Infrared conformal gauge theory on the lattice

Kari Rummukainen

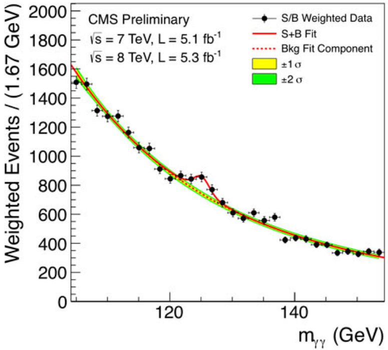
University of Helsinki and Helsinki Institute of Physics

DESY Theory Workshop 2013

Introduction:

- The Higgs particle has been found!
  - *the Standard Model is in excellent shape*
  - *Higgs field is centrally important: drives the EW symmetry breaking!*
  - *Scalar → theoretical problems:*

  ⇒ *naturalness, vacuum stability, unitarity bound . . .*
- There is still room (and need!) for BSM physics.
- Most BSM models aim to ameliorate the problems in the SM by e.g.
  - Pairing bosons with fermions (SUSY)
  - Cutoff (extra dimensions)
  - No scalars at all (**Technicolor** and related models)



Introduction: Conformal Window

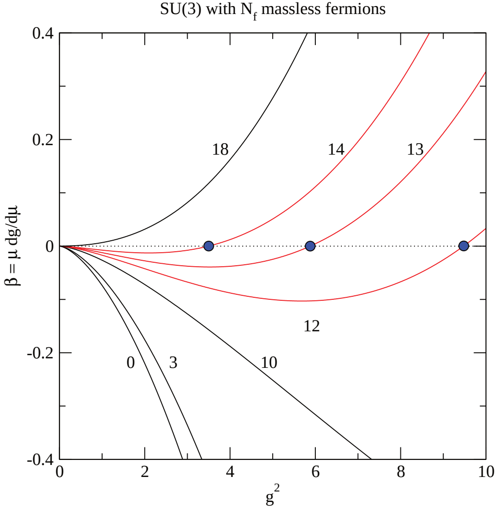
Consider *2-loop perturbative* β-function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

Generically 3 different behaviours:

- Small N_f: running coupling, confinement and χSB (QCD-like)
- Medium N_f: IR fixed point, no χSB [Banks,Zaks]
- Large N_f: Asymptotic freedom lost

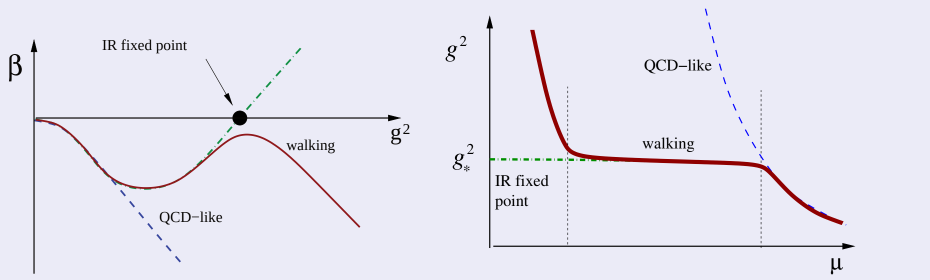
Conformal window: range of N_f where IRFP exists



Walking coupling

- What happens when we approach the lower edge of the conformal window from below?
- Competition between IR conformal behaviour and non-perturbative confinement/χSB
- The β-function may get close to zero at finite coupling

$\Rightarrow$ The coupling evolves slowly, *walks*.



- Strong coupling: perturbation theory not applicable, lattice simulations needed

Motivation

- Building block for **Walking Technicolor** models
- Theoretical curiosity: strongly coupled IR conformal/almost conformal phase, "unparticles", sQGP
- Lot of recent activity both on and off the lattice
- On LATTICE 2013 conference, 38 contributions on this topic!

Technicolor

- Technigauge + massless techniquarks Q
- Techniquarks have both technicolor and EW charge (exactly like quarks in the SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale: $\Lambda_{\rm TC} \sim f_{\rm TC} \sim \Lambda_{\rm EW}$
- After chiral symmetry breaking looks like SM:

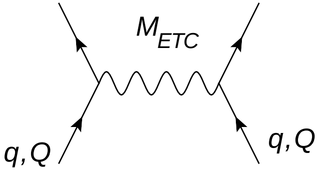
$\Rightarrow$

decay constant $f_{\rm TC}$	$\leftrightarrow$	Higgs expectation value v
scalar $\bar{Q}Q$ σ-meson	$\leftrightarrow$	Higgs particle
Goldstone pseudoscalars, "pions"	$\leftrightarrow$	W,Z -longitudinal modes
exotic technihadrons (observable!)		

- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Does *not* explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

Extended technicolor

- In addition to the "pure" technicolor, introduce a new higher-energy interaction coupling Standard Model fermions q (quarks, leptons) and techniquarks (Q): extended technicolor (ETC)
  Several options, e.g. massive gauge boson, $M_{\rm ETC}$:



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- $\frac{1}{M_{\rm ETC}^2}\bar{Q}Q\bar{q}q \longrightarrow$ SM fermion mass $m_q \propto \frac{1}{M_{\rm ETC}^2}\langle\bar{Q}Q\rangle_{\rm ETC}$
- $\frac{1}{M_{\rm ETC}^2}\bar{q}q\bar{q}q \longrightarrow$ extra FCNC's (harmful!)
- $\frac{1}{M_{\rm ETC}^2}\bar{Q}Q\bar{Q}Q \longrightarrow$ explicit χSB in the techniquark sector

$\langle\bar{Q}Q\rangle_{\rm ETC}$: condensate evaluated at the ETC scale
$\langle\bar{Q}Q\rangle_{\rm EW}$: condensate at TC∼EW) scale

Extended technicolor

I) In order to avoid unwanted FCNC's need to push $\Lambda_{\rm ETC} \approx M_{\rm ETC} \gtrsim 1000 \times \Lambda_{\rm EW} (\Lambda_{\rm TC} \approx \Lambda_{EW})$

II) For EWSB we must have $\langle \bar{Q} Q \rangle_{\rm EW} \propto \Lambda_{\rm EW}^3$

III) On the other hand, $\langle \bar{Q} Q \rangle_{\rm ETC} \propto m_q M_{\rm ETC}^2$ (top quark!)

- Using RG evolution

$$\langle \bar{Q} Q \rangle_{\rm ETC} = \langle \bar{Q} Q \rangle_{\rm EW} \exp \left[\int_{\Lambda_{\rm EW}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu \right]$$

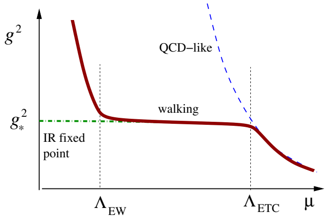
  where $\gamma(g^2)$ is the mass anomalous dimension.

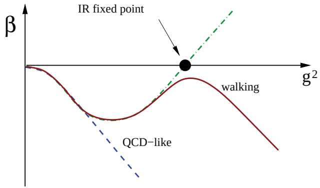
- In weakly coupled theory γ is small, and $\langle \bar{Q} Q \rangle$ is $\sim$ constant.
- *Thus, it is not possible to satisfy the constraints I), II), II) in a QCD-like theory, where the coupling is large only on a narrow energy range above χSB.*

Walking coupling

- If the coupling *walks*, i.e. if $g^2 \approx g_*^2$ (constant) over the range from TC to ETC, then we can solve $\langle \bar{Q}Q \rangle_{\mathrm{ETC}} \approx \left(\frac{\Lambda_{\mathrm{ETC}}}{\Lambda_{\mathrm{TC}}} \right)^{\gamma(g_*^2)} \langle \bar{Q}Q \rangle_{\mathrm{TC}}$ (condensate enhancement)
- Inserting II) and III) we obtain

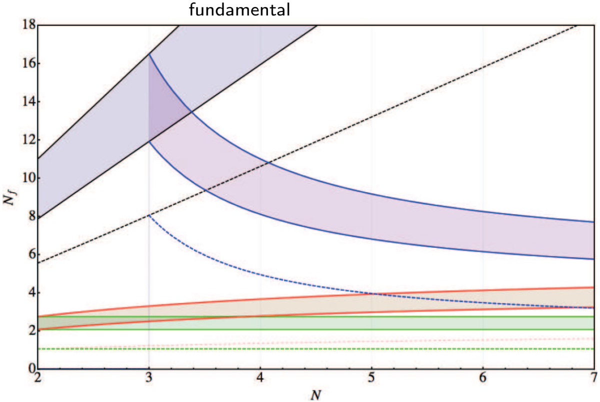
$$\gamma(g_*^2) \approx 1 - 2$$





- Walking → Higgs naturally light, "dilatonic"

Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

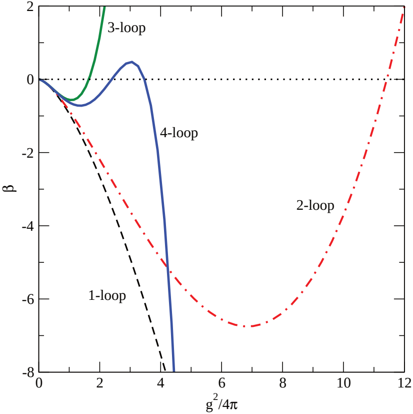
2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- *In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich]* → recent interest

Existence of the IRFP essentially non-perturbative

Example: Perturbative β-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions

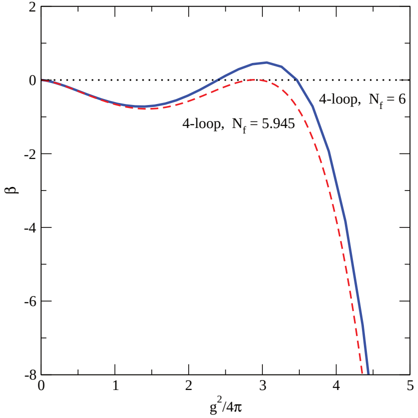


[4-loop MS: Ritbergen, Vermaseren, Larin]

Results from lattice: existence of IRFP inconclusive

"Walking" at $N_f \lesssim 6$

Interestingly, the fixed point vanishes from 4-loop MS beta function if N_f is slightly lowered from 6:



Goals:

Take SU(N) gauge theory with N_f fermions in some representation.

- Locate the lower edge of the conformal window
- Measure $\beta(g^2)$-function
- Measure $\gamma(g^2)$
- We want to find a theory which
  - is walking or
  - is just within conformal window (easy to deform into walking)
  - has large anomalous exponent γ near FP
    - AdS-QFT: Indications that $\gamma = 1$ at the lower edge of the conformal window [Järvinen et al.]
  - Has "light" scalar (Higgs) – walking helps!
  - Compatible with EW precision measurements (S,T,U -parameters) $\rightarrow$ small N_f preferred
- Technicolor phenomenology: SU(2) or SU(3) gauge theory with $N_f = 2$ adjoint or 2-index symmetric representation fermions are favoured.
- "Hadron" spectrum, chiral symmetry breaking pattern

Models studied

Red: conformal   Blue: χSB   Black: unclear

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: Appelquist et al; Deuzeman et al; Fodor et al; Jin et al; Aoki et al; Schaich et al; Gelzer et al
  - $N_f = 9$: Fodor et al
  - $N_f = 10$: Hayakawa et al; Appelquist et al
  - $N_f = 12$: Hasenfratz; Appelquist et al; Deuzeman et al; Xin and Mawhinney; Fodor et al; Okawa et al; Aoki et al; Cheng et al; Itou; Lin et al; Gelzer et al
  - $N_f = 16$: Damgaard et al; Heller; Hasenfratz; Fodor et al; Deuzemann et al
- SU(2) + fundamental rep fermions:
  - $N_f = 4$: Karavirta et al
  - $N_f = 6$: Del Debbio et al; Karavirta et al; Appelquist et al; Tomii et al; Voronov et al
  - $N_f = 8$: Iwasaki et al; Lin et al
  - $N_f = 10$: Karavirta et al

Models studied

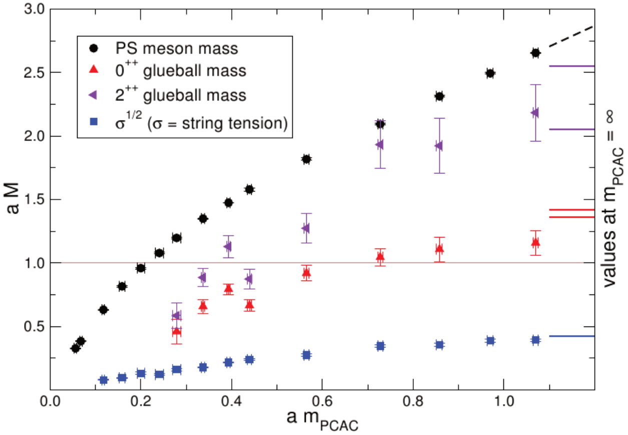
Red: conformal Blue: χSB Black: unclear

- SU(2) + $N_f = 1$ adjoint rep: Athenodorou et al
- SU(2) + $N_f = 2$ adjoint rep: Catterall et al; Bursa et al; Hietanen et al; Rantaharju et al; De Grand et al; Del Debbio et al; August and Maas; Arthur et al
- SU(3) + $N_f = 2$ 2-index symmetric rep: DeGrand et al; Sinclair and Kogut; Fodor et al
- SU(3) + $N_f = 2$ adjoint rep: DeGrand et al
- SU(4) + $N_f = 2$ 2-index symmetric rep: DeGrand et al
- SU(4) + $N_f = 6$ 2-index antisymmetric rep: DeGrand et al
- SO(4) + $N_f = 2$ vector rep: Hietanen et al

Classifying conformal / χ SB ?

- Measure β-function directly
  - Schrödinger functional
  - MCRG
  - Gradient flow methods [Talk by Ramos]
- Measure technihadron and glueball masses and string tension as functions of the techniquark mass m_Q:
  - Non-zero m_Q takes us away from the (possible) IRFP
  - Conformal: $M \propto m_Q^{1/(1+\gamma)}$, incl. string tension
  - χSB: $M_\pi \propto m_Q^{1/2}$, others remain massive
- Dirac operator eigenvalue distribution: scales with γ

Example: mass spectrum of SU(2)+$N_f = 2$ adjoint fermions

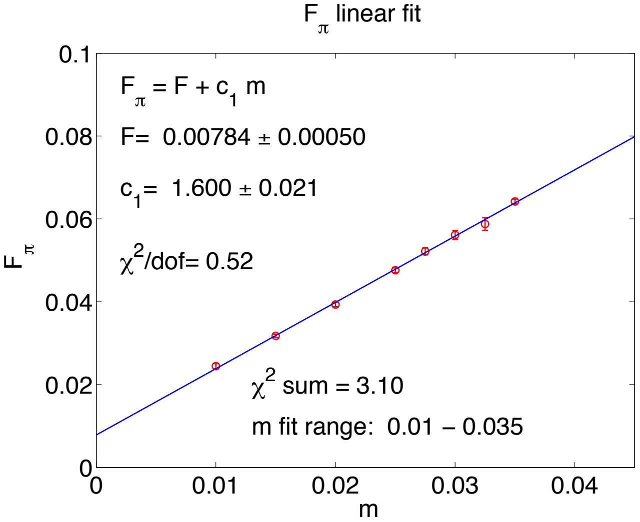


[Del Debbio et al 09]

Masses vanish with $M \propto m_Q^{1/(1+\gamma)}$ [Miransky]
More recent measurements, taking into account *Finite volume scaling:* $\gamma \approx 0.371$
[Del Debbio et al, Lattice 2013]

SU(3) $N_f = 12$ spectrum

F_π: non-zero intercept as $m_Q \to 0$? Looks QCD-like (χSB)



[Fodor, Holland, Kuti, Nogradi, Schroeder, 2011]

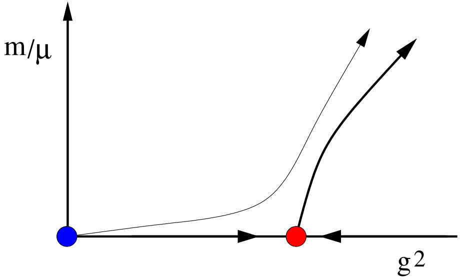
Mass spectrum measurement

- If the massless $m_Q = 0$ theory has an IRFP, excitation masses $M \propto m_Q^{1/(1+\gamma)}$
- Excitation size $\propto M^{-1}$, diverges as $m_q \to 0$ [Del Debbio and Zwicky]
- All excited states in a given channel become massless $\to$ excitation spectrum $\sim$ continuous, "unparticles".

$\to$ great care needed in mass spectrum measurment – not yet fully under control.

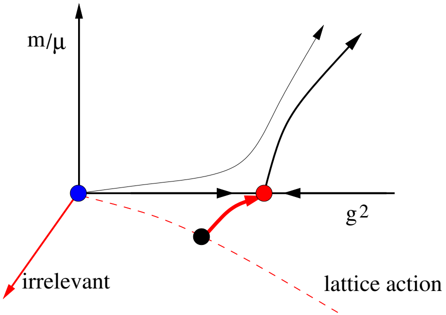
RG flow in the conformal case

- Relevant parameters at UV: g^2 and m_Q



- Only m_Q is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$

RG flow on the lattice



- Irrelevant operators (cutoff effects) die out as $(a/L)^x$, (x: some scaling exponent, L: IR scale)
- Evolution of g^2 along the physical axis very slow

$\Rightarrow$ irrelevant lattice effects can (and do!) mask the physical evolution

- Need either:
  - Very large lattices (large L/a) – impractical
  - Very high quality lattice action – small cutoff effects
    - Clover improvement, stout/nHYP smeared action

Dirac operator eigenvalues

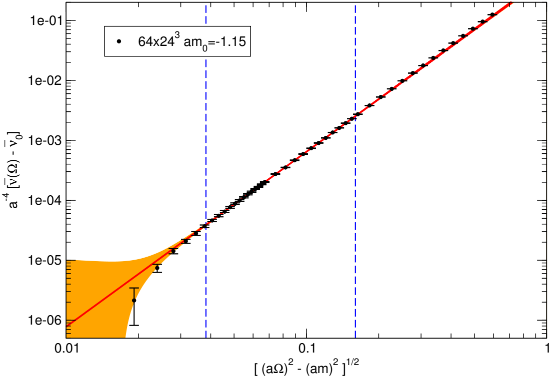
Eigenvalue density of the lattice $(D^\dagger D + m^2)$: [DeGrand; Del Debbio and Zwicky; Patella]

$$\langle \bar{Q} Q \rangle \propto m^\eta \Leftrightarrow \rho(\lambda) \propto \lambda^\eta$$

$\Rightarrow$ Mode number

$$\nu(\Omega) = C + (\Omega^2 - m^2)^{2/(1+\gamma)}$$

In SU(2) + $N_f = 2$ adjoint rep fermions, $\gamma = 0.37(2)$:



[Patella]

Measuring the coupling

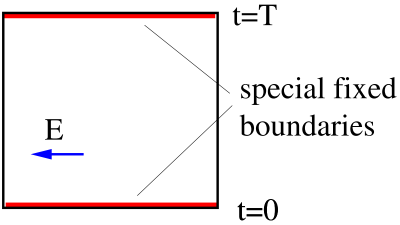
Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised with a **twist angle** η

At the classical level, we have

$$\frac{dS_{\rm class.}}{d\eta} = \frac{A}{g^2}$$

where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\frac{1}{g^2_{\rm SF}} = \left\langle \frac{1}{A}\frac{dS}{d\eta} \right\rangle$$



- Evaluates $g^2_{\rm SF}$ directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Step scaling function

- Step scaling: coupling when the lattice size is (e.g.) doubled

$$\Sigma(u, L/a) = g^2(g_0^2, 2L/a)_{u=g^2(g_0^2, L/a)}$$

- Continuum limit:

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, L/a)$$

- Step scaling is related to β-function:

$$-2\ln 2 = \int_u^{\sigma(u)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

- Close to the fixed point:

$$\beta(g) \approx \frac{g}{2\ln 2}\left(1 - \frac{\sigma(g^2)}{g^2}\right)$$

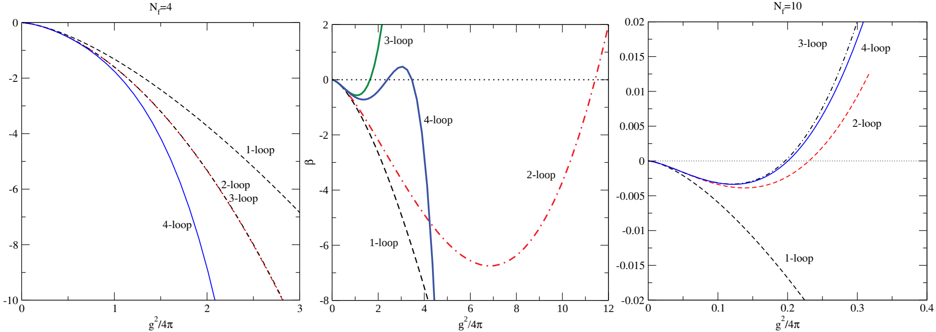
- 1-loop analysis indicates that finite lattice spacing effects large ($\sim 50\%$ at $L/a = 10$) $\Rightarrow$ improvement! [Alpha; Karavirta et al.]

Fundamental rep SU(2) with $N_f = 4, 6$ and 10

- Measure coupling using SF
- Measure also γ using SF (different boundary condition)
- Choose:
  - $N_f = 4$: QCD-like, chiral symmetry breaking
  - $N_f = 6$: close to lower edge of conformal window?
  - $N_f = 10$: upper edge of conformal window
- Wilson-clover action with 1-loop perturbative $c_{\rm SW}$, and with perturbative boundary improvement coefficients
- Data from [Karavirta et al, 2012]

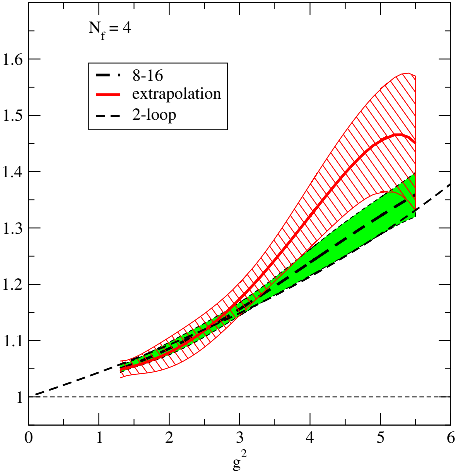
Fundamental rep: perturbation theory

Perturbative β-function w. $N_f = 4, 6, 10$ [3,4-loop MS: Ritbergen, Vermaseren, Larin]



$N_f = 4$ QCD-like, confining
$N_f = 6$ completely non-perturbative
$N_f = 10$ perturbative Banks-Zaks FP, "test case".

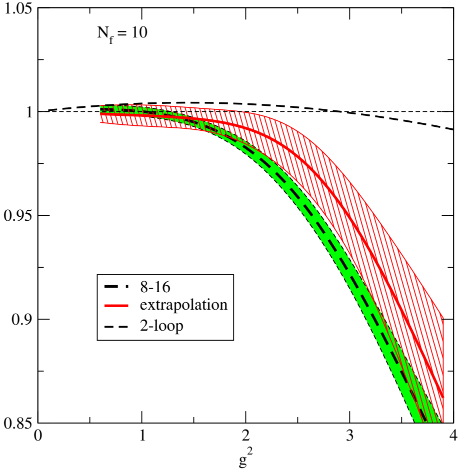
Step scaling function: $N_f = 4$



Step scaling > 1: coupling grows as length L grows: QCD-like behaviour

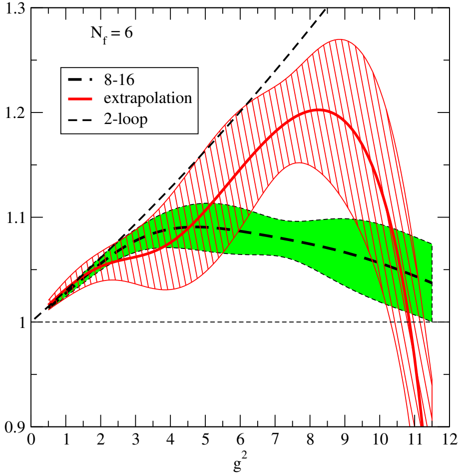
$$\beta(g) \approx \frac{g}{2\ln 2}\left(1 - \frac{\sigma(g^2)}{g^2}\right)$$

Step scaling function: $N_f = 10$



- Cannot resolve the slow evolution below $g^2_{\rm SF} \sim 2.5$ – not accurate enough
- Above this step scaling diverges from perturbative curve.
- Strong coupling, lattice artefact?

Step scaling function: $N_f = 6$

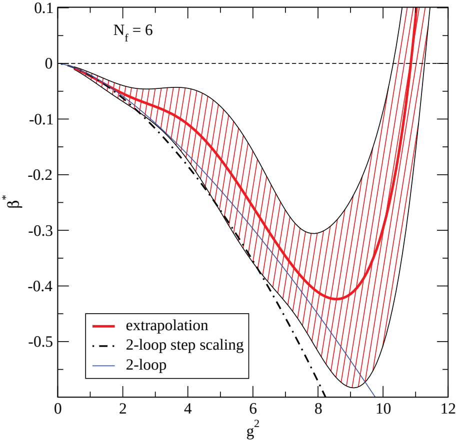


- Does this suggest IRFP at $g^2_{\mathrm{SF}} \gtrsim 12$ ($\alpha \gtrsim 1$)?
- However, control is lost at $g^2_{\mathrm{SF}} \sim 12 - 14$ ($\beta_L \approx 1.39$)

$\Rightarrow$ Result not reliable

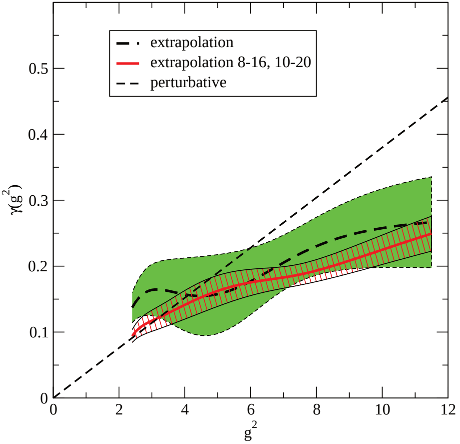
- Need to have actions which work at strong coupling

Estimate of the β-function: $N_f = 6$



Result: $N_f = 6$ mass anomalous exponent

- Determined using SF (different boundary conditions)

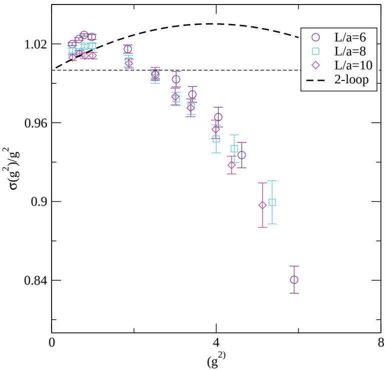


- More robust measurement than coupling
- Smaller than perturbative at strong coupling – generic feature?

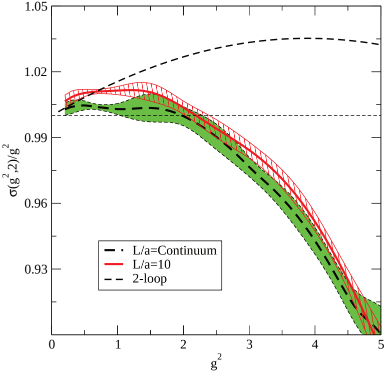
Schrödinger functional in SU(2)+$N_f = 2$ adjoint fermions

HEX-smeared Wilson-Clover [Rantaharju et al, LATTICE 2013; preliminary]
Step scaling with volume pairs $L/a = 6$-12, 8-16, 10-20

Raw step scaling



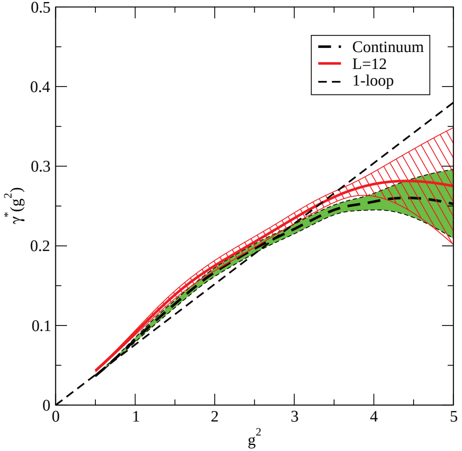
Interpolation + continuum extrap.



Questionable continuum extrapolation? Lattice artifacts on small volumes?
Largest volume: IRFP at $g^2_{\rm SF} = 2 - 2.5$. Compatible with previous results [Hietanen et al; Del Debbio et al; DeGrand et al].

Schrödinger functional in SU(2)+$N_f = 2$ adjoint fermions

- Mass scaling exponent γ:



- At IRFP $g_{\rm SF}^{*2} \approx 2.25$, $\gamma(g_{\rm SF}^{*2}) \approx 0.2$
- Smaller than the one from the eigenvalue spectrum $\gamma^* \approx 0.37$ [Patella]
- Caused by different estimates of the critical coupling

What do the results imply?

- Lattice analysis is significantly more difficult than in QCD-like theories:
  - Slow evolution $\rightarrow$ small signal
  - Slow evolution $\rightarrow$ strong bare coupling
  - $\Rightarrow$ Conflicting results, unknown systematics
- Improvement is needed: better actions, better methodology, understanding of limitations
- Schrödinger functional is running out of steam at $(L/a)^4 \approx 24^4$: Noisy signal, huge statistics required (several $\times 10^5$ trajectories per point)
- Gradient flow is a promising new tool
  - Can reach larger volumes than with SF - potentially less lattice artifacts
  - talk by Ramos
  - Preliminary studies in this context by Nogradi et al; Fritzsch and Ramos; Rantaharju; Cheng et al

Conclusions

- Status of the field: early days still. No full consensus yet of the “best practices” – getting there
- Mapping out the theory space: IRFP, χSB
- Walking has not been unambiguously observed (except in toy models)
- Not yet clear quantitative phenomenology: Higgs and exotica masses, branching ratios . . .
- Theories considered in isolation: coupling to EW?
  - Has an effect on the physics
  - Axial gauge coupling: we do not even know how to do it!

Walking in 2d O(3)

2-d O(3) model with topological charge [de Forcrand, Pepe, Wiese]

$$S = \frac{1}{2g^2} \int d^2x \, \partial_i u_a \partial_i u_a + i\theta Q$$

with $|u| = 1$

$$Q = \int d^2x \epsilon_{ij} \epsilon_{abc} U_a \partial_i U_b \partial_j U_c$$

- asymptotically free
- mass gap
- has a IR fixed point at $\theta = \pi$! (integrable model)
- Adjusting θ the degree of "walking" can be changed

Walking in 2-d O(3)