DESY THEORY WORKSHOP
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Nonperturbative QFT: Methods and Applications

DESY Hamburg, Germany

HELMHOLTZ ASSOCIATION

DESY

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Two-dimensional models on the world sheet of non-Abelian strings

with A. Yung ...

★ Hanany-Tong, 2003

★ ★ Auzzi et al., 2003 }

★ ★ ★ Shifman-Yung, 2003 - ...

⋮

❖ Gaiotto, 2012 & Gaiotto, Gukov, Seiberg, 2013 "surface defects"...

Outline: a) Non-Abelian BPS strings in SYM

b) World sheet models from $\mathcal{N}$=2 bulk

c) World sheet models from $\mathcal{N}$=1 bulk

★ ★ ★ ★ $\mathcal{N}=2 \to \mathcal{N}=(2,2)$

$\mathcal{N}=1 \to \mathcal{N}=(2,0)$ nonminimal (and minimal)

$\mathcal{N}=0 \to \mathcal{N}=(0,0)$ (no SUSY)

bulk   2D world sheet

Prototype model: N_f=N=2; $\mathcal{N}$ =2

$$S = \int d^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}|D_\mu a^a|^2 \right.$$

$$+ \quad \mathrm{Tr}\,(\nabla_\mu \Phi)^\dagger\,(\nabla^\mu \Phi) + \frac{g_2^2}{2}\left[\mathrm{Tr}\,\left(\Phi^\dagger T^a \Phi\right)\right]^2 + \frac{g_1^2}{8}\left[\mathrm{Tr}\,\left(\Phi^\dagger \Phi\right) - N\xi\right]^2$$

$$\left. + \quad \boxed{\frac{1}{2}\mathrm{Tr}\,\left|a^a T^a\,\Phi + \Phi\,\sqrt{2}M\right|^2} + \frac{i\,\theta}{32\,\pi^2}\,F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \right\},$$

$$\Phi = \begin{pmatrix} \varphi^{11} \varphi^{12} \\ \varphi^{21} \varphi^{22} \end{pmatrix}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}$$

U(2) gauge group, 2 flavors of (scalar) quarks

SU(2) Gluons A^a_μ + U(1) photon + gluinos+ photino

$$\Phi = \sqrt{\xi} \times I$$

Basic idea:

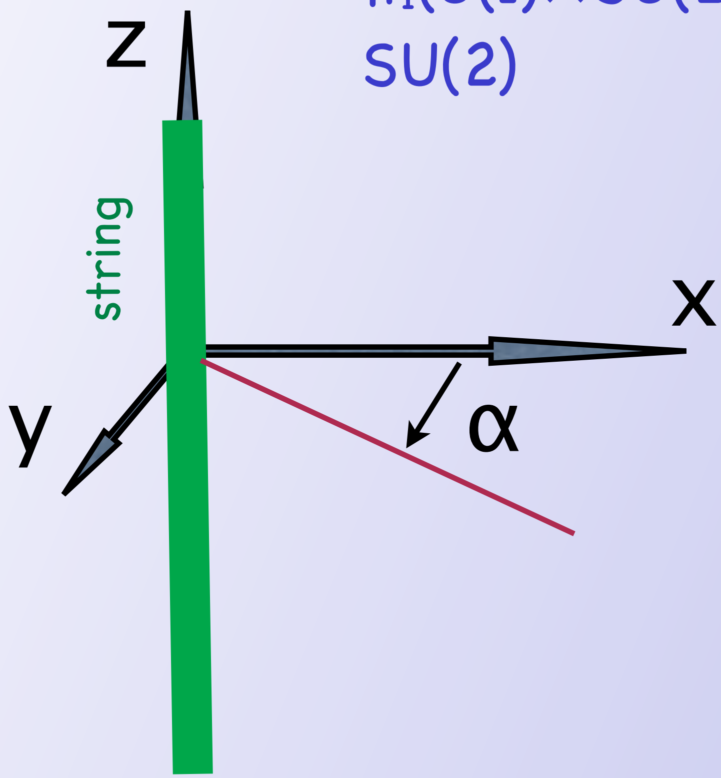
- **Color-flavor locking in the bulk → Global symmetry G;**
- **G is broken down to H on the given string;**
- G/H coset; **G/H sigma model on the world sheet.**

☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ -1; another -1 rotate by π in U(1)



$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)

ANO $\sqrt{\xi}\, e^{i\alpha}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$T = 4\pi\xi$

Non-Abelian $\sqrt{\xi}\begin{pmatrix}e^{i\alpha} & 0\\ 0 & 1\end{pmatrix}$

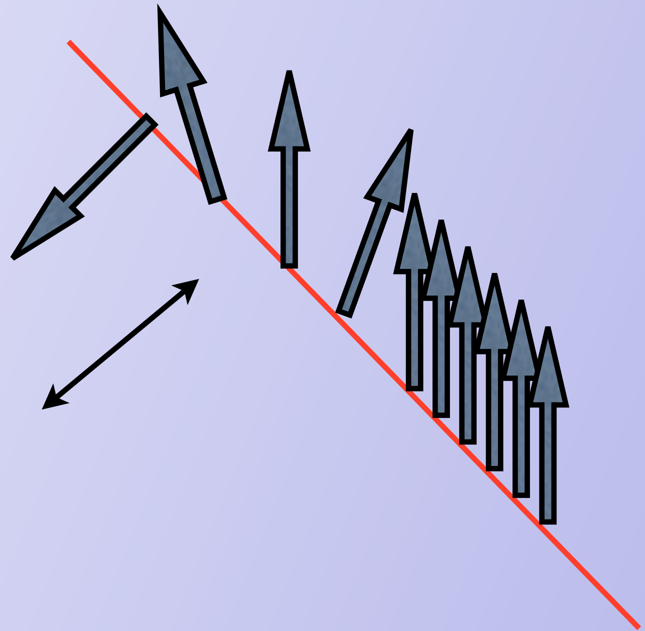
$T_{U(1)} \pm T^3{}_{SU(2)}$

x_0 ← string center in perp. plane

$T = 2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation



classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

✵ ✵

Versions of CP(N-1) models in 2D: nonsupersymmetric and supersummetric - with twisted mass and Z_N symmetry

$\mathcal{N}$ = (2.2) and (0,2) (2.2)

★ Gauged formulation ★ (Witten, 1979)

1. Non-SUSY bulk → no SUSY in 2D

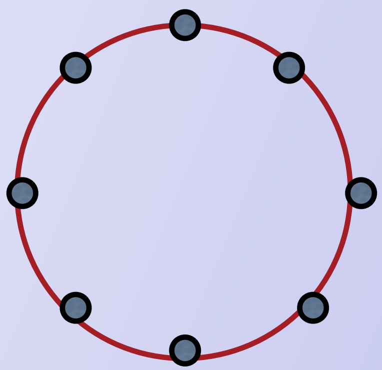
$$S^{(1+1)} = \int dt\, dz \left\{ 2\beta\, |\nabla_\alpha n|^2 + \frac{1}{4e^2} F^2_{\alpha\gamma} + \frac{1}{e^2} |\partial_\alpha \sigma|^2 \right.$$

$$\left. + \; 4\beta \left| \left(\sigma - \frac{m_\ell}{\sqrt{2}} \right) n^\ell \right|^2 + 2e^2\beta^2 \left(|n^\ell|^2 - 1 \right)^2 \right\}$$

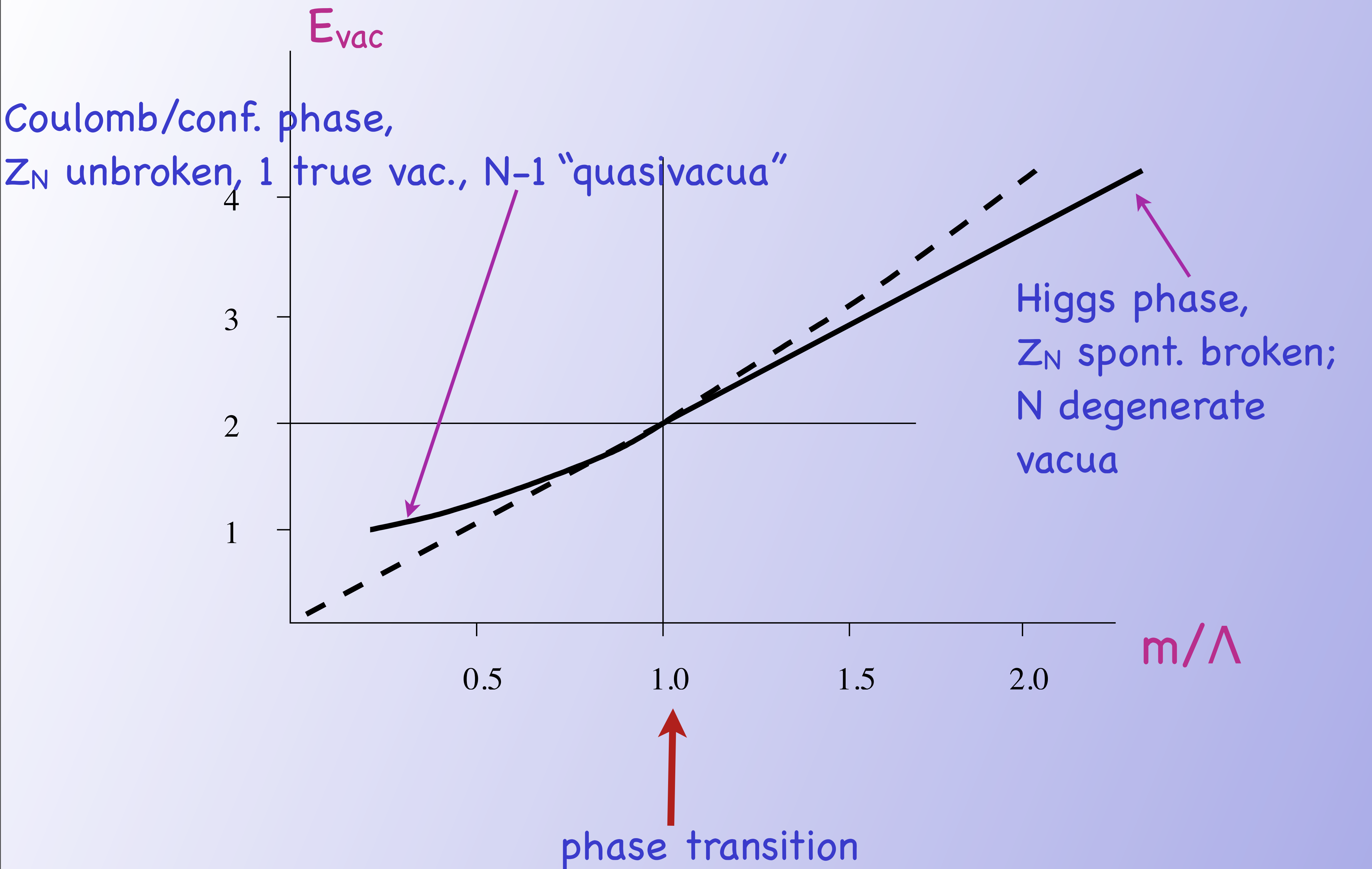
$$\nabla_\alpha = \partial_\alpha - i A_\alpha$$

m/Λ

Z_N symmetry



m~ $e^{2\pi i/N}, \; e^{4\pi i/N}, \; \ldots, \; e^{2(N-1)\pi i/N}, \; 1$


E_{vac}
Coulomb/conf. phase,
Z_N unbroken, 1 true vac., N-1 “quasivacua”
4
3
2
1
0.5
1.0
1.5
2.0
m/Λ
Higgs phase,
Z_N spont. broken;
N degenerate
vacua
phase transition

GSY hep-th/0601131

2. Introduction of 2D axion restores Z_N and eliminates Coulomb/confinement phase

$$\mathcal{L}_a = f_a^2\,(\partial_\mu a)^2 + \frac{a}{2\pi}\,\varepsilon_{\alpha\gamma}\partial^\alpha A^\gamma$$

Photon is (2D) Higgsed

3. $\mathcal{N} = 2$ SUSY bulk

Hanany-Tong 2004

SY hep-th/0403149

BSY ArXiv:1308.4494

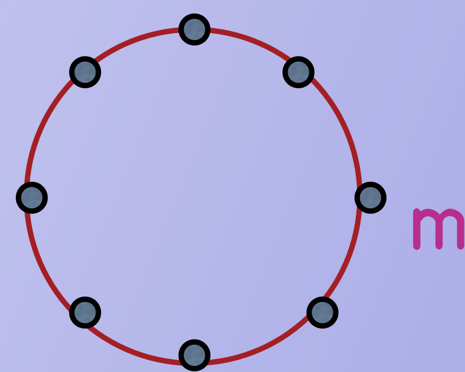
BSY arXiv:1001/1757

N = (2,2) CP(N-1) model

$$\mathcal{L} = \frac{1}{e_0^2}\left(\frac{1}{4}F_{\mu\nu}^2 + |\partial_\mu \sigma|^2 + \frac{1}{2}D^2\right) + i\,D\,\left(\bar{n}_i n^i - 2\beta\right)$$

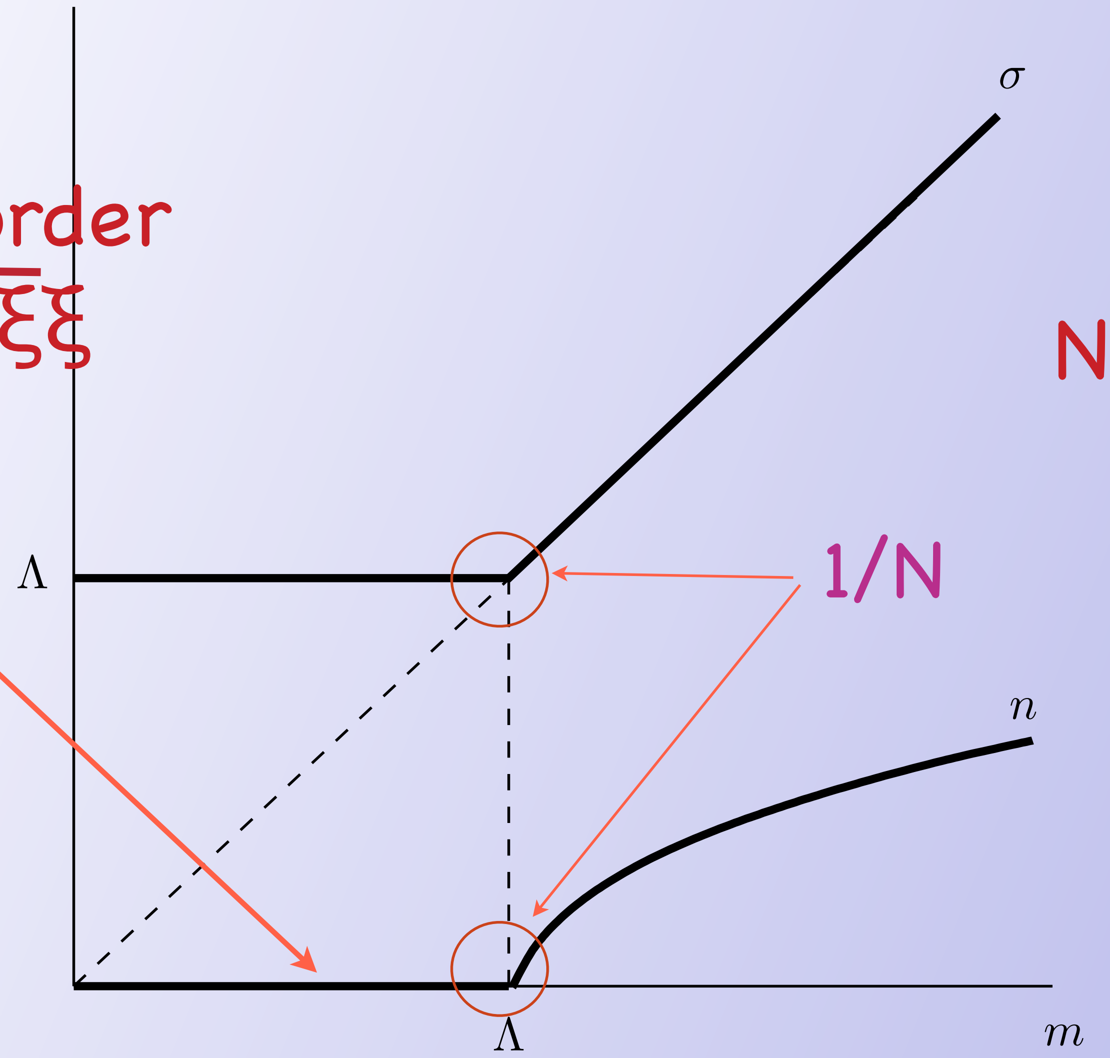
$$+ \left|\nabla_\mu n^i\right|^2 + 2\sum_i \left|\sigma - \frac{m_i}{\sqrt{2}}\right|^2 |n^i|^2$$

+ fermions



Bifermion order parameter $\bar{\xi}\xi$

No confinement



$E_{vac}=0$ always, SUSY unbroken,

Z_N always broken, (N degenerate vacua)

Crossover instead of phase transition

Strong-coupling ↔ weak coupling Higgs regime

☆ An interesting not yet fully resolved question

(SY + S. Gukov, in progress)

Direct (exact) large-N one-loop calculation:

SY arXiv:0803/0698

$$V_{\rm eff} \;=\; \int d^2x\, \frac{N}{4\pi}\left\{-\left(iD+2|\sigma|^2\right)\ln\frac{iD+2|\sigma|^2}{\Lambda^2}+iD\right.$$

$$\left.+\;\; 2|\sigma|^2\,\ln\frac{2|\sigma|^2}{\Lambda^2}+2|\sigma|^2\,u\right\},$$

vesrus exact Veneziano-Yankielowicz superpotential of σ log σ type

BSY ArXiv:1104.5241

BSY ArXiv:1202.5612

4. BPS Spectrum of SUSY CP(N-1) with Z_N twisted masses (curves of marginal stability)

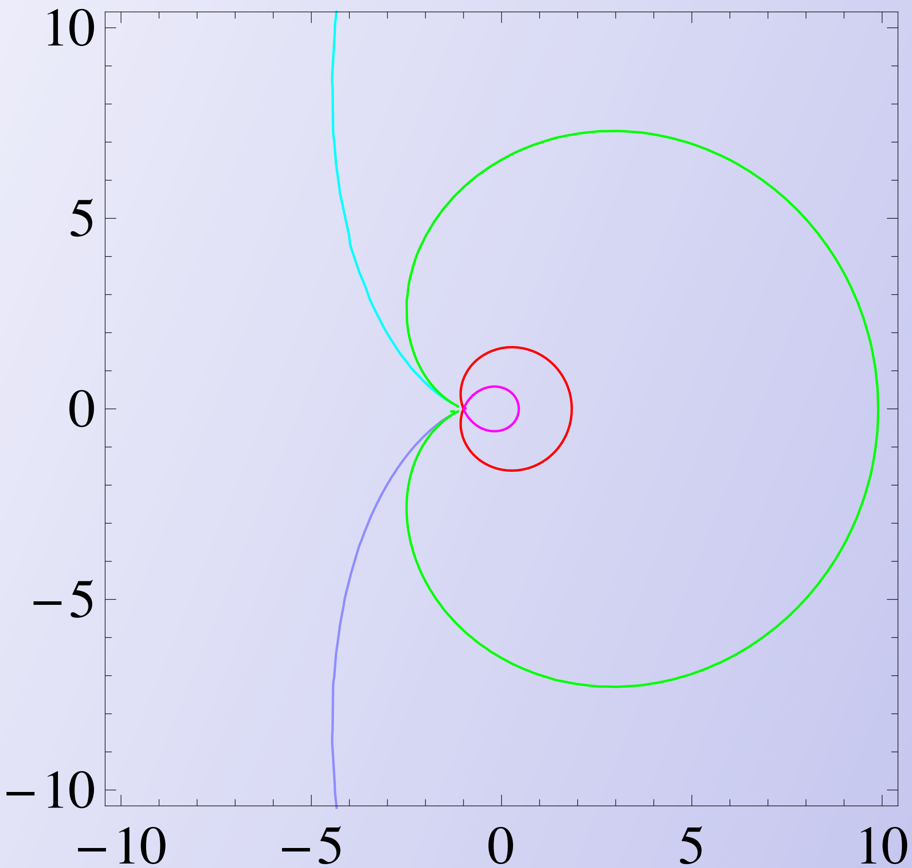


Figure 10: The decay curves of CP^2 in m_0^3 plane. The primary curve is shown in red. The two vertical whiskers are the initial coils of the two spirals

5. $\mathcal{N} = 1$ SUSY bulk

↓

$\mathcal{N} = (0,2)$ CP(N-1) model

Edalati-Tong

SY arXiv:0803/0158

SY arXiv:0803/0698

BSY arXiv:0901/4603

BSY arXiv:0907/2715

SY arXiv:1005/5264

BSY arXiv:1001/1757

Supersymmetry is broken, generally speaking !!!

Phase transitions possible and do occur * * *

All phase transitions are of the second kind!

Break $\mathcal{N}$ = 2 down to $\mathcal{N}$ = 1 in the bulk

Deformation of the bulk: ADD W= $\mu(A^a)^2 + \mu' A^2$

Heterotic deformation the of the World-sheet theory:

(2,2) supersymmetry is broken down to (0,2)

$$L_{heterotic} = \zeta_R^\dagger i\partial_L \zeta_R + [\gamma \zeta_R R (i\partial_L \phi^\dagger) \psi_R + H.c.] - g_0^2 |\gamma|^2 \left(\zeta_R^\dagger \zeta_R\right)\left(R \psi_L^\dagger \psi_L\right)$$

at small γ

ζ_R is Goldstino

$$\mathcal{E}_{vac} = |\gamma|^2 \left| \langle R \psi_R^\dagger \psi_L \rangle \right|^2$$

(0,2) supersymmetry is spontaneously broken!

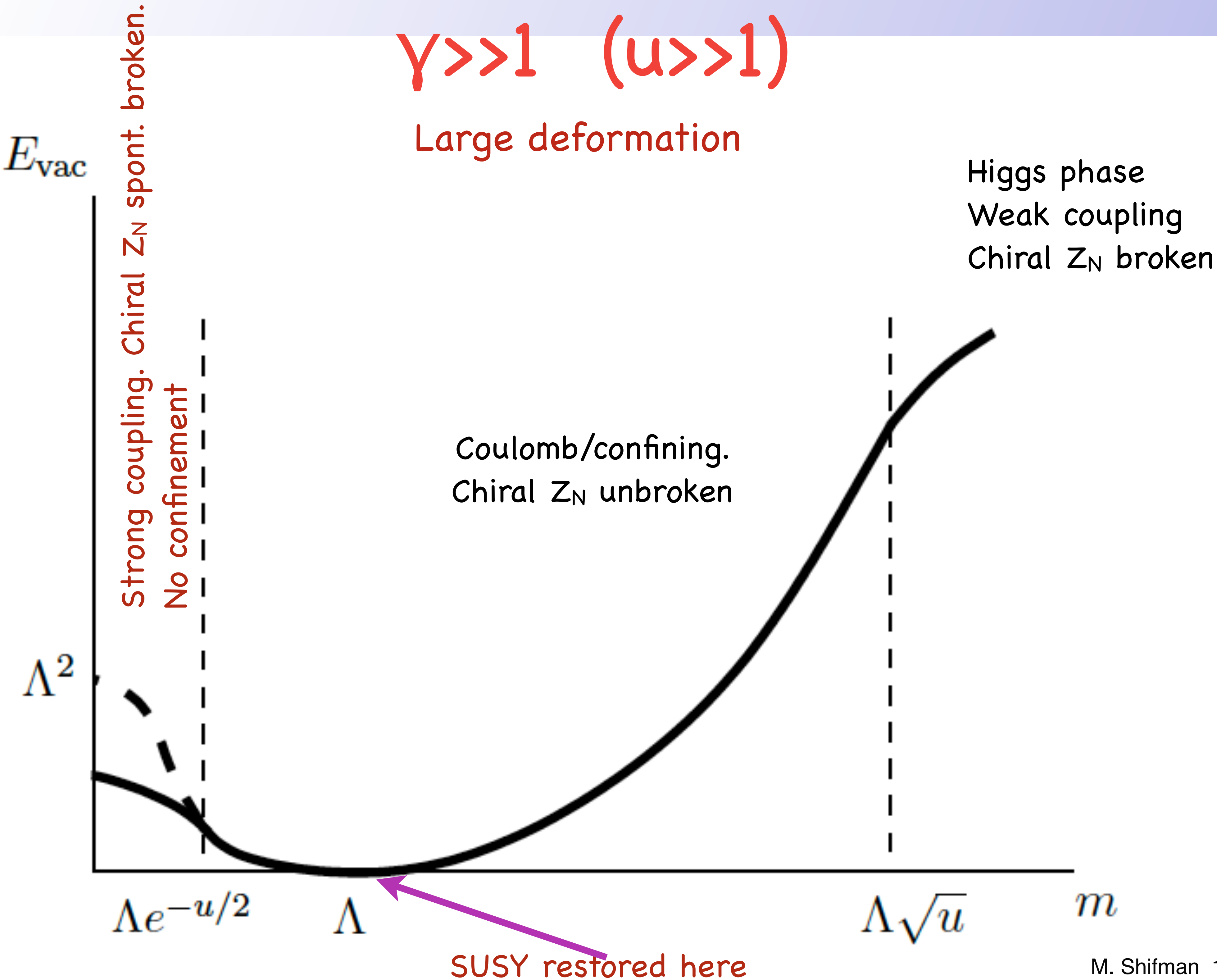
At large N heterotic CP(N-1) is also solvable (a là Witten) and presents a wealth of various phases

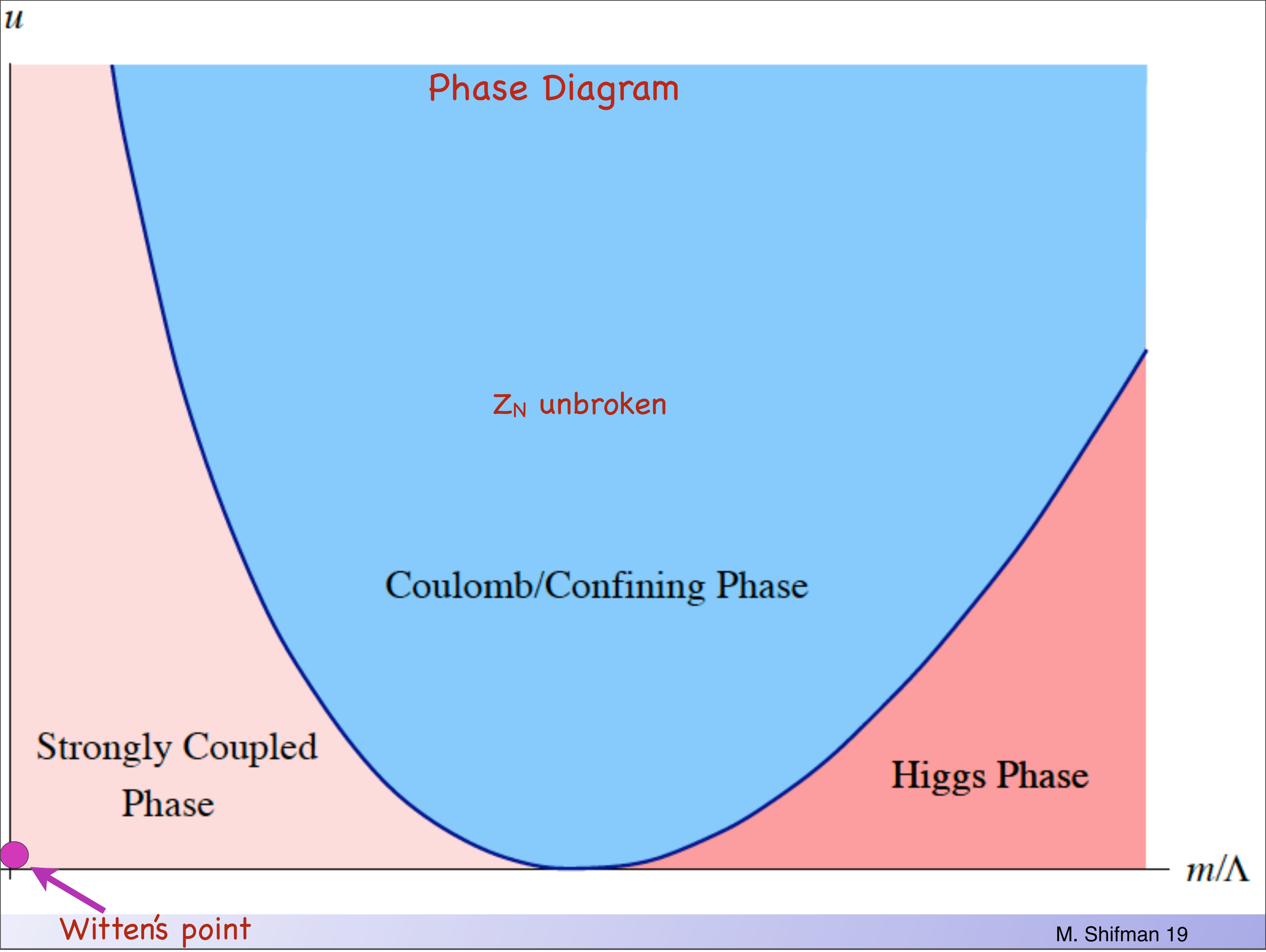
We have two parameters, γ and m, and a nontrivial phase diagram

With this choice of mass parameters we have Z_N symmetry, and phases with broken/unbroken Z_N.
SUSY is spontaneously broken

γ>>1 (u>>1)

Large deformation




u
Phase Diagram
Z_N unbroken
Coulomb/Confining Phase
Strongly Coupled
Phase
Higgs Phase
m/Λ
Witten's point

6. $\mathcal{N}$ = 1 or 2 SUSY bulk, @ Semilocal Strings

☆ Hanani-Tong model →Obtained from string/D brane consideration

☆ ☆ From field theory we get zn model: DIFFERNENT

☆ ☆ ☆ Large-N limit the same!!!

SY hep-th/0603134

SVY arXiv:1104/2077

KSVY arXiv:1107/3779

Hanani-Tong model

$$\mathcal{L}^{het}_{\mathbb{WCP}^{N_F-1}} = |\nabla_\mu n_i|^2 + |\tilde{\nabla}_\mu \rho_j|^2 .$$

$$-\sum_{i=0}^{N-1} |\sigma - m_i|^2 \, |n_i|^2 - \sum_{j=0}^{\tilde{N}-1} |\sigma - \mu_j|^2 \, |\rho_j|^2 - D\left(|n_i|^2 - |\rho_j|^2 - r_0\right)$$

$$-\, 2|\omega|^2|\sigma|^2$$

$$\nabla_\mu n_i = (\partial_\mu - iA_\mu) n_i, \quad \tilde{\nabla}_\mu \rho_j = (\partial_\mu + iA_\mu)\rho_j$$

$N_F = N + \tilde{N}$

$$m_k = m\, e^{2\pi i \frac{k}{N}}\,, \qquad k = 0, \ldots, N-1$$

$$\mu_l = \mu\, e^{2\pi i \frac{l}{\tilde{N}}}\,, \qquad l = 0, \ldots, \tilde{N}-1\,.$$

zn Model (MS+Vinci+Yung)

$$S_{\text{exact}} = \int d^2x \left\{ |\partial_k (z_j n_i)|^2 + |\nabla_k n_i|^2 + \frac{1}{4e^2} F_{kl}^2 + \frac{1}{e^2} \, |\partial_k \sigma|^2 \right.$$

$$\left. + \; |m_i - \tilde{m}_j|^2 \, |z_j|^2 |n_i|^2 + \left| \sqrt{2}\sigma + m_i \right|^2 |n_i|^2 + \frac{e^2}{2} \left(|n_i|^2 - r \right)^2 \right\},$$

$$i = 1, ..., N \,, \qquad j = 1, ..., \tilde{N} \,, \qquad \nabla_k = \partial_k - i A_k \,.$$

+ deform. + fermions

z_j of the opposite charge compared to n_i and unconstrained

Derived from the bulk theory in the limit $\ln(\xi L^2) >> 1$

- ☞ At $N \to \infty$ HT = zn

- ☞ BPS sectors the same at any N

- ☞ New type of renormalizability

Phase Diagram in the large-N solution of weighted CP(N-1) or, which is the same, zn model

From Koroteev-Monin-Vinci, 2010

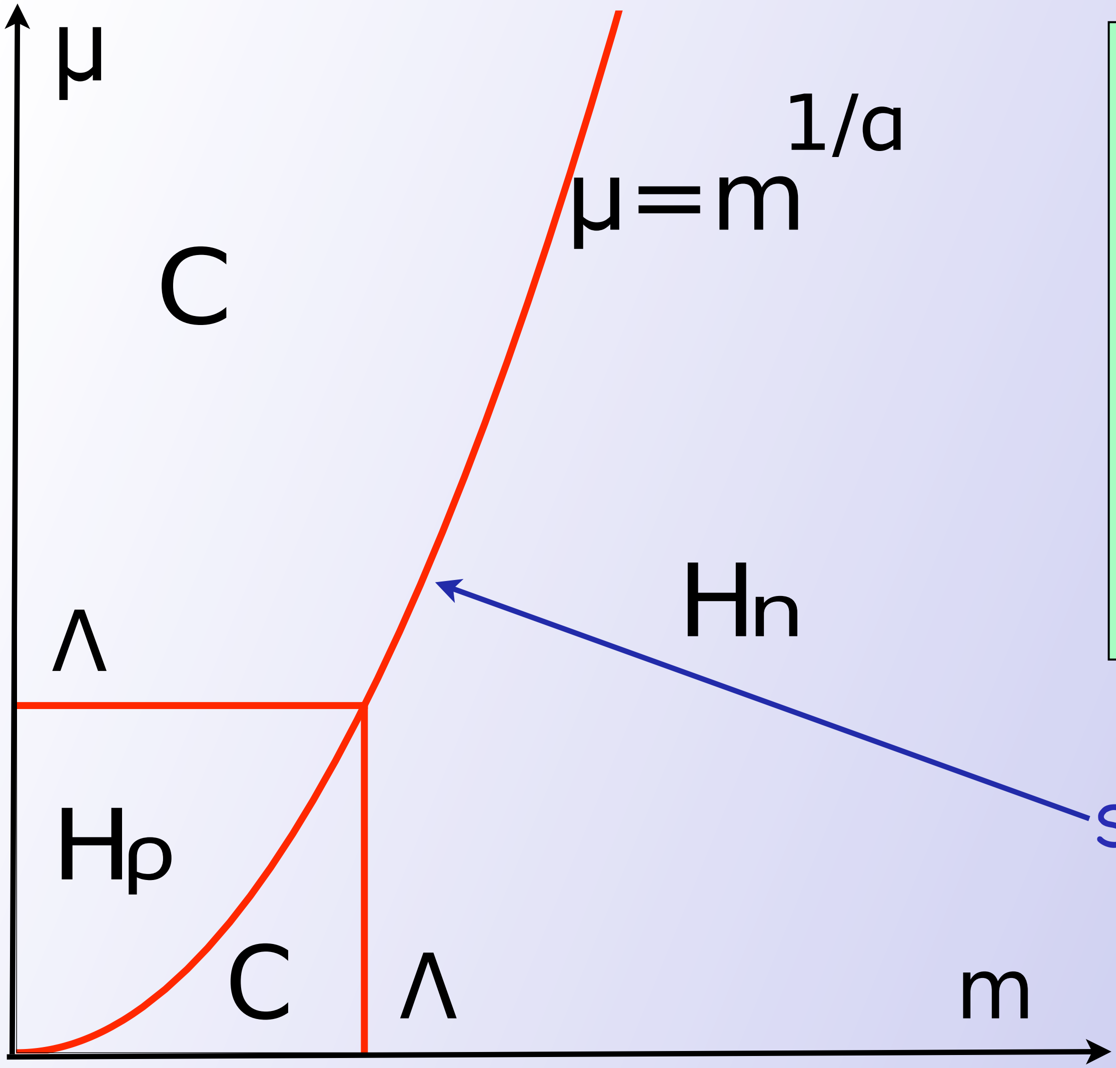


Figure 4: Phase Diagram of the weighted (2, 2) $\mathbb{CP}^{N-1}$ model in the large-N approach. There are four domains with different VEVs for σ: two Higgs branches $\mathbf{H}\rho$ and $\mathbf{H}n$, and two Coulomb branches $\mathbf{C}$. In the Coulomb phase $\mathbf{C}$ $r = 0$. The curve $\mu/\Lambda = (m/\Lambda)^{1/\alpha}$ together with horizontal and vertical lines starting from $\mu = \Lambda$ and $m = \Lambda$ respectively separates the $\mathbf{C}$ phases from the Higgs phases. In $\mathbf{H}n$ $r > 0$ and in $\mathbf{H}\rho$ $r < 0$. On the super-conformal line $\mu/\Lambda =(m/\Lambda)^{1/\alpha}$ a new branch described by a super-conformal theory opens up.

Two-dimensional Analog of $\mathcal{N}=1$ Yang-Mills Theory in Four Dimensions

In the (0,2) literature known as the heterotic minimal model at level k

with X. Cui

arXiv:1009.4421

arXiv:1105.5107

arXiv:1111.6350

$$\mathcal{L}_A = \frac{1}{g^2}\int d^2\theta_R \frac{A^\dagger i \overleftrightarrow{\partial}_{RR} A}{1+A^\dagger A}$$

$$= \quad G\left\{\partial^\mu\phi\partial_\mu\phi^\dagger + i\psi_L^\dagger \overleftrightarrow{\partial}_{RR}\psi_L - 2i\,\frac{1}{\chi}\,\psi_L^\dagger\psi_L\,\phi^\dagger\overleftrightarrow{\partial}_{RR}\phi\right\}$$

$$G = \frac{2}{g^2\,\chi^2}\,,$$

$$\chi \equiv 1 + \phi\,\phi^\dagger\,.$$

NR theorems

$$\beta(g^2) = -\frac{g^4}{2\pi}\frac{1 + \frac{N_f}{2}\gamma(B_i)}{1 - \frac{1}{4\pi}g^2}$$

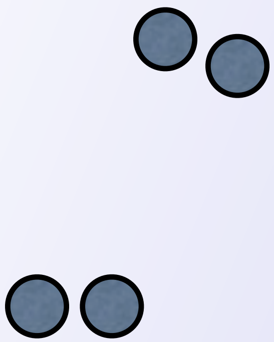
Full analog of NSVZ β function in 4D SYM

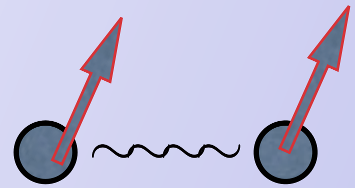
☞ Anomaly: J. Chen, AV + MS, in progress

Applications in Condensed Matter

^{3}He-B example

^{3}He atoms





Spin 1/2

P-wave paring

L=1, S=1 ➟ Cooper pair order parameter $e_{\mu i}$ ← 3×3 matrix

Spin-orbit small, symmetry of H is $G = U(1)_p \times SO_S(3) \times SO_L(3)$

In the ground state $U_p(1) \times SO_S(3) \times SO_L(3) \to H_{\rm B} = SO(3)_{S+L}$

Vectorial order parameter broken on the vortex

$$S_{\text{world sheet}} = (\mu^2/2\beta)\int d^2x\, (\partial_\mu S^i)\, (\partial_\mu S^i)$$

$$S^i S^i = 1$$

Clasically two "rotational" zero modes.

QMechanically may be lifted

◇ Assume χ^i is spin field!

◇◇ Add $\Delta L = \varepsilon\, (\partial_i \chi^i)(\, \partial_k \chi^k\,)$

If $\varepsilon \neq 0$ but small ⇒

$$\Delta_{CP(1)} S_{\text{world sheet}} = \varepsilon \int d^2x \, \{(\partial_z S^3)^2 - M^2[1-(S^3)^2]\}$$

$$\mathcal{L}_{x_\perp} = \frac{T}{2} (\partial_a \vec{x}_\perp)^2 - \tilde{M}^2 \left(S^3\right)^2 (\partial_z \vec{x}_\perp)^2 \, ,$$

★ EXTRA (quasi)gapless modes ★

★ ★ Translational (Kelvon) and orientational (spin) modes mix with each other ★ ★

Instead of conclusions

✷ A treasure trove of novel 2D models with intriguing dynamics!

✷ 4D ↔ 2D Correspondence

☛ World-sheet theory ↔ strongly coupled bulk theory inside

Dewar flask