HELMHOLTZ ASSOCIATION
DESY THEORY WORKSHOP
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Nonperturbative QFT: Methods and Applications
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Resurgence at work: the Principal Chiral Model

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What/Why PCM?

$$S = \frac{N}{2\lambda}\int_{\Sigma} \mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger$$

$\rightarrow U \in SU(N)$

Pros

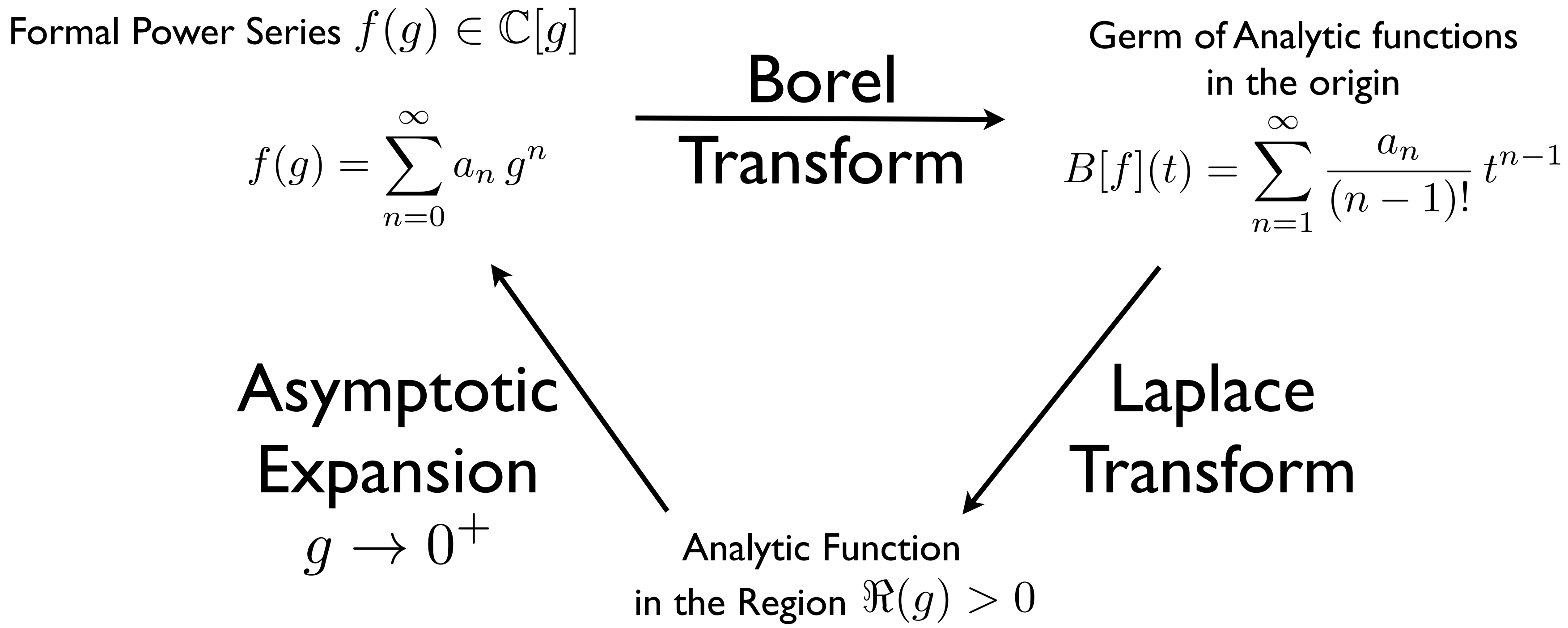
- Asymptotically free
- Mass gap
- Large-N Matrix Model
- Integrability (FKW)
- Susy

Cons

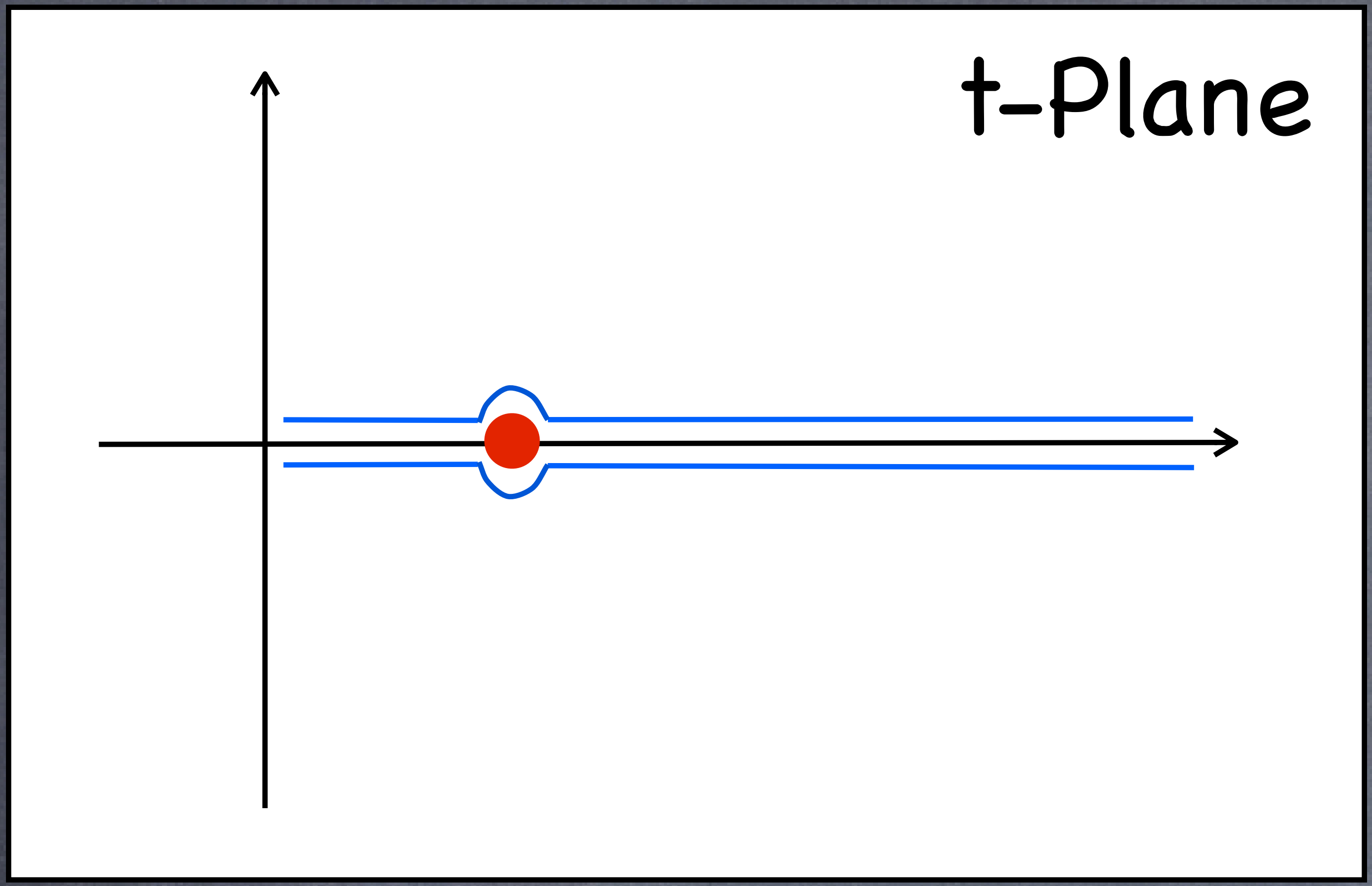
- No Large-N Sol.
- No Instantons
- Renormalon ambiguities

Ultra-crash course: Borel & Renormalon

See Gerald's talk (it's much better than this)



$$s[f](g) = a_0 + \int_0^{\infty} dt \, e^{-t/g} \, B[f](t)$$



Ambiguity for PCM:

$$f_+(g) - f_-(g) \sim i\, e^{-\frac{8\pi k}{g^2 N}}$$

k=1,2,...

Non-perturbative objects w/ action $S_0 = \dfrac{8\pi k}{g^2 N}$

Instantons: mappings from

$$\mathbb{R}^2 \cup \{\infty\} \sim S^2 \to SU(N)$$

Trivial

$$\Pi_2(SU(N))$$

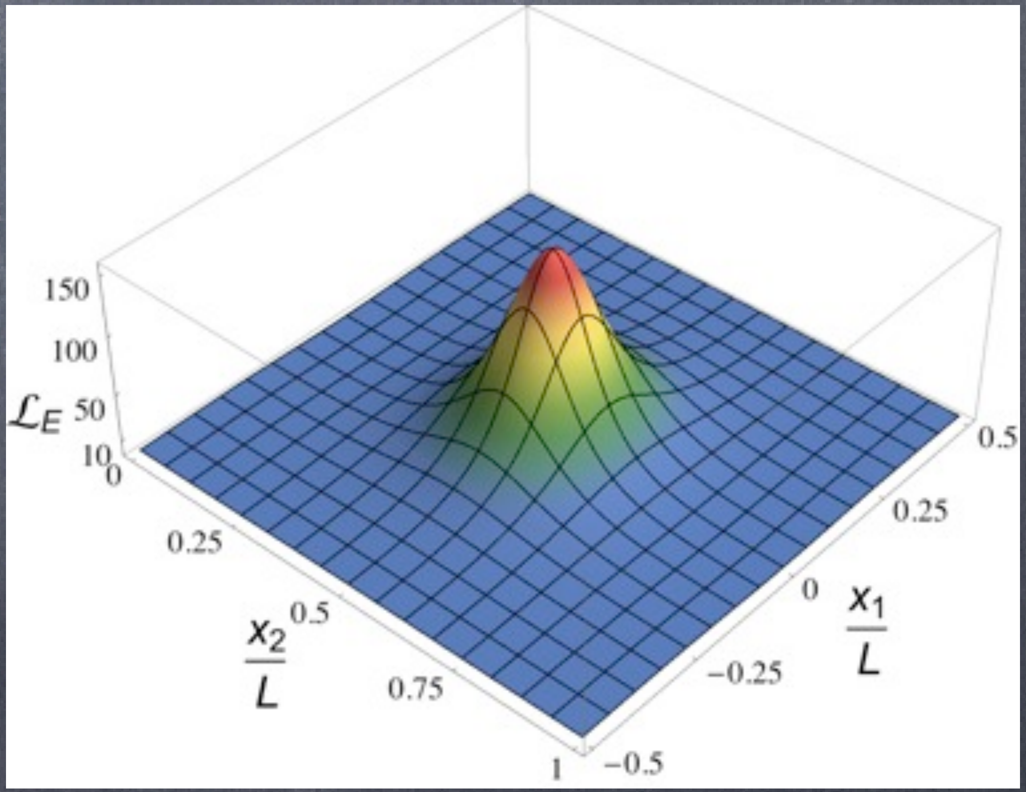
Unitons:

Uhlenbeck '89 → Full Characterization of soln. to PCM eom

$\mathbb{CP}^{N-1} \subset SU(N)$ geodesic embedding

$$U_{\rm uni} = e^{i\pi/N}(1_N - 2P),\ P_{ij} = \frac{v_i\, v_j^\dagger}{v^\dagger \cdot v}$$

$\mathbb{CP}^{N-1}$ lump embedded



$$S_{\rm uni} = 16\pi \times \frac{N}{2\lambda}$$

(and integer multiples)

S^1 Reduction

Twisted b.c. → $U(t, x+L) = e^{iH_L} U(t,x) e^{-iH_R}$

→ Chemical Potential $SU(N)_L \times SU(N)_R$

Fateev-Kazakov-Wiegmann

→ Background gauge Field $\partial_\mu U \to D_\mu U$

Unique choice such that small-L th.
continuously connected to $\mathbb{R}^2$

No phase transition or rapid cross-overs

Related to Volume independence and unbroken $\mathbb{Z}_N$

Kovtun-Yaffe-Unsal-Argyres-Dunne-...

Unitons w/ twisted b.c?

Brendel-Bruckmann-Janssen and Wipf

Take $\mathbb{CP}^{N-1}$ lump → Twist it **Step 1**

$$v(z) \to \Omega(z) v(z)$$

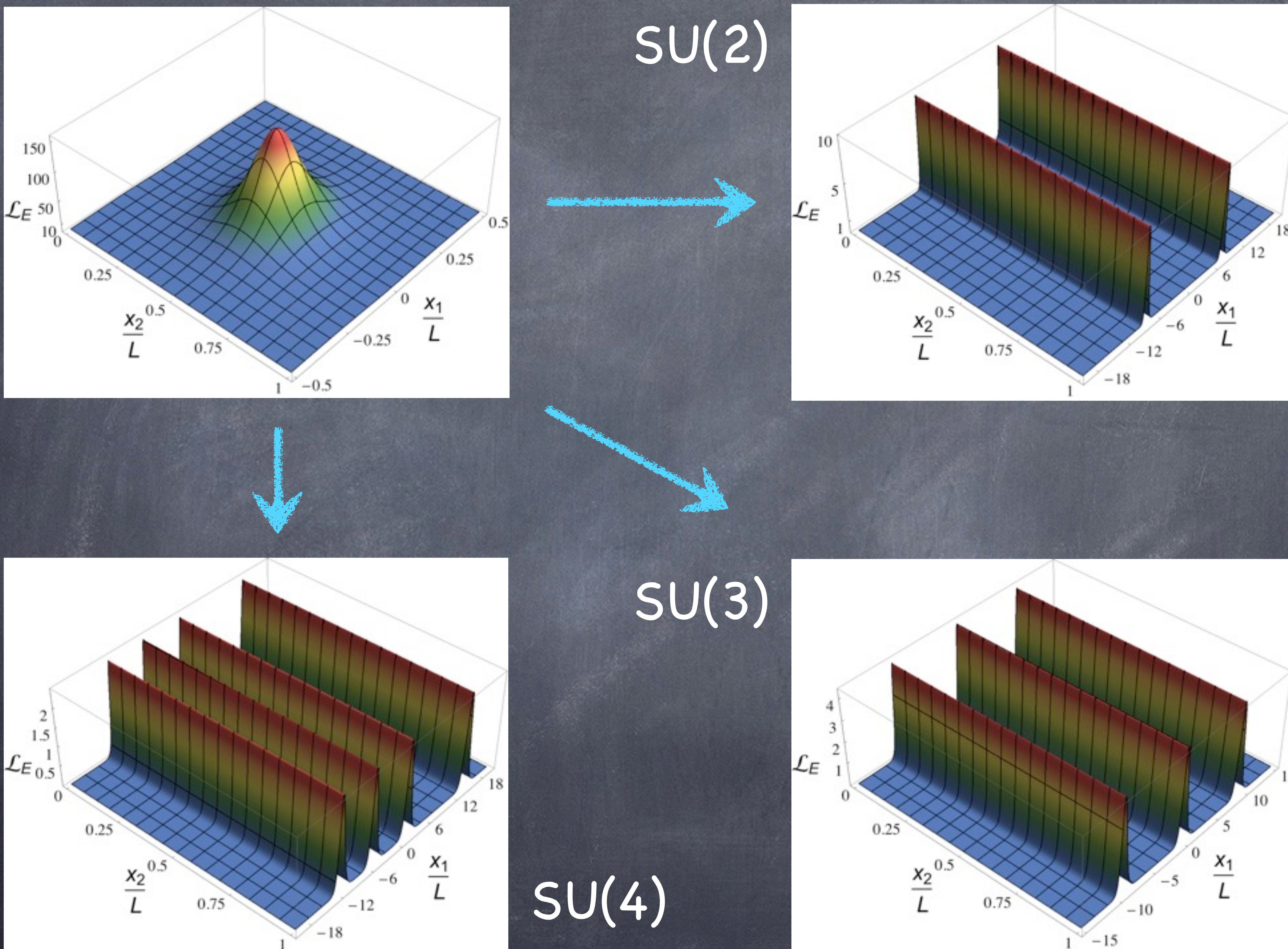
$$z = (x_1 + i\, x_2)/L$$

$$\Omega(z) = \begin{pmatrix} e^{i\mu_1 z} & & & \\ & e^{i\mu_2 z} & & \\ & & \ddots & \\ & & & e^{i\mu_N z} \end{pmatrix}$$

for PCM take the Uniton

$$U_{\rm uni} = e^{i\pi/N}(1_N - 2P),\ P_{ij} = \frac{v_i\, v_j^\dagger}{v^\dagger \cdot v}$$

→ Twist it (with Unique b.c.) **Step 2**



SU(2)
SU(3)
SU(4)
$\mathcal{L}_E$
$\frac{x_2}{L}$
$\frac{x_1}{L}$

At small-L theory reduces to QM

Continuously connected w/ $\mathbb{R}^2$ theory

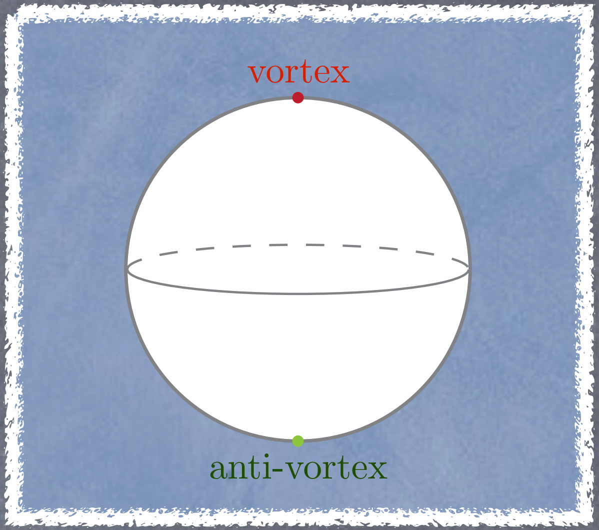
These “Fractons” : semiclassical realization of IR renormalons

Crucial twisted b.c. introducing potential on target space

Standard homotopy insufficient to classify saddle points in the path integral

Interesting Directions:

work in progress w/ Sungjay Lee



PCM can only be made $\mathcal{N}=(1,1)$

$\mathbb{CP}^{N-1}$ as GLSM is $\mathcal{N}=(2,2)$ ⟶ Localize on S^2

Benini,Cremonesi,
Doroud,Gomis,LeFloch,Lee

Expand $Z_{loc}(\xi,\theta)$ for F.I. big ⟶ Renormalon?

see i.e. Russo

- ABJM
- N=2*
- N=2, SU(2) w/ 4 Hyper

(Does $Z_{loc}(\xi,\theta)$ satisfies tt*?)

KEEP CALM AND THANKS FOR LISTENING

Backup Slides

Twisted b.c.:

$$\tilde{U} = e^{-iH_L \frac{x}{L}} U \, e^{iH_R \frac{x}{L}}$$ periodic field

w/ background gauge field

$Z(L; H_L, H_R)$ computed at 1Loop

REQUIRED:

- Order N^0 Free energy
- No Persistent currents

$$\frac{\partial \left[\log Z\right]}{\partial H_{V/A}} \sim \langle J_x^{V/A} \rangle_{H_V, H_A} = 0$$

Axial part set to 0

$$\Omega = e^{i \oint dx\, H_V} = e^{i\, L\, H_V}\,.$$

$$V_{1Loop}(\Omega) = (N_f - 1)\frac{1}{\pi L^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(|\mathrm{Tr}\,\Omega^n|^2 - 1\right)$$

Identical expression for 1loop potential for Polyakov loop in SU(N)

Thermal

$$e^{i\frac{2\pi k}{N}}\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

Spatial

$$\begin{pmatrix} 1 & & & \\ & e^{i\frac{2\pi}{N}} & & \\ & & \ddots & \\ & & & e^{i\frac{2\pi(N-1)}{N}} \end{pmatrix}$$