}
- kinematical analysis:
  $u_{1s} \to 1,\ u_{2s}, u_{3s} \to 0$ with $\tilde{u}_{2s} = \frac{u_{2s}}{1-u_{1s}} = \mathcal{O}(1), \quad \tilde{u}_{3s} = \frac{u_{3s}}{1-u_{1s}} = \mathcal{O}(1)$

Y-system in the MRL

[1207.4204]

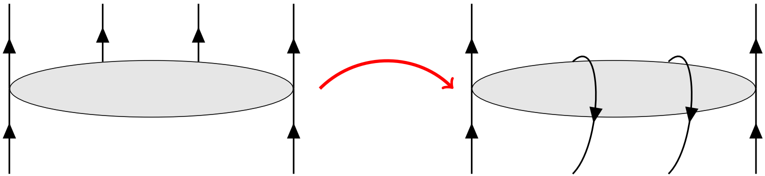
- $u_{as} = \frac{Y_{2s}}{1+Y_{2s}}\Big|_{\theta=i(k\pi/4-\varphi_s)}$
- demand that cross ratios show behaviour predicted by MRL
- MRL corresponds to choice m_s large, $\varphi_s \to -(s-1)\frac{\pi}{4}$, $C_s \to$ const.
- in this limit, integrals in Y-system can be neglected

$$\boxed{\log Y_{a,s}(\theta) \cong -m_s \cosh\theta \pm C_s + \mathcal{O}(e^{-m})}$$

- in this limit, R trivial

Multi-Regge regions

- 2^{n-4} regions, corresponding to the signs of $k_i^0 = E_i$
- different regions connected by analytic continuation in $s_i \to s_i e^{i\alpha}$



- for the above example:

$$s_{34} \to e^{i\pi} s_{34} \quad s_{56} \to e^{i\pi} s_{56}, \quad s_{345} \to e^{i\pi} s_{345}, \quad s_{456} \to e^{i\pi} s_{456}$$

$$\Rightarrow u_1 \to e^{-2\pi i} u_1, \quad u_2 \to u_2, \quad u_3 \to u_3$$

Multi-Regge regions in the Y-system

- continuation in $u_{as} \sim$ continuation in m, C, φ
- (numerical) inversion of $u_{as} = \frac{Y_{2s}}{1+Y_{2s}}\Big|_{\theta=i(k\pi/4-\varphi_s)}$ to find paths for parameters
- solutions of $Y_{as}(\theta) = -1$ can cross real axis

$$\log Y_{a,s}(\theta) = -m_s \cosh\theta \pm C_s + \sum_{a',s'} \int d\theta' \mathcal{K}^{a,a'}_{s,s'}(\theta - \theta' + i\varphi_s - i\varphi_{s'}) \log(1 + Y_{a',s'}(\theta'))$$

- crossing leads to contributions to R

$$R' = \int \frac{d\theta}{2\pi} |m_s| \cosh\theta \log\left[(1 + Y_{a,s}(\theta))\right] \pm i|m_s| \sinh(\theta_0) + \ldots$$

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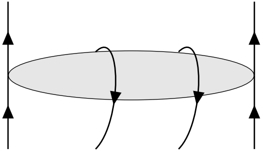
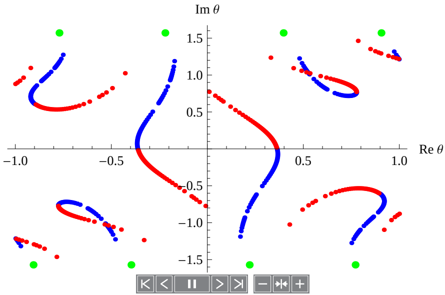
[Bartels/Kotanski/Schomerus]



$u_1 \to e^{-2\pi i} u_1\,,\ u_2 \to u_2\,,\ u_3 \to u_3$



Solutions of $Y_3(\theta) = -1$ along continuation

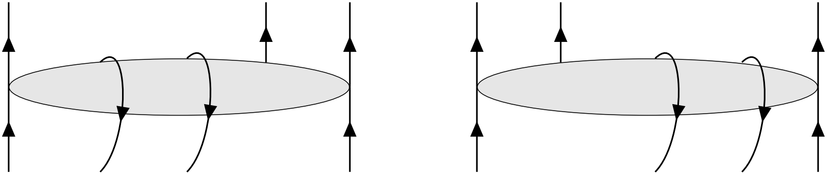
- Remainder function has Regge behaviour:

$$e^{-\frac{\sqrt{\lambda}}{2\pi} R_6} \sim \left((1 - u_1) \sqrt{\tilde{u}_2 \tilde{u}_3} \right)^{\frac{\sqrt{\lambda}}{2\pi} e_2}$$

- $e_2 = -\sqrt{2} + \frac{1}{2} \log\left(3 + 2\sqrt{2}\right) < 0$

7-point case

[1310.xxxx]



$$R_{7,--+}(u_{as}) = R_6(u_{12}, u_{22}, u_{32})$$
$$R_{7,+--}(u_{as}) = R_6(u_{11}, u_{21}, u_{31})$$

- strong coupling results consistent with weak coupling LLA identities!

Summary and Outlook

- studied scattering amplitudes in strongly coupled $\mathcal{N} = 4$ SYM
- showed that Y-system equations simplify in MRL
- calculated 6-point and 7-point remainder function for specific continuations $\rightarrow$confirmed weak coupling LLA prediction

next steps:

- remaining 7-point continuations
- weak coupling: new contribution for 8-point case
- make contact with weak coupling results