J.G. Körner at LHC-D Workshop 2008 - Topquark Physik (III), Bad Honnef,

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Helicity Content of W-Bosons from Top quark Decays at NNLO

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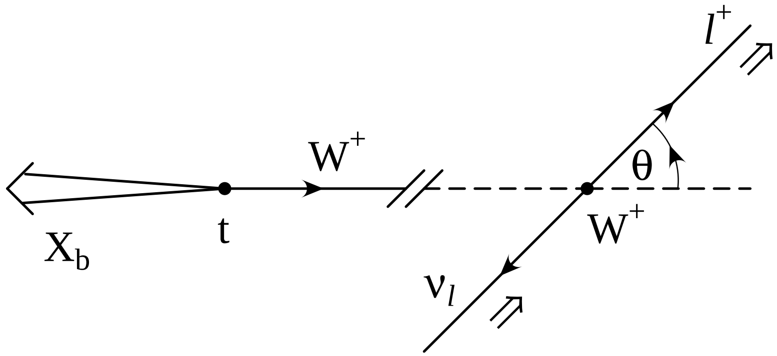
(much help from A. Kadeer (Mainz) and S. Groote (Univ of Tartu))

Some properties of the Top Quark

- The top quark is the heaviest heavy flavour quark
- The top quarks decay almost $100\,\%$ of the time into $(b + W^+)$
- Top quarks decay before they can hadronize. Top quark decays provide ideal setting to test perturbative QCD (no hadronization effects, negligible Λ_{QCD}/m_t effects)
- The decay $t \to b + W^+$ is weak $\Longrightarrow$ the W^+-boson is polarized
- The W^+ decays weakly ($\to (l^+ + \nu_l),\ (q_i + \bar{q}_j)$) $\Longrightarrow$ it is self-analyzing.
- LHC will produce a $(t\bar{t})$-pair every 4 (0.4) seconds in low (high) luminosity run

Helicity rates of the W–boson in $t \to b + W^+$



$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{4}\sin^2\theta\ \Gamma_L + \frac{3}{8}(1+\cos\theta)^2\Gamma_+ + \frac{3}{8}(1-\cos\theta)^2\Gamma_-$$

- Γ_L: longitudinal
- Γ_+: transverse plus
- Γ_-: transverse minus
- polar angle θ is measured in W–rest frame.

Integrating over $\cos\theta$ one recovers the total rate $\Gamma = \Gamma_L + \Gamma_+ + \Gamma_-$.

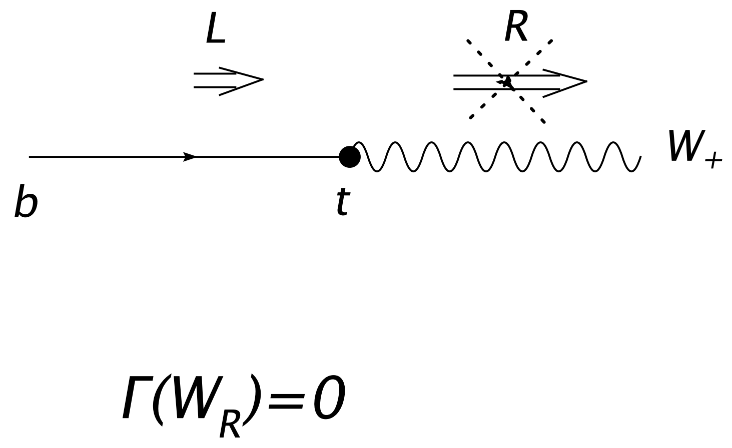
Leading Order (LO) Calculation

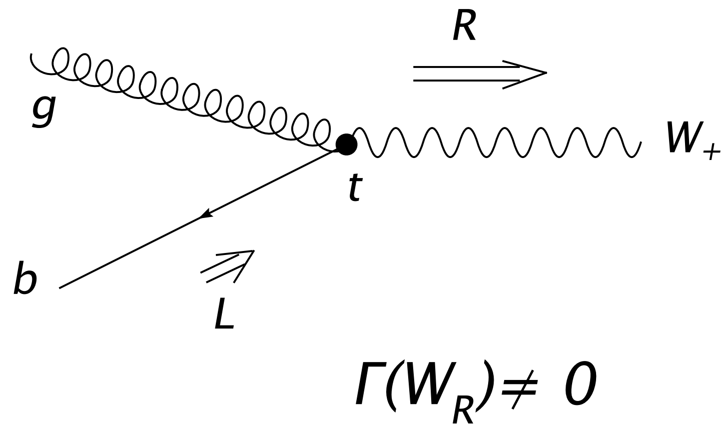
For the helicity fractions $\mathcal{G}_i = \Gamma_i/\Gamma$ the LO results are

$$\mathcal{G}_L : \mathcal{G}_+ : \mathcal{G}_- = \frac{1}{1+2x^2} : 0 : \frac{2x^2}{1+2x^2}$$

($\mathcal{G}_L + \mathcal{G}_+ + \mathcal{G}_- = 1$) where $x^2 = m_W^2/m_t^2 = 0.211$.
Numerically one has

$$\mathcal{G}_L : \mathcal{G}_+ : \mathcal{G}_- = 0.703 : 0 : 0.297$$





Anticipate: $\Gamma_+ \neq 0$ at NLO, NNLO

Next-to-Leading Order (NLO) Calculation

NLO corrections to the helicity rates have been calculated in H.S. Do, M. Fischer, S. Groote, B. Lampe, M.C. Mauser, J.G.K.(99,01,02,03)

Numerically one has ($\hat{\Gamma}_i = \Gamma_i^{LO+NLO}/\Gamma_{U+L}^{LO}$)

$$
\begin{aligned}
\hat{\Gamma} &= 1 - 0.0854 \\
\hat{\Gamma}_L &= 0.703(1 - 0.095) \\
\hat{\Gamma}_+ &= 0.000927 \\
\hat{\Gamma}_- &= 0.297(1 - 0.0656)
\end{aligned}
$$

NLO helicity fractions are

$$\mathcal{G}_L : \mathcal{G}_+ : \mathcal{G}_- = 0.696 : 0.001 : 0.303$$

compared to LO helicity fractions

$$\mathcal{G}_L : \mathcal{G}_+ : \mathcal{G}_- = 0.703 : 0 : 0.297$$

An early MC study quotes experimental sensitivities of $\Delta\mathcal{G}_L = 0.7\,\%$ and $\Delta\mathcal{G}_+ = 0.3\,\%$ for an integrated luminosity of 100 fb^{-1} at Tevatron II energies ($\approx 8 \cdot 10^6$ $(t\bar{t}$-pairs)). Compare this to NLO changes $\Delta\mathcal{G}_L = 0.7\,\%$ and $\Delta\mathcal{G}_+ = 0.1\,\%$. Much higher event rates can be reached at the LHC in one year.
A more recent MC study based on 10 fb^{-1} at the LHC quotes measurement uncertainties of $\Delta\mathcal{G}_L = 1.9\,\%$, $\Delta\mathcal{G}_+ = 0.21\,\%$ and $\Delta\mathcal{G}_- = 1.8\,\%$.

Experimentally, there has been a continuing interest in the measurement of the helicity fractions. Latest measurements:

$$\begin{aligned} \mathrm{CDF}(2007)\ : \qquad & \mathcal{G}_L = 0.85^{+0.15}_{-0.22}(\mathrm{stat}) \pm 0.06(\mathrm{syst}) \\ & \mathcal{G}_+ = 0.05^{+0.11}_{-0.05}(\mathrm{stat}) \pm 0.03(\mathrm{syst}) \\ \mathrm{DO}(2005)\ : \qquad & \mathcal{G}_L = 0.56 \pm 0.31 \end{aligned}$$

and, assuming that $\mathcal{G}_L$ is fixed at its SM value, they quote :

$$\mathrm{DO}(2007): \qquad \mathcal{G}_+ = 0.056 \pm 0.080(\mathrm{stat}) \pm 0.057(\mathrm{syst})$$

All of these measurements are well within SM predictions.
Latest: (Dec. 12th 2007; DO)
$\mathcal{G}_L = 0.425 \pm 0.16(\mathrm{stat}) \pm 0.053(\mathrm{syst})$
$\mathcal{G}_+ = 0.119 \pm 0.090(\mathrm{stat}) \pm 0.053(\mathrm{syst})$
Consistent with the SM only at the 30% confidence level.

Next-to-Next-to-Leading Order (NNLO) Calculation

Motivation :

- check on convergence properties of perturbative series; e.g. size of Γ_+ at NNLO.
- NNLO calculations are the call of the day
- develop new techniques which may be useful in other NNLO calculations

Use a technique developed by Blokland et al. which has been succesfully applied to the calculation of the total rate (Blokland, Czarnecki,Ślusarczyk,Tkachov 04,05). The main ideas are

- Reduce two-mass-scale problem (m_t, m_W) to one mass scale problem (m_t). Obtain results as an expansion in powers of m_W/m_t and logarithms $\ln m_W/m_t$.

- Optical theorem: Calculate three-loop diagrams and take imaginary parts

$$\Gamma(two - loop) = \frac{1}{m_t} \text{Im}\left(\Sigma(three - loop)\right) ,$$

- Use dimensional regularization to regularize UV and IR/M singularities

- Expansion by regions. Consider two regions:
  i) hard region
  All loop momenta are hard $\mathcal{O}(m_t)$. Can expand W-propagator

$$\frac{1}{q^2 - m_W^2} = \frac{1}{q^2} + \frac{m_W^2}{q^4} + \frac{m_W^4}{q^6} + \ldots$$

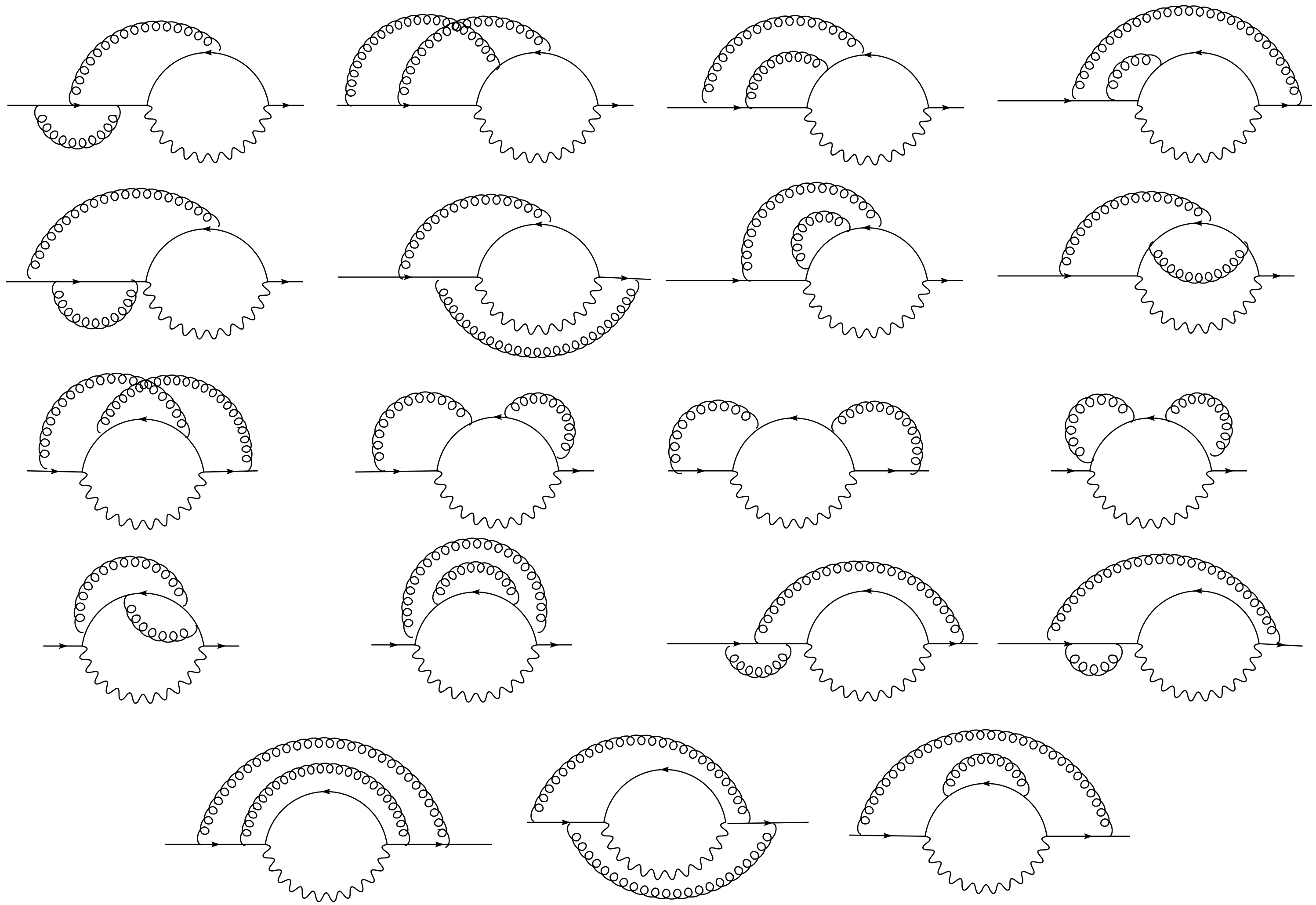
  Massive propagator has been converted into a sum of massless propagators.
  ii) soft region
  All momenta are hard but momentum flowing through W is soft. Cannot use above expansion of the W propagator. But integrals factorize into two-loop self-energy-type integrals and a one-loop vacuum bubble diagram.

- reduction to 23 master integrals by integration-by-parts identities. Use of Laporta's algorithm.

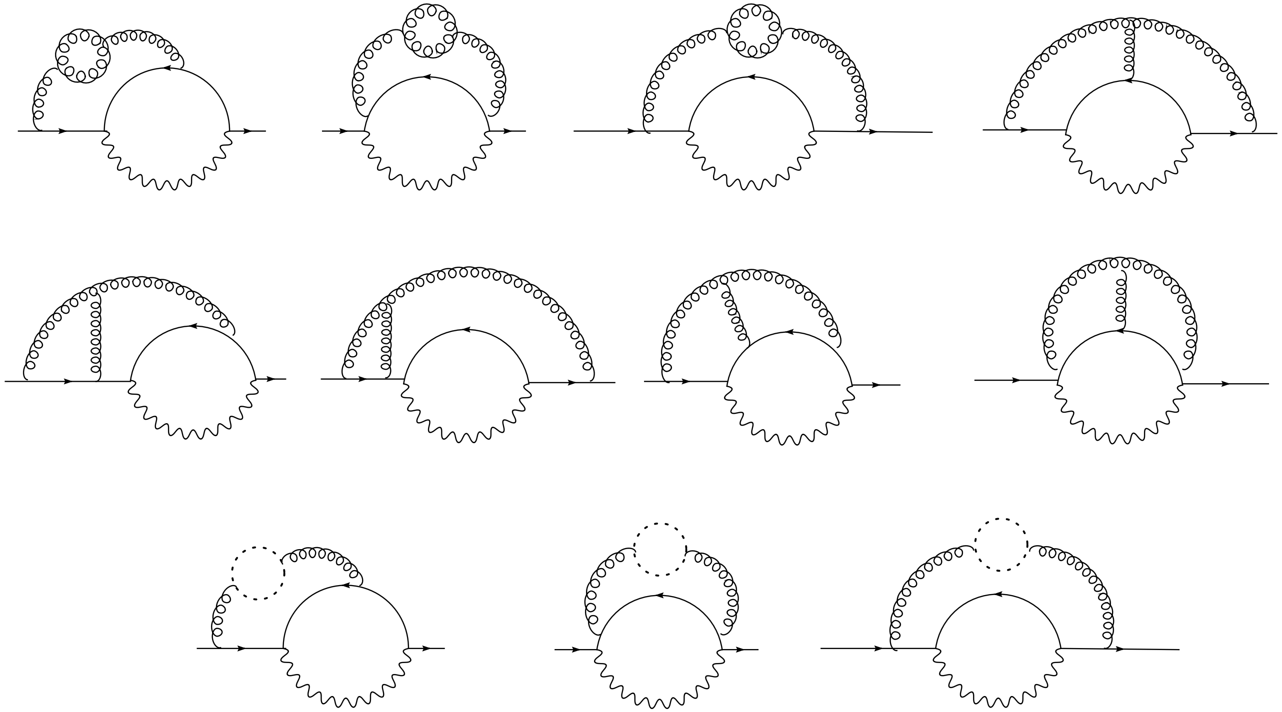
- Viability of method has been checked against known NLO result

36 $\mathcal{O}(\alpha_s^2)$ three-loop Feynman diagrams:

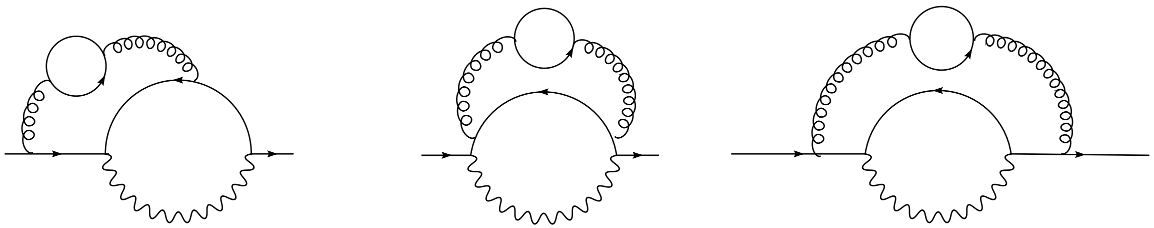
Abelian graphs:



Non-Abelian graphs:



Quark loop contributions



Two new features appear in the NNLO calculation of the helicity rates $\Gamma_{L,\pm}$:

- Treatment of γ_5 in dimensional regularization
  replace

$$\gamma_\mu \gamma_5 \rightarrow \frac{1}{3!} \epsilon_{\mu\alpha\beta\gamma} \gamma^\alpha \gamma^\beta \gamma^\gamma$$

  Need additional finite three-loop counter terms
  (Larin, Vermaseren 91,93)

- Replacement of the total rate projector

$$\mathbb{P}^{\mu\nu}_{U+L} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}$$

  by projectors $\mathbb{P}^{\mu\nu}_{L,\pm}$ onto the three helicity rates.

Viability of the treatment of the new features have been tested against known LO and NLO results.

Construction of Helicity Projectors $\mathbb{P}^{\mu\nu}_{L,\pm}$

It is convenient to define covariant projectors to project onto the helicity rates, *cif.*

$$\Gamma_i \sim \mathbb{P}^{\mu\nu}_i H_{\mu\nu} \qquad (i = L, +, -)$$

Start with

$$\begin{aligned} \mathbb{P}^{\mu\nu}_L &= \epsilon^{\mu}_L \, \epsilon^{*\nu}_L \\ \mathbb{P}^{\mu\nu}_\pm &= \epsilon^{\mu}_\pm \, \epsilon^{*\nu}_\pm \end{aligned}$$

In the rest frame of the W^+ one has

$$\begin{aligned} \epsilon^{\mu}_L &= (0; 0, 0, 1) \\ \epsilon^{\mu}_\pm &= \frac{1}{\sqrt{2}}(0; \mp 1, -i, 0) \end{aligned}$$

The covariant projectors can be constructed from the following three projectors

- Projector for the total rate

$$\mathbb{P}^{\mu\nu}_{U+L} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}$$

- Projector for the longitudinal helicity rate

$$\mathbb{P}^{\mu\nu}_{L} = \frac{m_W^2}{m_t^2}\frac{1}{|\vec{q}\,|^2}\left(p_t^\mu - \frac{p_t \cdot q}{m_W^2} q^\mu\right)\left(p_t^\nu - \frac{p_t \cdot q}{m_W^2} q^\nu\right).$$

- Projector for the forward-backward asymmetric helicity rate

$$\mathbb{P}^{\mu\nu}_{F} = -\frac{1}{m_t}\frac{1}{|\vec{q}\,|} i\epsilon^{\mu\nu\alpha\beta} p_{t,\alpha} q_\beta$$

Finally, the three projectors read ($\Gamma_i \sim \mathbb{P}_i^{\mu\nu} H_{\mu\nu}; \quad i = L, +, -,$)

$$\mathbb{P}_L^{\mu\nu} = \frac{m_W^2}{m_t^2} \frac{1}{|\vec{q}|^2} \left(p_t^\mu - \frac{p_t \cdot q}{m_W^2} q^\mu \right) \left(p_t^\nu - \frac{p_t \cdot q}{m_W^2} q^\nu \right)$$

$$\mathbb{P}_\pm^{\mu\nu} = \frac{1}{2} \left(\mathbb{P}_{U+L}^{\mu\nu} - \mathbb{P}_L^{\mu\nu} \pm \mathbb{P}_F^{\mu\nu} \right)$$

The denominator factors $|\vec{q}|^{-2}$ and $|\vec{q}|^{-1}$ are needed for the correct normalization of the projectors, cif.

$$g_{\alpha\beta} \mathbb{P}_i^{\mu\alpha} \mathbb{P}_j^{\beta\mu} = -\delta_{ij} \mathbb{P}_i^{\mu\nu}$$

The denominator factors $|\vec{q}|^{-2}$ and $|\vec{q}|^{-1}$ somewhat complicate the NLO and NNLO calculation of the helicity rates as compared to the total rate.

How to deal with the propagator-like factors $|\vec{q}\,|^{-2}$ and $|\vec{q}\,|^{-1}$ in the helicity projectors?

I) Hard region

Expand in inverse powers of the (large) propagator factor $N := (p_t + q)^2 - m_t^2$.

$$
\begin{aligned}
|\vec{q}\,|^2 &= q_0^2 - m_W^2 \\
&= \left(\frac{p_t q}{m_t}\right)^2 - m_W^2
\end{aligned}
$$

Expand in propagator factor $N := (p_t + q)^2 - m_t^2 = 2p_t q + q^2$, i.e. $p_t q = \frac{1}{2} N(1 - m_W^2 N^{-1}))$. One then has

$$
\frac{1}{|\vec{q}\,|^{\,2}} = \frac{4m_t^2}{N^2} \sum_{i=0}^{\infty} \left(\frac{2m_W^2 N^2 - m_W^4 + 4m_t^2 m_W^2}{N^2} \right)^i ,
$$

$$\frac{1}{|\vec{q}\,|} = \frac{2m_t}{N} \sum_{i=0}^{\infty} \binom{2i}{i} \left(\frac{2m_W^2 N - m_W^4 + 4m_t^2 m_W^2}{4\,N^2} \right)^i .$$

II) soft region

We cannot perform an expansion of $|\vec{q}\,|$, since $|\vec{q}\,|^2 = q_0^2 - m_W^2$ and q_0 is of order m_W in the soft region. However, in this region the W boson loop factorises. Therefore, we only have to replace the usual one-loop vacuum bubble integrals with integrals of the type

$$\int \frac{\mathrm{d}^d q}{(q^2 - m_W^2)\,(q_0^2 - m_W^2)^n} \,,$$

NNLO Results for the helicity rates

Colour structure of rates:

$$\Gamma_i = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi} \left\{ \hat{\Gamma}_i^{(0)} + \frac{\alpha_s}{2\pi} C_F \, \hat{\Gamma}_i^{(1)} \right.$$
$$\left. + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[C_F^2 \, \hat{G}_i^{FF} + C_F C_A \, \hat{G}_i^{FA} + C_F T_F \left(\hat{G}_i^{FH} + n_l \, \hat{G}_i^{FL} \right) \right] \right\},$$

The NNLO contributions have been calculated up to $x^{10} (x^{10} \ln x)$. As a sample result we show $\hat{G}_L^{FF}$.

$$
\begin{aligned}
\hat{G}_L^{FF} \quad = \quad & 20 - \frac{119}{12}\pi^2 + 19\pi^2 \ln 2 - \frac{53}{2}\zeta_3 - \frac{11}{180}\pi^4 \\
& + \left[384 - 32\pi^2 + \frac{35}{3}\pi^2 \ln 2 - \frac{32}{3}\pi^2 \ln^2 2 - \frac{225}{2}\zeta_3 - \frac{11}{60}\pi^4 + \right. \\
& \qquad \left. + \frac{8}{3}\ln^4 2 + 64\,\mathrm{Li}_4(1/2) \right] x^2 \\
& + \left[\frac{3761}{8} - \frac{145}{9}\pi^2 - \frac{55}{4}\pi^2 \ln 2 + \frac{32}{3}\pi^2 \ln^2 2 - \frac{2119}{24}\zeta_3 - \frac{211}{180}\pi^4 \right. \\
& \qquad \left. - \frac{8}{3}\ln^4 2 - 64\,\mathrm{Li}_4(1/2) + \left(\frac{1535}{108} - \frac{25}{18}\pi^2 \right) \ln x \right] x^4 \\
& + \left[-\frac{12651131}{540000} + \frac{53941}{9000}\ln x + \frac{285727}{7200}\pi^2 \right. \\
& \qquad \left. + \frac{7}{15}\pi^2 \ln x - \frac{155}{3}\pi^2 \ln 2 - \frac{19}{10}\pi^4 + \frac{911}{6}\zeta_3 \right] x^6
\end{aligned}
$$

$$
\begin{aligned}
&+\Big[-\frac{1011779148461}{2593080000}+\frac{11838863}{2315250}\ln x+\frac{1044164059}{8467200}\pi^2-\frac{11}{630}\pi^2\ln x \\
&\quad -173\pi^2\ln 2-\frac{19}{10}\pi^4+\frac{93829}{210}\zeta_3\Big]x^8 \\
&+\Big[-\frac{272829246010 1459}{1680315840000}+\frac{1866474913}{2000376000}\ln x+\frac{62340595343}{152409600}\pi^2+\frac{1}{42}\pi^2\ln x \\
&\quad -\frac{12289}{21}\pi^2\ln 2-\frac{19}{10}\pi^4+\frac{929779}{630}\zeta_3\Big]x^{10}+\mathcal{O}\left(x^{12}\right)
\end{aligned}
$$

where ζ_n denotes Riemann's zeta function with integer argument n.

Some features of the polarized NNLO calculation:

- 12 new three-loop master integrals
- Convergence of the x–expansion $\Longrightarrow$ Table
- Convergence of the perturbation series

Define helicity fractions up to $\mathcal{O}(n)$ by writing ($n = 0, 1, 2$ denote the contributions up to LO, NLO and NNLO , respectively)

$$\mathcal{G}_i^{(n)} = \frac{\sum_{i=0}^{n} \Gamma_i^{(n)}}{\sum_{i=0}^{n} \Gamma^{(n)}},$$

where $i = L, +, -$.

Results are presented in the form $\mathcal{G}_i = \mathcal{G}_i^{(0)} + \Delta\mathcal{G}_i^{(1)} + \Delta\mathcal{G}_i^{(2)}$ with increments $\Delta\mathcal{G}_i^{(n)} = \mathcal{G}_i^{(n)} - \mathcal{G}_i^{(n-1)}$ and also, if $\mathcal{G}_i^{(0)} \neq 0$, as $\mathcal{G}_i = \mathcal{G}_i^{(0)}(1 + \delta\mathcal{G}_i^{(1)} + \delta\mathcal{G}_i^{(2)})$.

$$\begin{aligned}
\mathcal{G}_L &= 0.6971 - 0.0075 - 0.0023 \\
&= 0.6971(1 - 0.0108 - 0.0034) \\
\mathcal{G}_+ &= 0 \qquad\quad + 0.00103 + 0.00023 \\
\mathcal{G}_- &= 0.3029 + 0.0065 + 0.0021 \\
&= 0.3029(1 + 0.0214 + 0.0070)
\end{aligned}$$

The perturbative expansion is well behaved.

Table 1: Numerical values for coefficients of $[x^n, x^n \ln x]$

	$\hat{\Gamma}_L$	$\hat{\Gamma}_+$	$\hat{\Gamma}_-$	$\hat{\Gamma}$
$[x^0]$	$-1.959 \cdot 10^{-2}$	0	0	$-1.959 \cdot 10^{-2}$
$[x^2]$	$4.756 \cdot 10^{-3}$	$3.875 \cdot 10^{-4}$	$-3.876 \cdot 10^{-3}$	$1.267 \cdot 10^{-3}$
$[x^4]$	$6.758 \cdot 10^{-4}$	$1.370 \cdot 10^{-4}$	$-9.983 \cdot 10^{-4}$	$-1.856 \cdot 10^{-4}$
$[x^5]$	0	$-5.388 \cdot 10^{-4}$	$5.388 \cdot 10^{-4}$	0
$[x^6]$	$-1.482 \cdot 10^{-4}$	$1.197 \cdot 10^{-4}$	$4.930 \cdot 10^{-4}$	$4.646 \cdot 10^{-4}$
$[x^7]$	0	$7.795 \cdot 10^{-5}$	$-7.795 \cdot 10^{-5}$	0
$[x^8]$	$-1.725 \cdot 10^{-5}$	$-2.366 \cdot 10^{-5}$	$-1.746 \cdot 10^{-5}$	$-5.837 \cdot 10^{-5}$
$[x^9]$	0	$3.464 \cdot 10^{-6}$	$-3.464 \cdot 10^{-6}$	0
$[x^{10}]$	$-1.296 \cdot 10^{-6}$	$-1.876 \cdot 10^{-6}$	$-2.214 \cdot 10^{-6}$	$-5.386 \cdot 10^{-6}$
Σ	$-1.433 \cdot 10^{-2}$	$1.613 \cdot 10^{-4}$	$-3.944 \cdot 10^{-3}$	$-1.811 \cdot 10^{-2}$