The Weyl Consistency Conditions & Standard Model Vacuum Stability

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CP^3 Origins
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based on arXiv:1306.3234, in collaboration with

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DESY Theory Workshop, 25th September 2013

Outline

◇ Weyl consistency conditions: definition

◇ Relations among the ß functions

◇ Example: vacuum stability in the Standard Model

The conformal symmetry

Consider a theory on a curved background $\gamma_{\mu\nu}(x)$, with classical conformal invariance

$$\mathcal{L} = \mathcal{L}_{CFT} + g_i\, \mathcal{O}^i$$

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free-field theory — marginal operators

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The Weyl (≡conformal) symmetry

$$\gamma_{\mu\nu} \to e^{2\sigma(x)}\gamma_{\mu\nu} \qquad g_i(\mu) \to g_i(e^{-\sigma(x)}\mu)$$

is broken by the scale dependence of the renormalized couplings

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Trick: promote the coupling constants to space-dependent, non-propagating fields

Conformal symmetry and renormalisation

In the presence of external sources (gravity and space-dependent couplings), additional counterterms are needed in the theory:

$$W \equiv \log\left[\int \mathcal{D}\Phi\, e^{i\int \mathrm{d}^d x\sqrt{-g}\mathcal{L}}\right]$$

regularized generating functional

$$\tilde{W} = W + \int \mathrm{d}^d x\sqrt{-g}\,\mu^{-\epsilon}\left[Z_a E(\gamma_{\mu\nu}) + Z_\chi^{ij}\partial_\mu g_i \partial_\nu g_j R^{\mu\nu} + \ldots\right]$$

renormalized generating functional

all possible dimension-four diffeomorphism-invariant operators, including:

- ◇ curvature tensors, e.g. Weyl tensor
  $E(\gamma_{\mu\nu}) = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
- ◇ 1, 2 and 4 derivatives of the couplings
  (no diff.-invariant terms with three derivatives)

Only valid around d = 4 space-time dimensions

The Weyl anomaly

The conformal symmetry is broken at the quantum level

Variation under Weyl transformation:

$$\Delta_\sigma \equiv \int \mathrm{d}^4x\, \sigma(x) \left(2\gamma_{\mu\nu} \frac{\delta}{\delta\gamma_{\mu\nu}} - \beta_i \frac{\delta}{\delta g_i} \right)$$

$$\Delta_\sigma \tilde{W} = \int \mathrm{d}^d x \sqrt{-g} \Big[\sigma \; \left(a\, E(\gamma_{\mu\nu}) + \chi^{ij}\, \partial_\mu g_i \partial_\nu g_j\, G^{\mu\nu} \right)$$
$$+ \partial_\mu \sigma\, \omega^i \partial_\nu g_j\, G^{\mu\nu} + \dots \Big]$$

In flat space and for constant couplings,

$$T^\mu_\mu = \beta_i [\mathcal{O}^i], \quad \text{with} \quad [\mathcal{O}^i] = \frac{\delta W}{\delta g_i}$$

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Einstein tensor $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu} R$

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Functions of the couplings g_i

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We neglect here anomalous flavor currents that can lead to limit cycles

Fortin, Grinstein, Stergiou (2012)
Luty, Polchinski, Rattazzi (2012)

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The Weyl consistency conditions

Jack, Osborn (1990), Osborn (1991)

The Weyl anomaly is abelian:

$$\Delta_\sigma \Delta_\tau \tilde{W} = \Delta_\tau \Delta_\sigma \tilde{W}$$

Gives a number of consistency relations among the functions $a,\ \chi^{ij},\ \omega^i,\ \ldots$

$$\frac{\partial \tilde{a}}{\partial g_i} = \chi^{ij}\beta_j + \left(\frac{\partial \omega^i}{\partial g_j} - \frac{\partial \omega^j}{\partial g_i}\right)\beta_j \qquad \tilde{a} = a - \omega^i \beta_i$$

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Note that $\frac{\mathrm{d}\tilde{a}}{\mathrm{d}\mu} > 0$ if $\chi^{ij} > 0$

$$\tilde{a} = a - \omega^i \beta_i$$

$\Longrightarrow$ $\tilde{a}$ theorem

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$\Longrightarrow \tilde{a}$ theorem

In general, ω^i is an exact one-form at the leading orders in perturbation theory

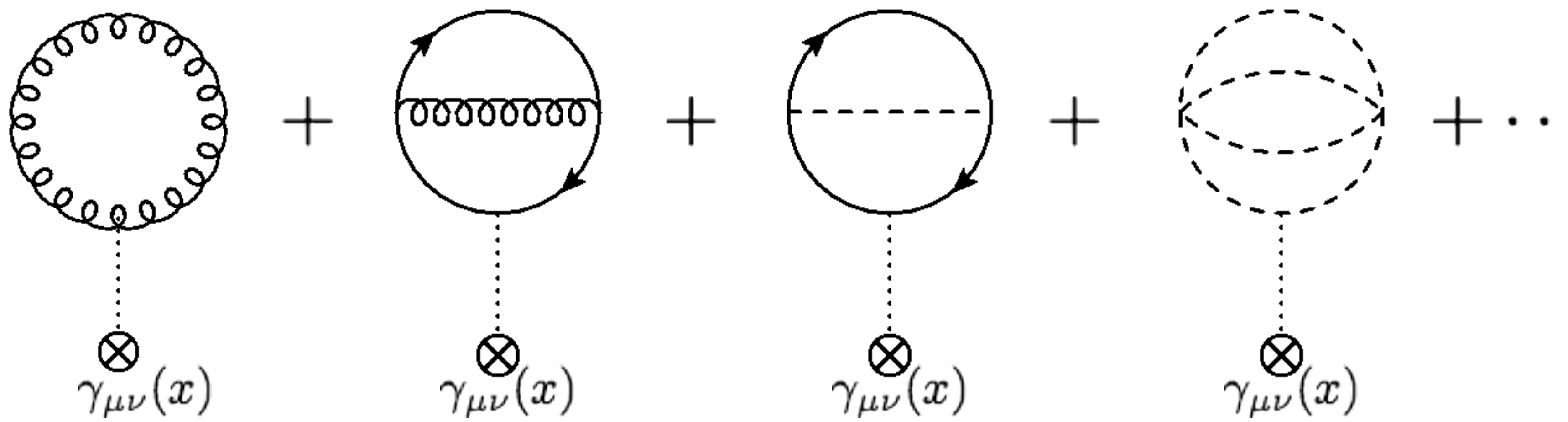
$$\frac{\partial \tilde{a}}{\partial g_i} \approx \chi^{ij}\beta_j \quad \Leftrightarrow \quad \beta_i \approx \chi_{ij}\frac{\partial \tilde{a}}{\partial g_j}$$

The RG flow is a gradient flow in a space with metric χ^{ij}

In terms of Feynman diagrams

a is equal to the trace of the energy-momentum tensor on a 4-sphere:

$$a = \left\langle T^{\mu}_{\mu} \right\rangle_{S^4} =$$



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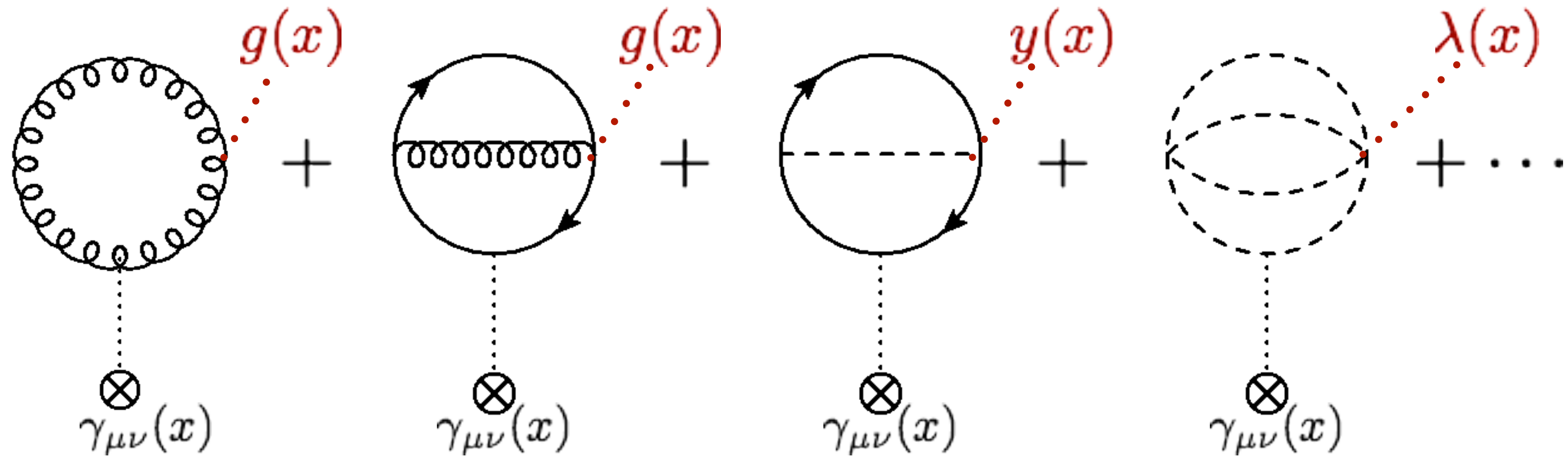

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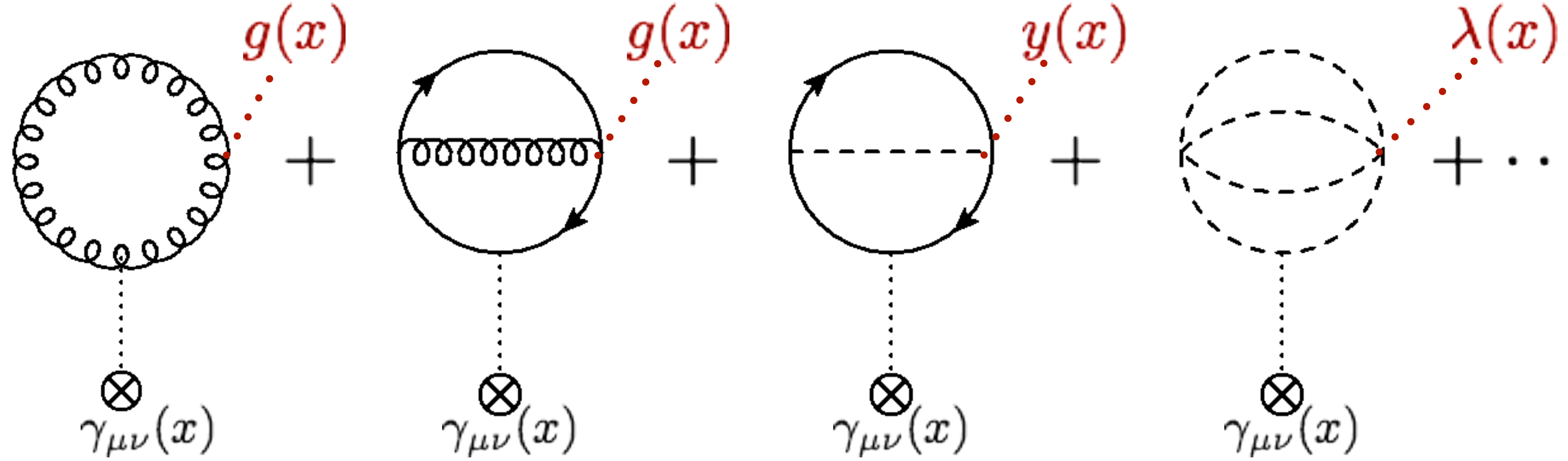
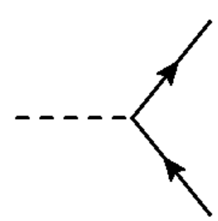


Partial derivatives are equivalent to removing one interaction vertex

$$\beta_i \approx \chi_{ij} \frac{\partial \tilde{a}}{\partial g_j} \qquad \frac{\partial}{\partial g_i} \to \frac{\delta}{\delta g_i(x)}$$

$\beta_y =$  $+\cdots$

$\beta_\lambda =$  $+\cdots$

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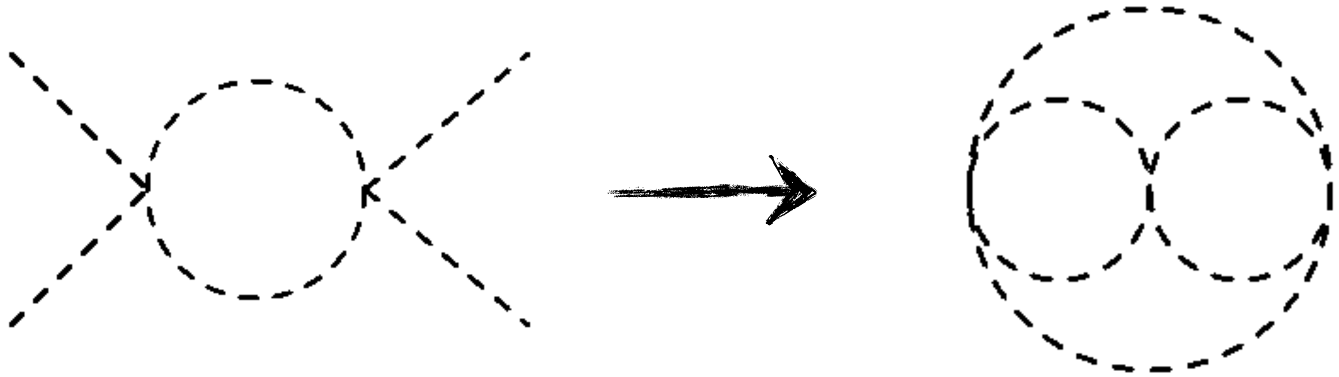
Counting loops

◇ One-loop β function of a scalar quartic interaction



4-loops diagram

Counting loops

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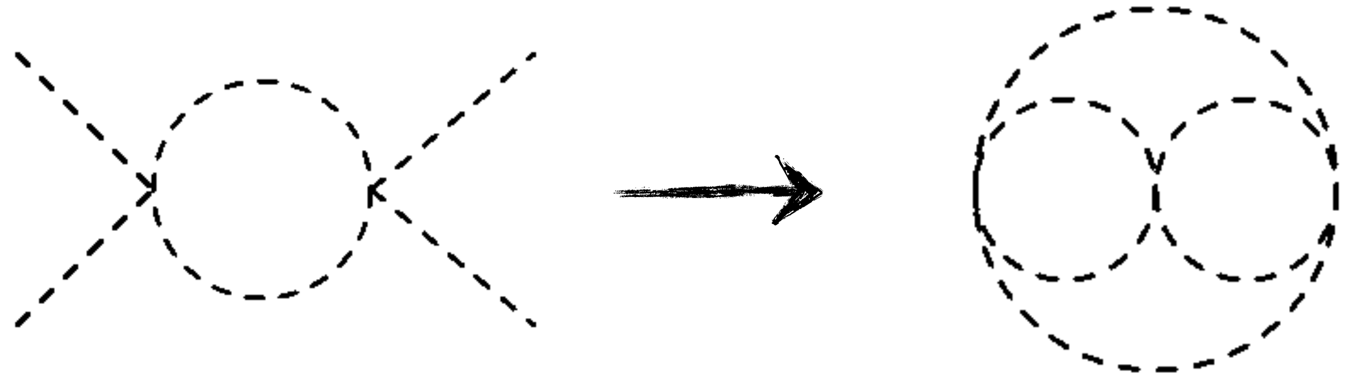
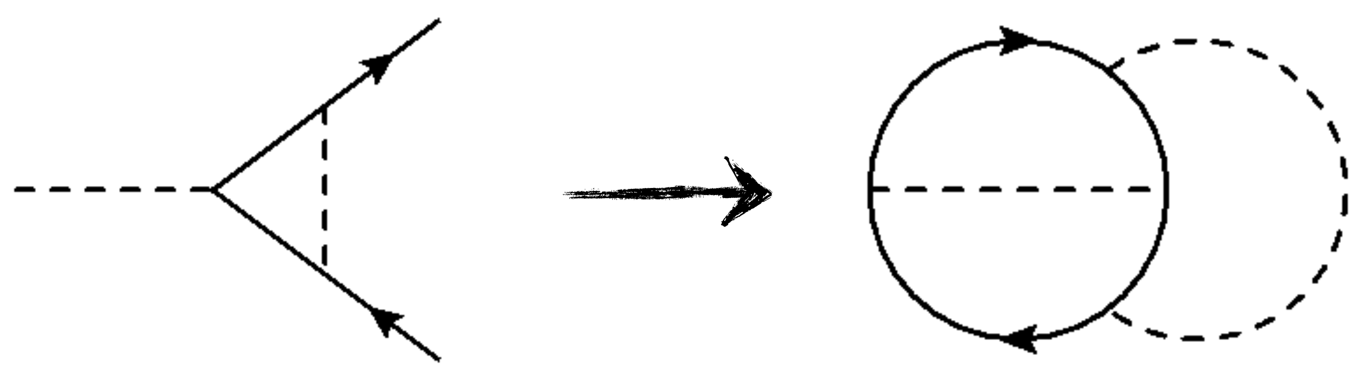


4-loops diagram

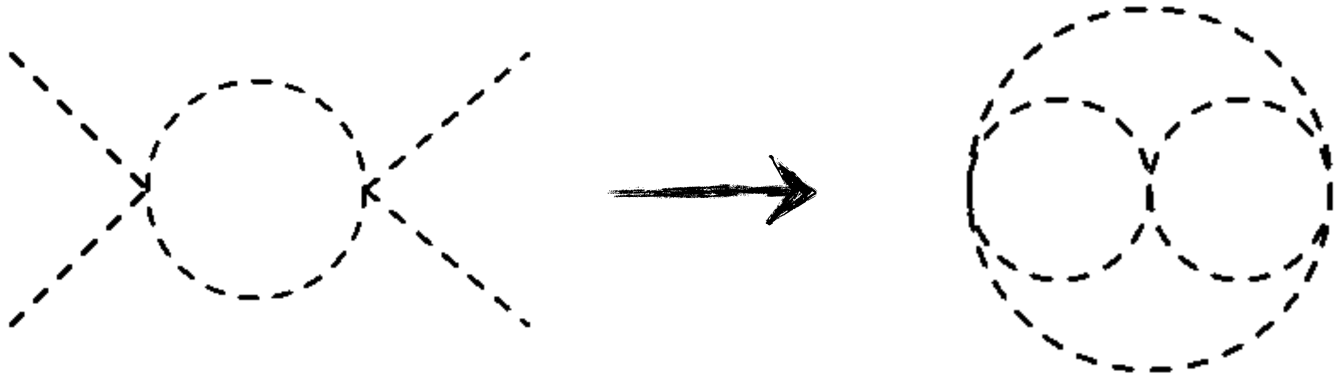
◇ One-loop β function of a Yukawa interaction



3-loops diagram

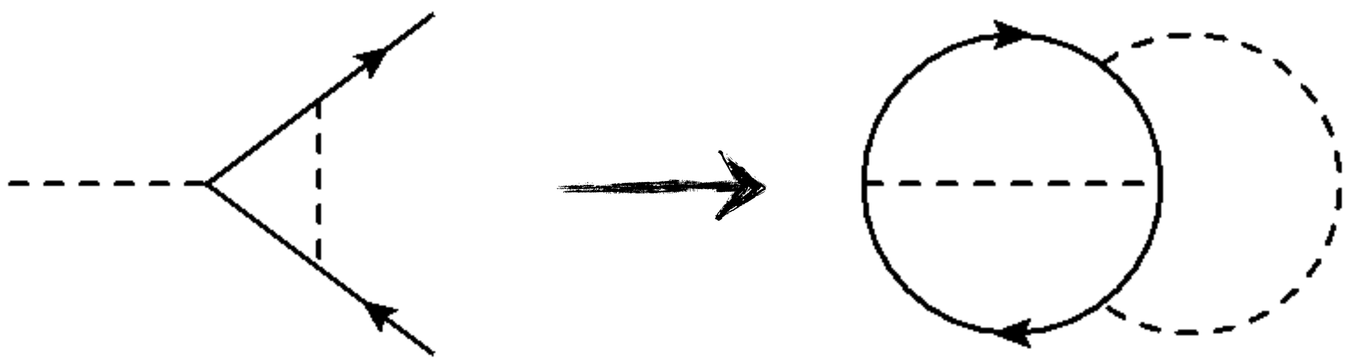
Counting loops

- One-loop β function of a scalar quartic interaction



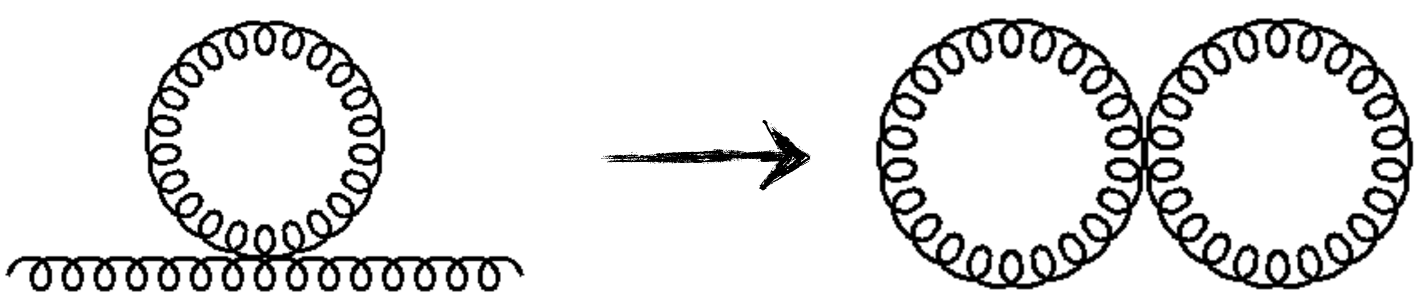
4-loops diagram

- One-loop β function of a Yukawa interaction



3-loops diagram

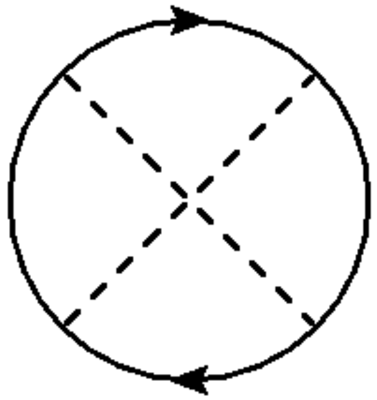
- One-loop β function of a gauge interaction



2-loops diagram

Multiple couplings

What about diagrams involving multiple couplings?



4-loops diagram with
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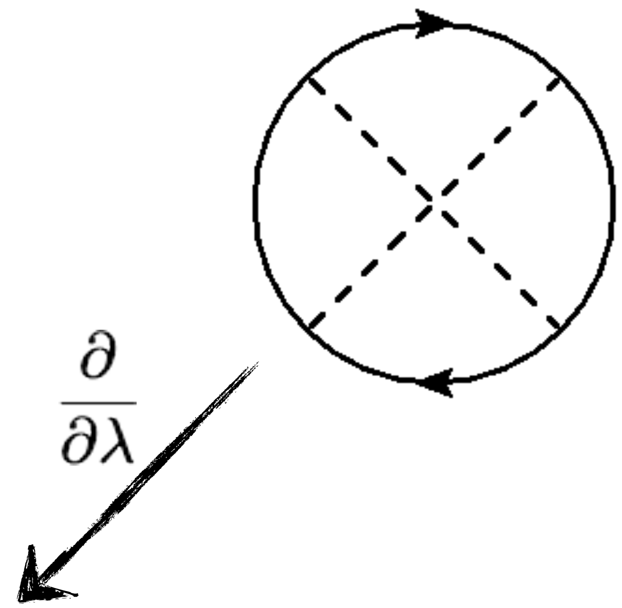

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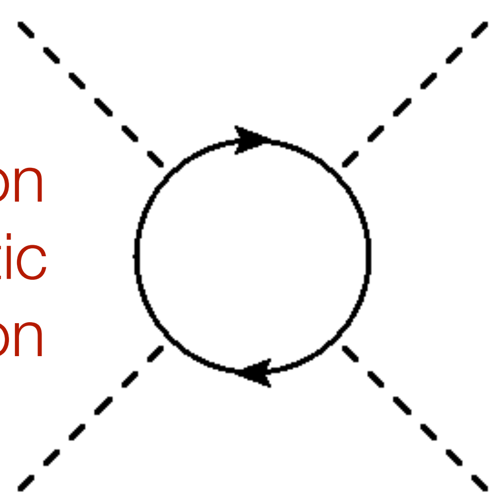

1-loop contribution to the quartic β function

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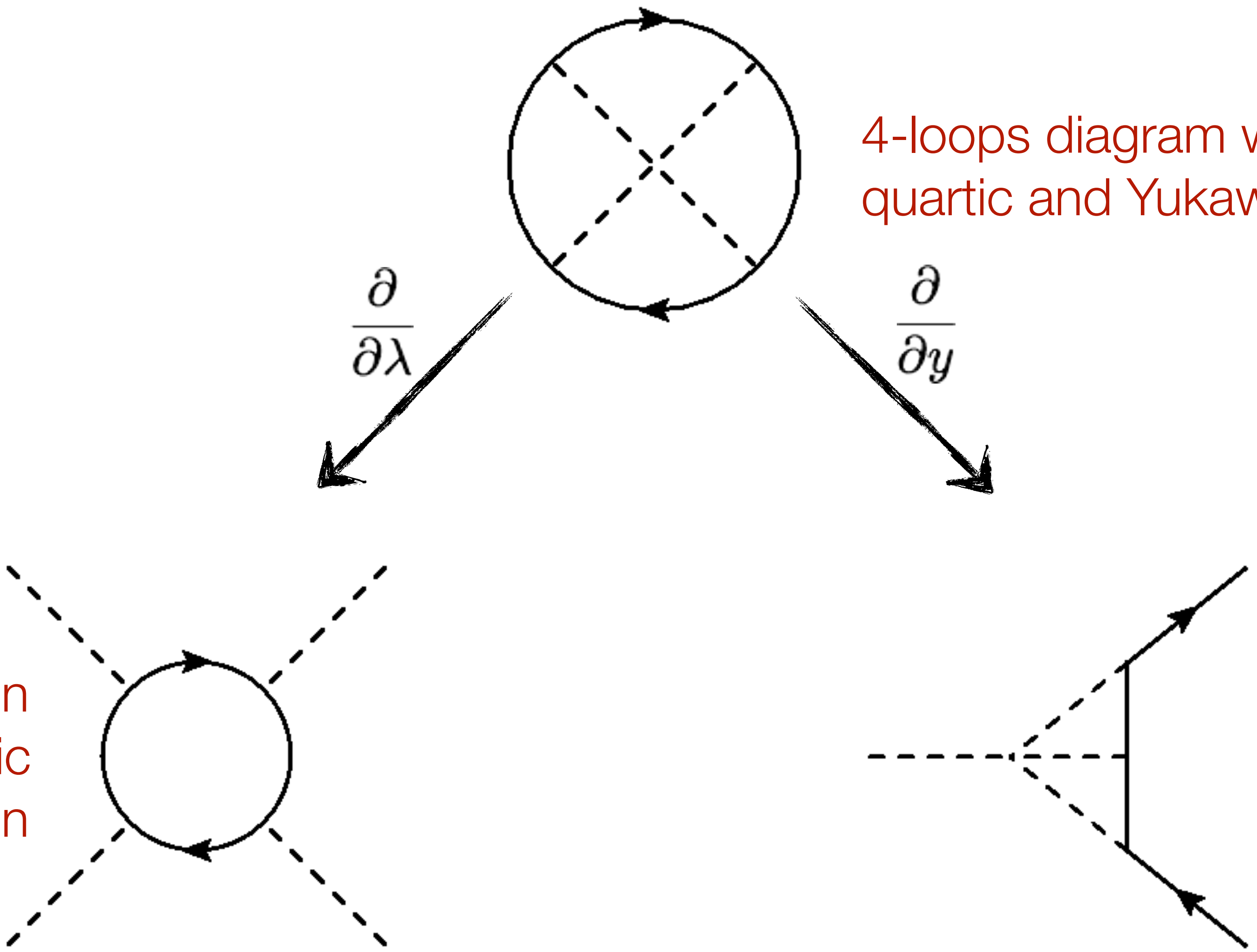

4-loops diagram with
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1-loop contribution
to the quartic
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2-loops contribution
to the Yukawa
β function

Multiple couplings

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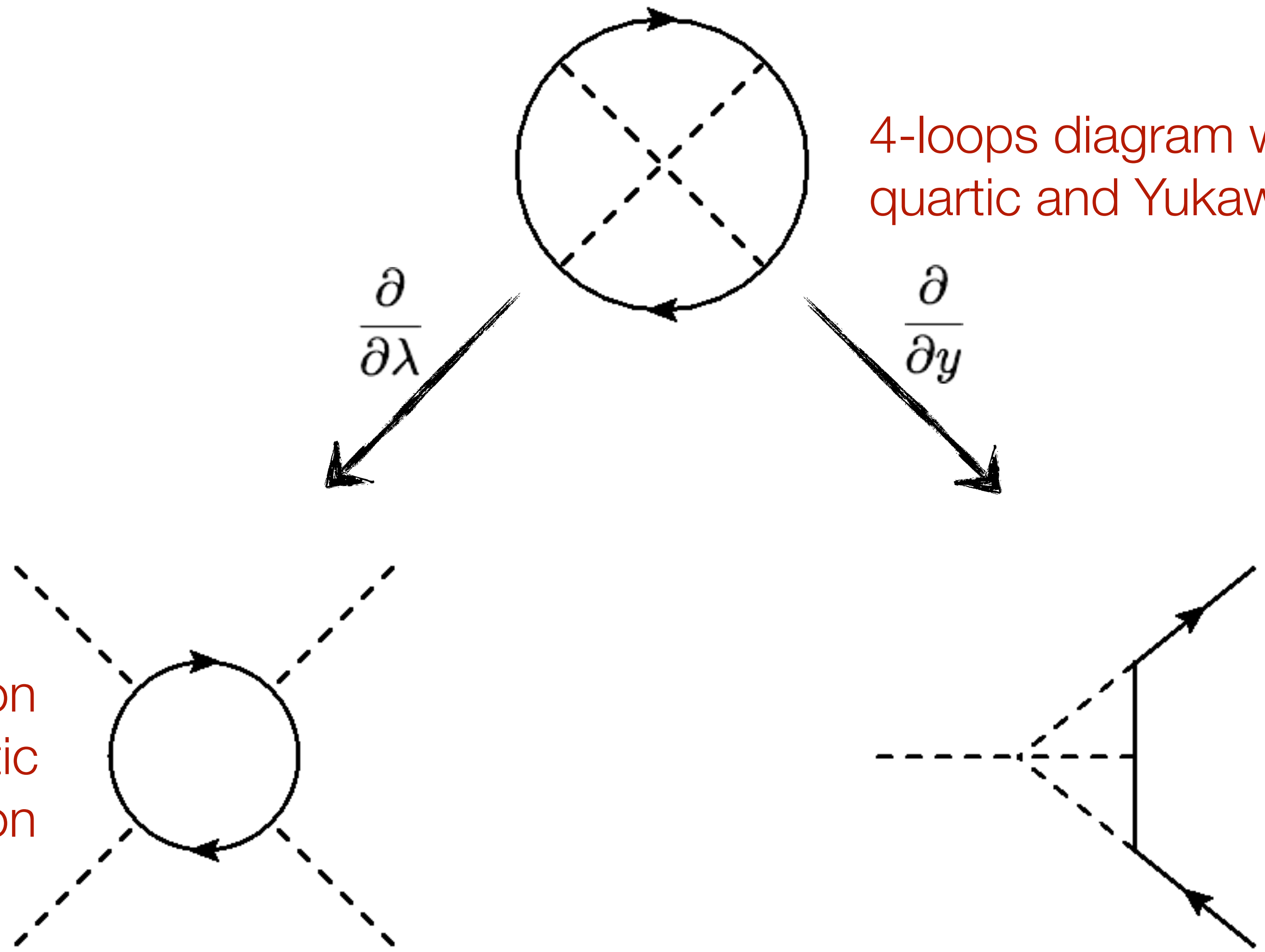

4-loops diagram with quartic and Yukawa couplings

1-loop contribution to the quartic β function

2-loops contribution to the Yukawa β function

$$\frac{\partial^2 \tilde{a}}{\partial g_i \partial g_j} \approx \frac{\partial}{\partial g_i}\left(\chi^{jk}\beta_k\right) \approx \frac{\partial}{\partial g_j}\left(\chi^{ik}\beta_k\right)$$

An example: the Standard Model

Neglecting all Yukawa coupling apart from the top one, the theory has five couplings:

$$\left\{\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_\lambda\right\} \equiv \left\{\frac{g_1^2}{(4\pi)^2}, \frac{g_2^2}{(4\pi)^2}, \frac{g_3^2}{(4\pi)^2}, \frac{y_t^2}{(4\pi)^2}, \frac{\lambda}{(4\pi)^2}\right\}$$

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Neglecting all Yukawa coupling apart from the top one, the theory has five couplings:

no square!

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The metric is diagonal at lowest order

Jack, Osborn (1990)

$$\chi^{ij} = \mathrm{diag}\left(\frac{1}{\alpha_1^2}, \frac{3}{\alpha_2^2}, \frac{8}{\alpha_3^2}, \frac{2}{\alpha_t}, 4\right)$$

matches the powers of α_i

Gives a set of relations among the β functions, e.g.

$$2\frac{\partial}{\partial\alpha_t}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_t}{\alpha_t}\right) + \mathcal{O}\left(\alpha_i^2\right),$$

1-loop

2-loop

$$\frac{3}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_2}{\alpha_2^2}\right) = \frac{\partial}{\partial\alpha_2}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right),$$

$$\cdots$$

The Standard Model β functions

$$\beta_1 = 2\alpha_1^2\Bigg\{\frac{1}{12}+\frac{10n_G}{9}+\left(\frac{1}{4}+\frac{95n_G}{54}\right)\alpha_1+\left(\frac{3}{4}+\frac{n_G}{2}\right)\alpha_2+\frac{22n_G}{9}\alpha_3+\left(\frac{163}{1152}-\frac{145n_G}{81}-\frac{5225n_G^2}{1458}\right)\alpha_1^2$$
$$+\left(\frac{87}{64}-\frac{7n_G}{72}\right)\alpha_1\alpha_2-\frac{137n_G}{162}\alpha_1\alpha_3+\left(\frac{3401}{384}+\frac{83n_G}{36}-\frac{11n_G^2}{18}\right)\alpha_2^2+\left(\frac{1375n_G}{54}-\frac{242n_G^2}{81}\right)\alpha_3^2-\frac{n_G}{6}\alpha_2\alpha_3$$
$$+\alpha_t\left[-\frac{17}{12}-\frac{2827}{576}\alpha_1-\frac{785}{64}\alpha_2-\frac{29}{6}\alpha_3+\left(\frac{113}{32}+\frac{101n_t}{16}\right)\alpha_t\right]+\alpha_\lambda\left(\frac{3}{4}\alpha_1+\frac{3}{4}\alpha_2-\frac{3}{2}\alpha_\lambda\right)\Bigg\}$$

$$\beta_2 = 2\alpha_2^2\Bigg\{-\frac{43}{12}+\frac{2n_G}{3}+\left(\frac{1}{4}+\frac{n_G}{6}\right)\alpha_1+\left(-\frac{259}{12}+\frac{49n_G}{6}\right)\alpha_2+2n_G\alpha_3+\left(\frac{163}{1152}-\frac{35n_G}{54}-\frac{55n_G^2}{162}\right)\alpha_1^2$$
$$+\left(\frac{187}{64}+\frac{13n_G}{24}\right)\alpha_1\alpha_2-\frac{n_G}{18}\alpha_1\alpha_3+\left(-\frac{667111}{3456}+\frac{3206n_G}{27}-\frac{415n_G^2}{54}\right)\alpha_2^2$$
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$$\vdots$$

$$\beta_\lambda = \frac{9}{16}\alpha_2^2-\frac{9}{2}\alpha_\lambda\alpha_2+\frac{3}{16}\alpha_1^2-\frac{3}{2}\alpha_\lambda\alpha_1+\frac{3}{8}\alpha_1\alpha_2+12\alpha_\lambda^2+6\alpha_\lambda\alpha_t-3\alpha_t^2+\ldots$$

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relations between the 2-loop gauge β functions

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$\vdots$

relations between the 3-loop gauge and 1-loop Higgs quartic β functions

$$\beta_\lambda = \frac{9}{16}\alpha_2^2-\frac{9}{2}\alpha_\lambda\alpha_2+\frac{3}{16}\alpha_1^2-\frac{3}{2}\alpha_\lambda\alpha_1+\frac{3}{8}\alpha_1\alpha_2+12\alpha_\lambda^2+6\alpha_\lambda\alpha_t-3\alpha_t^2+\ldots$$

Precision running in the Standard Model

Knowing the value of the Standard Model couplings at an arbitrary scale is important: vacuum stability, grand unification, cosmology...

The state-of-the-art computation makes use of the gauge, top Yukawa and Higgs quartic β functions at the 3-loops order

Degrassi et al. (2012), Buttazzo et al. (2013)

Inconsistent with the conformal symmetry at energies $E > v \sim 246$ GeV!

CP3 Origins

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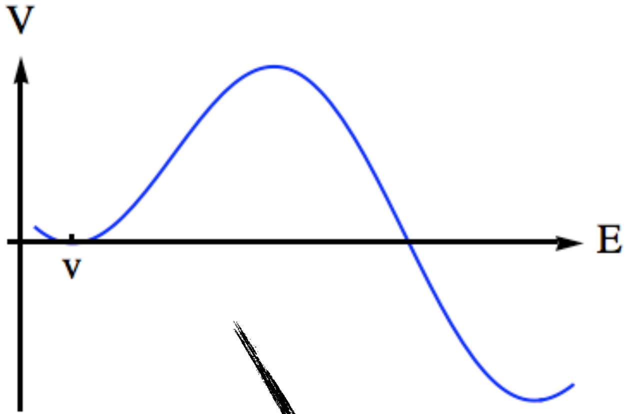
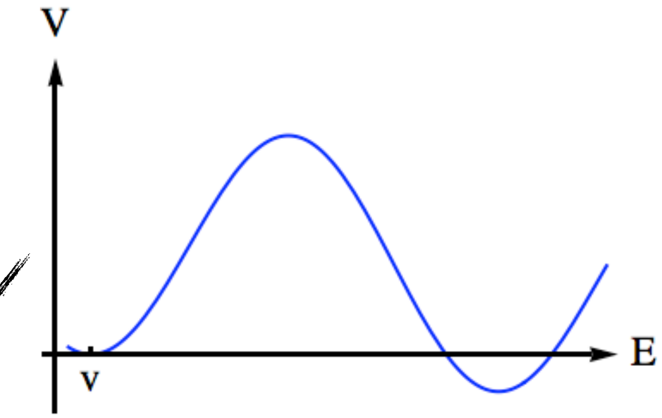
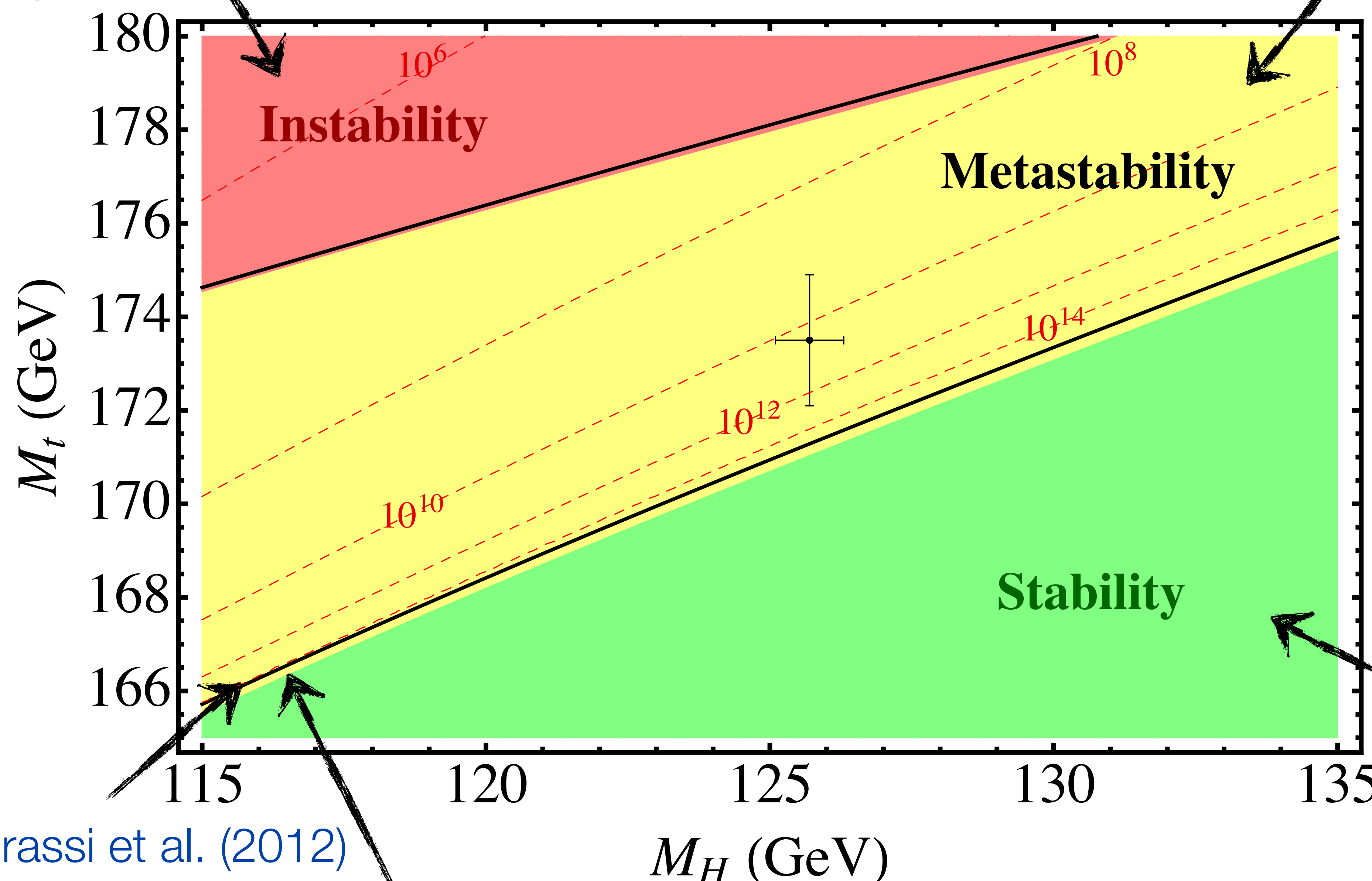
The best Weyl-consistent running based on the existing computations:

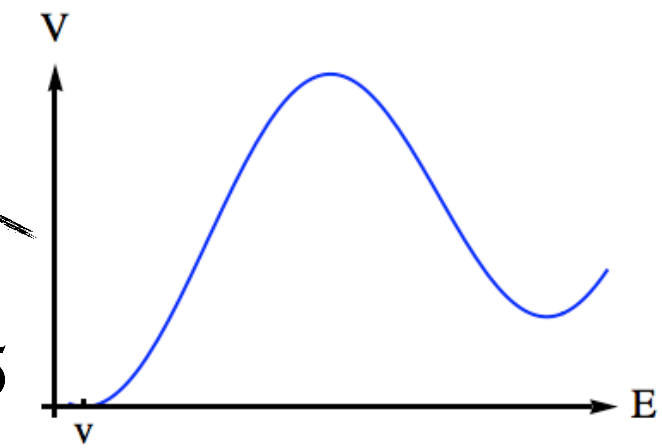
◇ 3 loops in the gauge β functions
◇ 2 loops in the top Yukawa β function
◇ 1 loop in the Higgs quartic β function

Standard Model Vacuum Stability





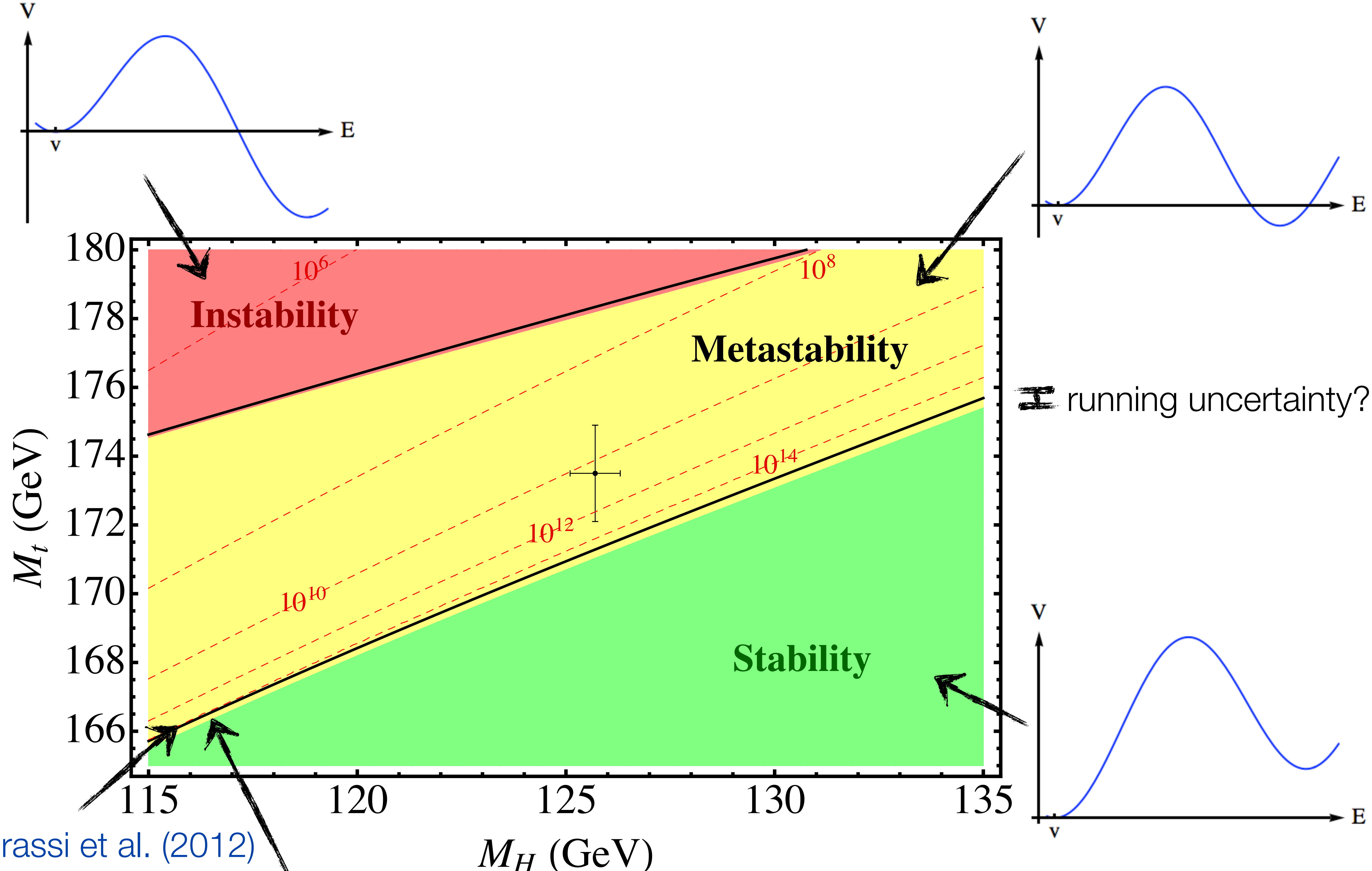




Degrassi et al. (2012)

3-2-1 counting scheme

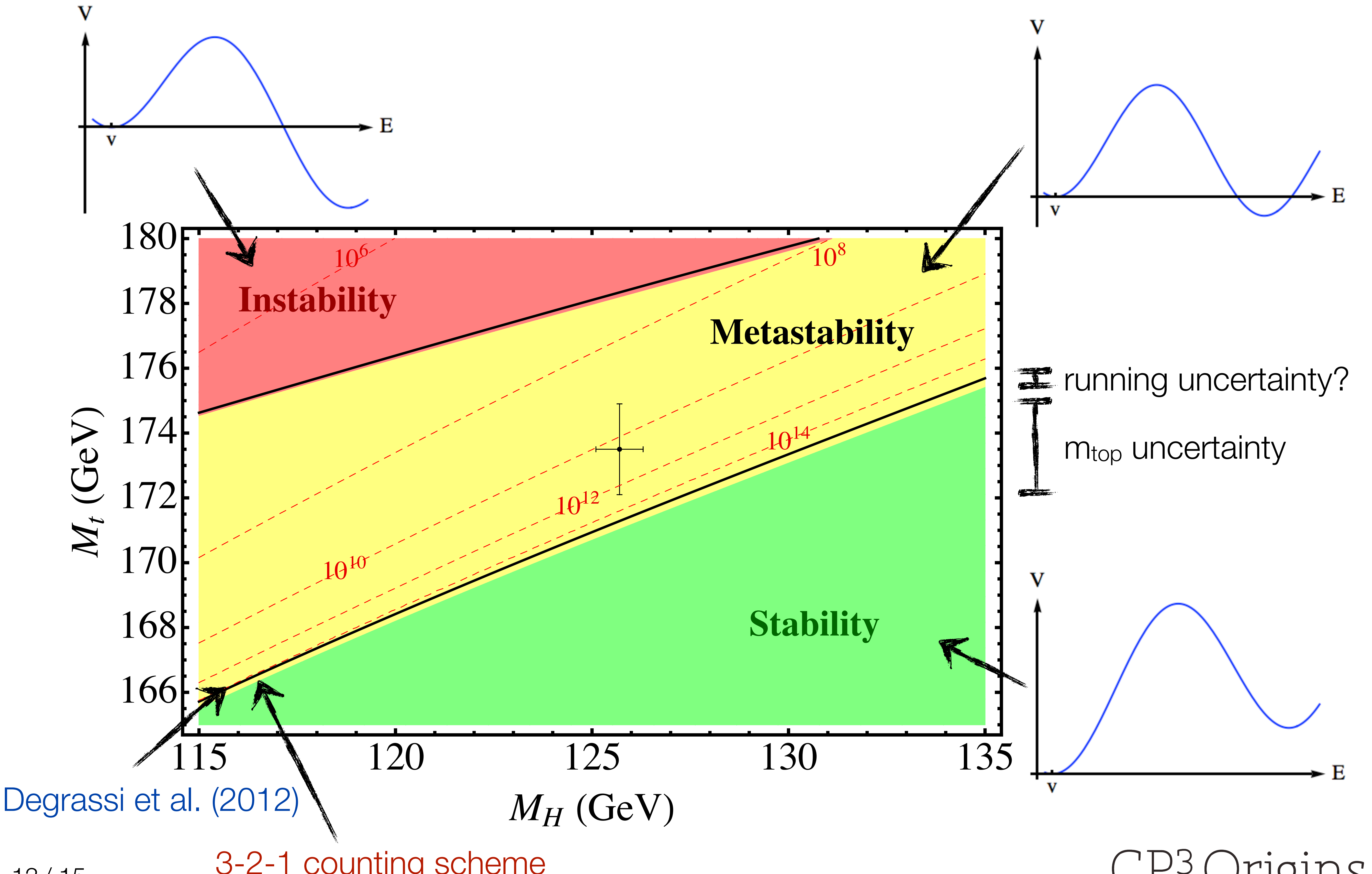
Standard Model Vacuum Stability



3-2-1 counting scheme

CP3 Origins

Standard Model Vacuum Stability



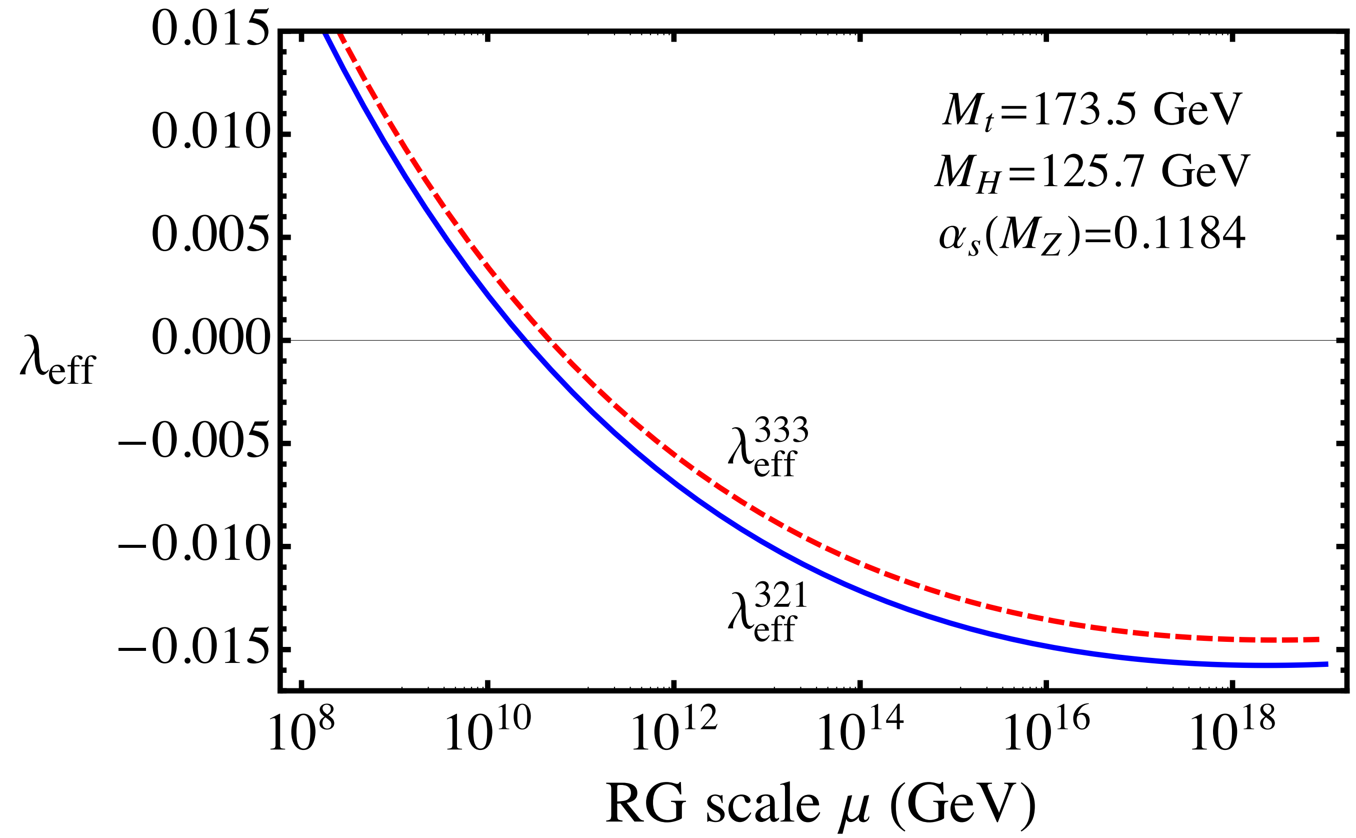
CP3 Origins

Importance of precision running

"Coincidence" in the SM:

$$\lambda(\mu) = 0 \quad \text{and} \quad \frac{d\lambda}{d\mu}(\mu) = 0$$

happen around the same scale
→ Higgs inflation?



CP³ Origins

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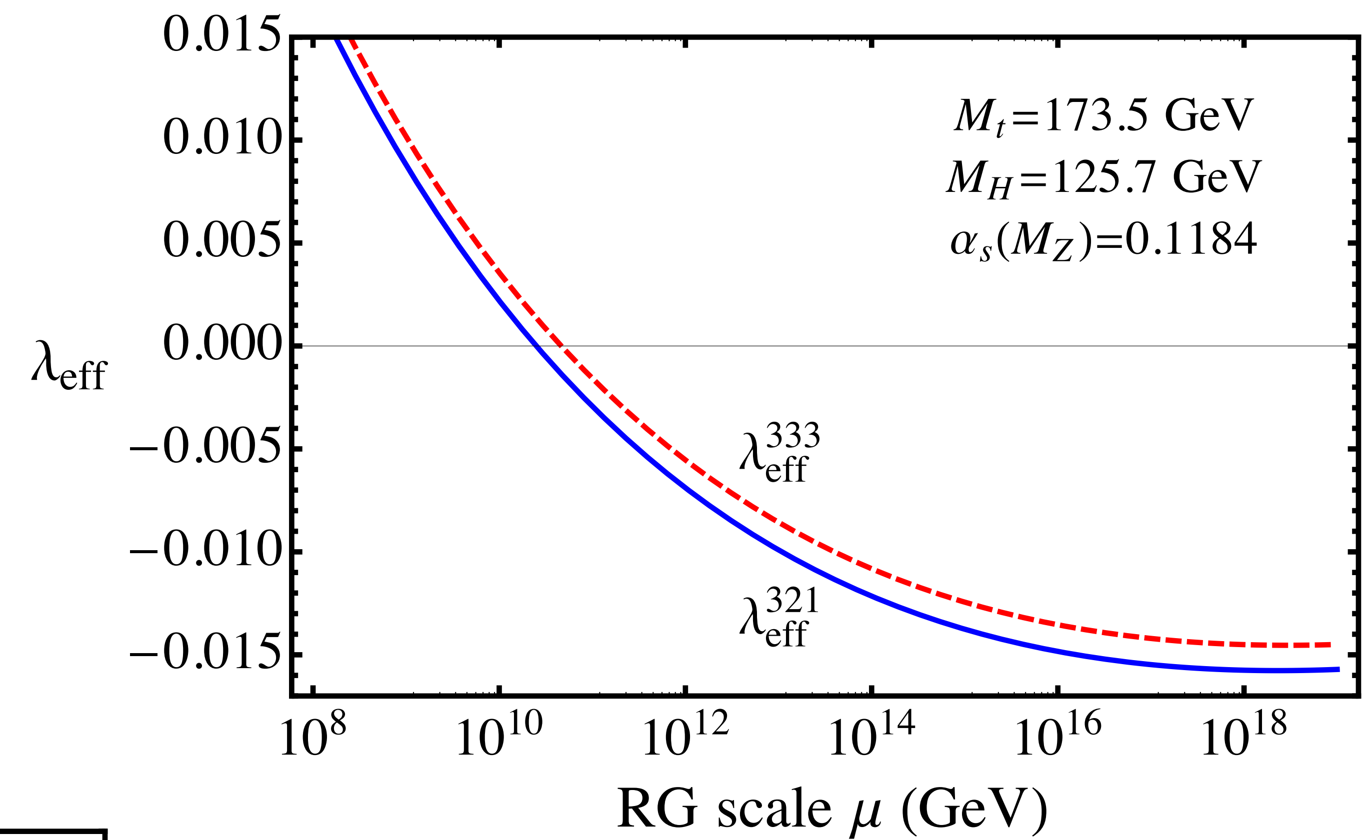

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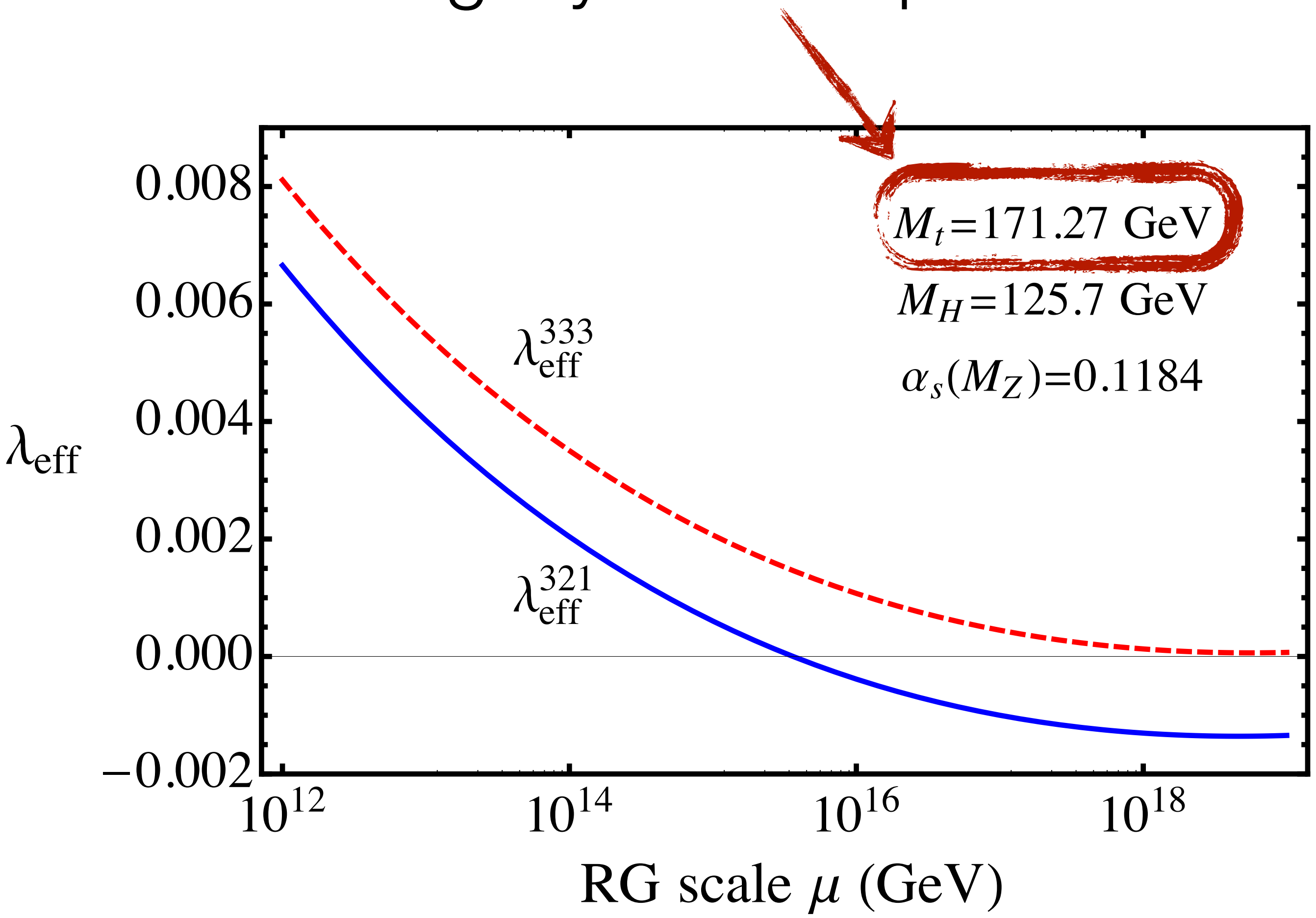

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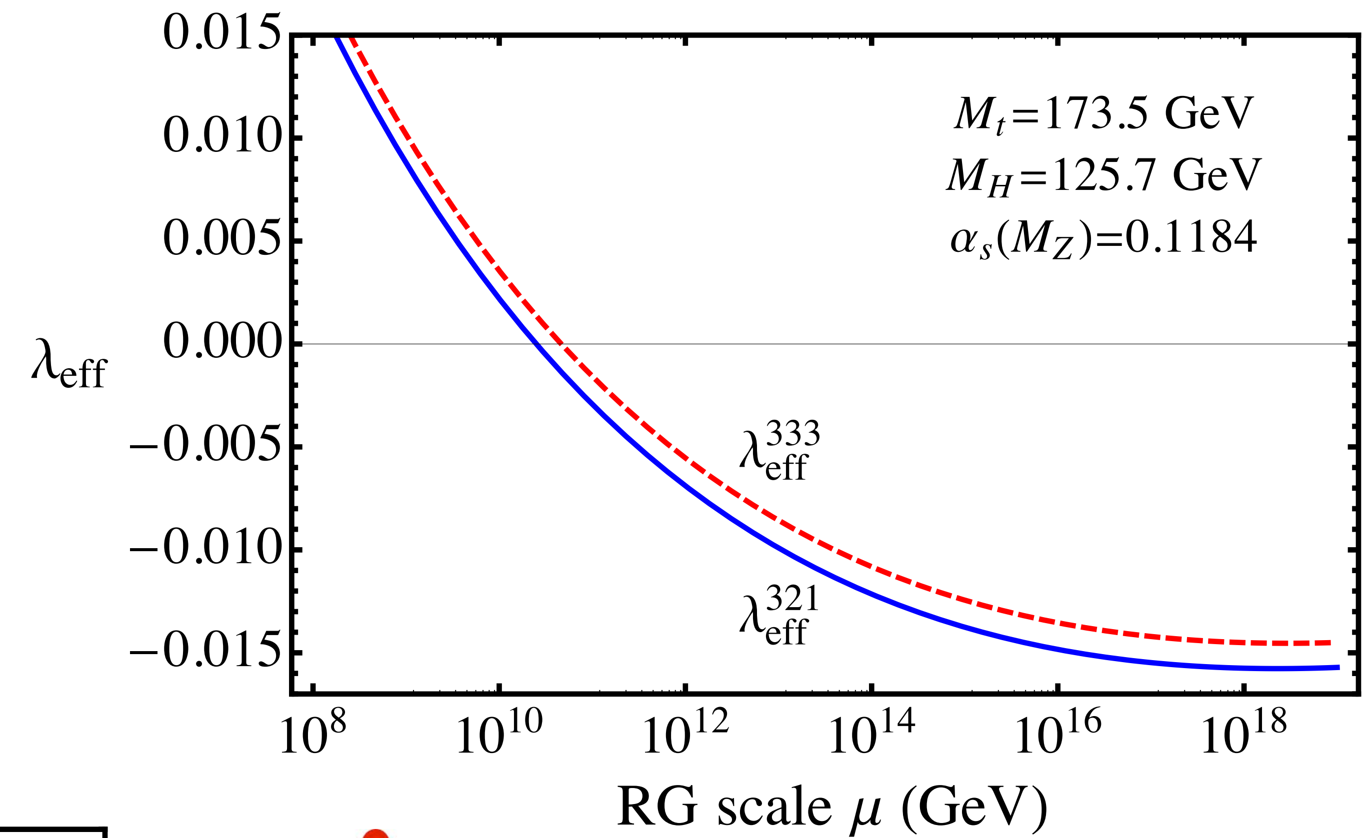

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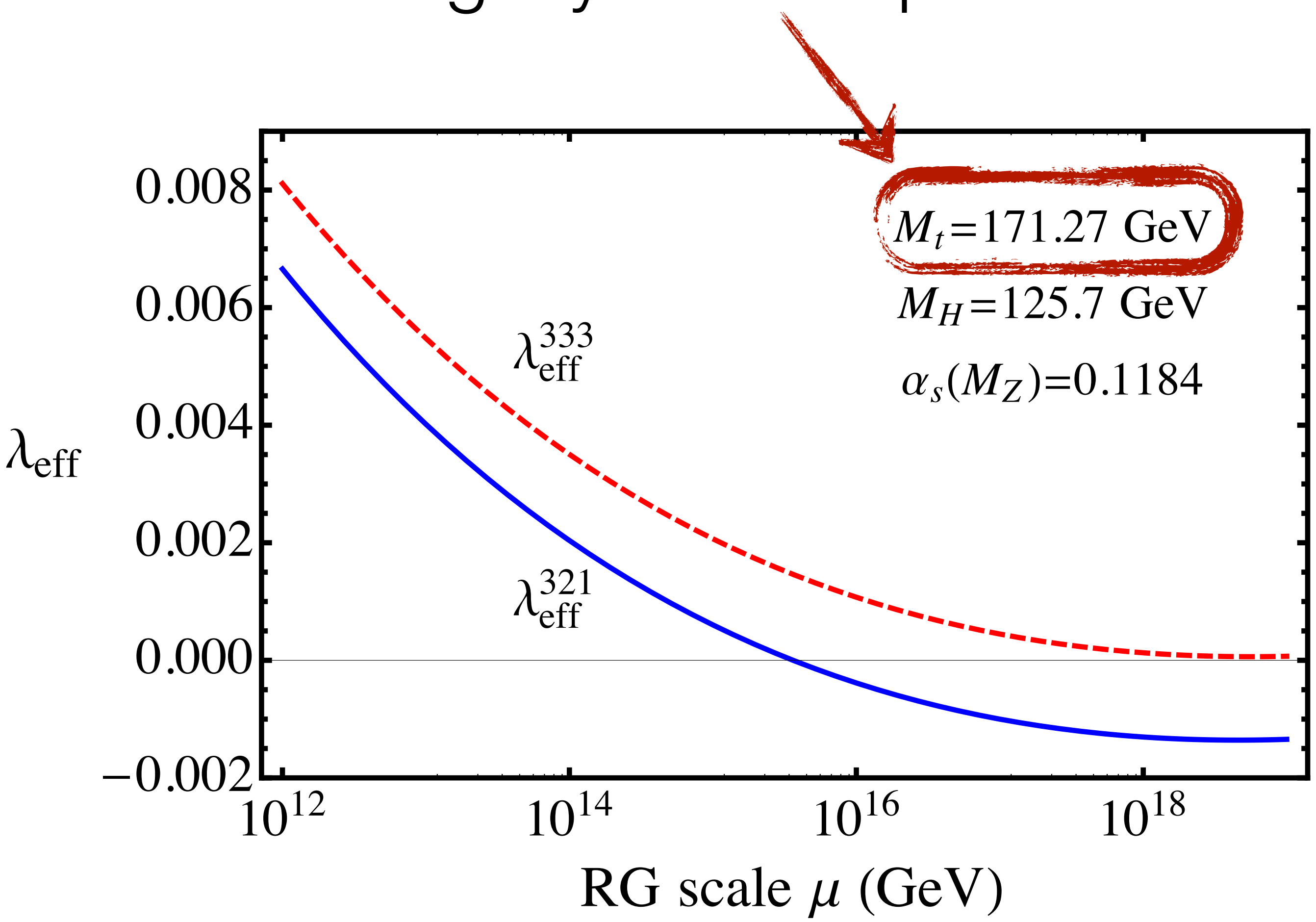

Important uncertainties also in:

- ◇ matching of $\overline{\text{MS}}$ parameters at the electroweak scale
- ◇ tunneling probability

Summary & Outlook

◇ The Weyl symmetry constrains the RG flow of a theory (more in the talks by M. Luty and R. Rattazzi tomorrow?)

◇ For theories with multiple couplings, there are relations among the β functions at different loop order

◇ A consistent scheme was established and used in the Standard Model to determine the vacuum stability

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◇ Important for the search of perturbative fixed points in gauge-Yukawa theories Antipin, Gillioz, Mølgaard, Sannino (2013)

◇ The Weyl consistency conditions could be used to determine the SM gauge β functions at 4-loop

CP3 Origins