An asymptotic safety scenario for gauged chiral Higgs-Yukawa models

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Motivations

Triviality of the Higgs sector:
Landau pole for the Higgs self-interaction

$$\beta_\lambda^{1\text{loop}} = a\lambda^2, \qquad \lambda_\Lambda = \frac{\lambda_R}{1 - a\lambda_R \log\frac{\Lambda}{m_R}} \qquad (a > 0)$$

$$\Lambda \to \infty \iff \lambda_R = 0 \text{ trivial QFT}$$
$$\Lambda < \infty \iff \lambda_R \neq 0 \text{ effective QFT}$$

confirmed by lattice for pure scalar theories

Hierarchy between Electroweak and Planck/GUT scales:
Quadratic running of the Higgs mass (vev^2, $v = \langle\phi\rangle$)

$$\beta_\kappa \equiv \Lambda\partial_\Lambda\left(\frac{v^2}{2\Lambda^2}\right) = -2\kappa + \text{ interactions}$$

@ Gaussian FP ($\kappa = \lambda = 0$) linearized RG flow with eigenvalues $\{-2, 0\}$

$$\text{light Higgs masses} \iff \text{fine tuning in the UV}$$

in SM: what happens beyond perturbative expansions?

Asymptotic safety

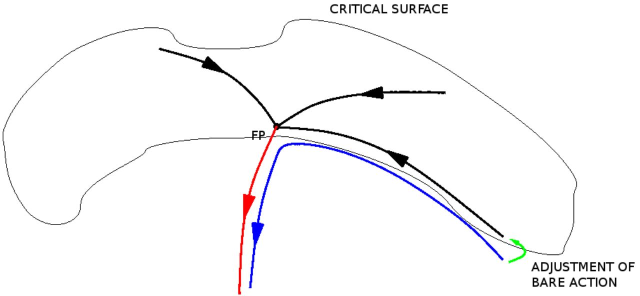
Building fundamental QFT's by means of RG FP G_{i*}

$$\Lambda \partial_\Lambda G_i(\Lambda)|_{G_*} = 0 \qquad G_i(\Lambda) - G_{i*} = e^{-\theta_i \log(\Lambda/\Lambda_0)} + \cdots$$

r) $\theta_r > 0$ RELEVANT = attractive
(local coordinates of the UV critical surface)

i) $\theta_i < 0$ IRRELEVANT = repulsive
(IR-dumped $\rightarrow$ IR universality)

m) $\theta_m = 0$ MARGINAL = nonlinear flow is needed



Asymptotic safety

Building fundamental QFT's by means of RG FP G_{i*}

$$\Lambda\partial_\Lambda G_i(\Lambda)\big|_{g_*} = 0 \qquad G_i(\Lambda) - G_{i*} = e^{-\theta_i \log(\Lambda/\Lambda_0)} + \cdots$$

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Asymptotic freedom: GFP $G_{i*} = 0$ with interacting relevant or marginally relevant directions

Asymptotic safety (Weinberg '76): NGFP $G_{i*} \neq 0$ with a finite number of relevant or marginally relevant directions
1) no triviality
2) hierarchy? if small critical exponents $\longrightarrow$ slow UV running
3) more predictive power than perturbation theory?
4) limited accessible window for IR parameters

The model

Yukawa systems as toy models of the SM and of GUT's

Triggering AS in SSB regime

$$\beta_\kappa = -2\kappa + \text{bosons' interactions} - \text{fermions' interactions}$$

quasi-conformal behaviour of the vev $\Longleftrightarrow$ boson dominance

Chiral Yukawa: $N_{\rm L} \times \psi_{\rm L}, \quad 1 \times \psi_{\rm R}, \quad 2N_{\rm L} \times \phi$
global $\mathrm{U}(1)_{\rm L} \times \mathrm{U}(1)_{\rm R}, \ \mathrm{SU}(N_{\rm L})_{\rm L}$

massless ϕ's do not decouple

Higgs: gauging $\mathrm{SU}(N_{\rm L})_{\rm L}$ to kill the Goldstones

$$S = \int d^d x \Big[\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) + U(\phi^\dagger \phi) + i(\bar{\psi}^a_{\rm L} \not{D}^{ab} \psi^b_{\rm L} + \bar{\psi}_{\rm R} \not{\partial} \psi_{\rm R}) + \bar{h}\, \bar{\psi}_{\rm R} \phi^{\dagger a} \psi^a_{\rm L} - \bar{h}\, \bar{\psi}^a_{\rm L} \phi^a \psi_{\rm R} \Big]$$

No $\mathrm{U}(1)_{\rm Y}$ gauge sector here!

fRG method

1PI functional RG (Wetterich '91)

1) Regularization: IR mass-like regulator $R_k(p^2)$

2) Average effective action Γ_k

$$k = 0 \longleftarrow \overset{k}{\longleftarrow} \longleftarrow k = \Lambda$$

1PI effective action — bare action

3) Flow equation $\partial_t \equiv k\partial_k$

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right]$$

4) Truncation

$$\Gamma_k[\Phi] = \sum_i \bar{G}_i(k)\mathcal{O}_i[\Phi] \longrightarrow \partial_t \bar{G}_i(k) = \bar{\beta}_i(\bar{G}_j, k)$$

e.g. derivative expansion $\mathcal{L}_k = U_k(\Phi) + Z_\Phi \partial_\mu \Phi \partial^\mu \Phi + \cdots$

5) Rescalings

$$G_i(t) \equiv \frac{k^{-D_i}}{Z_\phi^{n_\phi/2} \cdots Z_W^{n_W/2}} \bar{G}_i(k) \longrightarrow \partial_t G_i = \beta_i(G_j)$$

Threshold effects & β's

SSB: dynamically generated masses proportional to running couplings

$$\mu_W^2 = \frac{g^2\kappa}{2} \qquad \mu_t^2 = h^2\kappa \qquad \mu_H^2 = 2\lambda\kappa \qquad (N_L = 2)$$

ϕ^4 1-loop

$$\partial_t \lambda = \frac{1}{16\pi^2}\left\{12\lambda^2 + 4\lambda h^2 - 4h^4 - 9\lambda g^2 + \frac{9}{4}g^4\right\}$$

ϕ^4 fRG @NLO-derivative expansion

$$\partial_t \lambda = \frac{1}{16\pi^2}\left\{12\lambda^2\left(\frac{1}{4} + \frac{3}{4(1+\mu_H^2)^3}\right) + \frac{4\lambda h^2}{(1+\mu_t^2)^4} - \frac{4h^4}{(1+\mu_t^2)^3}\right.$$

$$\left. -9\lambda g^2\left(\frac{1}{2(1+\mu_W^2)} + \frac{1}{2(1+\mu_W^2)^2}\right) + \frac{9}{4}\frac{g^4}{(1+\mu_W^2)^3} + \cdots\right\}$$

Nonperturbative β's for:	λ	κ	h^2	g^2	and higher ϕ^{2n}
that is, for:	μ_H^2	μ_W^2	μ_t^2	g^2	and higher ϕ^{2n}

Non-Gaußian fixed points

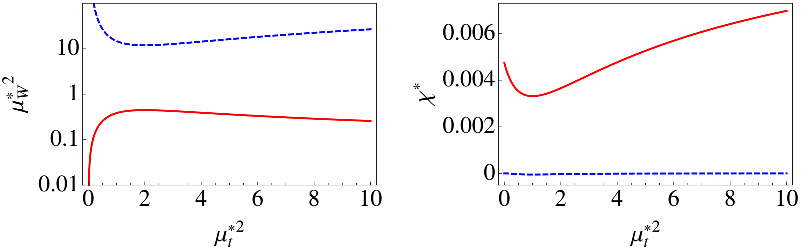
For $g^2 = 0$ also $\partial_t g^2 = 0$
Strategy: solve FP eqs. for matter sector at $g^2 \neq 0$ (and $\partial_t g^2 \neq 0$)
At least one solution exists

when $g^2 \to 0$, $\kappa_* \sim 1/g^2$ $\quad h_*^2 \sim g^2$ $\quad \lambda_* \sim g^4$

$\mu_{\mathrm{H}*}^2 \sim g^2$ $\quad 0 < \{\mu_{\mathrm{t}*}^2 , \mu_{W*}^2 , \chi \equiv \frac{\mu_{\mathrm{H}*}^2}{g^2}\} < \infty$

$\partial_t \mu_{\mathrm{t}}^2 = \frac{\mu_{\mathrm{t}}^2}{\mu_W^2}(\partial_t \mu_W^2)$ line of fixed points

higher $\phi^{2n} \sim g^{2n}$ flat potential



UV FP-regime

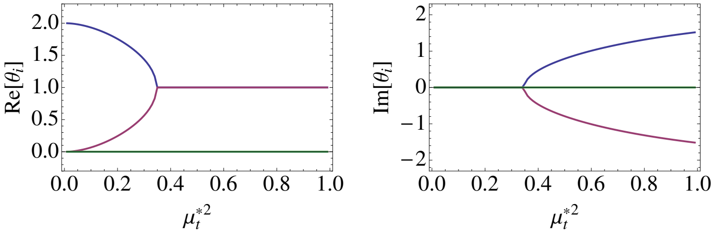
Linearized RG flow close to FP's

$$\partial_t G_i = B_i{}^j (G_j - G_j^*) + \dots \,, \qquad B_i{}^j = \left.\frac{\partial \beta_i}{\partial G_j}\right|_*$$

Critical exponents $\qquad \theta_i = $ eigenvalues of $(-B_i{}^j)$

For $G_i \equiv \{g^2, \mu_W^2, \mu_t^2, \chi\}$

$\theta_3 = \theta_4 = 0$ (marginally relevant+exactly marginal)
$\theta_1 = \theta_2^* \neq 0$ (relevant)



partial smoothening of hierarchy
no improved predictive power

higher ϕ^{2n} have canonical exponents (irrelevant)

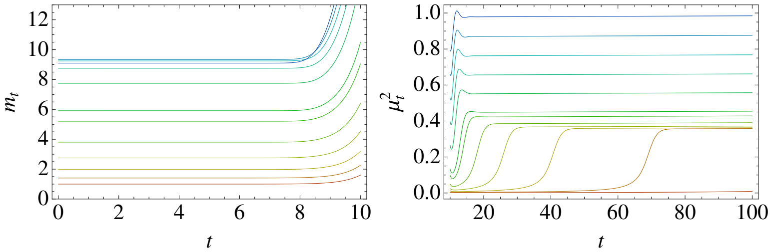
No triviality

@FP's all couplings vanish, the vev $\kappa \to \infty \quad \Longleftrightarrow \quad$ finite μ's

Interacting theory @NGFP's

1) @GFP massless gauge bosons and chiral fermions
   @NGFP dynamically generated masses
2) critical exponents different from GFP

Asymptotically safe trajectories connected to the Higgs phase

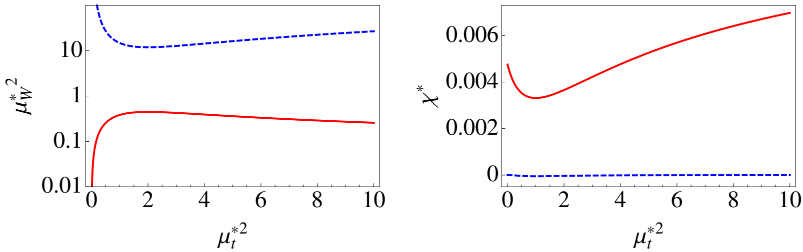


IR dimensionful freeze-out   UV dimensionless FP-regime

Flows to the electroweak scale

We neglect higher ϕ^{2n} & start the flow close to NGFP

@FP $\mu^2_{W*} \sim \mu^2_{t*}$ $\chi_* \sim 10^{-2}\mu^2_{t*}$



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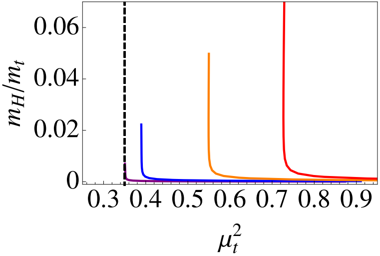
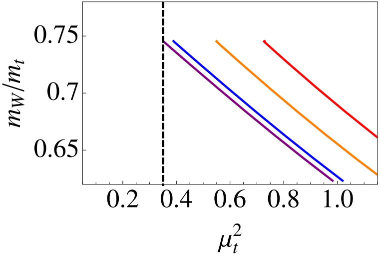
For initial conditions close to a specific FP we can get $m_H/m_t \sim 0.7$





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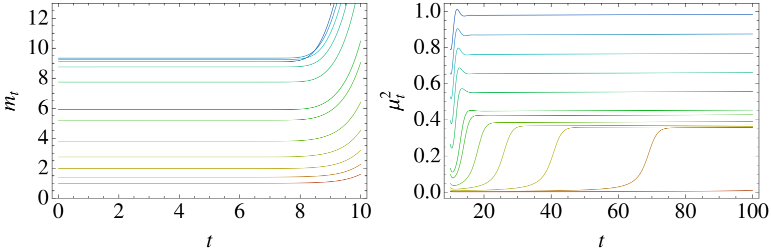
These tuned trajectories exhibit a 'walking' regime



Conclusions

1) We studied the nonperturbative RG flow of gauged chiral Higgs-Yukawa models, interesting as toy models of the Higgs-top sector of the SM or of GUT's, by means of the 1PI fRG

2) We found a line of NGFP's, characterized by a flat scalar potential and finite dynamically generated masses, which seems solid against the relaxation of the required nonperturbative approximations

3) These FP's allow for the construction of asymptotically safe theories corresponding to RG trajectories interpolating between a standard Higgs phase and a UV FP-regime

4) The corresponding theories would show no triviality problem, would have a milder hierarchy problem, and would contain as many free parameters as in the conventional perturbative definition of the model

5) We have performed non-exhaustive scans of the mass spectrum emerging from the flow to the Higgs phase along these renormalizable trajectories