WZ term in permutation coset

Based on

A. Cagnazzo and K. Zarembo, "B-field in AdS(3)/CFT(2) Correspondence and Integrability," arXiv:1209.4049 [hep-th].

Plan

* String theories as coset sigma models
* Permutation cosets and WZ term
* AdS3 theory with a mixed flux

String theory	Supercoset
GS action	Sigma model
$AdS_5 \times S^5$	$\dfrac{PSU(2,2\|4)}{SO(1,4) \times SO(5)}$
$AdS_4 \times CP^3$	$\dfrac{OSp(6\|4)}{SO(1,3) \times U(3)}$
$AdS_3 \times S^3 \times T^4$	$\dfrac{PSU(1,1\|2)^2}{SU(1,1) \times SU(2)}$

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Supercoset sigma models can be used to describe a Superstring in an AdS background

SEMI-SYMMETRIC COSETS

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$G \longrightarrow$ global symmetry

$H_0 \longrightarrow$ subgroup of G

$$\mathfrak{g} = \mathfrak{h}_0 \oplus \mathfrak{h}'$$

$\mathfrak{h}_0$ is a subalgebra

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Under which $\mathfrak{h}_0$ elements are invariant

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purely bosonic case

Ω is parity

$$\Omega^2 = \mathrm{Id}$$

$$[\mathfrak{h}', \mathfrak{h}'] \subset \mathfrak{h}_0$$

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Under which $\mathfrak{h}_0$ elements are invariant

supersymmetric case

Ω is Z4 symm $\qquad \Omega^4 = \mathrm{Id}$

$$\mathfrak{h}' = \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

$\mathfrak{h}_0, \mathfrak{h}_2$ bosonic

$\mathfrak{h}_1, \mathfrak{h}_3$ fermionic

$$[\mathfrak{h}_n, \mathfrak{h}_m] \subset \mathfrak{h}_{m+n, \mathrm{mod} 4}$$

Why is it useful to describe superstring with a coset sigma model?

Z4 Coset sigma models possess the property to be **INTEGRABLE**

Bena,Polchinski,Roiban'03

Classical Integrability $\Leftrightarrow$ Flat Lax connection

$$dL + LL = 0$$

To describe a string theory the sigma model has to be
CONFORMAL

zero beta function

good cosets classified by K.Zarembo in 2009

Two objects in string theory:

- Closed strings
- branes

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- Closed strings → NSNS flux
- branes → RR fluxes

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$$AdS_3 \times \mathcal{M}^7$$

GS superstring with RR and NSNS Fluxes

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Supercoset sigma model + WZ term

Z4 Supercoset

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Berkovits, Bershadsky, Hauer, Zhukov and Zwiebach
For AdS3: Babichenko, Stefanski and Zarembo

Semi-symmetric $\frac{\mathcal{G}}{\mathcal{H}_0}$ superspace, invariant under Ω automorphism

$$\Omega(J_n) = i^n J_n \qquad [T_n, T_m\} = T_{m+n,\mathrm{mod}4}$$

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + J_1 \wedge J_3\right)$$

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$$\sqrt{-h}h^{\mu\nu} J_{2\mu} J_{2\nu} \qquad \epsilon^{\mu\nu} J_{1\mu} J_{3\nu}$$

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Z4 forbidden

bosonic coset
Hull, Spence '89
Witten '92

Permutation Supercoset

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$$\frac{G \times G}{G_B} \qquad\qquad \Omega = \begin{pmatrix} 0 & \mathrm{id} \\ (-1)^F & 0 \end{pmatrix}.$$

Babichenko, Stefanski and Zarembo

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Babichenko, Stefanski and Zarembo

$$G = PSU(1,1|2) \qquad\qquad G = D(2,1;\alpha)$$

$$AdS_3 \times S^3 \times T^4 \qquad\qquad AdS_3 \times S^3 \times S^3 \times S^1$$

A.C., K.Zarembo

$$S = \frac{1}{2}\int_{\mathcal{M}} \mathrm{Str}\left(J_2 \wedge *J_2 + \kappa J_1 \wedge J_3\right)$$

Introduced because of integrability,
Kappa symmetry and conformality.

$$+\chi \int_{\mathcal{B}} \left(\frac{2}{3} J_2 \wedge J_2 \wedge J_2 + J_1 \wedge J_3 \wedge J_2 + J_3 \wedge J_1 \wedge J_2\right)$$

Equations of motion

$$\begin{aligned} D * J_2 - \kappa\, J_1 \wedge J_1 + \kappa\, J_3 \wedge J_3 - 2\chi J_2 \wedge J_2 - \chi J_1 \wedge J_3 - \chi J_3 \wedge J_1 &= 0 \\ (\kappa J_1 + *J_1) \wedge J_2 + J_2 \wedge (\kappa J_1 + *J_1) + \chi\,(J_2 \wedge J_3 + J_3 \wedge J_2) &= 0 \\ (\kappa J_3 - *J_3) \wedge J_2 + J_2 \wedge (\kappa J_3 - *J_3) + \chi\,(J_2 \wedge J_1 + J_1 \wedge J_2) &= 0 \end{aligned}$$

\+ Maurer-Cartan equations

to find flat Lax Connection

$$dL + LL = 0$$

$$L = J_0 + \alpha_1 J_2 + \alpha_2 * J_2 + \beta_1 J_1 + \beta_2 J_3$$

$$dL + LL = 0$$

$$\kappa^2 = 1 - \chi^2$$

ensure to find a flat Lax connection

$$\begin{aligned} \alpha_2 &= \chi \pm \sqrt{-1 + \alpha_1^2 + \chi^2} \\ \beta_1 &= \pm\sqrt{\alpha_1 - \kappa\,\alpha_2} \\ \beta_2 &= \pm\sqrt{\alpha_1 + \kappa\,\alpha_2} \end{aligned}$$

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$$\alpha_1 = \kappa\,\frac{\mathrm{x}^2+1}{\mathrm{x}^2-1}$$

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- $\chi = 0 \;\; \kappa = 1$ supercoset Lax connection
- $\chi = 1 \;\; \kappa = 0$ WZ point

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Same constraint from Integrability, Conformality and Kappa Symmetry

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S-Matrix by Hoare and Tseytlin

Conclusions

- We have built the WZ term for a particular class of cosets, the Permutation cosets in order to Study superstring theory on AdS_3 backgrounds, supported by both type of fluxes RR and NSNS.
- We have shown that this does not spoil Integrability, Kappa Symmetry and Conformality, though we have to introduce a relation between the coupling constants of the theory.
- We have computed the BMN spectrum for the theory.

- S-matrix with deformation: proposal by Tsytlin and Hoare.
- Direct string worldsheet calculation by Wulff and Sundin
- To study the case $\chi = 1$ $\kappa = 0$ could provide a link between integrability technique and CFT worldsheet methods.
- Study of the finite gap equation to understand how to modify the receipt in absence of Z4 symmetry
- Integrability of the full theory: non supersymmetric and MASSLESS MODES in the light cone gauge.

Thank
you

We want to see if the theory is conformal.

Let's expand the action around a classical background

$$\bar{g}_{L,R} \in G^B$$

$$\bar{J}_{L,R} = \underbrace{A}_{\text{Grading zero}} \pm \underbrace{K}_{\text{Grading two}}$$

MC

$$\begin{aligned} \underbrace{F}_{dA + A \wedge A} + K \wedge K &= 0 \\ DK &= 0, \end{aligned}$$

EoM

$$D * K - 2\chi K \wedge K = 0$$

$$g_{L,R} = \bar{g}_{L,R}\, \mathrm{e}^{X_{L,R}}$$

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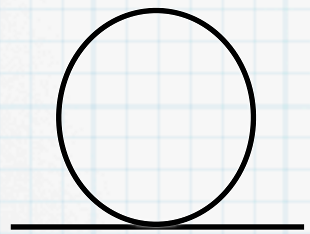
$$J = \bar{J} + \frac{1 - \mathrm{e}^{-\operatorname{ad} X}}{\operatorname{ad} X}\,\mathcal{D}X = \bar{J} + \mathcal{D}X - \frac{1}{2}\,[X, \mathcal{D}X] + \ldots$$

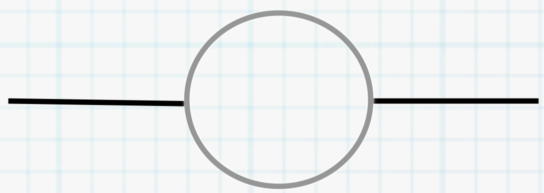
$\mathcal{D}X = dX + [\bar{J}, X]$

Beta-function: conformality?

Log-divergent contribution to the effective action, responsible for the renormalization of the dimension two operator $K \wedge *K$.



bosonic



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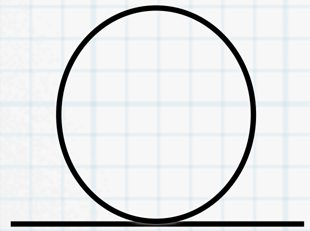

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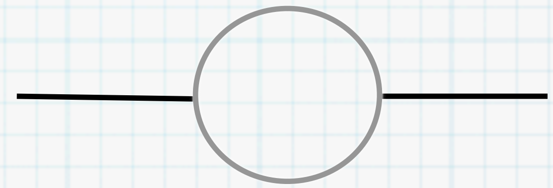

fermionic

Summing the two contributions

$$I = -\frac{\kappa^2}{8\pi} \ln \Lambda \int \mathrm{Str}\, \mathrm{ad}K \wedge *\mathrm{ad}K$$

differs from the supercoset case by a factor κ^2

coset vanishing conditions+ a fixed point in $\chi = 1 \;\; \kappa = 0$

As happen for a generic coset, the beta function is proportional to the Killing form. When the Killing form vanishes the theory is conformal. This in particular happens for

$PSU(n|n)$ and $OSp(2n+2|2n)$

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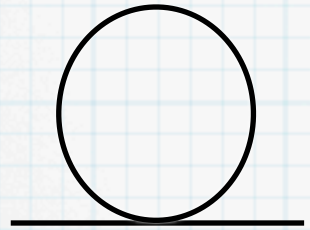

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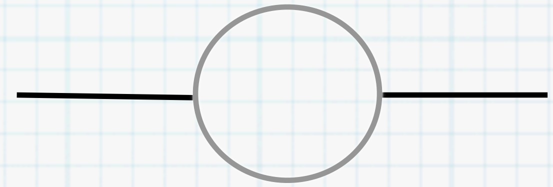

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$$S_F^{(2)} = \frac{1}{2}\int \mathrm{Str}\, X_I\, \underbrace{\left(D + \sigma_1\, \mathrm{ad}\, K\wedge\right)^{IJ} \left(* - \kappa\sigma_3 - \chi\sigma_1\right)^{JL}}_{\text{Hermitian Dirac Operator}}\, \mathrm{ad}\, K\, X_L$$

$I = (1,3)$

has solutions if:

$$1 - \kappa^2 - \chi^2 = 0$$

SAME CONSTRAINT
FROM INTEGRABILITY
AND KAPPA-SYMMETRY

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Kappa-symmetry

A consistent WZW has not to spoil the Kappa-symmetry, so we expect that its introduction do not affect the rank of the Kappa-symmetry.

$$\mathrm{rank}_\kappa = \dim\ker\mathrm{ad}\, K_+|_{\mathfrak{h}_1} + \dim\ker\mathrm{ad}\, K_-|_{\mathfrak{h}_3}$$

$$[K_\pm, \epsilon^\pm] = 0 \qquad\qquad \delta X_I = C_I^\pm \epsilon^\pm$$

$$\delta S_F^{(2)} = \int \mathrm{Str}\, X\, (D + \sigma_1\, \mathrm{ad}\, K\wedge)\, (\pm 1 - \kappa\sigma_3 - \chi\sigma_1)\, C^\pm\, \mathrm{ad}\, K\, \epsilon^\pm$$

$$\Downarrow$$

$$(\pm 1 - \kappa\sigma_3 - \chi\sigma_1)\, C^\pm = 0$$

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SAME CONSTRAINT
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BMN limit

Without WZW: Babichenko, Stefanski and Zarembo

Background: point-like string moving along a light-like geodesic

$$\bar{g}_{L,R} = e^{i(D+J)\tau}$$

Dilaton generator   Rotation generator   Worldsheet time

$$A = 0, \ \ K = i(D+J)d\tau$$

$$S_B^{(2)} = \frac{1}{2}\int \mathrm{Str}\left\{D_{\chi * K} X_2 \wedge * D_{\chi * K} X_2 - \boxed{\left(1-\chi^2\right)[K, X_2] \wedge * [K, X_2]}\right\}$$

After a rotation

$$e^{-\frac{i}{2}s\sigma_2}$$

$$\cos s = \kappa \qquad \sin s = \chi$$

$$S_F^{(2)} = \frac{1}{4}\int \mathrm{Str}\, X\left\{-\partial_+(1-\sigma_3) + \partial_-(1+\sigma_3) + 2(\chi - i\kappa\sigma_2)\,\mathrm{ad}\,K\right\}[K, X].$$

Field redefinition

$$X \longrightarrow \mathrm{e}^{\,i\chi(D+J)\sigma}\, X\, \mathrm{e}^{\,-i\chi(D+J)\sigma}$$

$$\mathcal{M}_B^2 = -\kappa^2(\mathrm{ad}K)^2 \qquad \mathcal{M}_F = i\kappa\,\mathrm{ad}K$$