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Time Evolution of the Large-Scale Tail of Primordial Magnetic Fields

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Contents

EGMF - Observation Status

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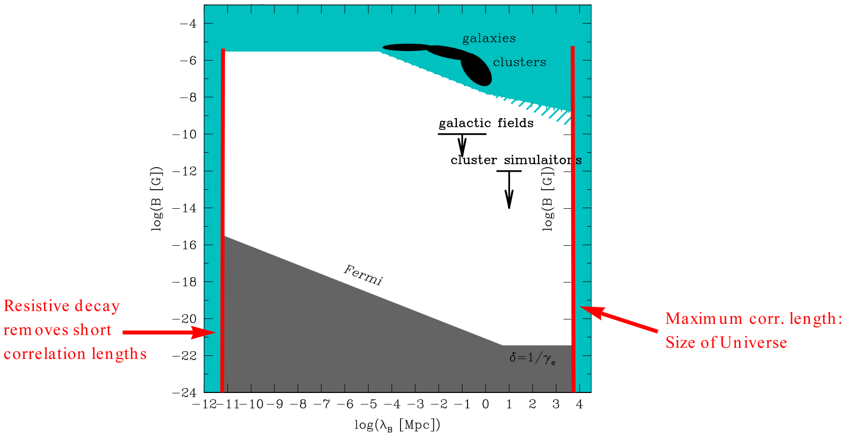

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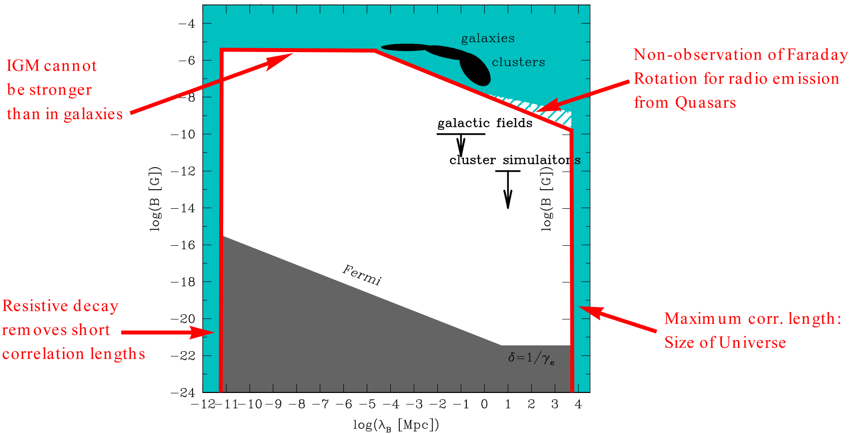

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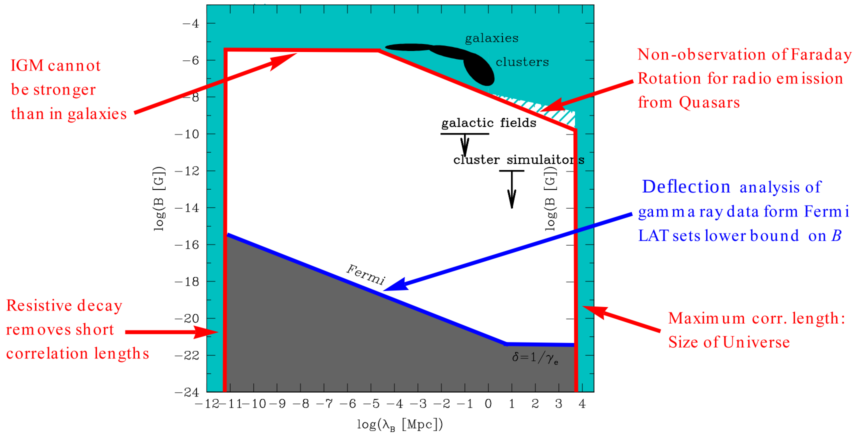

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- Cosmological scenario: Strong seed magnetic fields are generated in the Early Universe, e.g. at a phase transition (QCD, electroweak) [Sigl et al., 1997] or during inflation [Turner and Widrow, 1988], and some of the initial energy content is transfered to larger scales.

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- Basics for the time evolution: Homogeneous and isotropic magnetohydrodynamics in an expanding Universe.

Primordial Magnetic fields - Basic MHD

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- Navier-Stokes equations:
  $\rho \left(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\, \mathbf{v}\right) = -\nabla p + \mu \Delta \mathbf{v} + (\lambda + \mu) \nabla \left(\nabla \cdot \mathbf{v}\right) + \mathbf{f}$

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For the magnetic field and the turbulent fluid it follows therefore

$$\partial_t \mathbf{B} = \frac{1}{4\pi\sigma}\Delta \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})$$
$$\partial_t \mathbf{v} = -\left(\mathbf{v} \cdot \nabla\right)\mathbf{v} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho} + \mathbf{f}_v .$$

Primordial Magnetic fields - Basic MHD

The aspect of interest is the distribution of energies on different scales k, i.e. the magnetic spectral energy density M of the magnetic fields and the kinetic magnetic spectral energy density U

$$\epsilon_B = \frac{1}{8\pi V}\int \mathrm{d}^3x\, \mathbf{B}^2(\mathbf{x}) = \int \frac{\mathrm{d}^3k}{8\pi}\, |\hat{\mathbf{B}}(\mathbf{k})|^2 \equiv \rho \int \mathrm{d}k\, M_k$$

$$\epsilon_K = \frac{\rho}{2V}\int \mathrm{d}^3x\, \mathbf{v}^2(\mathbf{x}) = \frac{\rho}{2}\int \mathrm{d}^3k\, |\hat{\mathbf{v}}(\mathbf{k})|^2 \equiv \rho \int \mathrm{d}k\, U_k$$

$$h_B = \frac{1}{V}\int \mathrm{d}^3x\, \mathbf{A}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}) = i \int \mathrm{d}^3\mathrm{k}\, \left(\frac{\mathbf{k}}{\mathrm{k}^2} \times \widehat{\mathbf{B}}(\mathbf{k})\right) \cdot \widehat{\mathbf{B}}(\mathbf{k})^*$$

$$\equiv \rho \int \mathrm{d}k\, \mathcal{H}_k$$

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In Fourier space this means that the most general Ansatz is [von Kármán and Howarth, 1938, Junklewitz and Enßlin, 2011]

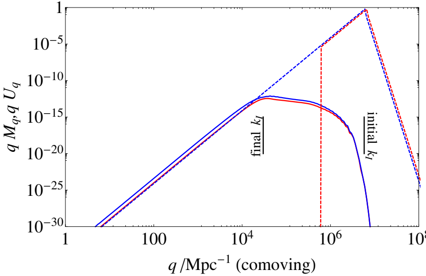
$$\langle \hat{B}_l(\mathbf{k})\hat{B}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm} - \frac{k_l k_m}{k^2})M(k) - \frac{i}{8\pi}\epsilon_{lmj}k_j\mathcal{H}(k)]$$

$$\langle \hat{v}_l(\mathbf{k})\hat{v}_m(\mathbf{k}')\rangle \sim \delta(\mathbf{k}-\mathbf{k}')[(\delta_{lm} - \frac{k_l k_m}{k^2})U(k) + i\alpha\epsilon_{lmj}k_j\mathcal{H}_f(k)]$$

Results on the Time Evolution of Primordial Magnetic Fields with Back-Reaction

[Saveliev et al., 2012]

- Starting either with an initial power-law ...



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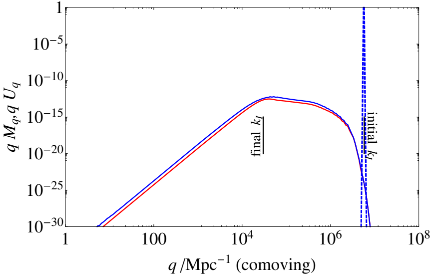
- Starting either with an initial power-law ...
- ... or a concentration of the spectral energies on a single scale the qualitative result is similar: a tendence to equipartition and both $M_q \sim q^4$ (i.e. $B \sim q^{\frac{5}{2}}$) and $U_q \sim q^4$ at large scales.



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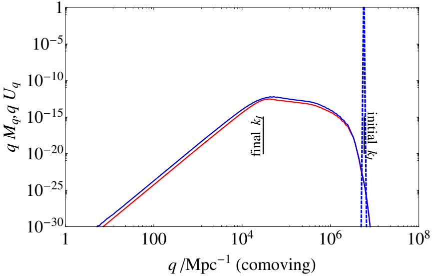

- A rough estimate for B (for the QCD phase transition) is given by $B(200\,\mathrm{pc}) \lesssim 5 \times 10^{-12}\,\mathrm{G}$

Results on the Time Evolution of Primordial Magnetic Fields with Back-Reaction

- Including magnetic helicity for the same initial conditions results in an inverse cascade, a fast transport of big amounts of magnetic energy to large scales. This is due to helicity conservation.

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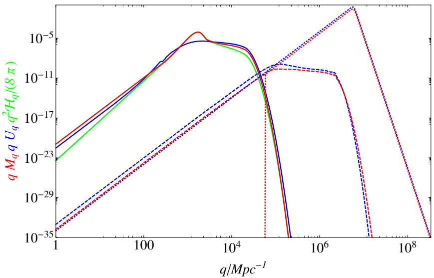

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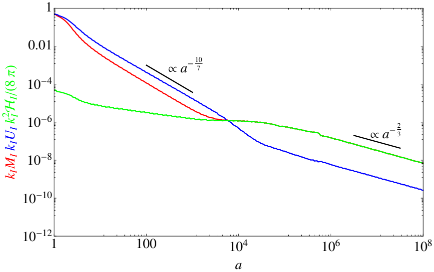

  - Two regimes are visible: When helicity is small, the considerations of the non-helical case are valid; once helicity reaches its maximal value, the behaviour changes dramatically

Conclusions and Outlook

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