Non-perturbative QFT for collider processes in the intense electromagnetic fields at the IP

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DESY

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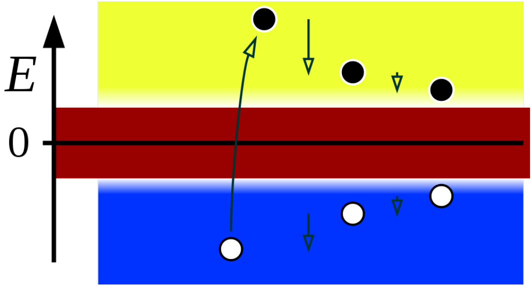
Preamble: Intense fields can polarise the vacuum



" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

- The Schwinger limit ($E_{\text{cr}} = 10^{18}$ V/m)
- Particles in future linear colliders will see $E \to E_{\text{cr}}$
- How do we incorporate a strong external field in QFT?

(W.H.) Furry Picture

- Separate gauge field into external A_μ^{ext} and quantum A_μ parts

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}A\!\!\!/\,\psi^{\text{FP}}$$

- Euler-Lagrange equation $\rightarrow$ new equations of motion requires exact (w.r.t. A^{ext}) solutions ψ^{FP}

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

- For certain classes of external fields (plane waves, Coloumb fields and combinations) exact solutions exist **[Volkov Z Physik 94 250 (1935), Bagrov and Gitman Exact solutions of Rel wave equations (1990)]**
- A QFT which is non-perturbative wrt external gauge field A^{ext} and perturbative wrt ψ^{FP}, A

assorted Furry Picture features

- External field makes space-time inhomogeneous so propagator depends on separate space-time points rather than on the difference between them **[Berestetski Lifshitz Pitaevski, QED $\S 109$]**
- Normalised IN and OUT states can be formed and LSZ extended to include such states **[Meyer, J Math Phys 11 312 (1970)]**
- Vanishing field strength at $t = \pm\infty \rightarrow$ stable vacuum
- Vacuum can be polarised so must include tadpole diagrams **[Schweber Relativistic QFT $\S$15g]**
- Operator and path integral representations for generating functional **[Fradkin, QED in an unstable vacuum]**
- Anomalous magnetic moment (one-loop) in a const crossed field varies from $\frac{\alpha}{2\pi}$ **[Ritus, JETP 30 1181 (1970)]**

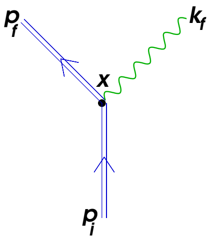
$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi\, dx}{(1+x)^3} \left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$

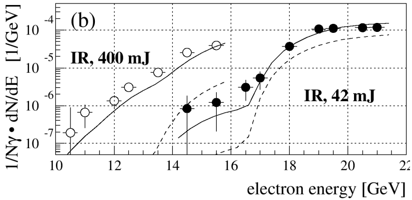
Synopsis

This is not a talk about the general structure of the Furry Picture, rather...applications

1. The Furry picture predicts distinct phenomenology
2. We are in an era of experimental tests of this phenomenology
3. The next generation of linear colliders will yield FP phenomena
4. We need solutions of the equations of motion in the particular field configuration
5. We need to apply the solutions in transition probabilities

Strong field experiments - SLAC E144 - 1990s

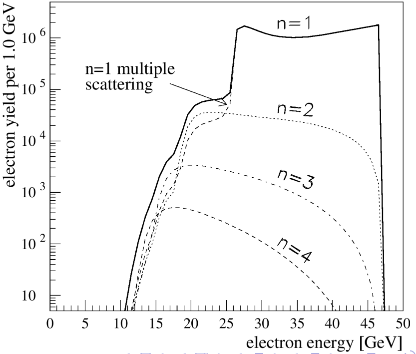




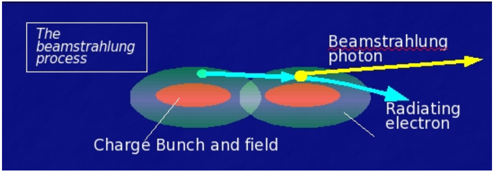
- Collided intense laser (10^{18} W/cm^2) with 46.6 GeV electrons
- effective momentum $q = p - \dfrac{e^2 a^2}{2k \cdot p} k$

$$\left(\sum_n\right) \quad q_i + nk \to q_f + k_f$$

- Compton-like scattering (HICS)
- Compton edge shifted by multiphoton effects



Strong fields at the collider Interaction Point



$\Upsilon \approx 1$ sets the strong field scale.

$$\Upsilon = \frac{e|\vec{a}|}{mE_{\text{cr}}}(k \cdot p)$$

- Υ depends on collider bunch parameters and the pinch effect
- Future linear colliders will have "strong" IP fields
- **All** collider processes are potentially "strong field processes"

Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
Υ_{av}	0.00015	0.001	0.24	4.9

(Volkov) Solution of the FP Dirac equation

Solution of the 2nd order Dirac equation with external 4-potential A_μ^{ext}

$$[D^2 + m^2 + \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}]\psi^{\text{FP}} = 0, \quad D_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$$

$$\psi^{\text{FP}} = e^{-i[p\cdot x + \$^p(k\cdot x)]}\, u_p$$

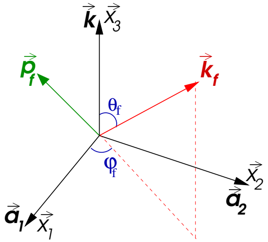
$$\$^p(k\cdot x) = \frac{1}{2(k\cdot p)}\int^{kx} \boxed{2eA^{\text{ext}}\cdot p - e^2 A^{\text{ext}\,2}} - \boxed{e\not{A}^{\text{ext}}\not{k}}$$

Volkov phase

Volkov spinor

Lorenz gauge with condition

$A^0 = 0 \implies \vec{a}_1 \perp \vec{a}_2 \perp \vec{k}$



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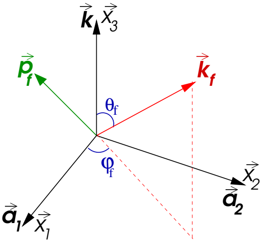
$$\$^p(k\cdot x) = \frac{1}{2(k\cdot p)}\int^{kx} \underbrace{2eA^{\text{ext}}\cdot p - e^2 A^{\text{ext}\,2}}_{\text{Volkov phase}} - \underbrace{e\not{A}^{\text{ext}}\not{k}}_{\text{Volkov spinor}}$$

Volkov phase

Volkov spinor



Orthornormality and Completeness of Volkov solutions **[Ritus, Ann Phys 69 552 (1971), Bergou and Varro, J Phys A 13 2823 (1980), Zakowicz JMathPhys 46 032304 (2005)]**

$$\int \frac{d^4x}{(2\pi)^4}\; e^{i[S^p(kx) - S^q(kx)]} = \delta^{(4)}(q-p)$$

$$\int \frac{d^4p}{(2\pi)^4}\; e^{i[S^p(kx) - S^p(ky)]} = \delta^{(4)}(x-y)$$

Volkov-type solutions

known solutions

- Single plane wave field **[Volkov, Z Phys 1935]**
- Circ/Linearly polarised field, constant field **[Nikishov and Ritus, JETP 1964]**
- Elliptically polarised field **[Lyulka, JETP 40 p815 1975]**
- 2 collinear orthogonal fields **[Lyulka 1975, Pardy 2004]**
- Coulomb fields + combinations **[Bagrov Gitman, Exact sols of Rel wave eqns 1990]**

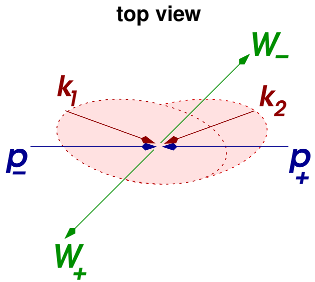
General procedure

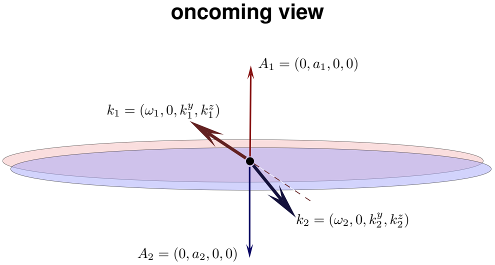
Klein-Gordon: $\left(D^2 + m^2\right) \phi_e = 0 \quad \rightarrow$ Volkov phase

2nd order Dirac: $\left(D^2 + m^2 \pm \frac{ie}{2} F^{\mu\nu}\sigma_{\mu\nu}\right) \psi_e = 0 \quad \rightarrow$ Volkov spinor

Dirac: $\left(i\not{D} - m\right) \psi_e = 0 \quad \rightarrow$ particular solution

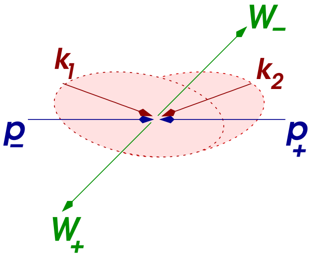
Solution of the FP Dirac equation in two fields



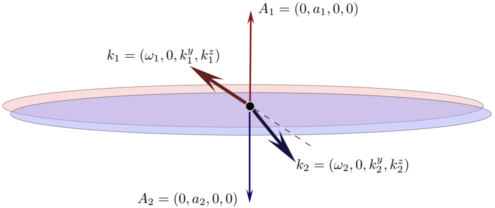


Solution of the FP Dirac equation in two fields

top view



oncoming view



- Transition probabilities are covariant, so choose collinear $\vec{k}_1 \,||\, \vec{k}_2$ reference frame
- external field is a superposition; rewrite as orthogonal components

$$A_\mu = A_{1\mu}(k_1 \cdot x) + A_{2\mu}(k_2 \cdot x) \to A_{+\mu} + A_{-\mu} \quad \text{where} \quad A_+ \cdot A_- = 0$$

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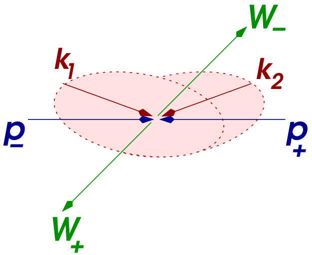

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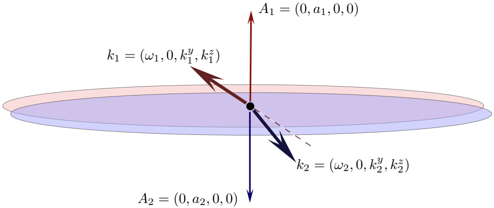

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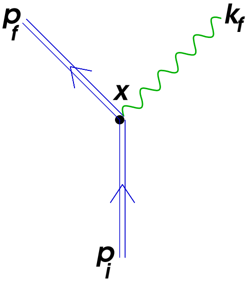
- solution is a product of Volkov solutions

$$[i\partial\!\!\!/ - eA\!\!\!/_+ - eA\!\!\!/_- - m]\,\psi^{\mathsf{FP}} = 0 \implies \psi^{\mathsf{FP}} = e^{-i\left[p\cdot x + \$^p_+ + \$^p_-\right]} u_r(p)$$

$$\text{where} \quad \$^p_+ = \int \frac{2eA_+(\phi)\cdot p - e^2 A_+(\phi)^2 - eA\!\!\!/_+(\phi)k\!\!\!/_1}{2k_1 \cdot p}\, d\phi$$

!st order Furry picture process and dressed vertex



FP Feynman diagrams only require a dressed vertex

$$\gamma_\mu^{\mathrm{FP}}(p_f,p_i) = e^{i\left[\$_+^{p_f}+\$_-^{p_f}\right]}\gamma_\mu\, e^{-i\left[\$_+^{p_i}+\$_-^{p_i}\right]}$$

$$\gamma_\mu^{\mathrm{FP}}(p_f,p_i) \to \int dr_1 dr_2\ \mathcal{F}^{-1}\left[\gamma_\mu^{\mathrm{FP}}(p_f,p_i)\right]\ e^{i(r_1k_1+r_2k_2)x}$$

contribution r_1k_1, r_2k_2 from external field enters into the conservation of momentum, allowing 1 vertex process

$$\delta^4(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

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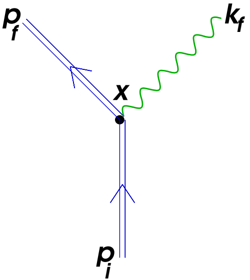

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two constant crossed fields leads to BesselK functions

$$A^{\text{ext}}_\mu = a_{1\mu}(k_1\cdot x)+a_{2\mu}(k_2\cdot x):\quad \mathcal{F}^{-1}\left[\gamma^{\text{FP}}_\mu(p_f,p_i)\right] \propto K_{\frac{1}{3},\frac{2}{3}}(z)$$

Traces are more complicated, and integration over final states needs care **[Hartin and Moortgat-Pick EPJC (2011)]**

$$\frac{|M_{fi}|^2}{VT} = -e^2\int dr_1 dr_2\;\text{Tr}[..r_1..r_2..]\;\frac{d\vec{p}_f\,d\vec{k}_f}{4\omega_f\epsilon_f}\;\delta^{(4)}(p_f+k_f-p_i-r_1k_1-r_2k_2)$$

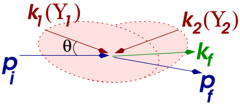
Beamstrahlung transition probability

We get a modification to the standard beamstrahlung transition probability

$$W = -\frac{e^2 m}{2\epsilon_i}\int_0^\infty \frac{du}{(1+u)^2}\left[\int dz + \frac{1+(1+u)^2}{1+u} X \frac{d}{dz}\right] \mathrm{Ai}(z), \quad u = \frac{\omega_f}{\epsilon_i - \omega_f}$$

1 field: $z = \frac{u^{2/3}}{(k_2\cdot p_i)^{2/3}}, \quad X = \frac{(k_2\cdot p_i)^{2/3}}{u^{2/3}}, \quad k_2 \equiv \Upsilon_2 \hat{k}_2$

2 fields: $z = \frac{u^{2/3}}{[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{1/3}}, \quad X = \frac{(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2 + 2a_1\cdot a_2 (k_1\cdot p_i)(k_2\cdot p_i)}{u^{2/3}\,[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{2/3}}$



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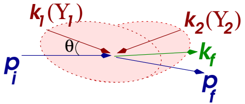
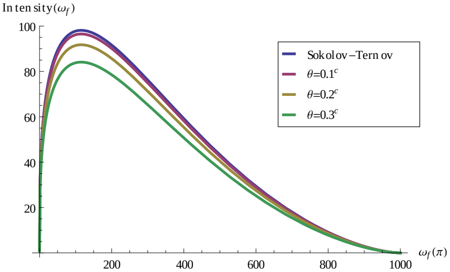
$$\text{2 fields:} \quad z = \frac{u^{2/3}}{[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{1/3}}, \quad X = \frac{(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2 + 2a_1\cdot a_2 (k_1\cdot p_i)(k_2\cdot p_i)}{u^{2/3}\,[(k_1\cdot p_i)^2 + (k_2\cdot p_i)^2]^{2/3}}$$



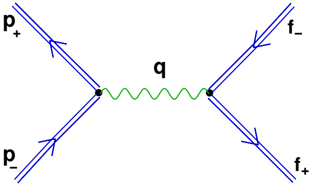
- Radiation power spectrum alters with the angle between the fields
- For Colliders there is little change $\theta_i \approx \pi$
- Can perform the analysis with laser fields where θ_i can be large
- **Treatment of higher orders beckons**



(e.g.) Generic two vertex Furry picture S channel

$$M_{fi} = g_1 g_2 \int dr_1 dr_2 ds_1 ds_2 \; \bar{v}_{p_+} \gamma^{\mathsf{FP}\mu} u_{p_-} \bar{\epsilon}_{f_+} \gamma^{\mathsf{FP}}_{\mu} \epsilon_{f_-} \frac{\delta(F - I - (r_1+s_1)k_1 + (r_2+s_2)k_2)}{(I + r_1 k_1 + r_2 k_2)^2}$$

- final states momentum $F \equiv f_- + f_+$ initial state momentum $I \equiv p_- + p_+$
- spin and polarisation sums as usual
- two dressed vertices γ^{FP}
- r_1, r_2, s_1, s_2 momentum contribution from two external fields at two vertices
- Phase integral not (much) more complicated than for 1 vertex process



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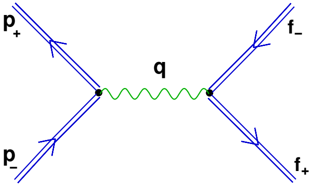

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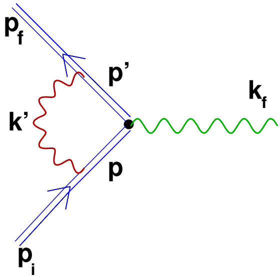
The pole structure depends on r_1, r_2 and is not standard need careful consideration of loops

Vertex function in (one) external field

$$\Gamma^{\text{FP}} = 2ie^2 \int drdsdl \int \frac{d^4k'}{k'^2}\, \gamma^{\text{FP}\nu}\, \frac{\not{p}' + m}{(q_f - k' - rk)^2 - m_*^2}\, \gamma_\mu^{\text{FP}}\, \frac{\not{p} + m}{(q_i - k' + sk)^2 - m_*^2}\, \gamma_\nu^{\text{FP}}\, \delta(q_f + k_f - q_i - lk)$$



- Examine pole structure of the vertex function

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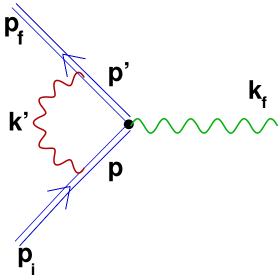

- Examine pole structure of the vertex function

- We combine denominators using Feynman parameters as normal,

$$\int \frac{d^4k'}{k'^2[(q_f - k' - rk)^2 - m_*^2][(q_i - k' + sk)^2 - m_*^2]}$$
$$= \int_0^1 dxdydz\, \frac{d^4k'}{(k'^2 - \Delta)^3}\, \delta(x + y + z - 1)$$

- Numerator more complicated than the usual case - need new tricksbut apart from the usual divergences we end up with additional poles in the residual

$$\frac{1}{\Delta(r, s, x, y, z)}$$

Vertex function in (one) external field

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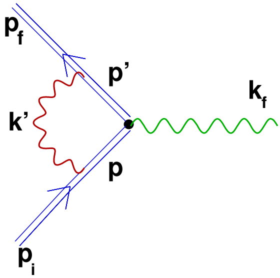

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- Examine pole structure of the vertex function

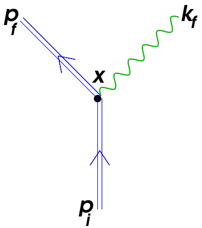
- Additional poles in the residue which match those in the tree level FP processwa
- Vertex function can be same order as tree-level diagram - must include!

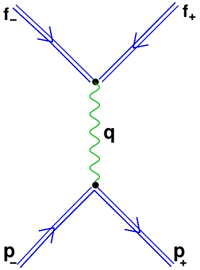
Summary

- The Furry picture is a semi-classical nonperturbative QFT which treats external electromagnetic fields exactly
- The Furry picture has several interesting features, non-vanishing vacuum currents, propagators that depend on separate space-time points
- For field vanishing at $t = \pm\infty$ LSZ can be extended to the FP and phenomenology can be calculated
- We are in the era of measurable effects, future linear colliders and intense lasers produce ”strong fields”
- New exact solutions for charged particles in two external fields applied to the photon radiation process - power spectrum changes
- **ALL** collider processes are potentially FP processes, need to extend this analysis to second and higher orders
- FP has a unique pole structure, need to include loops and show cancellation to all orders

Backup Slides

Strong field processes at Collider IP





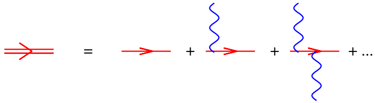
- **1st order:**
  - Beamstrahlung & coherent pair production
  - beam-beam simulations (CAIN, Guinea-PIG)
  - basis of ISR/FSR simulations
  - 1-vertex permitted $p_i + rk - p_f - k_f = 0$

- **ALL** processes at the IP are "strong field" processes

- **2nd order:**
  - "normal processes" in limit $E \to 0$
  - Need Volkov solution in fields of both bunches
  - Need to obtain the cross-section for a generic 2nd order process

Equiv Photon Approx and Perturbation expansion

- decompose external field into n equivalent photons
- sum the series to desired order of accuracy



$$G^e = G + G\hat{V}G + G\hat{V}G\hat{V}G + \ldots$$

$$G = (p^2 - m^2)^{-1}$$

$$\hat{V} = 2eA^e \cdot p - e^2 A^{e\,2}$$

- within certain constraints:
  - scalar particle
  - monochromatic photons

  the summation can be performed (Reiss Eberly 1966)

- Can the entire summation be performed in general ?

- The alternative is the Furry/Feynman method...

Requirements for a strong field event generator

REQUIREMENTS:

To simulate a charge bunch collision and calculate the field strength at each point of production

To have a finely scaled simulation in order to accurately model disruption, hour glass effect etc.

To perform a relatively complex cross-section calculation at each point of production

To have full spin tracking

To be flexible enough to include new higher order processes

SOLUTION:

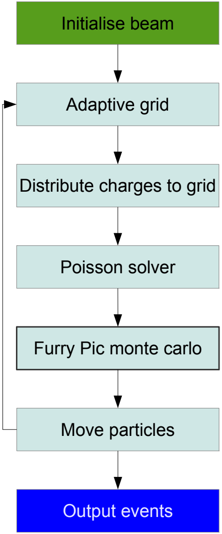
A PIC code using an efficient method for modeling the electrodynamics – crosscheck with CAIN/GP

MPI using openMPI or GPU programming

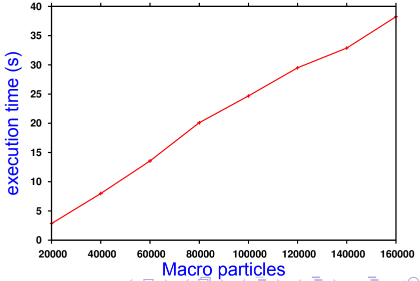
T-BMT with higher order corrections to AMM, Sokolov-Ternov and higher order helicity amplitudes

Allow new processes to be loaded externally

IPstrong - towards a strong field event generator



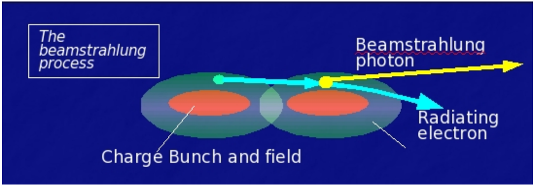
- Fortran 2003 with openMPI (Fortran 2008 has inbuilt gpu)
- 3D electrostatic poisson solver (MPI)
- Furry picture processes replace all other processes
- output in multiple formats (stdhep, lcio)
- cross-checks with existing programs



Infinite momentum frame

- QED can be formulated in a Lorentz frame moving at the limit of the speed of light (Kogut & Soper Phys Rev D 1(10) 2901 (1970))
- regular coordinates (t, x, y, z) can be expressed in light cone coordinates $x_\pm = \dfrac{1}{2}(t \pm z)$; $x_\perp = (x, y)$
- light cone dirac matrices separate into sub-algebras whose members anti-commute $\gamma_\pm \gamma_\perp = -\gamma_\perp \gamma_\pm$
- light cone scalar products are $a.b = 2a_+ b_- + 2a_- b_+ - a_\perp . b_\perp$

Collider strong field physics

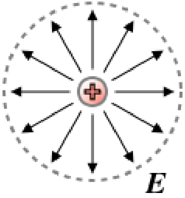


" Strong field processes are physics processes calculated simultaneously in the normal perturbation theory as well as exactly with respect to a strong electromagnetic field. "

" Such calculations are necessary when the external field seen by a particle approaches or exceeds E_{cr}. "

Strong fields at the collider IP

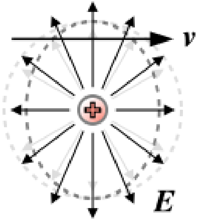
- moving charge has longitudinal length contraction



- relativistic charge bunch produces constant crossed plane wave field

$$A_\mu = a_{1\mu}(k \cdot x)$$

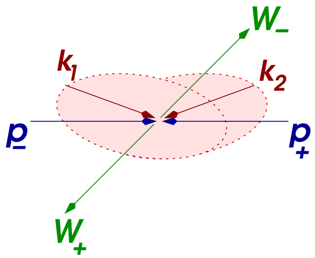
$$a_{1\mu} = (0, \vec{a})$$



- particle p sees a field strength parameter Υ

$$\Upsilon = \frac{e|\vec{a}|}{m E_{\text{cr}}}(k \cdot p)$$

Volkov-type solutions in two external fields



- both incoming bunches contribute external fields
- external field wavevectors are generally anti-collinear
- Need new Volkov-type solution

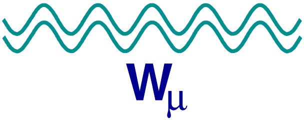
- strategy is to first solve Klein-Gordon equation $(D^2 + m_W^2)\phi_e^{\pm}$

$$\phi_e^{\pm} = \frac{1}{\sqrt{2\epsilon_p V}} \int dr \, \exp\left[-ib\, p \cdot x - ireA_e - \frac{(r-f)^2}{2|z|} \right]$$

- For constant crossed field Dirac equation solution proceeds from the Klein-Gordon solution

W boson Volkov Solution

- Equation of motion for the W boson



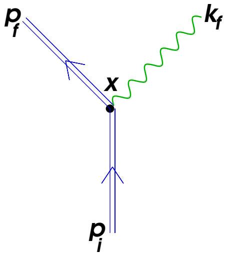
$$(D^2 + m_W^2)W_\nu + i2eF^\mu{}_\nu W_\mu = 0, \quad D^\mu W_\mu = 0$$

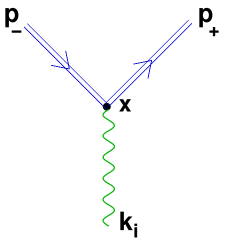
- with solution $W_\mu = E_p^W \, e^{-ip\cdot x} \, w_p$ where

$$E_p^W = \left(g_{\mu\nu} + \frac{e}{k\cdot p} \int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{e2} k_\mu k_\nu \right)$$
$$\cdot \exp\left[-\frac{i}{2(k\cdot p)} \left(2e(A^e \cdot p) - e^2 A^{e2} \right) \right]$$

- similar solutions can be found for other particles that couple to A^e

Beamstrahlung, incoherent/coherent pair production

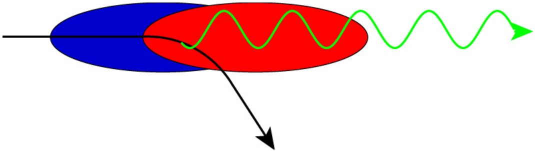




- IP beam-beam simulators - CAIN, Guinea-Pig
- beamstrahlung & coherent pair production calculated via quasi-classical approx
- incoherent pairs calculated with beamstrahlung photon and equivalent photon approx (EPA)
- **more exactly** these are 1st and 2nd order Furry picture processes

bkgd pairs	current	proposed
coherent	quasi-classical	1 vertex Furry picture
incoherent	EPA	2 vertex Furry picture

Formation length



" distance travelled by a charged particle while a radiated photon moves one wavelength in front of it "

A bad argument: *" If the bunch is sufficiently short we dont need to worry about strong field effects"*

- classical argument that only applies to the beamstrahlung
- strong field propagator integrated over all length scales