CFT and Hermitian Symmetric Superspaces

Václav Tlapák

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Based on 1310.xxxx with C. Candu and V. Schomerus

Introduction

- GKO coset construction yields rich class of 2D CFTs from WZNW models [Goddart, Kent, Olive]
- Many relativistic QFTs can be considered as deformations
- Kazama-Suzuki (KS) showed that hermitian symmetric GKO cosets with $\mathcal{N} = 1$ SUSY actually have $\mathcal{N} = 2$ SUSY
- Perturbations by chiral primary fields preserve conformal and $\mathcal{N} = 2$ structures [Dixon]

Introduction

- GKO and KS constructions remain valid if G, H are replaced by **supergroups** [Creutzig, Ronne, Schomerus]
- Resulting theories are **non-unitary**
- String Theory and statistical surface physics: models with $c \leq 0 \rightarrow$ never unitary
- Not well studied
- For $\mathcal{N} = 0$: Six families with continously varying exponents [Candu, Schomerus]
- We will extend this to KS and find three families of $\mathcal{N} = 2$ theories with exactly margnial deformation

Review of GKO and KS constructions

- Current algebra
  $\hat{G}_{k-g^\vee} \Rightarrow c^G = \mathrm{sdim}\,\mathfrak{g}\frac{k-g^\vee}{k}, \qquad \mathrm{sdim}\,\mathfrak{g} = \dim \mathfrak{g}_{\bar{0}} - \dim \mathfrak{g}_{\bar{1}}$
- $H \subset G$: equal rank, invariant under a $\mathbb{Z}_2$ automorphism
- $\mathfrak{g}/\mathfrak{h} = \mathfrak{m} = \mathfrak{m}^+ \oplus \mathfrak{m}^-$ two conjugate $\mathfrak{h}$ representations
- Bases labelled by positive (negative) roots $(-)\bar{\alpha} \in \bar{\Delta}$
- Add "fermionic" sector $\chi^{\pm\bar{\alpha}}$:
  - $2p$ copies of complex fermions $\psi, \bar{\psi} \leftrightarrow \bar{\alpha}$ bosonic
  - $2q$ copies of bosonic $\beta\gamma$-system $\leftrightarrow \bar{\alpha}$ fermionic
  - Embedd $\mathfrak{h} \rightarrow$ transform in the representation $\mathfrak{m}$
  - $h_\chi = 1/2$, $c_{\psi\bar{\psi}} = +1$, $c_{\beta\gamma} = -1$

Review of GKO and KS constructions

- Total $\mathfrak{h}$ current algebra at level $k - h^\vee$
- $c^{G/H} = c^G + \mathrm{sdim}\, G/H - \mathrm{sdim}\, \mathfrak{h} \frac{k-h^\vee}{k}$
- States in G/H theory obtained from

$$\mathscr{H}_G \otimes \mathscr{H}_\chi = \bigoplus \mathscr{H}_{\mathrm{G/H}} \otimes \mathscr{H}_{\mathrm{H}}$$

- Sectors labeled by $\big(\Lambda, (S,\sigma), \lambda\big)$
  - Λ - $\mathfrak{g}$ representation
  - $(S,\sigma) \in \mathbb{Z} \times \{0,1\}$ - sectors of χ theory
  - λ - $\mathfrak{h}$ representation

Review of GKO and KS constructions

- Energy and $U(1)$-charge of primary states:

$$h\big(\Lambda,(S,\sigma),\lambda\big) = \frac{1}{k}C^{(2)}_{\mathfrak{g}}(\Lambda) + h_F(S,\sigma) - \frac{1}{k}C^{(2)}_{\mathfrak{h}}(\lambda)$$

$$q\big(\Lambda,(S,\sigma),\lambda\big) = q_F(S,\sigma) - \frac{2}{k}\big(\rho^G - \rho^H\big)\cdot\lambda$$

with

$$h_F(S,\sigma) = \frac{S^2}{16}\,\mathrm{sdim}\,\mathfrak{g}/\mathfrak{h} + \frac{1}{4}\big(1+(-1)^S\big)\sigma$$

$$q_F(S,\sigma) = \frac{S}{4}\,\mathrm{sdim}\,\mathfrak{g}/\mathfrak{h} + \sigma$$

- Charge does not depend on the $\mathfrak{g}$ representation

Marginal Chiral Primaries

- Field with $h = \pm q/2$ is called (anti-)chiral primary
- Chiral Primaries of dimension $(h, \bar{h}) = (1/2, 1/2)$ generate an exactly marginal deformation [Dixon]
- Consider

$$|\Phi_\pm\rangle = \sum_{\bar{\alpha} \in \bar{\Delta}^+} |E_{\pm\bar{\alpha}}\rangle_{\mathrm{G}} \otimes \chi^{\mp\bar{\alpha}}_{-1/2}|0\rangle_F = |(\mathfrak{g}, (0,1), 0)\rangle_\pm \otimes |0\rangle^{\mathrm{H}}$$

$$h(\Phi_\pm) = \frac{1}{k} C^{(2)}_{\mathfrak{g}}(\mathfrak{g}) + \frac{1}{2}, \qquad q(\Phi_\pm) = \pm 1$$

- $\Phi_\pm$ is (anti-)chiral $\iff$ $g^\vee = C^{(2)}_{\mathfrak{g}}(\mathfrak{g}) = 0$

The KS models with an exactly marginal deformation

- Three families: [Serganova] [Zirnbauer]

$$
\begin{aligned}
&(A) \quad \mathrm{H} = \mathrm{PS}\big(\mathrm{U}(r|s) \times \mathrm{U}(n-r|n-s)\big) \subset \mathrm{G} = \mathrm{PSU}(n|n) \\
&(B) \quad \mathrm{H} = \mathrm{OSP}(2n|2n) \times \mathrm{SO}(2) \subset \mathrm{G} = \mathrm{OSP}(2n+2|2n) \\
&(C) \quad \mathrm{H} = \mathrm{SO}(2) \times \mathrm{SU}(n+1|n) \subset \mathrm{G} = \mathrm{OSP}(2n+2|2n)
\end{aligned}
$$

- With central charges

$$
c^{G/H} = \frac{3}{2}\,\mathrm{sdim}\, G/H = \begin{cases} -3(r-s)^2 & (A) \\ 0 & (B), (C) \end{cases}
$$

Conclusions and Outlook

- Constructed three series of KS-like supercosets that possess an exactly margnial deformation
- These have large multiplicity of fields with $h = q = 0$
  $\rightarrow$ large number of deformations?
- G/H coset models at $k \rightarrow \infty$ are belived to approach the sigma model on G/H at infinite radius [Fendley, Lerche]
  Perturbation brings the sigma model to finite radius
  $\Rightarrow$ sigma models are conformal
- Find Landau-Ginzburg description