Hagedorn instability versus large N volume independence

Aleksey Cherman
University of Minnesota

with Gokce Basar (SUNY Stony Brook),
Daniele Dorigoni (DAMTP, Cambridge),
and Mithat Ünsal (San Francisco State)

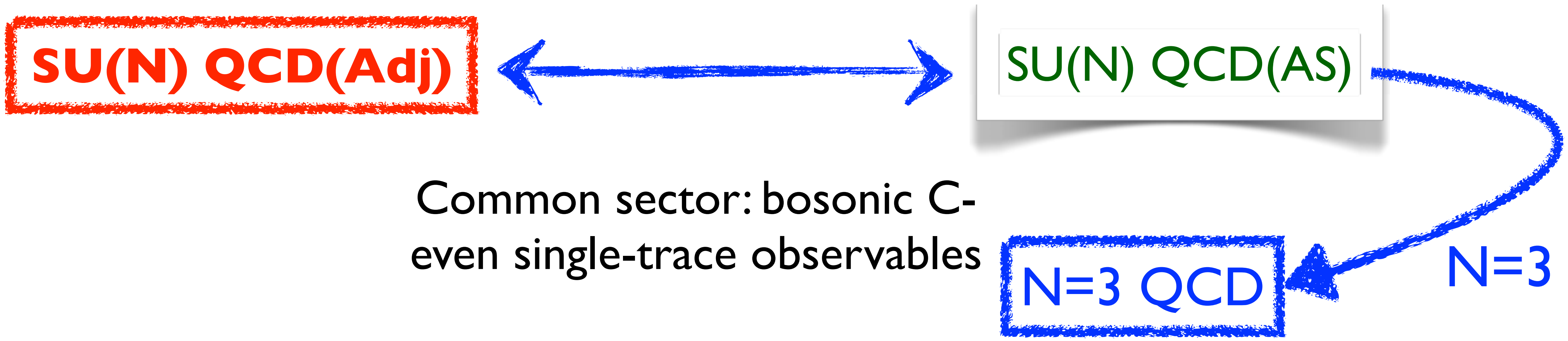
The star of the show: QCD(Adj)

SU(N) QCD(Adj) = SU(N) YM theory + N_f massless adjoint Weyl fermions

Why should you care about it?

On R^4 tied to large N QCD via orbifold/orientifold equivalence!

Armoni, Shifman, Veneziano



QCD(AS) is a phenomenologically viable large N limit of real QCD!

Armoni, Shifman, Veneziano;
AC, Cohen, Lebed

Preview of conclusion

There is evidence that QCD(Adj) has two important properties:

(1) Hagedorn spectrum of hadronic states — Hagedorn; Fubini, Veneziano...

Believed to apply to any confining large N theory

(2) Spatial volume independence (VI) when on e.g. $\mathbb{R}^3 \times S^1_L$

Special to QCD(Adj)! — Kovtun, Unsal, Yaffe

VI implies there are **no phase transitions** as a function of $L \sim N^0$

But Hagedorn seems to **force** transition at $L \sim N^0$; deconfinement...

VI and Hagedorn are in tension. — T. Cohen; M. Shifman

For tension to be resolved while keeping (1) and (2), would need degeneracies between bosonic and fermionic states at large N

Appears to require an emergent fermionic symmetry at large N

Why is this not obviously silly?

Coleman-Mandula Theorem (+ Haag-Lopuzhansky-Sohnius extension) says:

SUSY is the ONLY non-trivial extension of Poincare algebra of symmetries of S-matrix of a relativistic QFT.

When $N_f > 1$, QCD(Adj) is not supersymmetric!

But there is no conflict: at $N = \infty$, S-matrix becomes trivial.

`Glueball' decay amplitude $\sim 1/N$, scattering $\sim 1/N^2$

CM does **not** forbid emergent fermionic symmetry at large N!

But CM implies 1/N corrections would have to give explicit breaking

But there was no reason to expect any such symmetry, until noticing implication of VI and Hagedorn properties of QCD(Adj)...

Hagedorn spectrum

Widely believed that number of hadronic states
in confining large N gauge theories behaves as

$$\rho(M) \to \frac{1}{M}\left(\frac{T_H}{M}\right)^a e^{M/T_H}, \quad T_H \sim \Lambda_{\rm QCD}$$

Why believe it?

Heuristic reason: expect highly excited hadrons to behave like relativistic open or closed effective strings at large N.

Relativistic strings famously have a Hagedorn density of states!

Experimental data consistent with Hagedorn...

Recently, also argued that Hagedorn directly follows from standard large N features of QCD

T. Cohen...

Both heuristic and direct arguments apply to QCD(Adj)

Hagedorn instability

Put confining large N theory on $\mathbb{R}^3 \times S^1_\beta$

$$Z(\beta) = \mathrm{Tr}\, e^{-\beta H} = \int dM\, \rho(M)\, e^{-\beta M}$$

Now increase temperature from zero

Once $\beta > \beta_H = 1/T_H$, partition function diverges!

Implies phase transition should take place at or below Hagedorn scale, to phase where the density scales differently.

This is the deconfinement transition
to a quark-gluon plasma phase!

Large N volume independence

Basic idea found in lattice gauge theory by Eguchi and Kawai 1982

Statement: compactify pure SU(N) gauge theory on e.g. $\mathbb{R}^3 \times S^1_L$

Pick up a *topological* global symmetry - Z_N center symmetry

$$\Omega = \mathcal{P}e^{i\int_{S_1} A_{S_1}} \qquad \langle \mathrm{Tr}\ \Omega \rangle \longrightarrow \omega \langle \mathrm{Tr}\ \Omega \rangle, \quad \omega = e^{2\pi i/N}$$

Then, so long as center symmetry is unbroken...

... there will be no volume (L) dependence in expectation values of connected correlators of topologically trivial single-trace observables, up to 1/N corrections

Sounds great, and surprising... Can envision using it to reduce 4D YM theory to low-dim models, which may be easier to solve!

So why isn't it in all the textbooks?

Volume independence vs deconfinent

Problem: small S^1 ~ high T, so at small S^1 expect deconfinement!

Center symmetry should break!

To see it, compute perturbative effective potential for order parameter, the Wilson loop wrapping S^1...

$$V_{\text{pure YM}}(\Omega) = (-1)\frac{2}{\pi^2 L^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\,|\mathrm{Tr}\ \Omega^n|^2\,,$$

Minimized at $\Omega = 1$, so Tr $\Omega > 0$.

Perturbation theory reliable at $L << 1/\Lambda_{QCD}$, so we can be sure center breaks for small L.

Center-breaking leads to VI failure for general L in YM, and QCD(F)!

Bhanot, Heller, Neuberger 1982

Volume independence vs deconfinent

Center-breaking leads to VI failure for general L in YM, and QCD(F)!

Bhanot, Heller, Neuberger 1982

Several clever attempts in 80s to fix it -
quenched EK (82), twisted EK (83), etc

Bhanot, Heller, Neuberger; Gonzalez-Arroyo, Okawa

Unfortunately, these tricks didn't work.

Bringoltz-Sharpe; Teper-Vairinhos; Azeyanagi-Hanada-Hirata-Ishikawa; others...

(A. Gonzalez-Arroyo's plenary talk has all the history, and very recent working TEK proposal!)

Volume independence vs deconfinent

Center-breaking leads to VI failure for general L in YM, and QCD(F)!

Roadblock for ~25 years...

Volume independence in QCD(Adj)

In 2007, Kovtun, Unsal, Yaffe noticed that VI changes radically in **QCD(Adj)** on **spatial** circle:

$$V_{\rm eff}(\Omega) = (N_f - 1)\frac{2}{\pi^2 L^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\,|{\rm Tr}\ \Omega^n|^2$$

When N_f>1, minimum at center-symmetric Ω, with ${\rm Tr}\ \Omega^n = 0$!

At N_f=1, theory is supersymmetric, $V_{\text{eff, all loops}}$ vanishes; but non-perturbative effects force center-symmetric Ω

Unlike before, no center breaking seen at small L in QCD(Adj)

KUY proposed that QCD(Adj) gives first working VI realization!

Volume independence in QCD(Adj)

Subtlety: KUY weak-coupling calculation justified only in a non-'t Hooft large N limit with L~1/N.

Consistent with VI, which can only hold for $L \sim N^0$, 't Hooft limit

Otherwise we'd be able to trivially solve QCD using by working at weak coupling L~1/N....

But this means that to understand center symmetry realization and fate of VI, must use a non-perturbative method!

Only available such method is *numerical lattice simulations*.

VI from the lattice

Many simulations, one consensus: QCD(Adj) has VI at large N

Bringoltz, Sharpe 2009+; Azeyanagi et al 2010;
Narayanan-Hietanen 2009+;
Galvez et al 2011;
Gonzalez-Arroyo, Okawa 2011;
...

But isn't this is in direct conflict with the Hagedorn scaling of the hadron density of states?

Volume independence versus Hagedorn

Hagedorn forces a phase transition for thermal compactification.

VI only expected for spatial compactification!

Periodic boundary conditions for fermions: Euclidean path integral now calculates twisted partition function

$$\tilde{Z}(L) = \mathrm{Tr}\ (-1)^F e^{-LH}$$

$$= \int dM[\rho_{\mathcal{B}}(M) - \rho_{\mathcal{F}}(M)]e^{-LM}$$

For SUSY $N_f=1$ case, this is a supersymmetric index; L-independent

For $N_f>1$, not an index, but still sharply different from thermal partition function for QCD(Adj)

$$Z(\beta) = \mathrm{Tr}\ e^{-\beta H} = \int dM[\rho_{\mathcal{B}}(M) + \rho_{\mathcal{F}}(M)]e^{-\beta M}$$

Volume independence versus Hagedorn

The thermal and twisted partition functions
are not always different on practical level

Take YM theory + complex rep. fermions. Ex: QCD(F), QCD(AS).
Then $M_B \sim N^0$, but $M_F \sim N^1$ or N^2.

Fermionic Hilbert space not populated for $L \sim N^0$

$$L \sim N^0 \Rightarrow \tilde{Z}(L) = Z(\beta = L)$$

But QCD(Adj) is special! $M_B \sim N^0$, $M_F \sim N^0$

$$\tilde{Z}(L) \neq Z(\beta = L)$$

Thermal and twisted partition
functions are different in QCD(Adj)

For QCD(Adj) expect some cancelation in $\tilde{Z}$, but none for Z.

Volume independence versus Hagedorn

We take numerical experiments seriously:

assume QCD(Adj) has VI for all $L \sim N^0$ and $N_f \geq 1$

Expect QCD(Adj) to have Hagedorn scaling for both ρ_B and ρ_F

Then to avoid Hagedorn instability...

All exponential parts of ρ_B and ρ_F must cancel in twisted partition function!

But that appears to require degeneracies between infinite number of bosonic and fermionic states at $N=\infty$

Calls for an emergent fermionic symmetry!

At $N_f=1$, this symmetry is already known - it's SUSY!

`Happens' to work away from $N=\infty$ as well

At $N_f>1$, emergent symmetry can not be supersymmetry

Games with a stringy toy model

Don't know yet how to show symmetry emerges in QCD(Adj)

How plausible in it?

Are there any examples where a Hagedorn instability can be evaded without supersymmetry?

Define a `stringy' **toy** model - doesn't have any sharp connection to QCD!

Spectrum: $\boxed{M^2 \equiv N/\alpha'}$

$$N \equiv \sum_{n\in\mathbb{N}} n \underbrace{a_n^\dagger a_n}_{\text{bosonic}} + \sum_{i=1}^{N_f} \sum_{n\in\mathbb{N}} n \underbrace{f_{i\,n}^\dagger f_{i\,n}}_{\text{fermionic}}$$

Point of considering it is to illustrate point of principle: Hagedorn growth can cancel even in the absence of SUSY

Hagedorn growth in usual density of states

This toy model has a thermal Hagedorn density of states.

To see it, define combinatorial generating function to count states, all with same sign

$$\mathrm{Tr}\ q^N = \prod_{n=1}^{\infty} \frac{(1+q^n)^{N_f}}{1-q^n} = \sum_{n=0}^{\infty} d(n)\, q^n$$

number of states at level n

$$d(n) \sim \exp\left(\sqrt{2\pi^2(1+N_f/2)n/3}\right), \qquad n \gg 1$$

But then since $M^2 \sim n$, we have $M \sim n^{1/2}$, so d(n) scaling implies $\rho = \rho_B + \rho_F \sim e^{L_H M}$, as advertised.

No Hagedorn growth in twisted density of states

Now examine twisted density of states $\tilde{\rho} = \rho_B - \rho_F$

$$\mathrm{Tr}\ \left[(-1)^F q^N\right] = \prod_{n=1}^{\infty} (1 - q^n)^{(N_f - 1)} = \sum_{n=0}^{\infty} c(n) q^n$$

number of bosonic states minus number of fermionic states at level n

$N_f = 1$ case, SUSY

$$\mathrm{Tr}\ \left[(-1)^F q^N\right] = 1$$

$N_f = 2$ case, ~~SUSY~~

$$\mathrm{Tr}\ \left[(-1)^F q^N\right] = 1 - q - q^2 + q^5 + q^7 - q^{12} - q^{15} + q^{22} + \ldots .$$

Exact cancellation except at `generalized pentagonal numbers' $p_n^{\pm} = \frac{3n^2 \pm n}{2}$

No Hagedorn growth, but why?

Fermionic symmetry

Reason: there are N_f conserved fermionic charges

$$Q_i = \sum_{n \in \mathbb{N}} \sqrt{n}\, a_n^\dagger f_{i\,n}$$

Possible because model is free; but so is QCD(Adj) at large N!

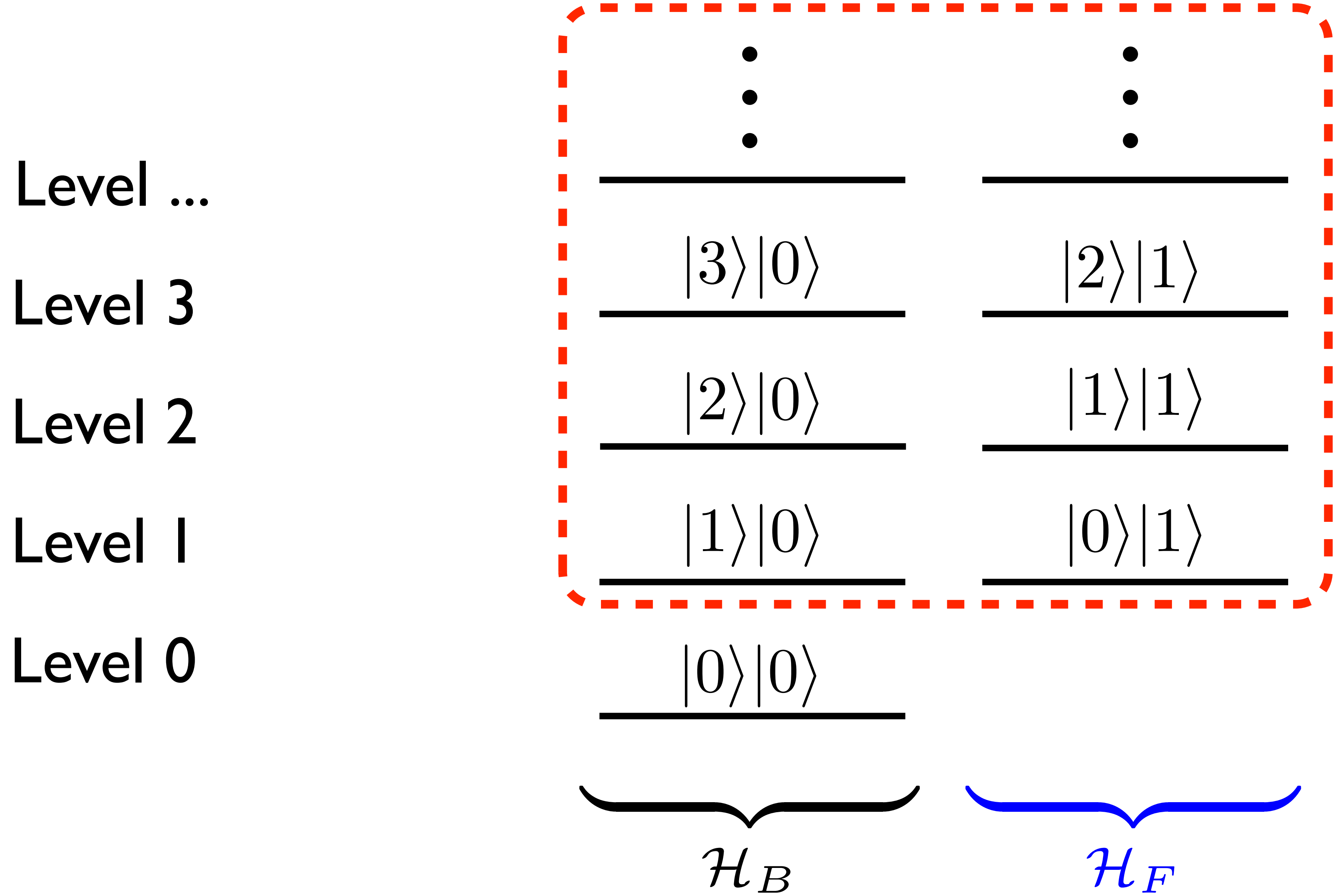
Conserved charges give spectral degeneracies, lead to huge amount of cancellation in twisted density of states

Can show that there is no Hagedorn scaling for any N_f

To see it in more detail, focus on set of oscillators with one fixed energy, one bosonic set, N_f fermionic ones

$$H \sim a^\dagger a + \sum_{i=1}^{N_f} f_i^\dagger f_i$$

N_f=1 SUSY at work

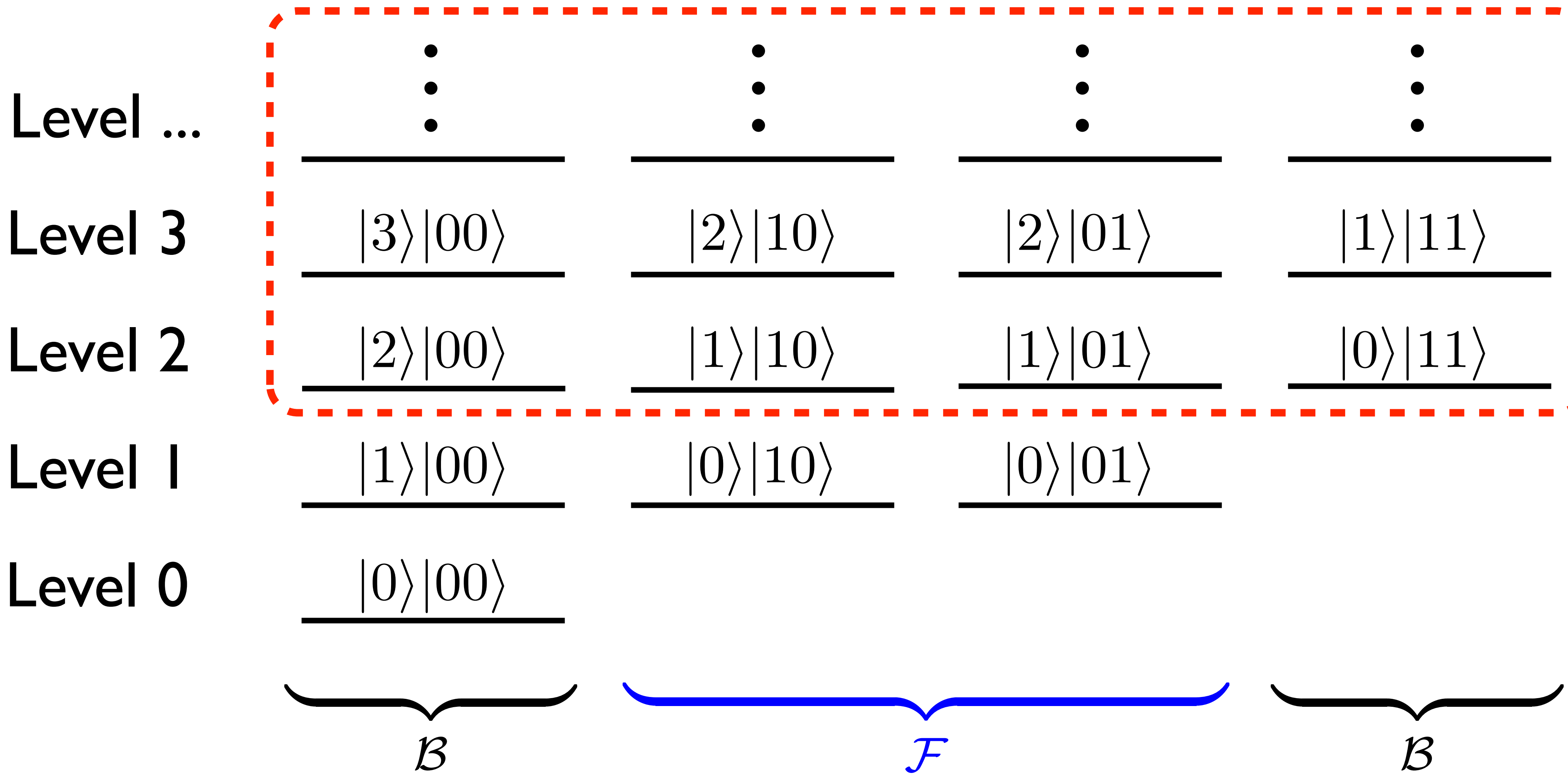


All states contribute to thermal partition function

In twisted partition function, states in the box all cancel each other

Only states outside the box, which are in cohomology of Q_i=Q, contribute to twisted partition function

Non-SUSY N_f=2 fermionic symmetry at work



All these states contribute to a thermal partition function

In twisted partition function, states in the box all cancel each other

Only states outside the box, which are in cohomology of Q_i, contribute to twisted partition function

Conclusions

If large N QCD(Adj) has both Hagedorn spectrum and volume independence, it should apparently have an emergent large N fermionic symmetry.

This may be the first exciting theoretical consequence of VI!

(applications of VI envisioned so far have been to save numerical costs)

Lattice calculations are continuing to look at VI; should also look for spectral degeneracies!

Can a microscopic realization of this conjectured symmetry be found?

Fermionic symmetries are extremely useful. If emergent non-SUSY fermionic symmetries do indeed exist, what can we do with them?

Backup: weak coupling in QCD(Adj)

Subtlety: V_{eff} calculation valid once theory becomes weakly coupled. Must happen for small enough L by asymptotic freedom.

But for QCD(Adj) on spatial circle,
small enough means $NL\Lambda << 1$.

Reason: perturbation theory defined with respect to choice of vacuum

Non-trivial weak-coupling holonomy = non-trivial background field

Changes regime of validity of perturbation theory!

Consistent with VI, which can only hold for $L \sim N^0$, 't Hooft limit

Otherwise we'd be able to trivially solve QCD using VI....

But this means that to understand center symmetry realization and fate of VI for $L \sim N^0$, must use a non-perturbative method!