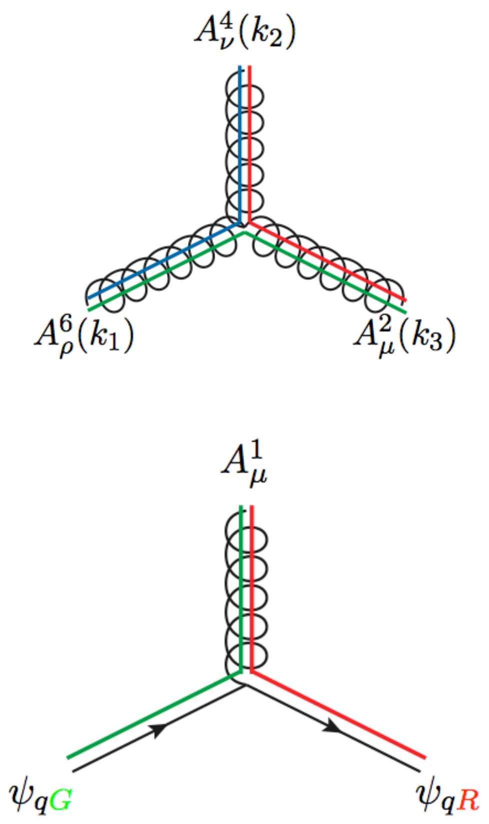
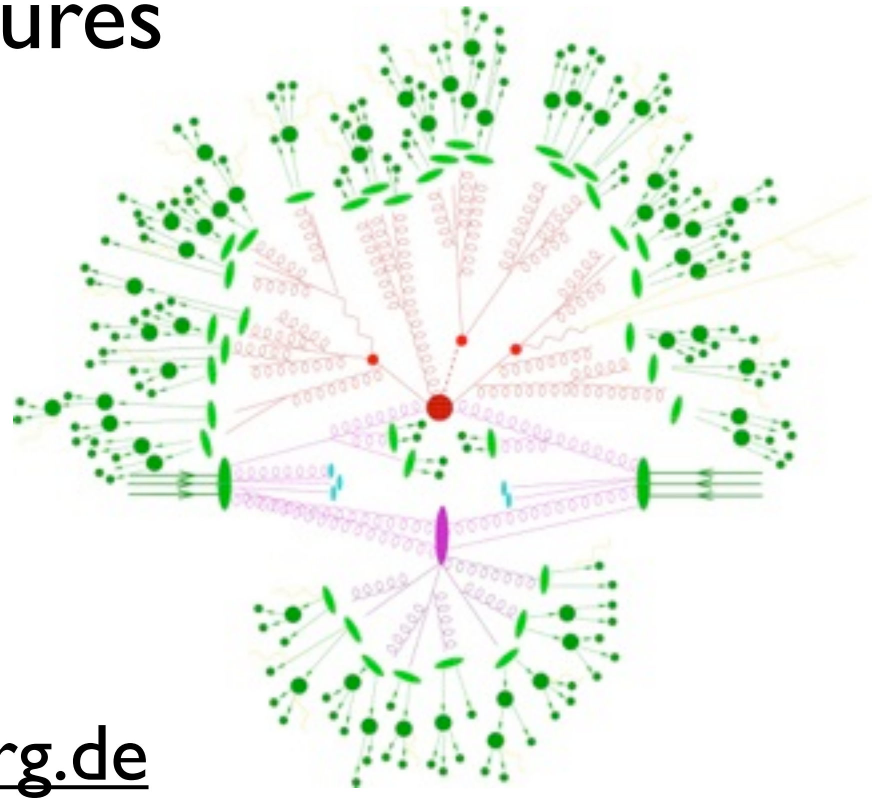
Quantum Chromodynamics



DESY Summer Student Lectures

August 9, 2013



Roman Kogler

University of Hamburg

roman.kogler@physik-uni.hamburg.de

Overview

Part 1 - Setting the Stage

The Static Quark Model

Deep-Inelastic Scattering

Discovery of quarks and colour

The QCD Lagrangian

Discovery of gluons

Part 2 - Working with QCD

Renormalisation

Perturbative QCD

Jets

Factorisation and Parton Distribution Functions

Part 2

QCD

Non-abelian gauge theory with $SU(3)$ symmetry, describes the interaction between coloured particles (quarks and gluons).

The Feynman rules can be derived from the QCD Lagrangian

$$\mathcal{L} = \sum_{f}^{n_f} \bar{q}_f (i\gamma^\mu \mathcal{D}_\mu - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{ghost}}$$

Very similar to the QED Lagrangian, except for the additional summation over a, which are the 8 colour degree of freedoms ($SU(3)$ instead of $U(1)$)

Covariant derivative: $\mathcal{D}_\mu = \partial_\mu - i g_s t_a A^a_\mu$

Field strength tensor for spin-1 gluons:

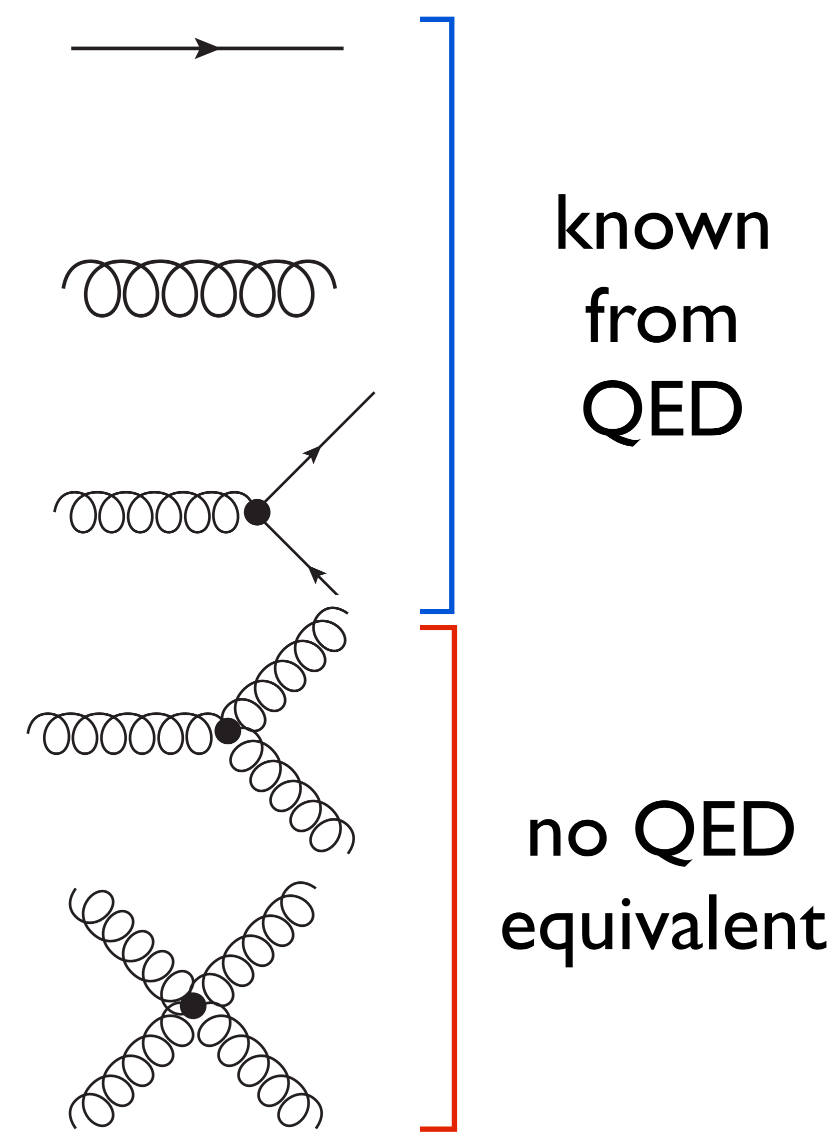
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \underline{g_s f_{abc} A^b_\mu A^c_\nu}$$

Non-abelian term, different from QED. Leads to gluon self-interaction.

The QCD Lagrangian

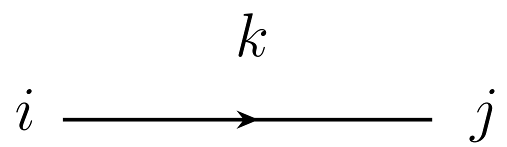
Let's plug the expressions for $\mathcal{D}_\mu$ and $G^a_{\mu\nu}$ into the Lagrangian:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_f^{n_f} \bar{q}_f (i\gamma^\mu \partial_\mu - m_f) q_f \\
& - \frac{1}{4} (\partial^\mu A^\nu_a - \partial^\nu A^\mu_a)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) \\
& + g_s A^a_\mu \sum_f^{n_f} \bar{q}_f \gamma^\mu t_a q_f \\
& - \frac{g_s}{2} f^{abc} (\partial^\mu A^\nu_a - \partial^\nu A^\mu_a) A^b_\mu A^c_\nu \\
& - \frac{g_s^2}{4} f^{abc} f_{ade} A^\mu_b A^\nu_c A^d_\mu A^e_\nu
\end{aligned}
$$

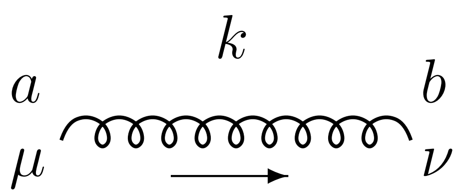


These terms can then be used to obtain the Feynman rules for QCD

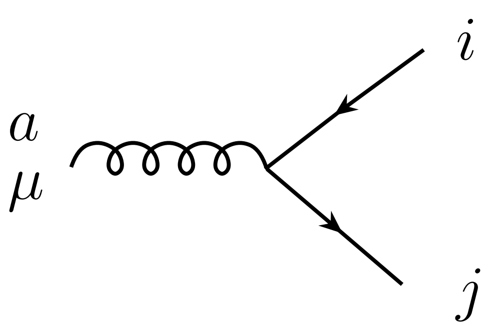
Feynman Rules For QCD



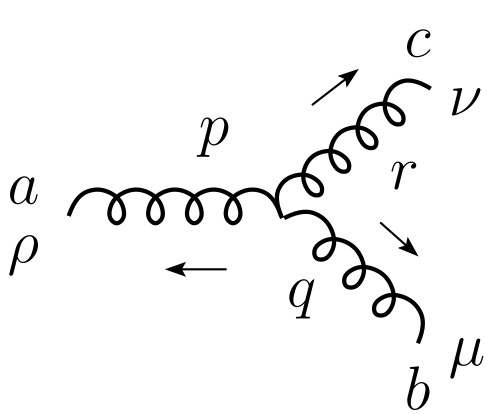
$$i\,\delta_{ij}\,\frac{(\not{k}+m)}{k^2-m^2+i\epsilon}$$



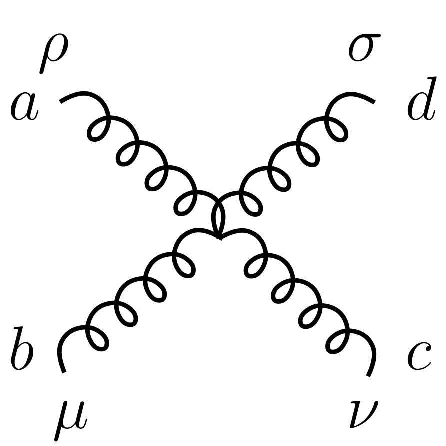
$$\frac{-i\,\delta_{ab}}{k^2+i\epsilon}\left[g_{\mu\nu}-(1-\eta)\frac{k_\mu k_\nu}{k^2}\right] \qquad \eta=\begin{cases}1, & \text{Feynman gauge}\\ 0, & \text{Landau gauge}\end{cases}$$



$$ig_s\gamma_\mu T^a_{ji}$$



$$-g_s f^{abc}\left[(p-q)_\nu\, g_{\rho\mu}+(q-r)_\rho\, g_{\mu\nu}+(r-p)_\mu\, g_{\nu\rho}\right]$$



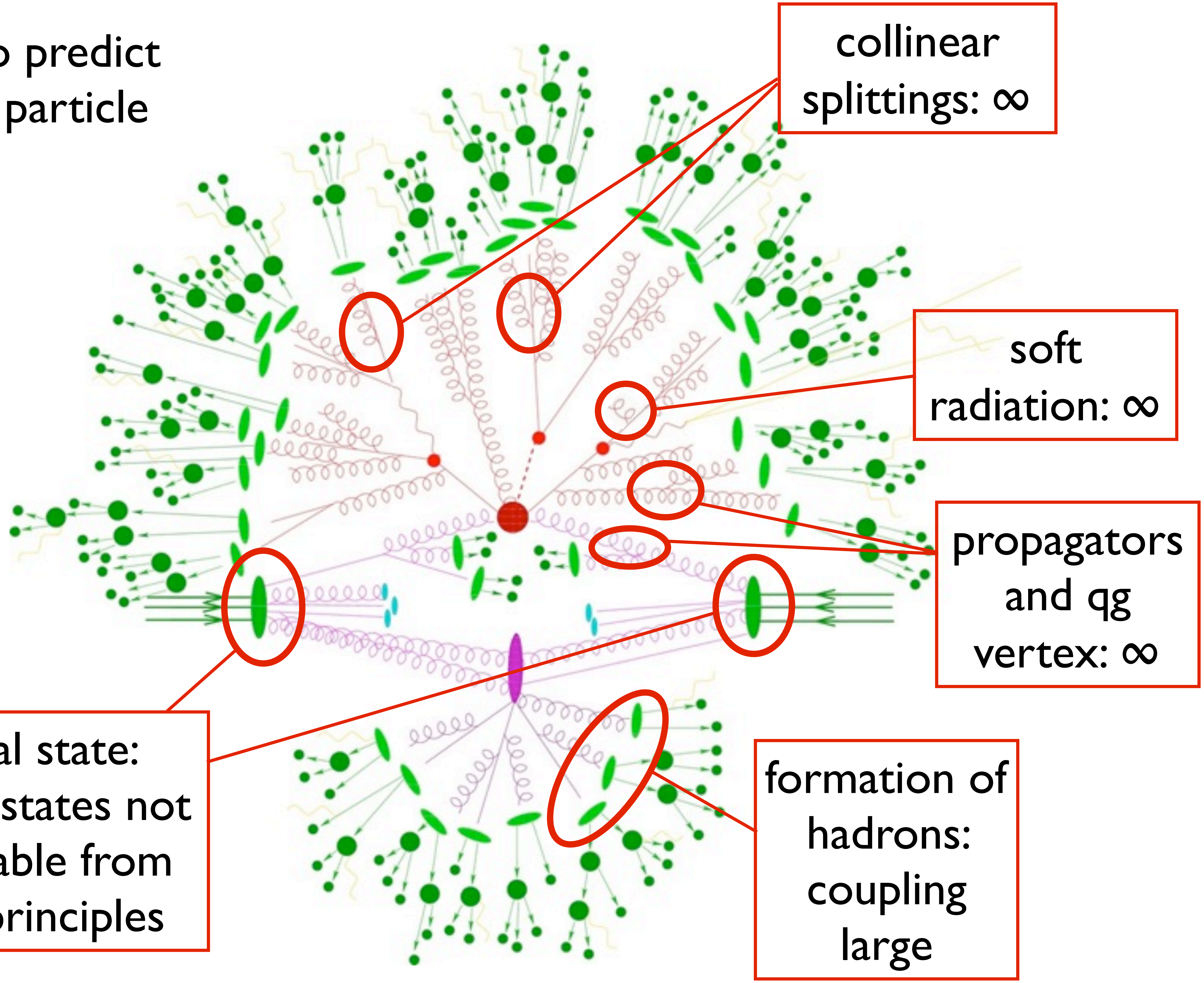
$$\begin{aligned}&-ig_s^2 f^{abe} f^{cde}\,(g_{\rho\nu}g_{\mu\sigma}-g_{\rho\sigma}g_{\mu\nu})\\ &-ig_s^2 f^{ace} f^{bde}\,(g_{\rho\mu}g_{\nu\sigma}-g_{\rho\sigma}g_{\mu\nu})\\ &-ig_s^2 f^{ade} f^{cbe}\,(g_{\rho\nu}g_{\mu\sigma}-g_{\rho\mu}g_{\sigma\nu})\end{aligned}$$

Using QCD

We would like to predict what happens at particle collisions at high energies

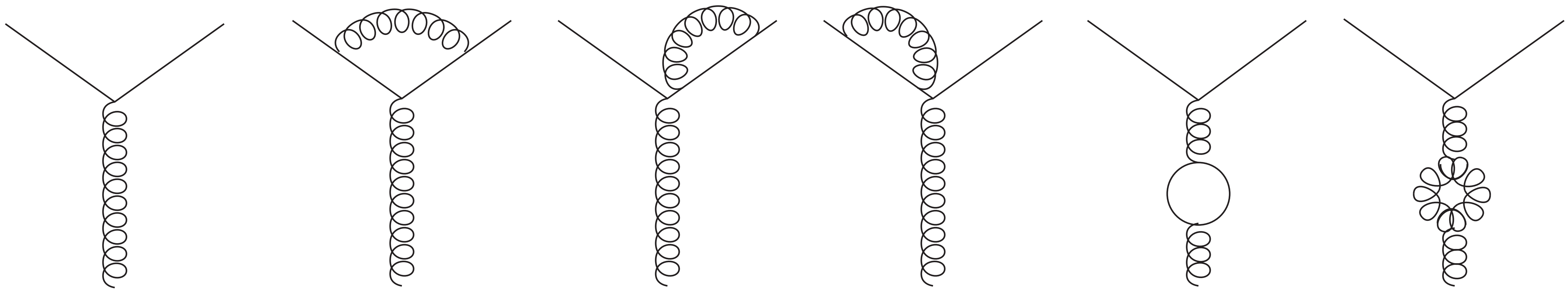
BUT

QCD is full of divergencies (and other difficulties)!



Singularities in QCD

Divergencies appear when constructing the first-order corrections to the quark-gluon interaction



leading order

vertex-correction and self energies

vacuum polarisation graphs

finite (?)

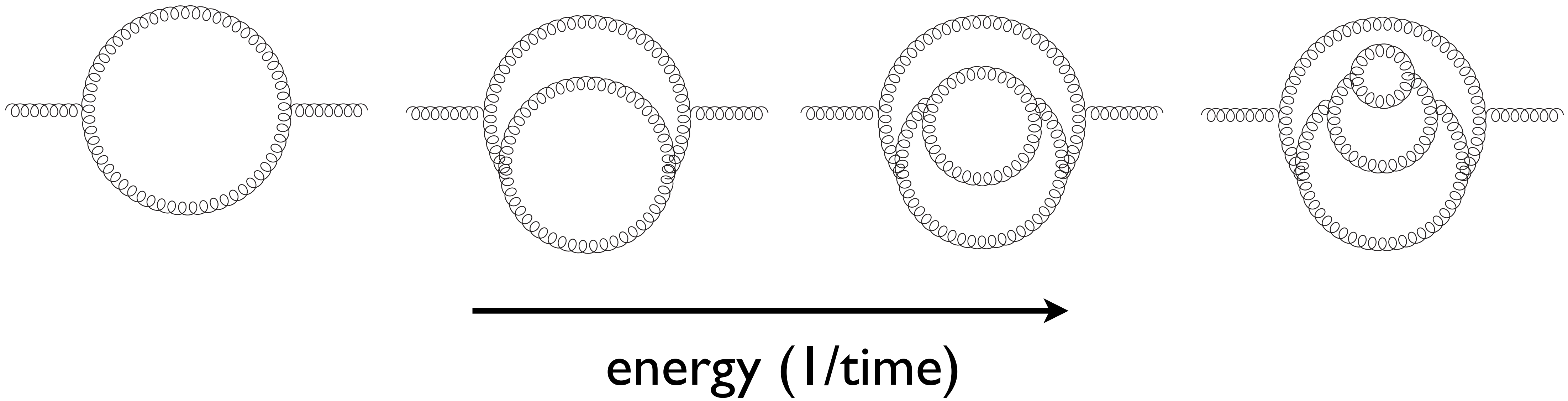
Integrals are infinite, due to unconstraint loop momenta: ultra-violet (UV) divergencies

Known from QED: redefinition of fields and masses will remove the vertex-correction and self energy divergencies (to all orders)

Renormalisation

Ultra-violet (UV) divergencies can be interpreted as virtual fluctuations on very small time scales (high energies)

Renormalisation: absorb virtual fluctuations in the definition of the bare coupling, this introduces a new scale parameter μ_R



μ_R has the dimension of energy (mass) and defines the point where the subtraction is performed (ultraviolet cut-off scheme)

More often used but less intuitive: dimensional regularisation, perform integration in $4\text{-}2\varepsilon$ dimensions

Renormalisation Group Equation

The dimensional parameter μ_R is arbitrary - no general observable $\Gamma(p_i, \alpha_s)$ should depend on it. (strong coupling: $\alpha_s = g_s^2/4\pi$)

Require: $$\left(\mu_R \frac{\partial}{\partial \mu_R} + \mu_R \frac{\partial \alpha_s}{\partial \mu_R} \frac{\partial}{\partial \alpha_s} + \gamma_\Gamma(\alpha_s)\right) \Gamma(p_i, \alpha_s) = 0$$

Renormalisation Group Equation (RGE)

⇒ a change in μ_R has to be compensated by a change in α_s

Running coupling: $\alpha_s = \alpha_s(\mu_R)$

The quantity $\beta(\alpha_s) = \mu_R \frac{\partial \alpha_s}{\partial \mu_R}$ is known as the QCD beta-function which can be computed.

QCD cannot predict the absolute value of $\alpha_s(\mu_R)$, but its scale dependence.

The Running Coupling

Expansion of the β-function: $\beta(\alpha_s) = -\alpha_s \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{(n+1)}$

Where the terms β_n are known up to four loops:

$$
\begin{aligned}
\beta_0 &= 11 - \frac{2}{3} n_f \\
\beta_1 &= 102 - \frac{38}{3} n_f \\
\beta_2 &= \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2 \\
\beta_3 &= \frac{149753}{6} + 3564\zeta_3 - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right) n_f \\
&+ \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right) n_f^2 + \frac{1093}{729} n_f^3
\end{aligned}
$$

In Fact...

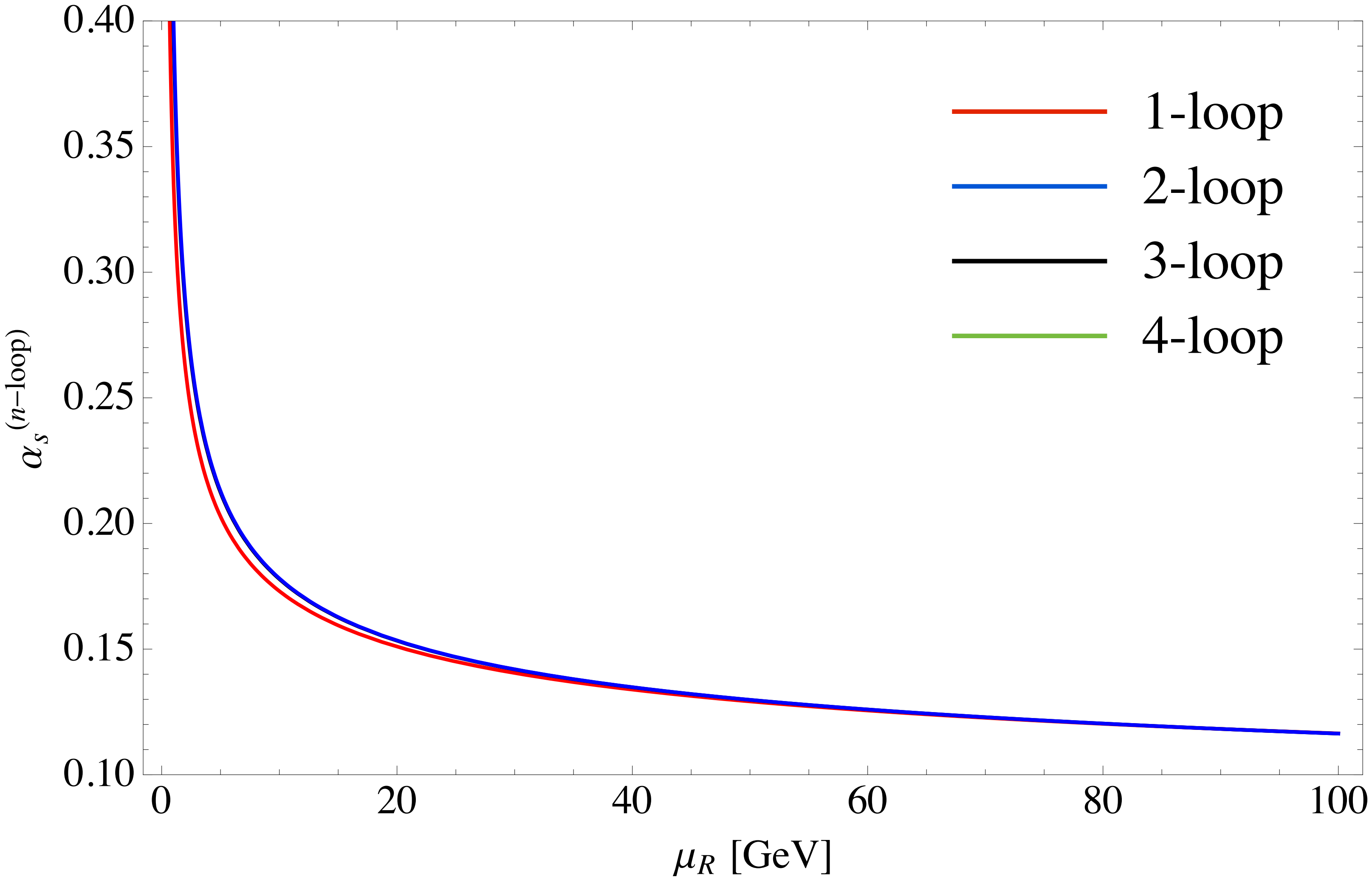
$$\frac{\partial a_s}{\partial \ln \mu_R} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + \mathcal{O}(a_s^6) \qquad (a_s = \alpha_s/4\pi)$$

$$\begin{aligned}
\beta_0 &= \frac{11}{3}C_A - \frac{4}{3}T_F n_f\,, \quad \beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f \\
\beta_2 &= \frac{2857}{54}C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9}C_F C_A T_F n_f \\
&\quad - \frac{1415}{27}C_A^2 T_F n_f + \frac{44}{9}C_F T_F^2 n_f^2 + \frac{158}{27}C_A T_F^2 n_f^2 \\
\beta_3 &= C_A^4\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right) + C_A^3 T_F n_f\left(-\frac{39143}{81} + \frac{136}{3}\zeta_3\right) \\
&\quad + C_A^2 C_F T_F n_f\left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right) + C_A C_F^2 T_F n_f\left(-\frac{4204}{27} + \frac{352}{9}\zeta_3\right) \\
&\quad + 46 C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2\left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right) + C_F^2 T_F^2 n_f^2\left(\frac{1352}{27} - \frac{704}{9}\zeta_3\right) \\
&\quad + C_A C_F T_F^2 n_f^2\left(\frac{17152}{243} + \frac{448}{9}\zeta_3\right) + \frac{424}{243}C_A T_F^3 n_f^3 + \frac{1232}{243}C_F T_F^3 n_f^3 \\
&\quad + \frac{d_A^{abcd} d_A^{abcd}}{N_A}\left(-\frac{80}{9} + \frac{704}{3}\zeta_3\right) + n_f\frac{d_F^{abcd} d_A^{abcd}}{N_A}\left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right) \\
&\quad + n_f^2\frac{d_F^{abcd} d_F^{abcd}}{N_A}\left(-\frac{704}{9} + \frac{512}{3}\zeta_3\right)
\end{aligned}$$

evaluation of
~ 50 000 diagrams

T. van Ritbergen, *et al.*, Phys. Lett. B400, 379 (1997)

The Running Coupling in QCD



Asymptotically free for $\mu_R \to \infty$

Confinement for $\mu_R \to 0$

Good convergence (expansion parameter $a_s \approx 0.01$)

No visible difference between 3-loop and 4-loop solution

The scale dependence $\alpha_s(\mu_R)$ of is one of the best known quantities in QCD

$\Rightarrow$ Possibility for stringent tests of QCD!

Perturbation Theory

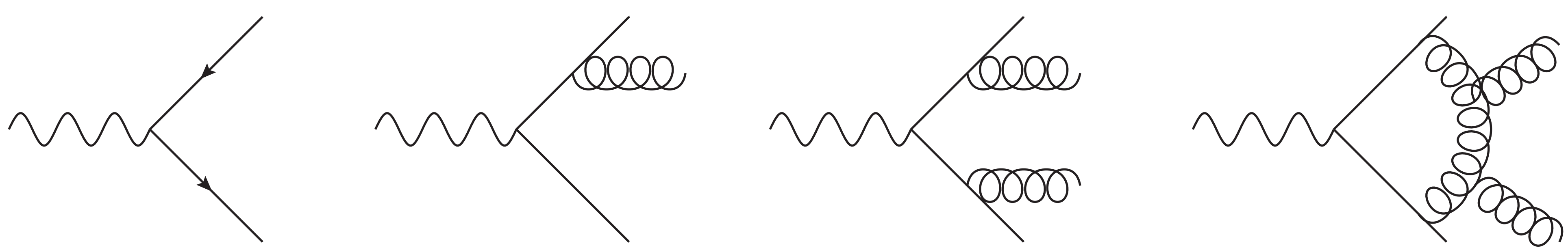
Smallness of $\alpha_s(\mu_R)$ at large scales allows for a series expansion in terms of α_s

Some observable O can be expressed as $O = \sum_{n=0}^{\infty} \alpha_s(\mu_r)^n C_n(\mu_r)$

Relies on the idea $O = \alpha_s c_1 + \underbrace{\alpha_s^2}_{\text{small}} c_2 + \underbrace{\alpha_2^3}_{\text{smaller}} c_3 + \underbrace{\ldots}_{\text{negligible?}}$

Coefficients c_n become very complex very quickly, so you don't want to deal with too many powers of α_s

$e^+e^- \to \text{had}:$



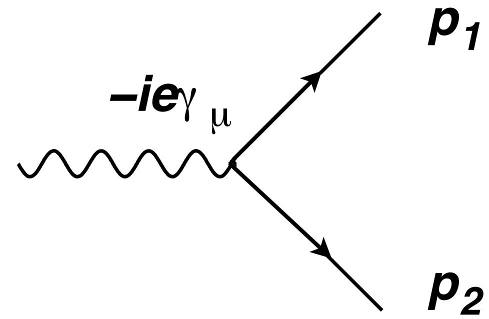
$|\mathcal{M}|^2 \propto \quad \alpha_s^0 \qquad \alpha_s^1 \qquad \alpha_2^2 \qquad \alpha_s^3$

Example Calculation

$e^+e^- \longrightarrow \text{had}$

Start with $\gamma^* \rightarrow q\bar{q}$

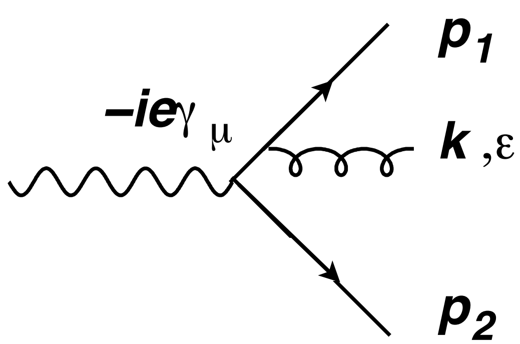
$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$

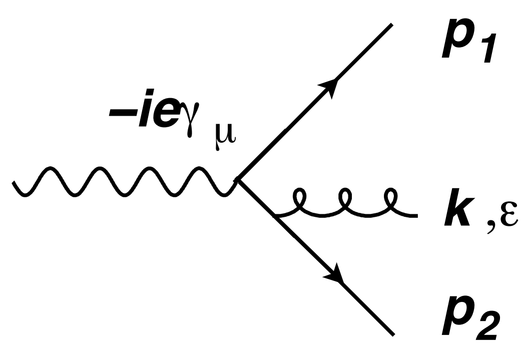


Remember: $\sigma_{q\bar{q}} = \frac{4\pi N_c}{3} \frac{\alpha^2 e_q^2}{s}$

Emit a gluon

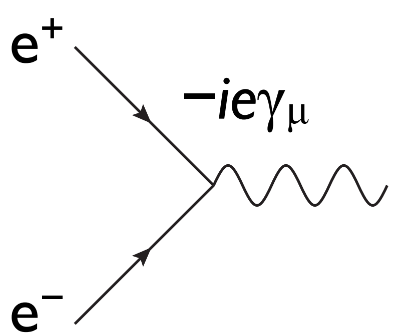
$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^a \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) + \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^a v(p_2)$$





For each piece, add the lepton current:

$$\frac{\bar{u}(e^+)(-ie\gamma_\mu)u(e^-)}{s}$$



Example Calculation

$e^+e^- \longrightarrow \text{had}$

and we get $|\mathcal{M}|^2 = \frac{4e^4 e_q^2 g_s^2}{s^2} L^{\mu\nu} Q_{\mu\nu}$

with
$$L^{\mu\nu} = |\bar{u}(e^+)\gamma^\mu u(e^-)|^2 = 4\left(p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu} p_+ \cdot p_-\right)$$
$$Q_{\mu\nu} = \left| \bar{u}(p_1) \left[\frac{\not{\epsilon}\,(\not{p}_1 + \not{k}\,)\gamma^\mu}{(p_1+k)^2} + \frac{\gamma^\mu (\not{p}_2 + \not{k}\,)\,\not{\epsilon}}{(p_2+k)^2} \right] v(p_2) \right|^2$$

simplify it by using energy fractions

$x_i = \frac{2E_i}{\sqrt{s}}$ which satisfy $p_i \cdot p_j = \frac{s(1-x_k)}{2}$ and $x_1 + x_2 + x_3 = 2$

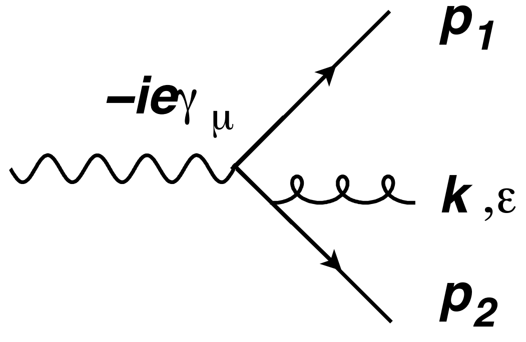
and we find $|\mathcal{M}|^2 = \frac{32 e^4 e_q^2 g_s^2}{s^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$

This needs to be integrated over the full three-particle phase space (together with phase space factors and δ-functions for momentum conservation).

More Divergencies!

$e^+e^- \longrightarrow \text{had}$

$$|\mathcal{M}|^2 = \frac{32e^4e_q^2g_s^2}{s^2}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$



This expression diverges for $x_1 \rightarrow 1$ and $x_2 \rightarrow 1$

Since $s(1-x_1) = 2p_2 \cdot k = 2E_2E_k(1-\cos\theta_{2,k})$

The divergencies appear for

- $E \longrightarrow 0$: infrared (or soft) divergence
- $\theta \longrightarrow 0$ and $\theta \longrightarrow \pi$: collinear divergence

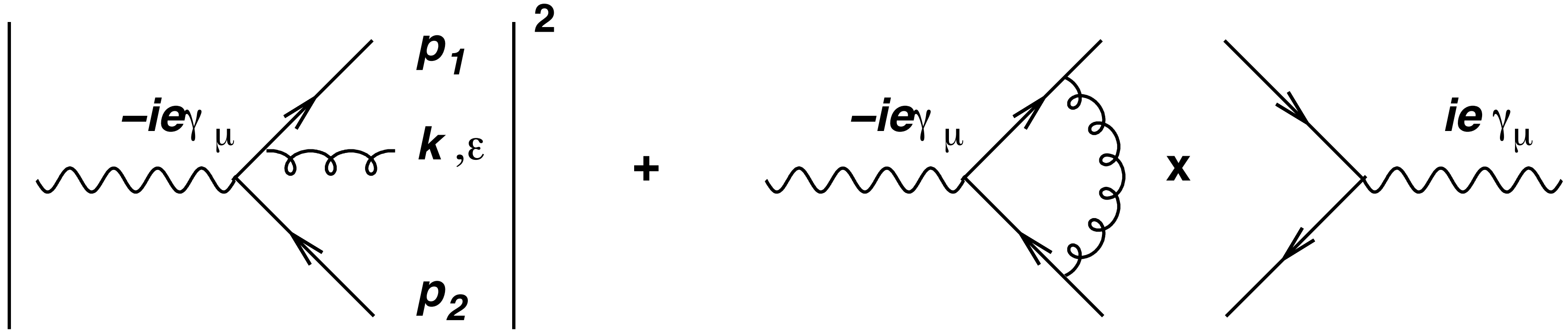
The divergencies here appeared in the context of $e^+e^- \rightarrow q\bar{q}$

But they are a very general property of QCD!

Real-Virtual Cancellations

$e^+e^- \to$ had: total cross section

Total cross section: sum of all real and virtual diagrams



Real part given by $\mathcal{R}(E,\theta)$ and virtual corrections $\mathcal{V}(E,\theta)$

So the total cross section is

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{dE}{E} \int \frac{d\theta}{\sin\theta} (\mathcal{R}(E,\theta) - \mathcal{V}(E,\theta)) \right)$$

Doing the calculation, we find

$$\lim_{E\to 0} (\mathcal{R}(E,\theta) - \mathcal{V}(E,\theta)) = 0 \quad \text{and} \quad \lim_{\theta\to 0,\pi} (\mathcal{R}(E,\theta) - \mathcal{V}(E,\theta)) = 0$$

Total Cross Section

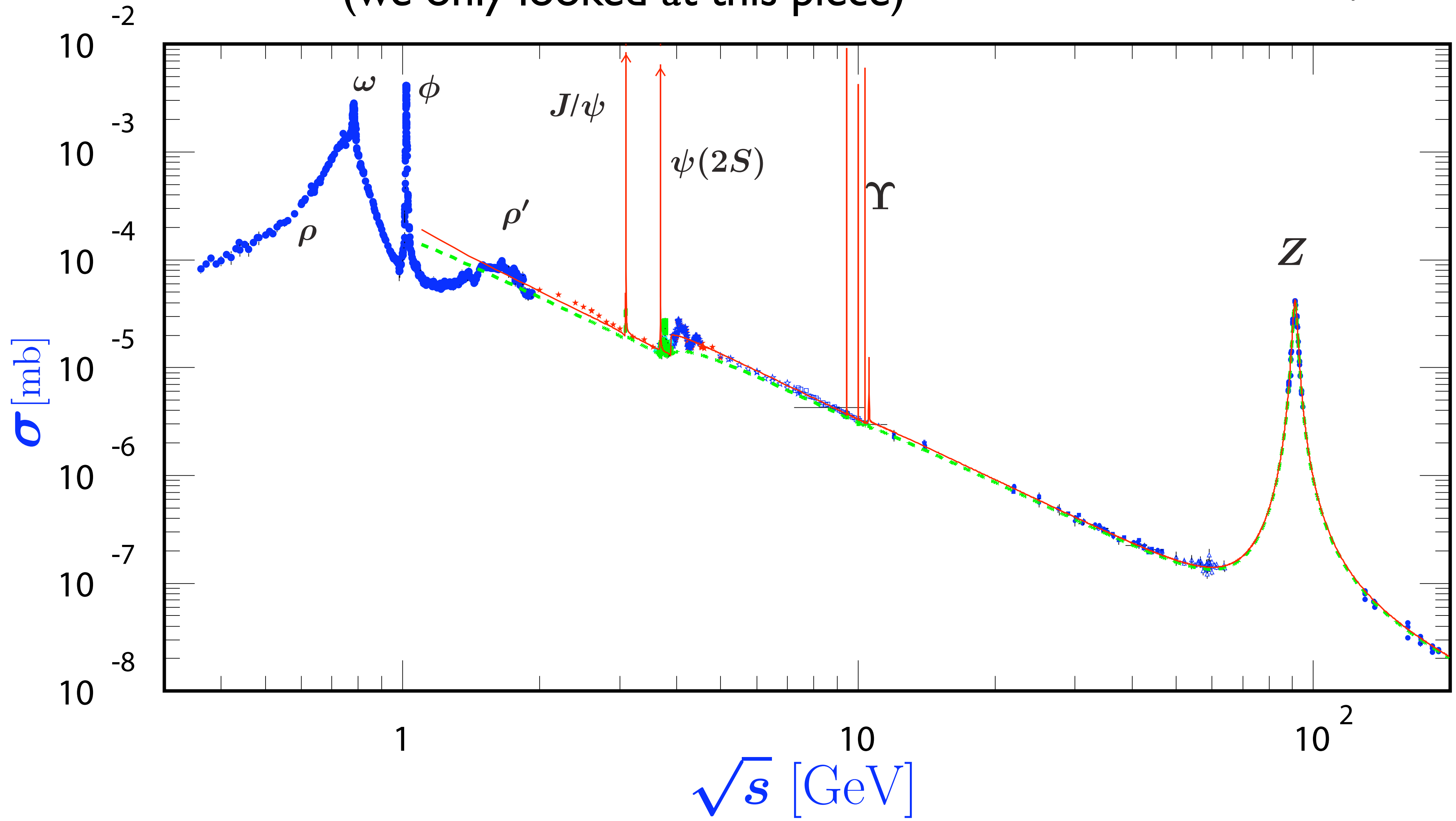
$e^+e^- \longrightarrow$ had: total cross section

Finally, including all real and virual corrections:

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + 1.045 \frac{\alpha_s(\mu_r)}{\pi} + 0.94 \left(\frac{\alpha_s(\mu_r)}{\pi} \right)^2 - 15 \left(\frac{\alpha_s(\mu_r)}{\pi} \right)^3 + \ldots \right)$$

(we only looked at this piece)

(numbers given for $n_f = 5$)



Perfect agreement with the data!

What does this mean?

What's the reason for

$$\lim_{E\to 0}(\mathcal{R}(E,\theta)-\mathcal{V}(E,\theta))=0 \quad \text{and} \quad \lim_{\theta\to 0,\pi}(\mathcal{R}(E,\theta)-\mathcal{V}(E,\theta))=0 \quad ?$$

Total cross section must be finite so the divergencies have to cancel

- Essence of the Kinoshita-Lee-Nauenberg and Bloch-Nordsiek theorems
- Generalises for an arbitrary number of gluons (and photons)

In other words:

Corrections to leading order result only come from hard gluon emission

Soft gluons do not matter:

- they are emitted on a long timescale ~$1/(E\theta)$ relative to collision ~$1/Q$
  → cannot influence the total cross section
- transition to hadrons also occurs on long timescale ~$1/\Lambda$ - can also be ignored (in this case)

What can we calculate?

Does the previous result mean we can only calculate total cross sections ?

No, it just means we have to be careful how we define our observables

Consider a measurement $\mathcal{I}$, which is determined by the function $\mathcal{S}_n$

$$\begin{aligned}
\mathcal{I} \;=\; & \frac{1}{2!}\int d\Omega_2 \frac{d\sigma[2]}{d\Omega_2}\, \mathcal{S}_2(p_1^\mu, p_2^\mu) \\
& + \frac{1}{3!}\int d\Omega_2 dE_3 d\Omega_3 \frac{d\sigma[3]}{d\Omega_2 dE_3 d\Omega_3}\, \mathcal{S}_3(p_1^\mu, p_2^\mu, p_3^\mu) \\
& + \frac{1}{4!}\int d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4 \frac{d\sigma[4]}{d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4}\, \mathcal{S}_4(p_1^\mu, p_2^\mu, p_3^\mu, p_4^\mu) \\
& + \cdots .
\end{aligned}$$

If $\mathcal{S}_n$ is collinear and infrared safe, the divergencies will cancel through the KLN theorem

In general:

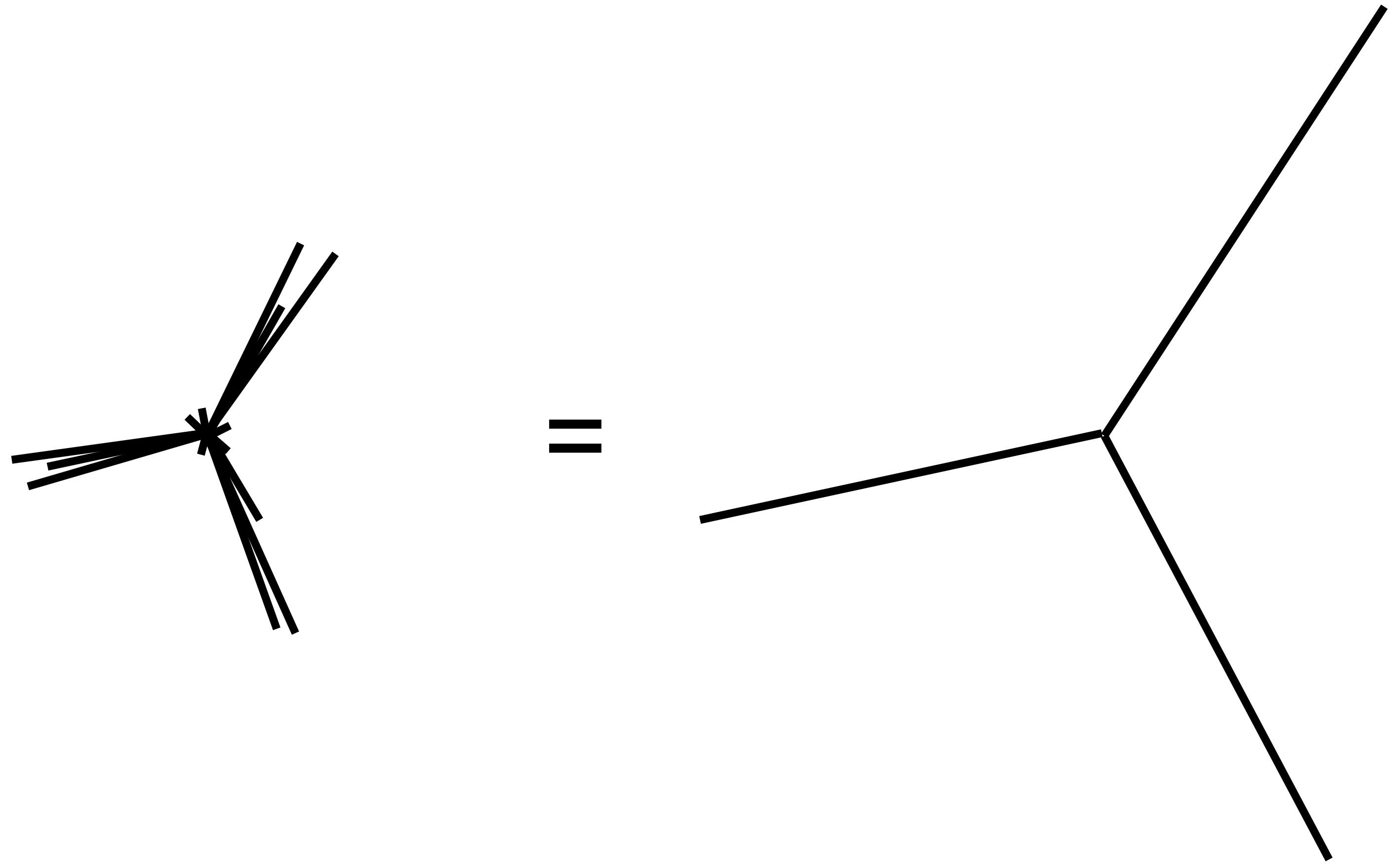
$$\mathcal{S}_{n+1}(p_1^\mu, \ldots, (1-\lambda)p_n^\mu, \lambda p_n^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu)$$

Infrared and Collinear Safety

The requirement $\mathcal{S}_{n+1}(p_1^\mu, \ldots, (1-\lambda)p_n^\mu, \lambda p_n^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu)$ means:

The measurement should not distinguish between a final state which contains:

- two collinear particles; or one with the sum of the momenta of the two
- a soft particle; or the same final state without it

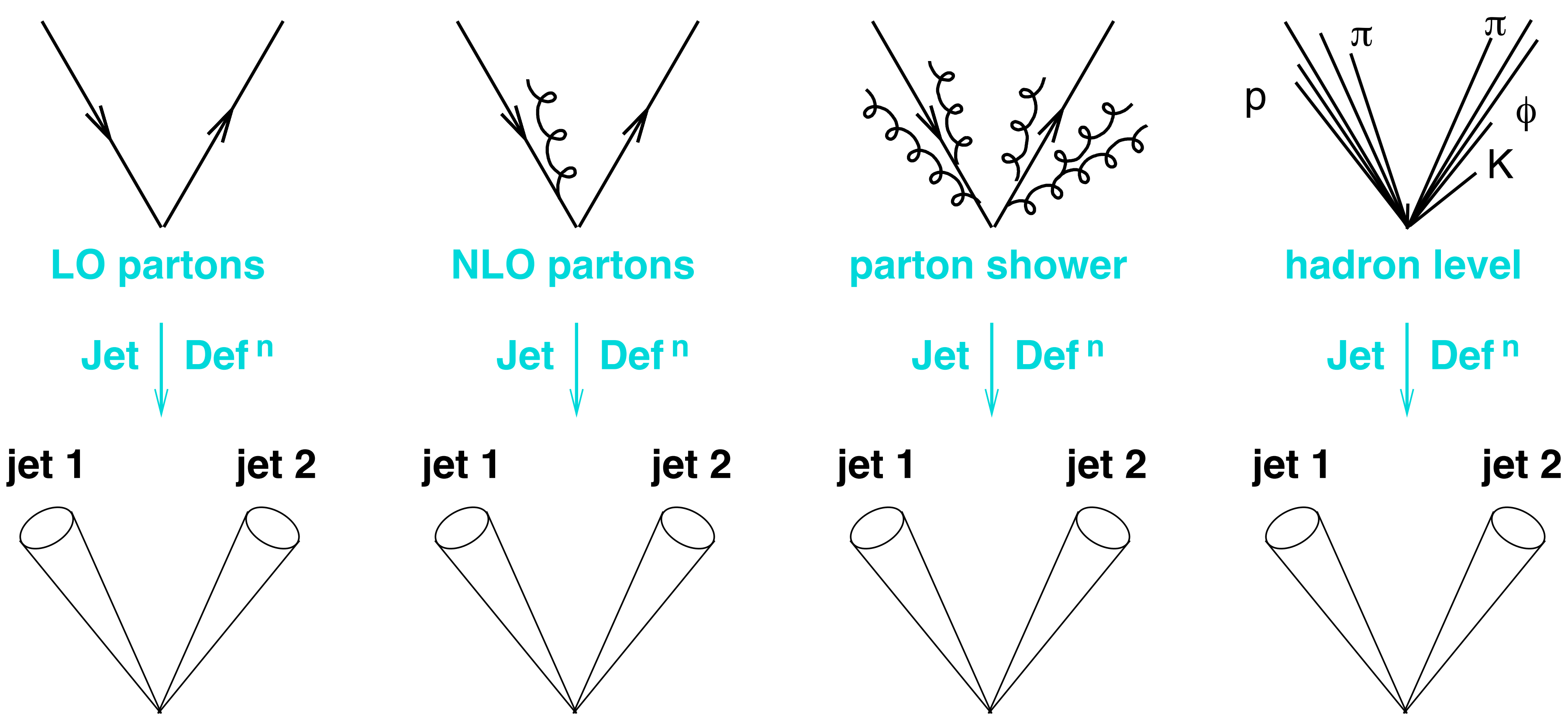


Examples: total cross sections (S_n=1), Thrust, Sphericity, Energy flows, jets...

Jets

A jet algorithm combines objects (partons, hadrons, detector deposits) which are "close" together

Different choices for infrared and collinear (IRC) safe jet algorithms exist, with different distance definitions, but the working principle is:

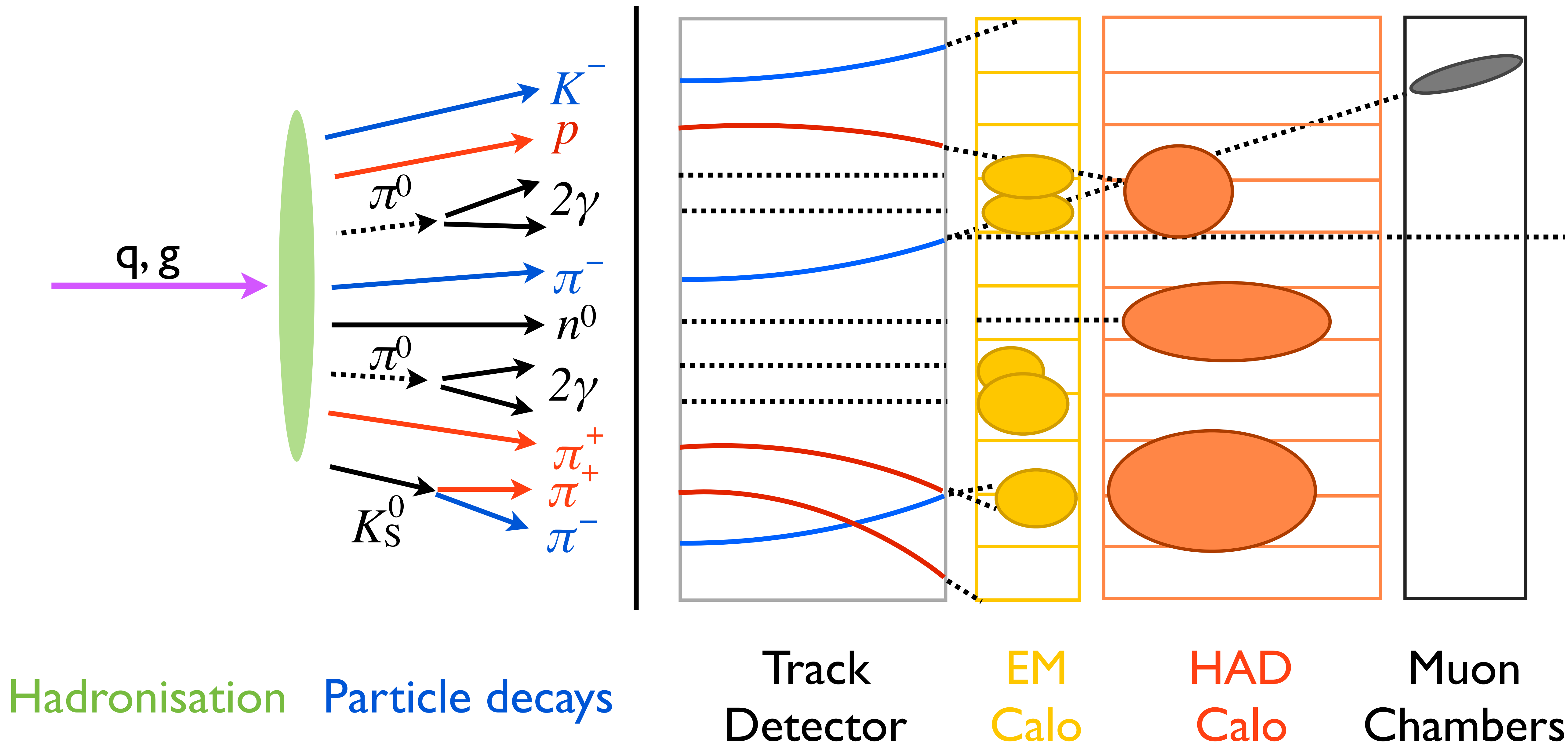


Projection to jets should be resilient to QCD and detector effects

Jets help us to study the underlying parton dynamics

(courtesy of Gavin Salam)

What Can We Measure?

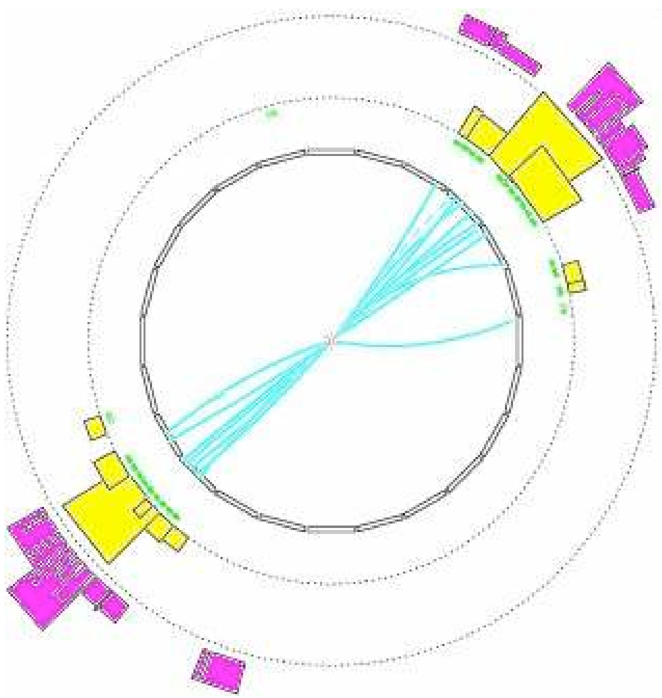


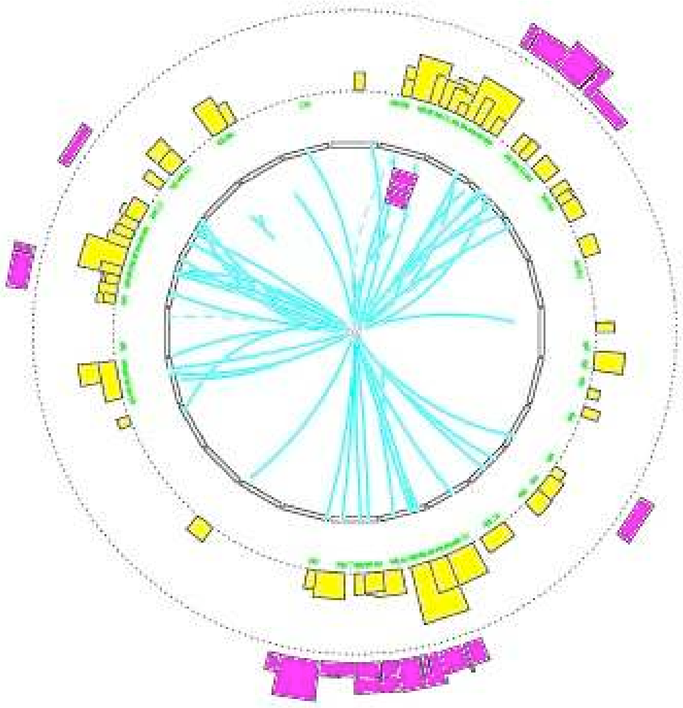
After the hadronisation and the detector effects it is virtually impossible to reconstruct all particles which originated from a single quark or gluon

The total deposited energy can be well measured

Jets

How many jets do you see?





A jet algorithm provides exact rules on how to combine particles to form a jet, mainly two approaches:

<u>Cone</u>

- top-down: centred around the idea of energy flow

<u>Sequential recombination</u>

- bottom-up: successively undoes QCD branching

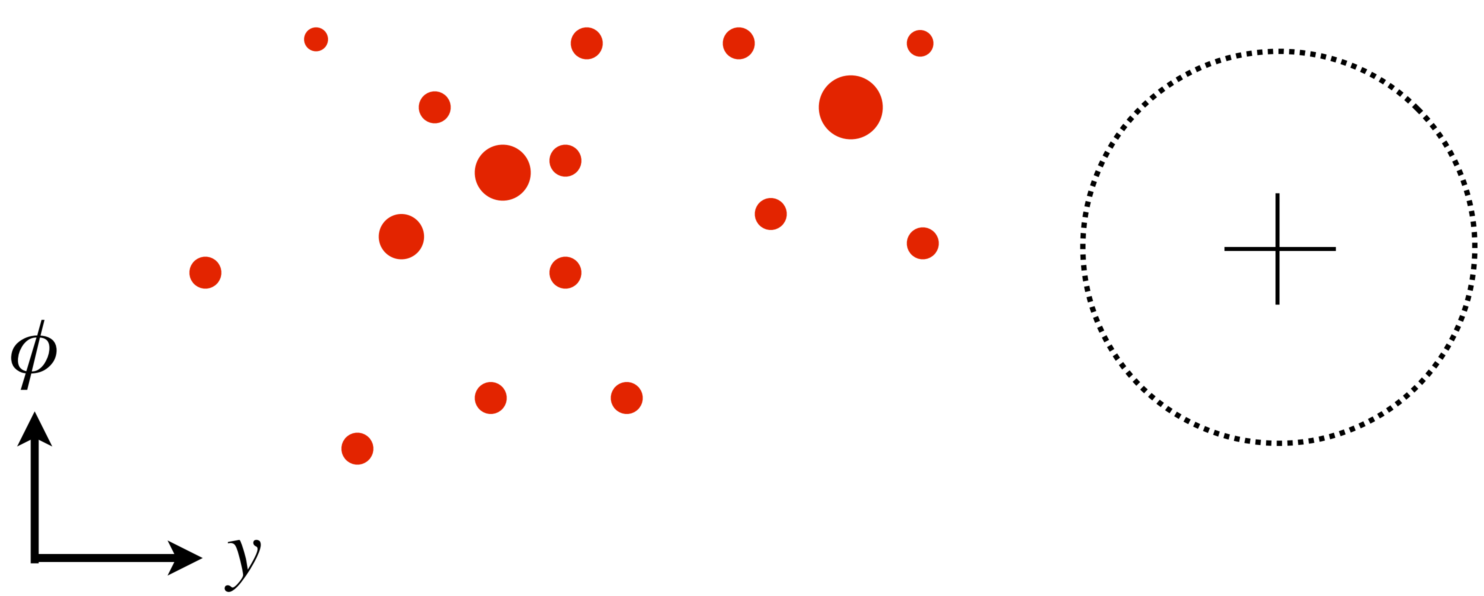
Cone Algorithms

Basic principle of cone algorithms:

- Cones are circles in rapidity y and azimuth ϕ
- A particle i is within the cone of radius R around the axis a if

$$(y_i - y_a)^2 + (\phi_i - \phi_a)^2 < R^2$$

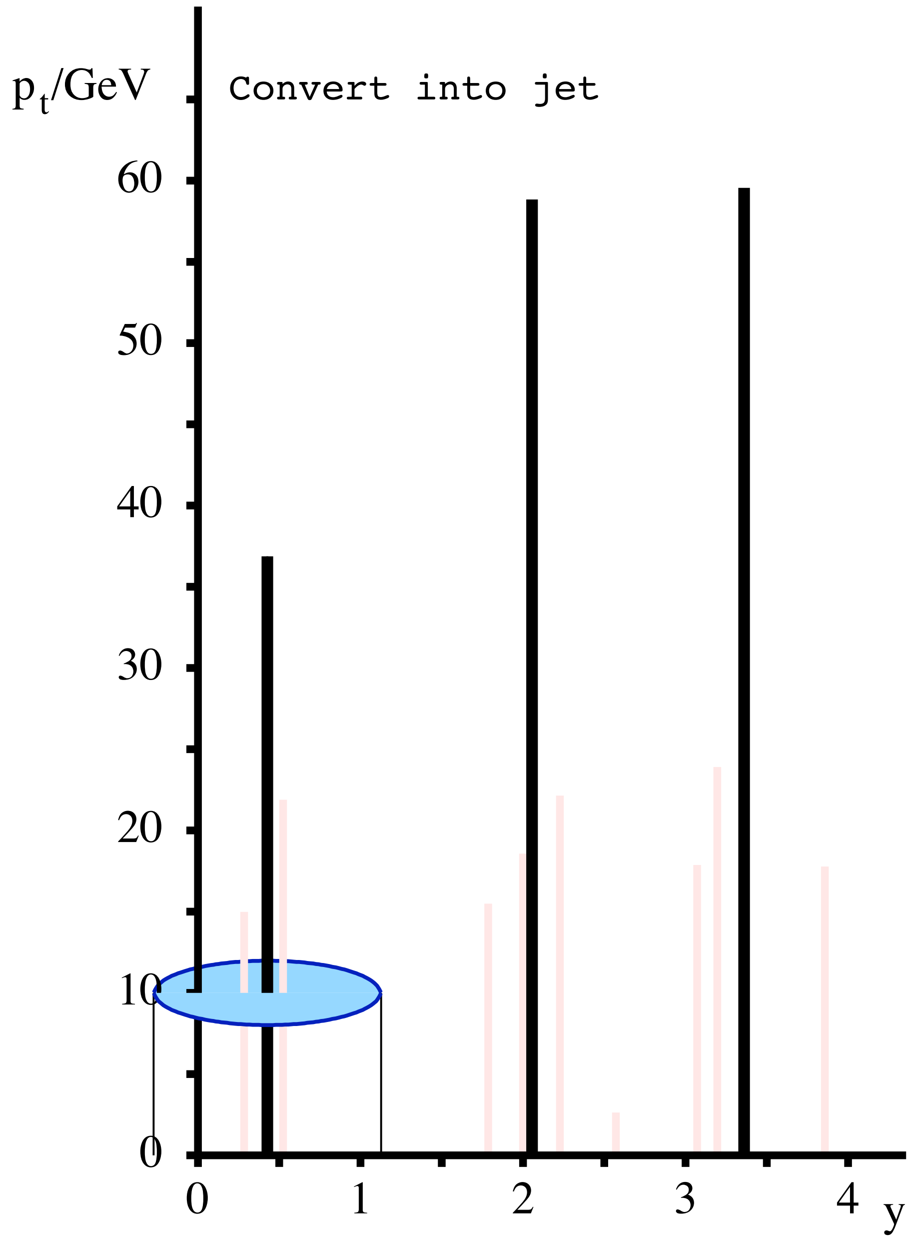
- Choice of R depends on the use-case
- Cone jet algorithms try to find the axis a which maximises the energy within the cone - easy?



Many different variants have been thought of

Cone Algorithms

An example for an IRC unsafe algorithm: Iterative Cone algorithm



Next-simplest of the cones

e.g. CMS iterative cone

- Take hardest particle as seed for cone axis
- Draw cone around seed
- Sum the momenta use as new seed direction, iterate until stable
- Convert contents into a "jet" and remove from event

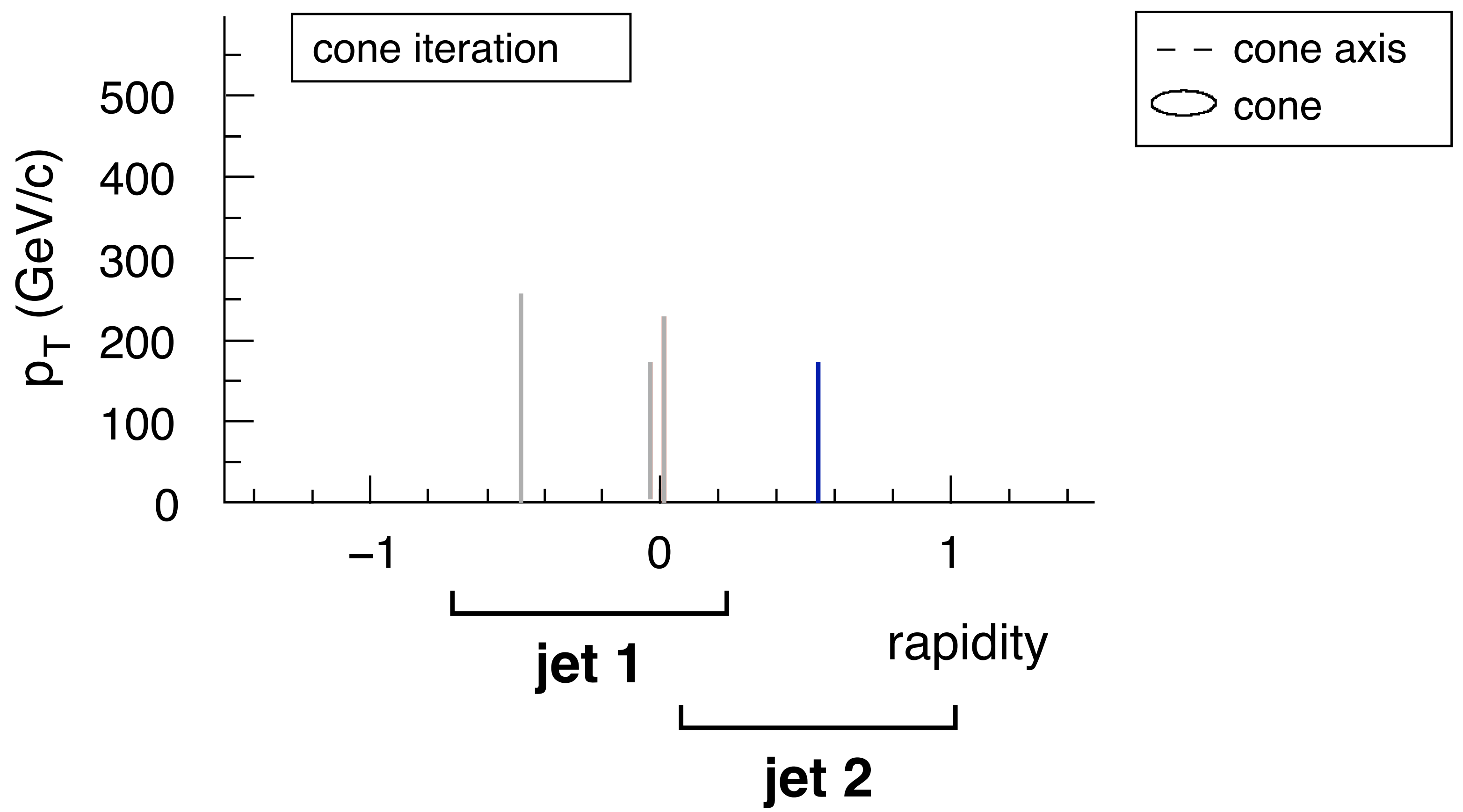
Notes

- "Hardest particle" is collinear unsafe more right away...

(courtesy of Gavin Salam)

Cone Algorithms

Why is it IRC unsafe?

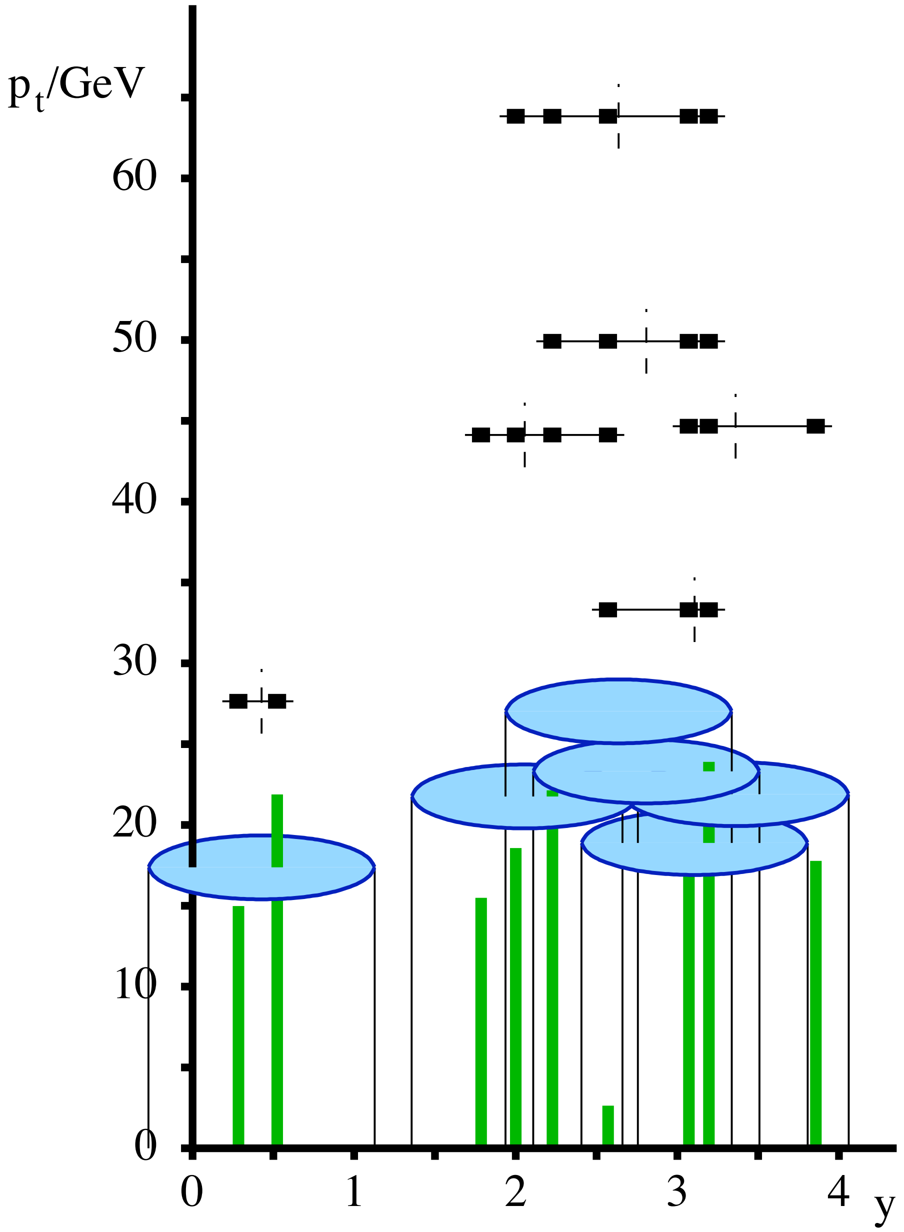


Collinear splitting can modify the hard jets: ICPR algorithms are collinear unsafe $\Longrightarrow$ perturbative calculations give ∞

(courtesy of Gavin Salam)

An IRC Safe Cone Algorithm

"Hardest particle" is collinear unsafe: only seedless cone algorithms can be IRC safe: development of SISCone algorithm



Aim to identify *all* stable cones, independently of any seeds

Procedure in 1 dimension (y):

- find all distinct enclosures of radius R — by repeatedly sliding a cone sideways until edge touches a particle
- check each for stability
- then run usual split–merge

In 2 dimensions (y,ϕ) can design analogous procedure **SISCone**
GPS & Soyez '07

This gives an IRC safe cone alg.

(courtesy of Gavin Salam)

Sequential Recombination Algorithms

Try to undo the QCD branching:

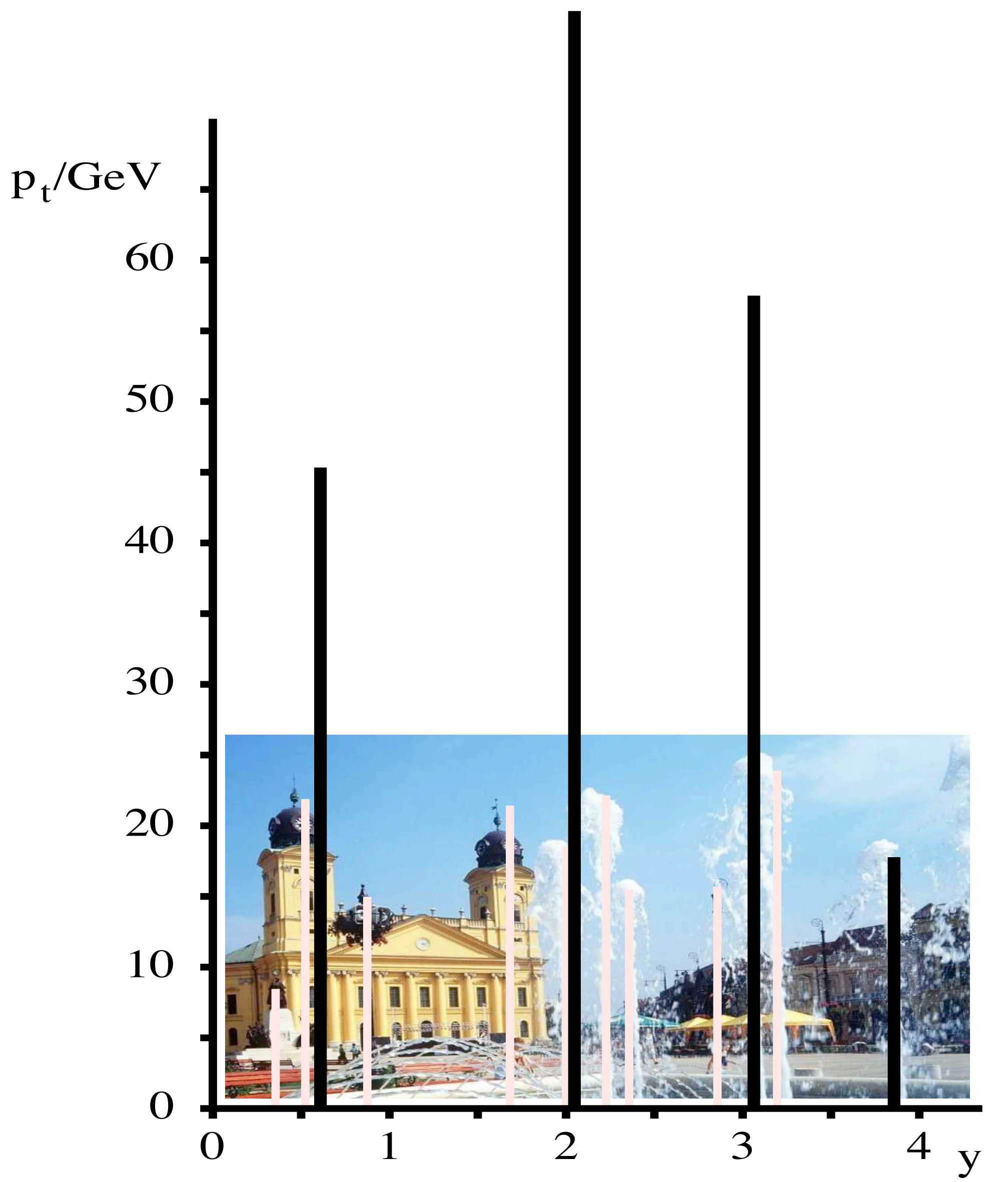
- Take pair of particles with strongest divergence between them and combine them
- Calculate distance d_{ij} between all particles and distance to beam d_{iB}

$$d_{ij} = \min(p_{t,i}, p_{t,j})\frac{\Delta R^2}{R^2} \quad \text{with} \quad \Delta R^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$
$$d_{iB} = p_{t,i}^2$$

1. Find smallest of d_{ij} and d_{iB}
2. If smallest is d_{ij}, combine particles i with j
3. If smallest is d_{iB}, call i a jet and remove from list of particles
4. Repeat from step 1 until no particles left

= longitudinally invariant inclusive k_t-algorithm

The k_t Algorithm



k_t alg.: Find smallest of

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2/R^2, \quad d_{iB} = k_{ti}^2$$

If d_{ij} recombine; if d_{iB}, i is a jet
Example clustering with k_t algorithm, $R = 0.7$

ϕ assumed 0 for all towers

(courtesy of Gavin Salam)

Sequential Recombination Algorithms

Different classes of jet algorithms

- Generalisation of the kt-algorithm:

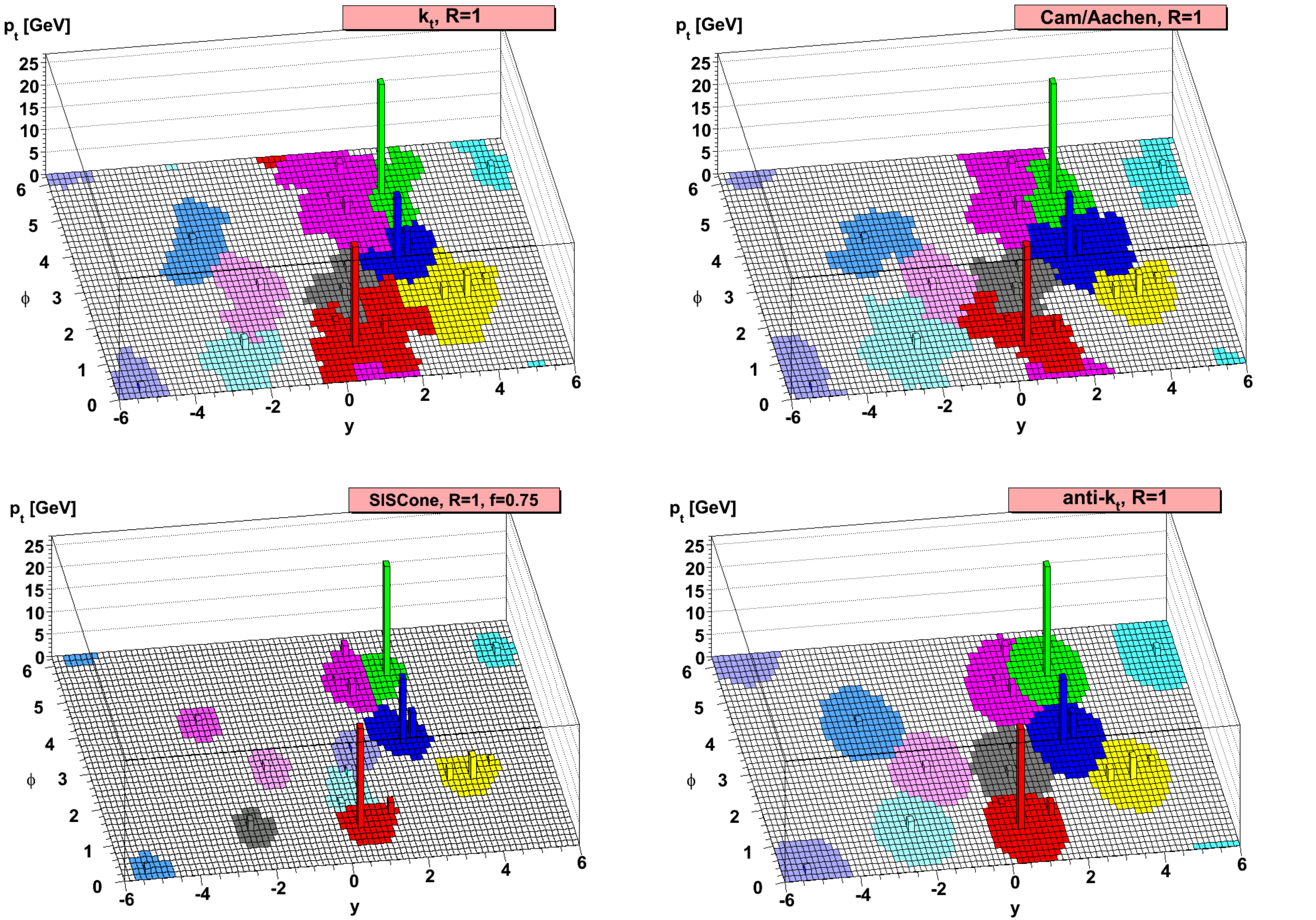
$$d_{ij} = \min(p_{t,i}^{2k}, p_{t,j}^{2k}) \frac{\Delta R^2}{R^2} \quad \text{with} \quad \Delta R^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = p_{t,i}^{2k}$$

1. k = 1: k_T-algorithm, combines softest particles first, very flexible jet boundaries
2. k = 0: Cambridge-Aachen algorithm: purely geometrical, combines closest particles first
3. k = −1: anti-k_T algorithm: combines hardest particles first, very spherical jets if no other hard particles are closer than R

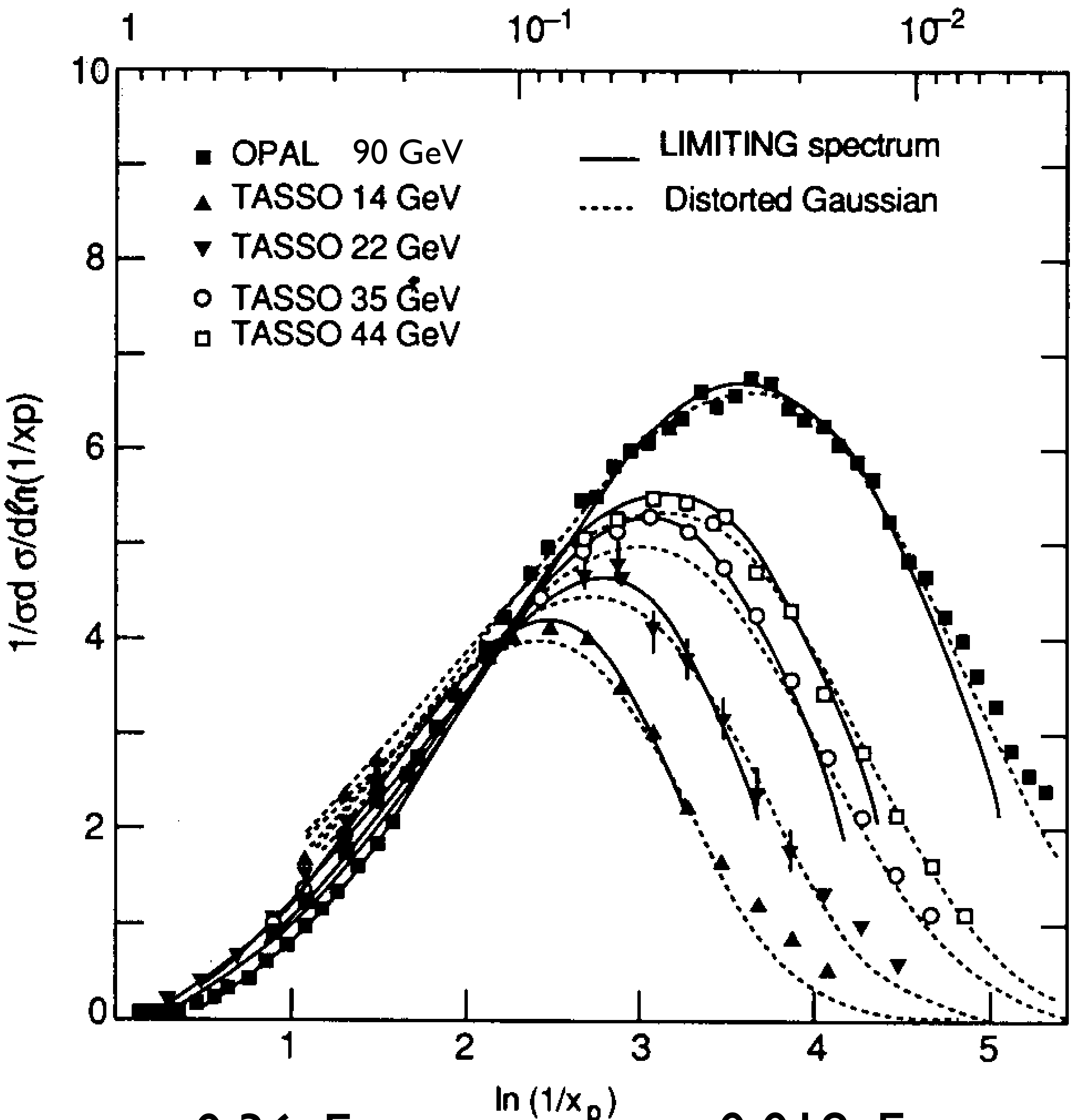
- Different recombinations of particles possible to calculate the jet axis:
  - E-scheme: massive jets
  - p_T scheme: massless jets

The Shape of Jets



area obtained with 'ghost particles'

How Does a Jet Look Like?



$0.36 \cdot E_{jet}$ $0.018 \cdot E_{jet}$

Rough approximation: particle content in a jet: $\pi^+ : \pi^- : \pi^0 = 1 : 1 : 1$ (+10% Kaons, Protons...)

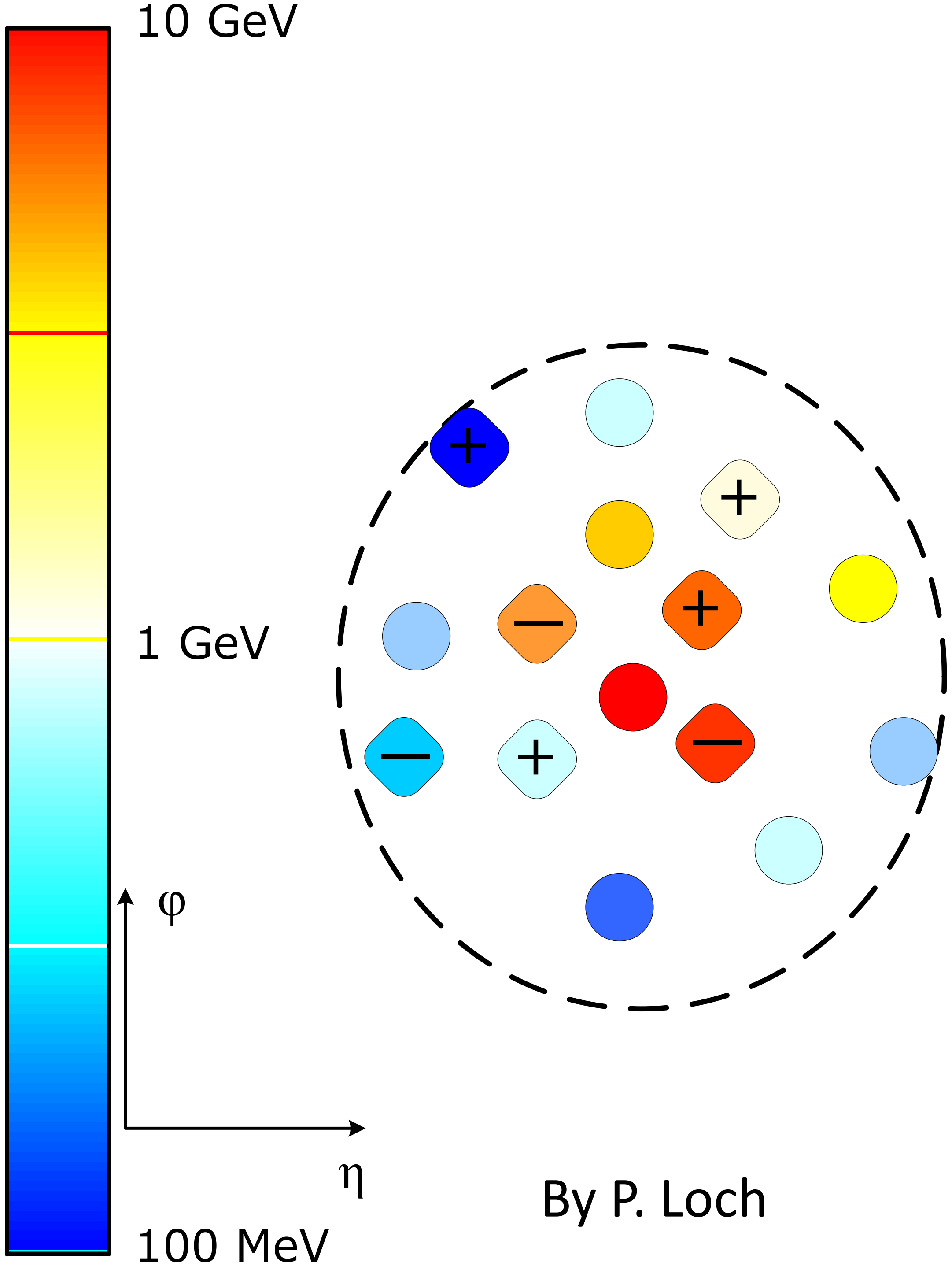
Shown here: charged particle spectra ($\pi^\pm$) in jets from e^+e^- collisions

$x_p = 2P / \sqrt{s}$

$E_{jet} \approx \sqrt{s} / 2$

More energy → higher multiplicity and more soft particles (compared to jet momentum)

Detector Effects On Jets



Change of composition

- Radiation and decay inside detector volume
- "Randomization" of original particle content

Defocusing changes shape in lab frame

- Charged particles bend in solenoid field

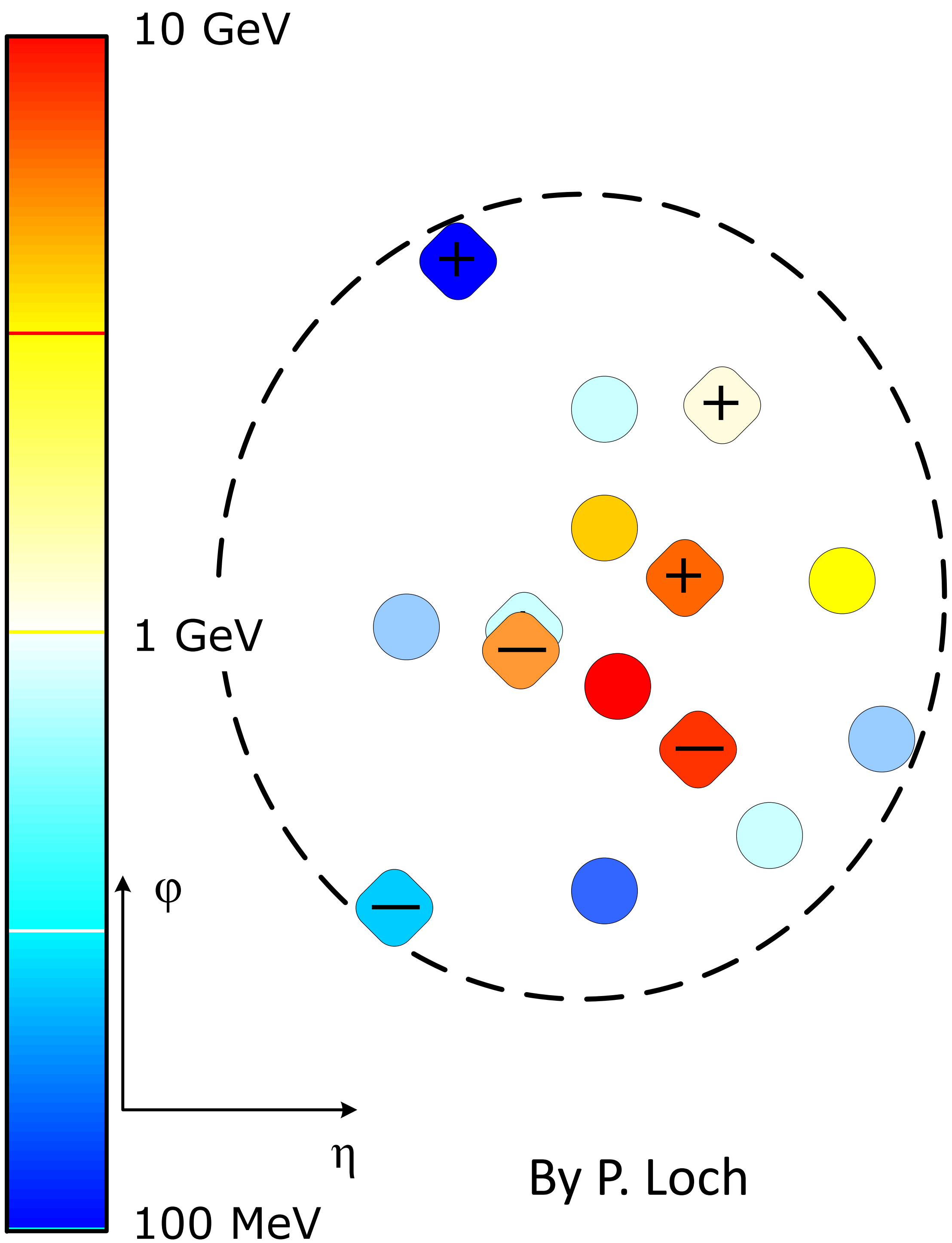
Attenuation changes energy

- Total loss of soft charged particles in magnetic field
- Partial and total energy loss of charged and neutral particles in inactive upstream material

Hadronic and electromagnetic cacades in calorimeters

- Distribute energy spatially
- Lateral particle shower overlap

Detector Effects On Jets



By P. Loch

Change of composition

- Radiation and decay inside detector volume
- "Randomization" of original particle content

Defocusing changes shape in lab frame

- Charged particles bend in solenoid field

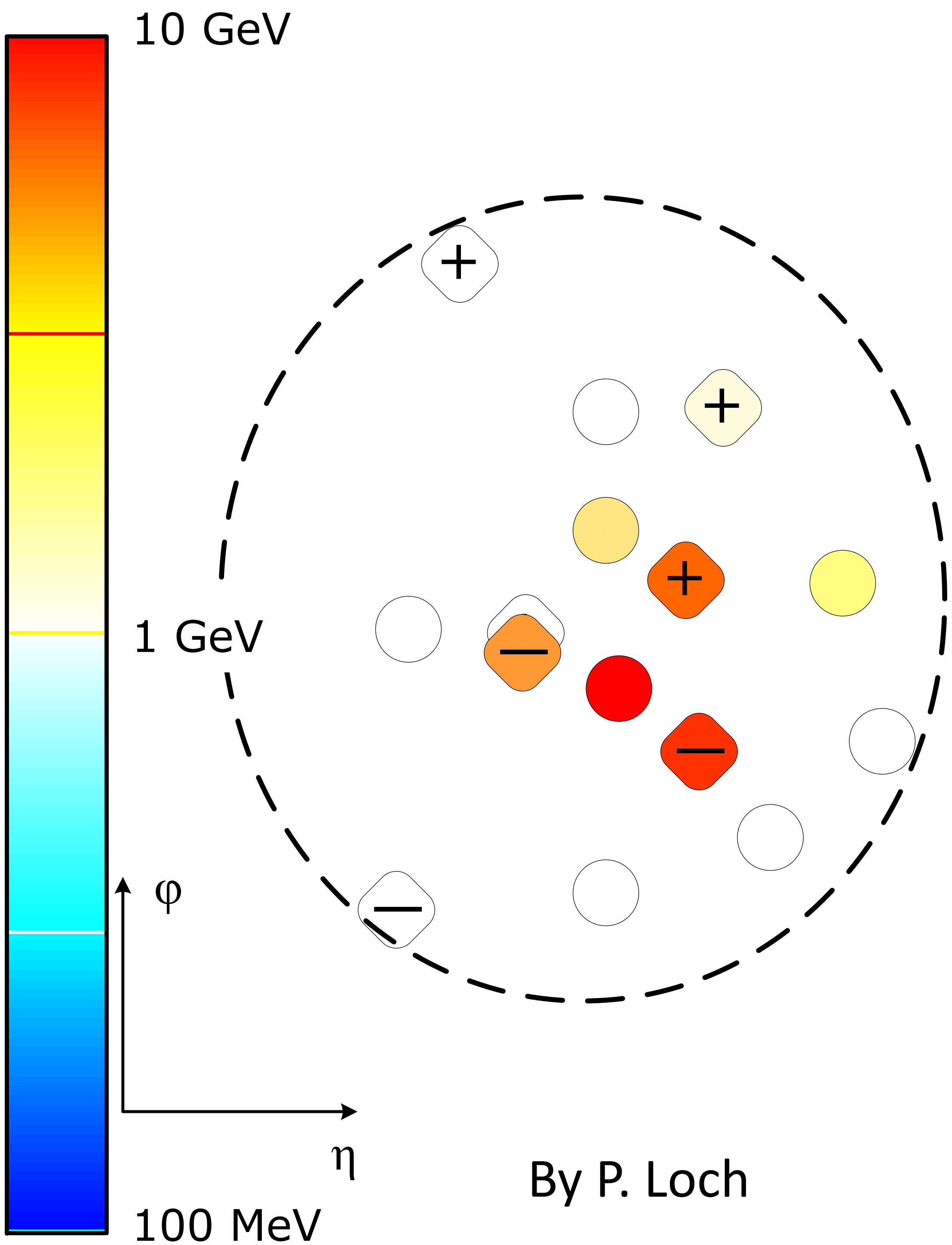
Attenuation changes energy

- Total loss of soft charged particles in magnetic field
- Partial and total energy loss of charged and neutral particles in inactive upstream material

Hadronic and electromagnetic cacades in calorimeters

- Distribute energy spatially
- Lateral particle shower overlap

Detector Effects On Jets



Change of composition

- Radiation and decay inside detector volume
- "Randomization" of original particle content

Defocusing changes shape in lab frame

- Charged particles bend in solenoid field

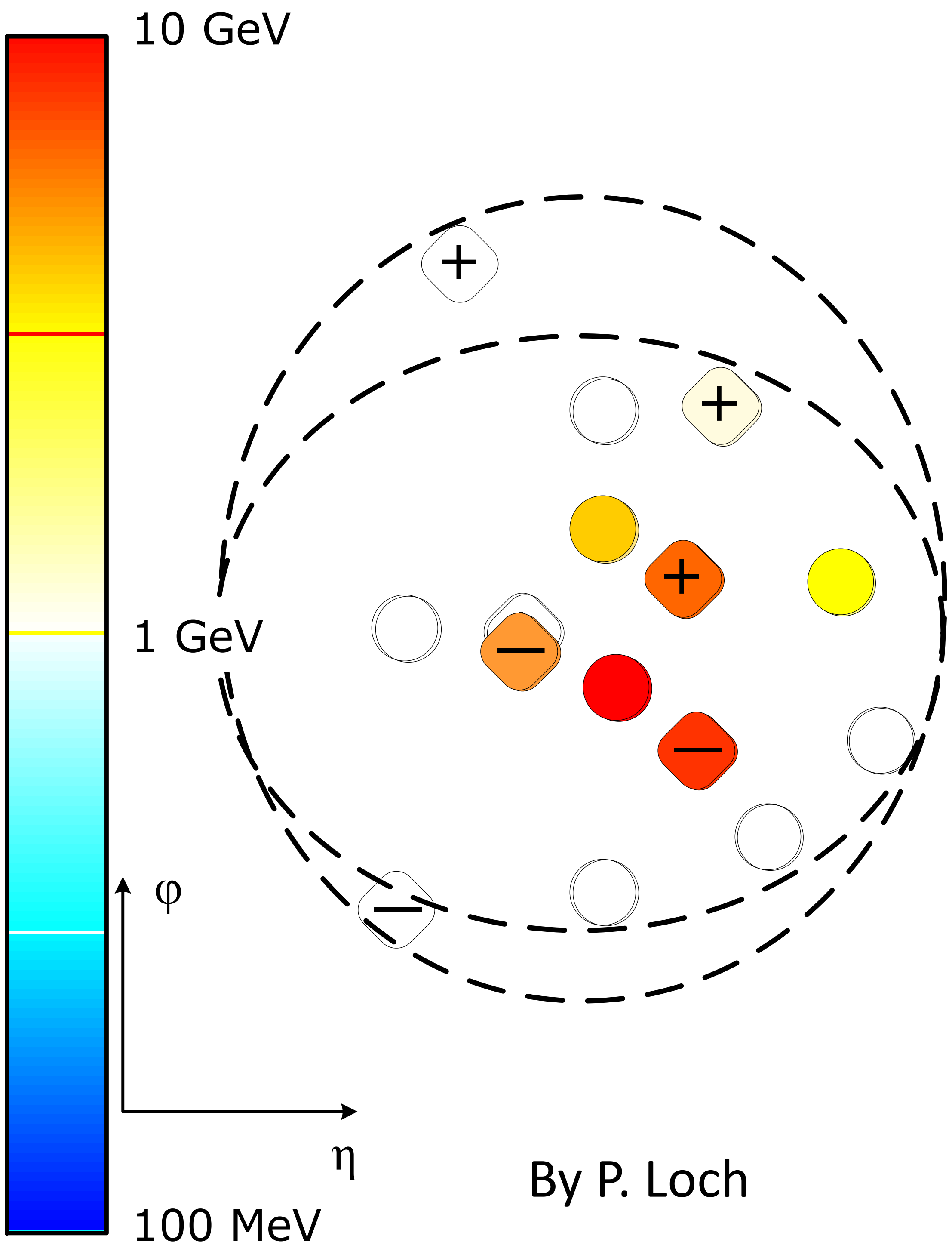
Attenuation changes energy

- Total loss of soft charged particles in magnetic field
- Partial and total energy loss of charged and neutral particles in inactive upstream material

Hadronic and electromagnetic cacades in calorimeters

- Distribute energy spatially
- Lateral particle shower overlap

Detector Effects On Jets



Change of composition

- Radiation and decay inside detector volume
- "Randomization" of original particle content

Defocusing changes shape in lab frame

- Charged particles bend in solenoid field

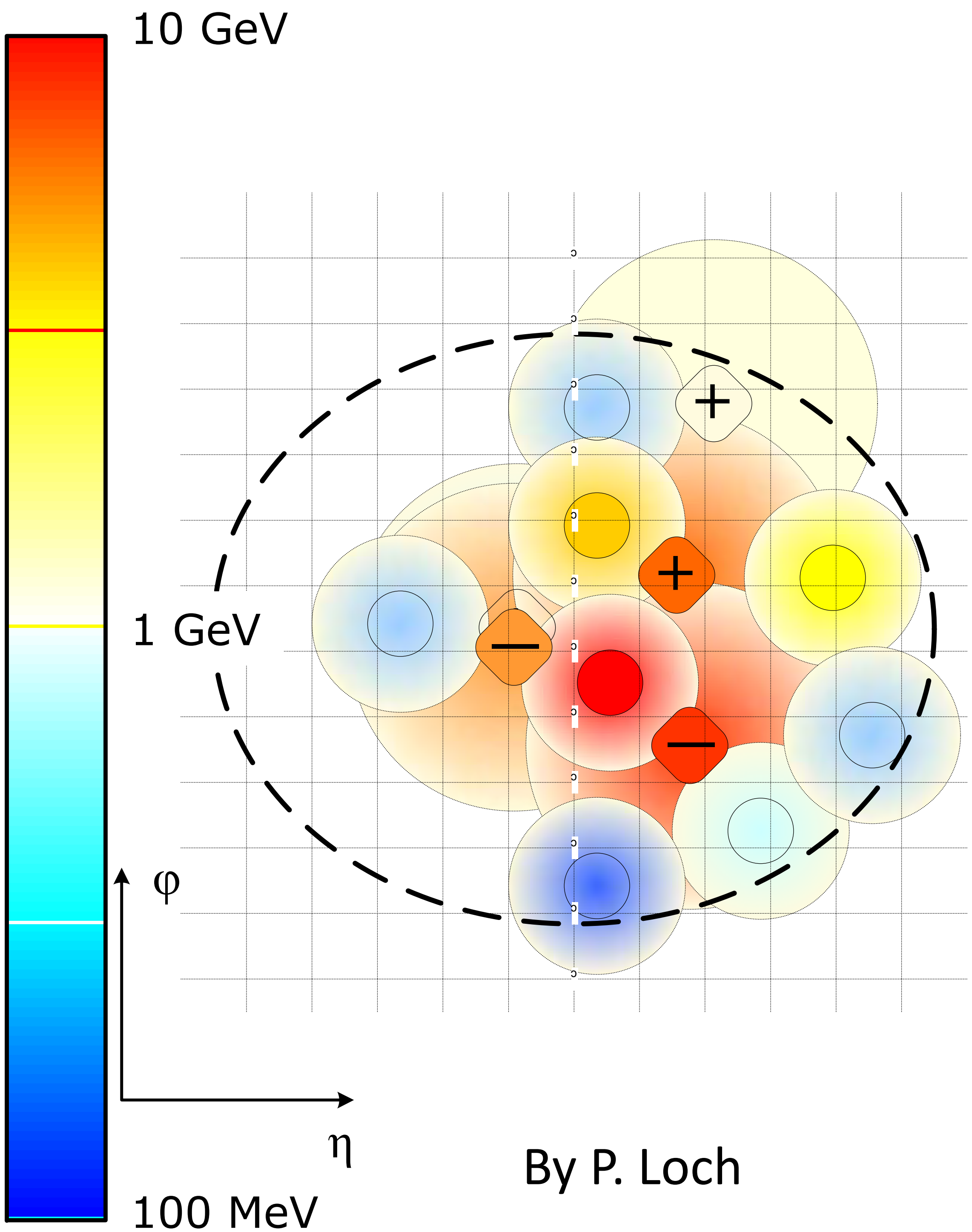
Attenuation changes energy

- Total loss of soft charged particles in magnetic field
- Partial and total energy loss of charged and neutral particles in inactive upstream material

Hadronic and electromagnetic cacades in calorimeters

- Distribute energy spatially
- Lateral particle shower overlap

Detector Effects On Jets



Change of composition

- Radiation and decay inside detector volume
- "Randomization" of original particle content

Defocusing changes shape in lab frame

- Charged particles bend in solenoid field

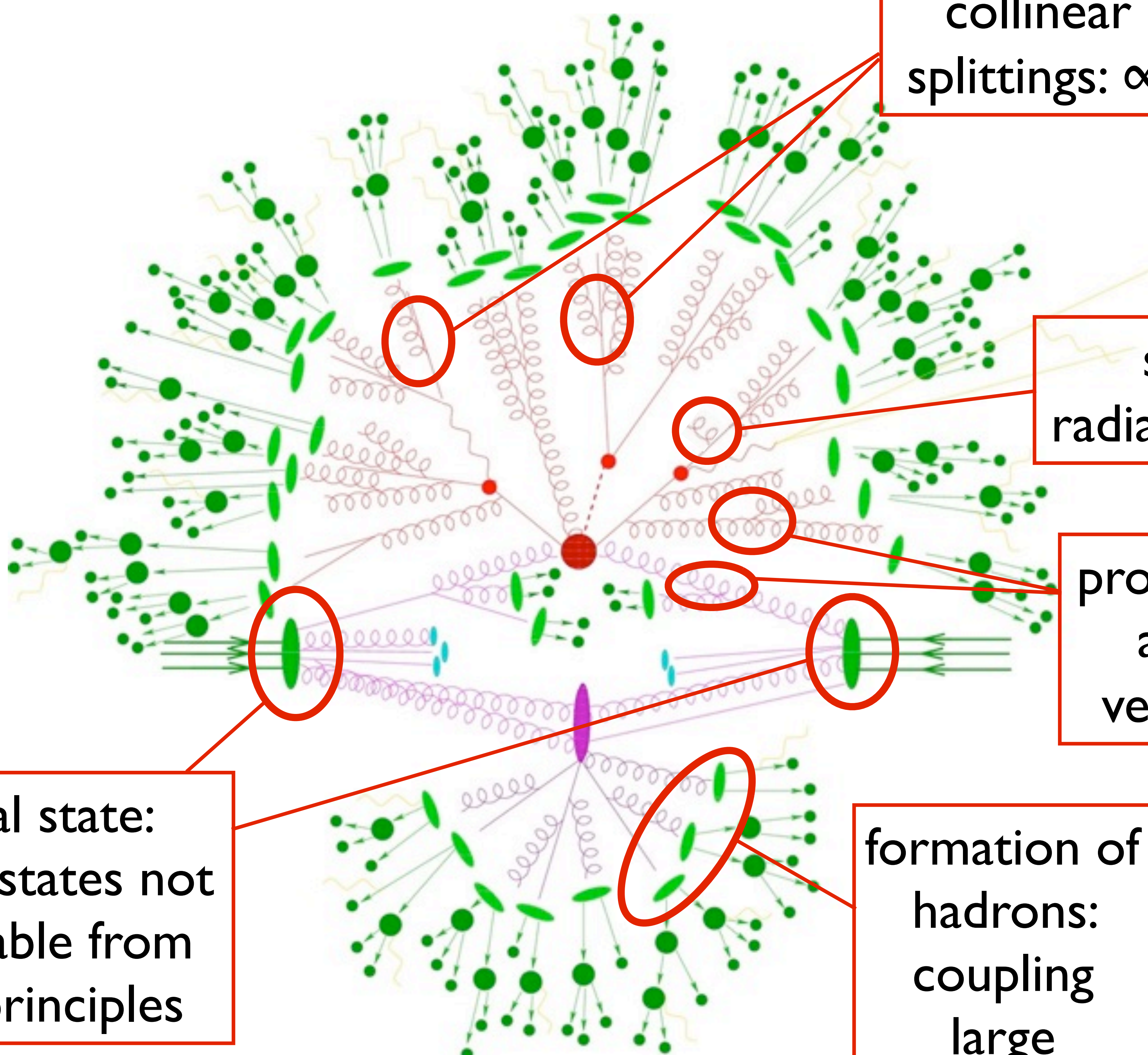
Attenuation changes energy

- Total loss of soft charged particles in magnetic field
- Partial and total energy loss of charged and neutral particles in inactive upstream material

Hadronic and electromagnetic cacades in calorimeters

- Distribute energy spatially
- Lateral particle shower overlap

Where Are We?



collinear splittings: ∞

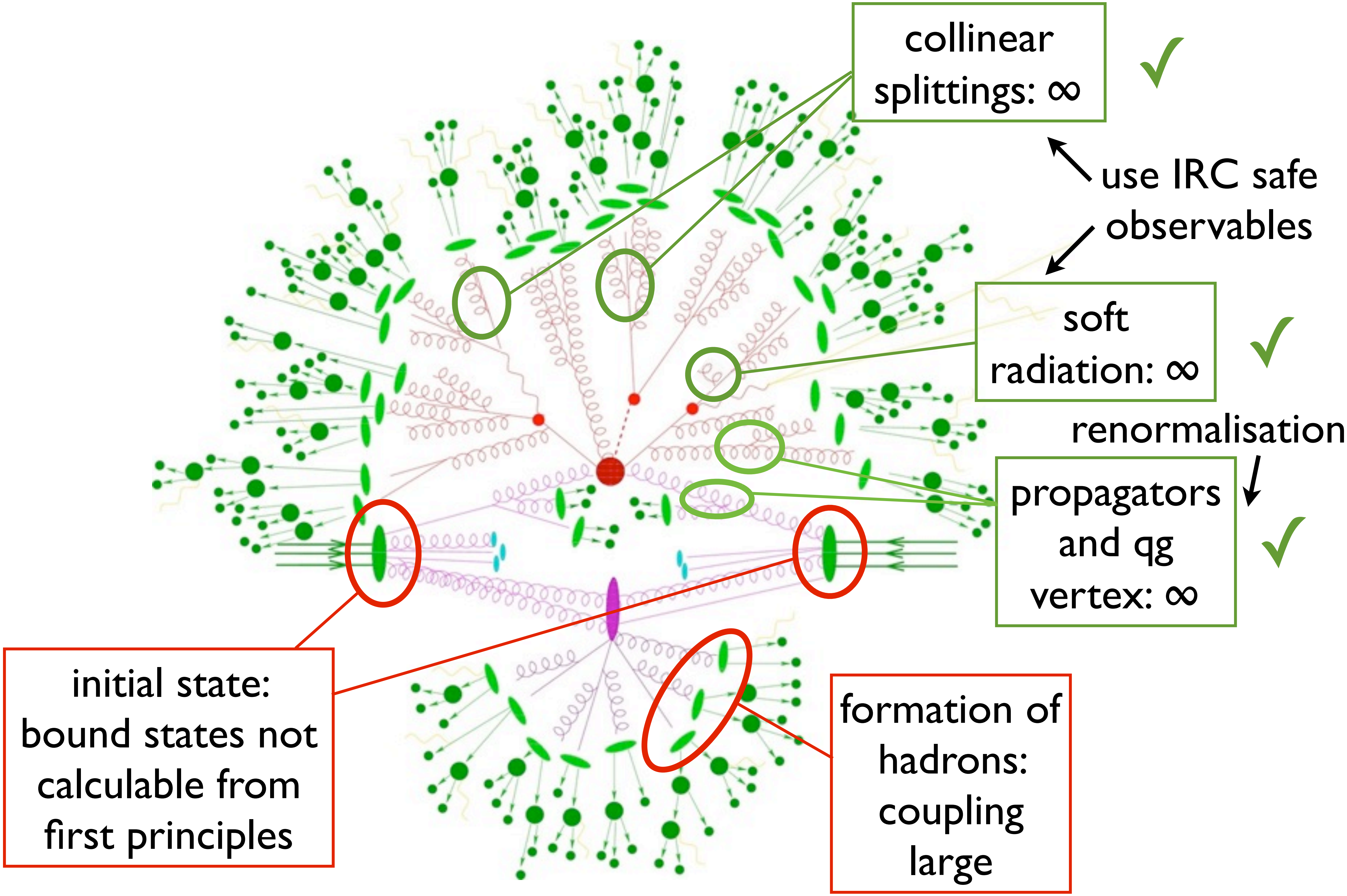
soft radiation: ∞

propagators and qg vertex: ∞

initial state: bound states not calculable from first principles

formation of hadrons: coupling large

Where Are We?

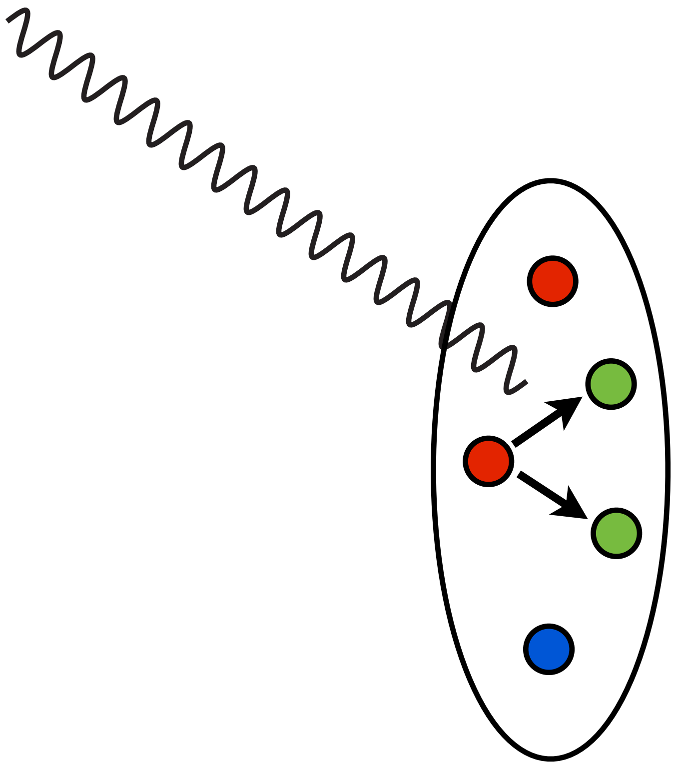

collinear splittings: ∞
use IRC safe observables
soft radiation: ∞
renormalisation
propagators and qg vertex: ∞
initial state: bound states not calculable from first principles
formation of hadrons: coupling large

Another Challenge

The small values of $\alpha_s(\mu_R)$ at large scales allows the application of perturbation theory

But as $\mu_R \to 0$, $\alpha_s(\mu_R)$ becomes large and higher order corrections become increasingly important ⇒ diagram techniques fail for bound states in QCD

How can we calculate anything with hadrons in the initial / final state involved?



Answer: different time / length scales!

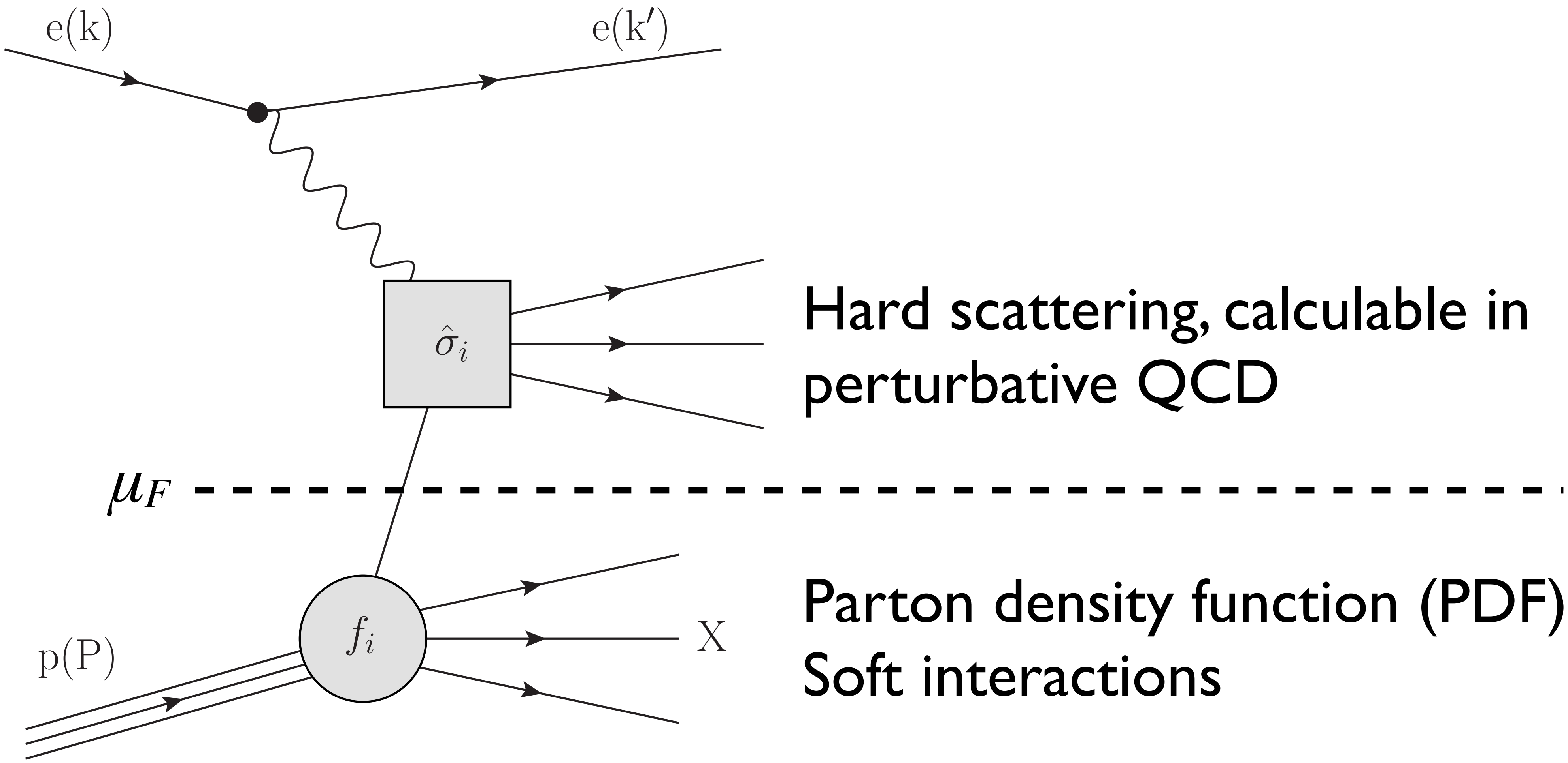
Timescale of proton fluctuation: $t = \frac{1}{\Delta E} \approx \frac{2xP}{k_T^2}$

Timescale of interaction: $\tau = \frac{1}{E_\gamma} = \frac{2xP}{Q^2}$

$\frac{t}{\tau} \approx \frac{Q^2}{k_T^2} \gg 1$ proton is 'frozen' during the interaction

Factorisation

Absorb long time (small scale) effects in the proton structure



Hard scattering, calculable in perturbative QCD

Parton density function (PDF), Soft interactions

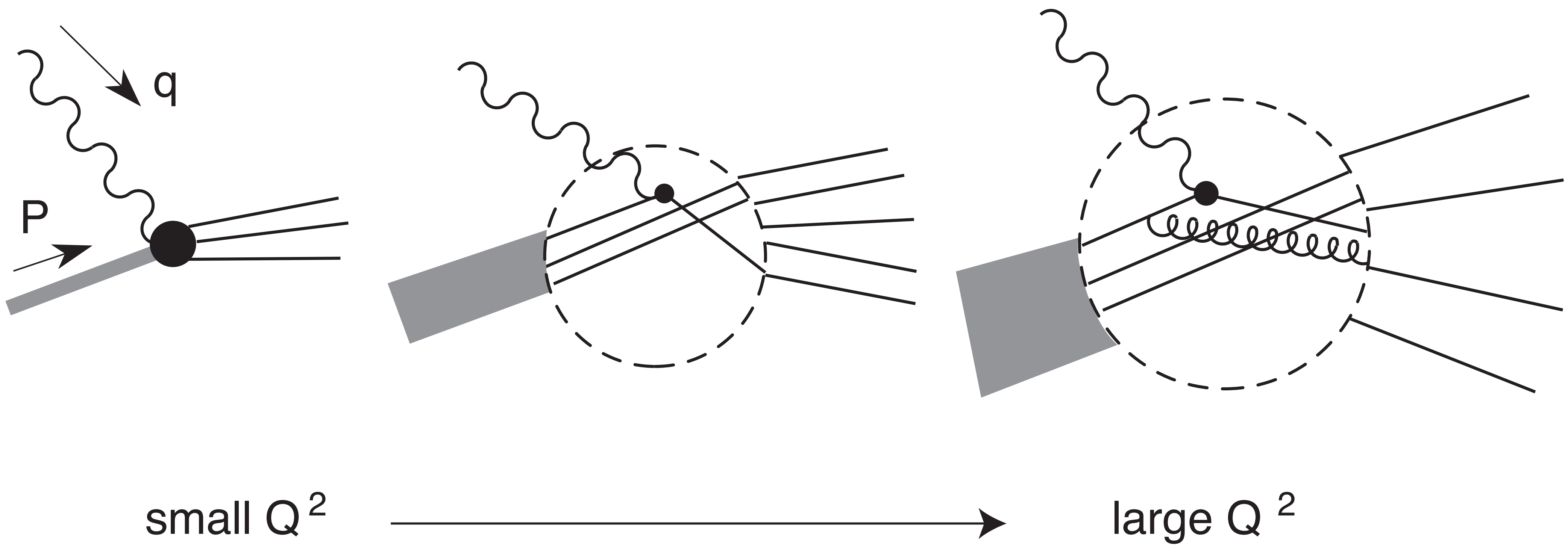
The factorisation scale μ_F gives the separation between long and short time physics

- PDFs acquire a scale dependence
- PDFs can not be predicted by QCD

Parton Evolution

Intuitive picture: the number of partons changes with scale $\mu_F = Q^2$

The virtual photon as probe with resolving power $Q^2 \sim 1 / \lambda$



many partons with large x

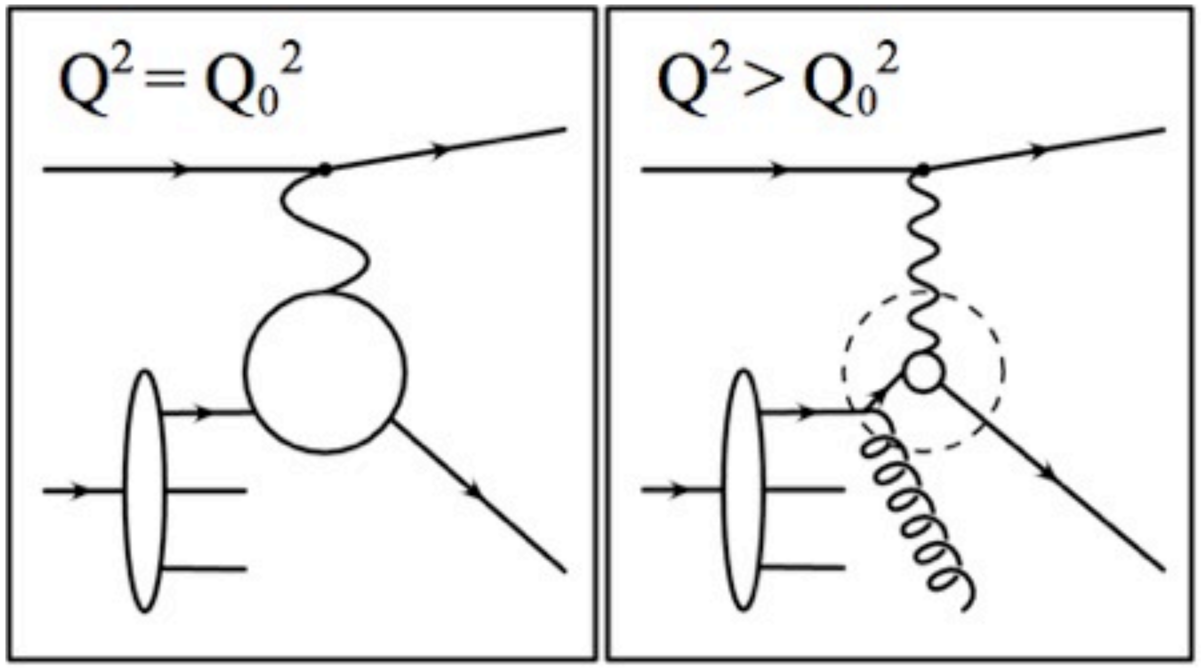
we 'see' parton radiation, branchings

⇒ many partons with small x

Drawing from A. Pich, arXiv:hep-ph/9505231 (1995)

Scaling Violations

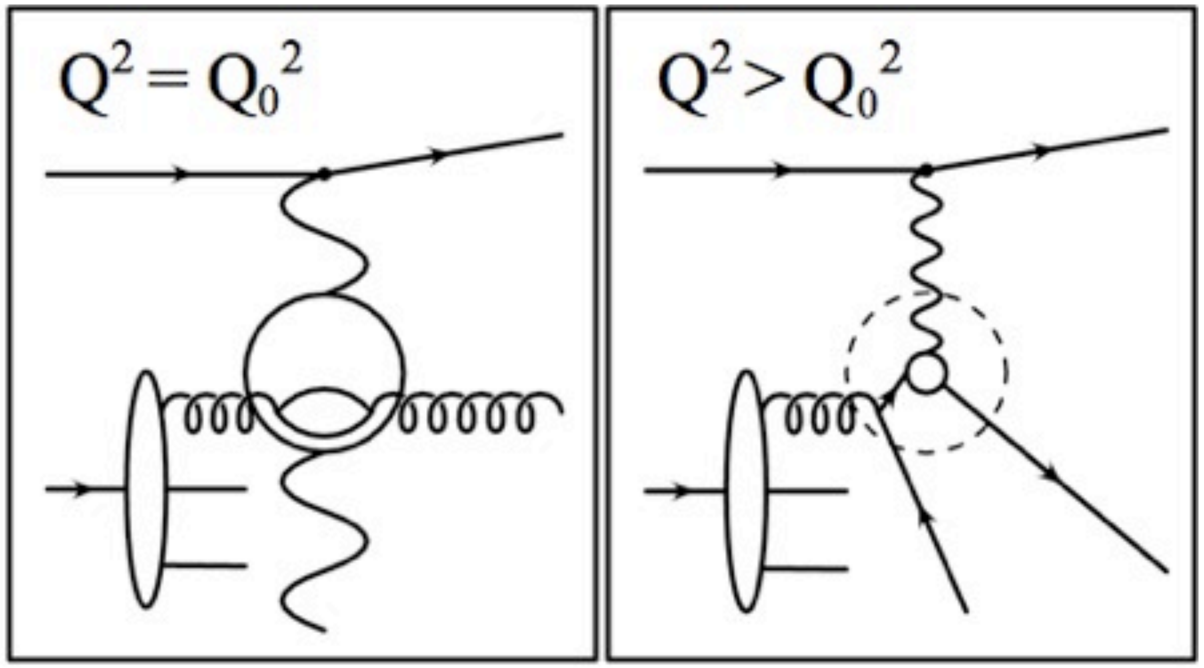
Large x



With increasing Q^2, the valence quarks radiate more and more gluons, so the studied x decreases

F_2 decreases with increasing Q^2

Small x



Gluons split into sea quarks, which can be resolved with increasing Q^2, more quarks become visible

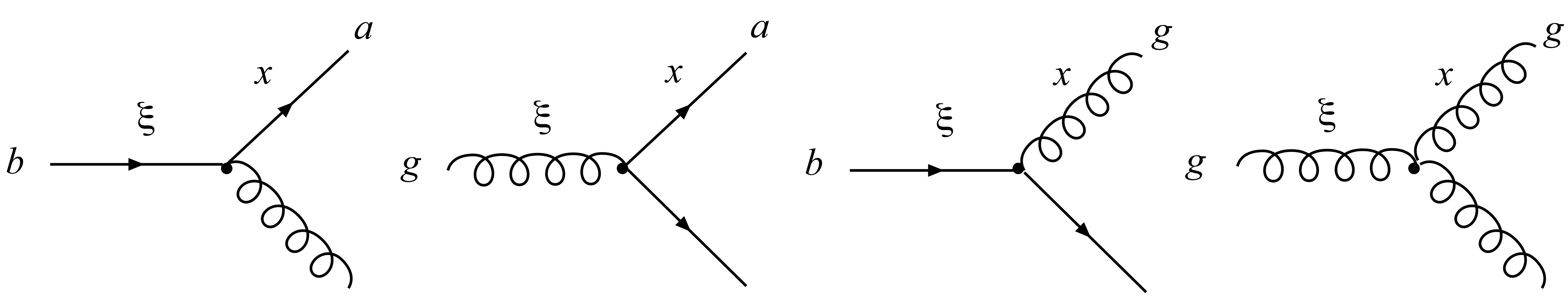
F_2 increases with increasing Q^2

DGLAP Equations

It is possible to calculate the evolution of partons in QCD:
DGLAP equations (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{\partial}{\partial \ln \mu_F^2}\begin{pmatrix} q_i(x,\mu_F^2) \\ g(x,\mu_F^2) \end{pmatrix} = \frac{\alpha_s(\mu_R)}{2\pi}\sum_j \int_x^1 \frac{\mathrm{d}\xi}{\xi}\begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}\right) & P_{q_i g}\left(\frac{x}{\xi}\right) \\ P_{g q_j}\left(\frac{x}{\xi}\right) & P_{gg}\left(\frac{x}{\xi}\right) \end{pmatrix}\begin{pmatrix} q_j(\xi,\mu_F^2) \\ g(\xi,\mu_F^2) \end{pmatrix}$$

Splitting functions $P_{ab}(x/\xi)$: meaning (in LO) of an emission probability:



$P_{ab}^{(0)}\left(\frac{x}{\xi}\right)$ $P_{ag}^{(0)}\left(\frac{x}{\xi}\right)$ $P_{gb}^{(0)}\left(\frac{x}{\xi}\right)$ $P_{gg}^{(0)}\left(\frac{x}{\xi}\right)$

We can predict the scale dependence of the quark $q(x, \mu_F)$ and gluon $g(x, \mu_F)$ distributions!

F_2 Revisited

In the QPM we had: $F_2(x) = x \sum_i e_i^2 q_i(x)$

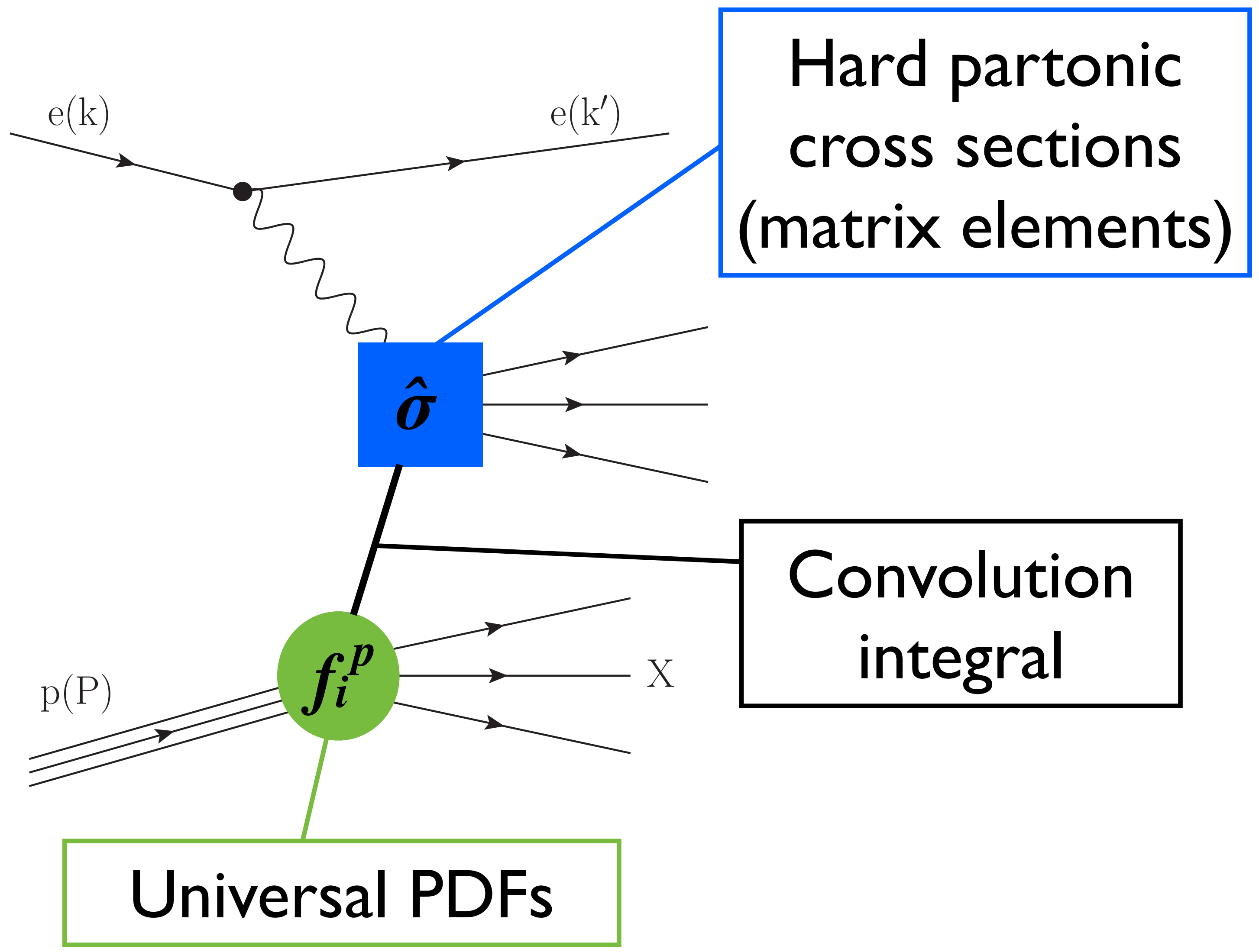
Now we have ($\overline{\text{MS}}$-scheme used) (in DIS use $\mu_F = Q^2$)

$$F_2(x,Q^2) = x \sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{\mathrm{d}\xi}{\xi} q(\xi,Q^2) \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}\left(\frac{x}{\xi}\right) + \ldots \right]$$
$$+ x \sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{\mathrm{d}\xi}{\xi} g(\xi,Q^2) \left[\frac{\alpha_s}{2\pi} C_g^{\overline{\text{MS}}}\left(\frac{x}{\xi}\right) + \ldots \right]$$

($C_q^{\overline{\text{MS}}}$ and $C_g^{\overline{\text{MS}}}$ are scheme-dependent coefficient functions)

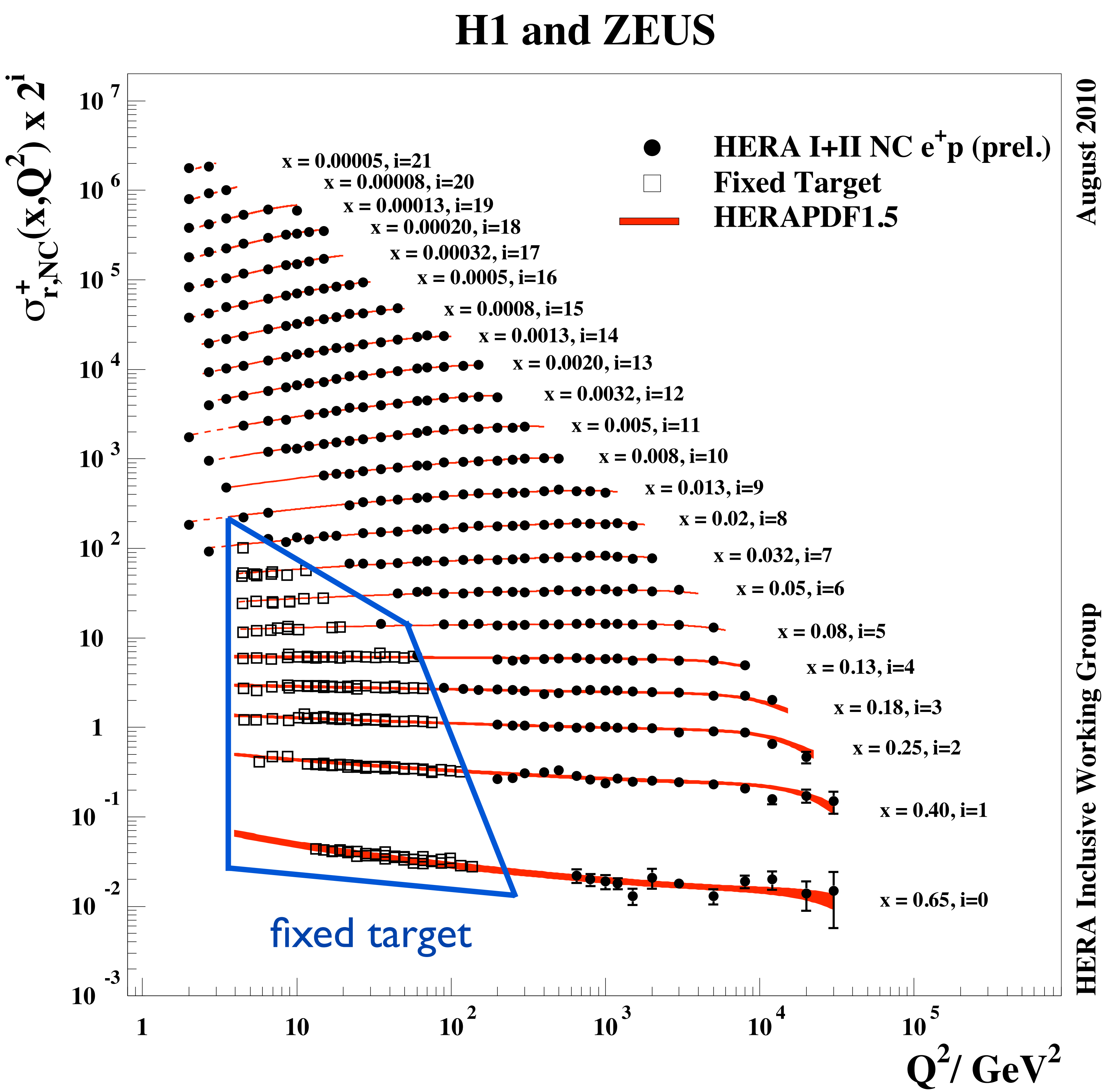
- In leading order (LO) we get back to the QPM
- F_2 obtained an explicit Q^2 dependence
- In next-to-leading order (NLO) F_2 is sensitive to the gluon component

Structure Functions and PDFs



We can obtain the Parton Distribution Functions by measuring structure functions

F_2 From HERA



$$\sigma^{+}_{\mathrm{r,NC}}(x,Q^2) = \frac{\mathrm{d}^2\sigma^{e^+p}_{\mathrm{NC}}}{\mathrm{d}Q^2\mathrm{d}x}\frac{xQ^4}{2\pi\alpha^2 Y_+}$$
$$\approx F_2(x,Q^2)$$

HERA data cover 5 orders in Q^2 and 4 orders in x

Approximate scaling at intermediate x

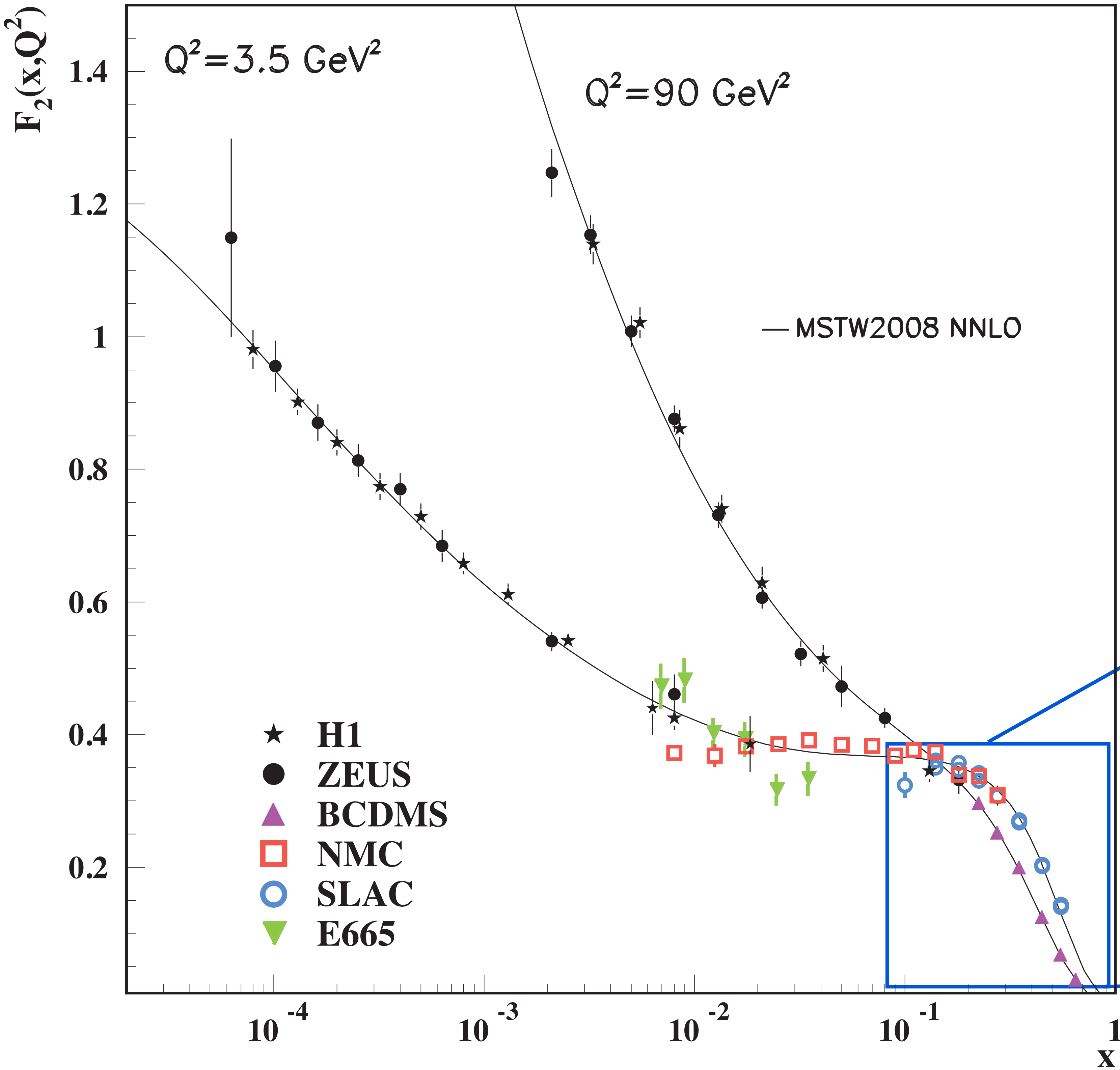
Clear scaling violations at small and large x

DGLAP works!

Huge success of QCD

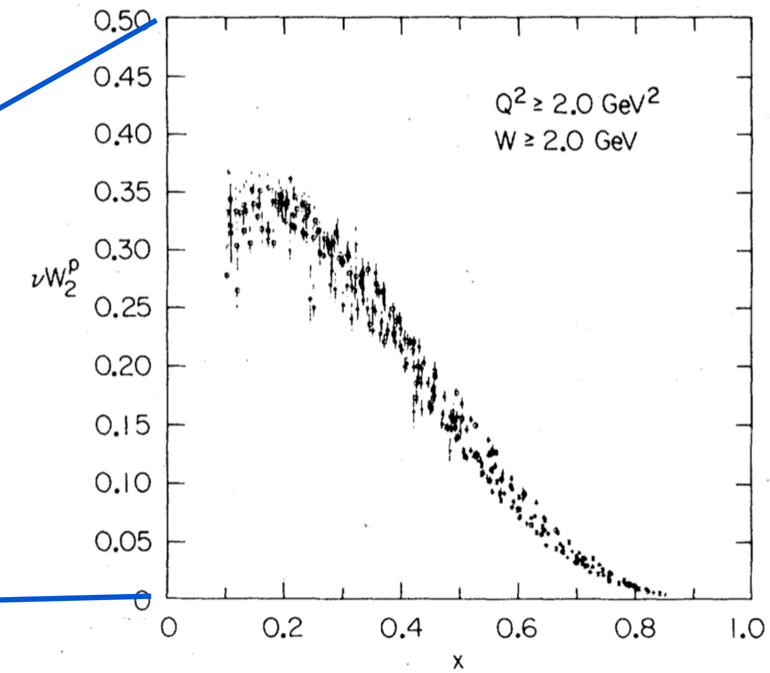
What Happens At Low x?

K. Nakamura, *et al.* (PDG), J. Phys. G37, 075021 (2010)



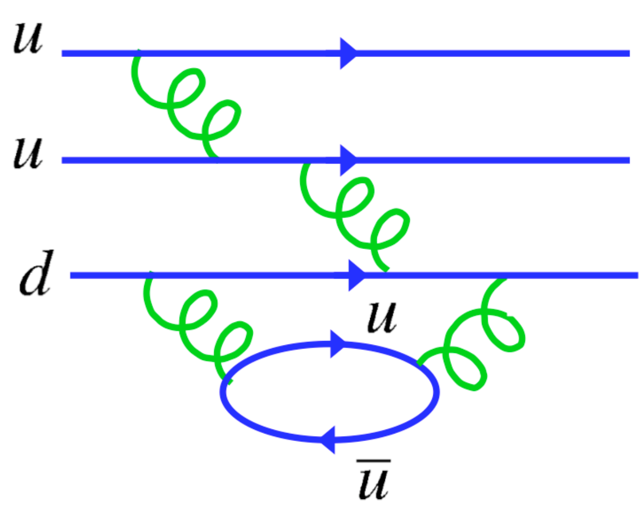
Early HERA data compared to fixed-target experiments

Strong rise of F_2 towards small x, becoming steeper with increasing Q^2



Parton Distribution Functions (PDFs)

Modify the simple QPM picture, where the proton was only made up of two up and one down quark



The up- and down-quark distributions obtain contributions from the valence quarks and the virtual sea quarks

$u(x) = u_v(x) + u_s(x)$ and $d(x) = d_v(x) + d_s(x)$

anti-quarks originate only from the sea

$\bar{u}(x) = \bar{u}_s(x)$ and $\bar{d}(x) = \bar{d}_s(x)$

The proton consists of two up quarks and one down quark:

$$\int_0^1 u_v(x)\mathrm{d}x = 2 \quad \text{and} \quad \int_0^1 d_v(x)\mathrm{d}x = 1$$ (quark number sum rules)

No a-priori expectation for the number of sea quarks and gluons.

Constituents Of The Proton

In general we have 10 quark and anti-quark densities and the gluon:

$$u, \bar{u}, d, \bar{d}, s, \bar{s}, c, \bar{c}, (b, \bar{b}), g$$

Distinguish only between up-type and down-type quarks:

$$U = u\,(+\,c), \quad D = d + s\,(+b)$$
$$\bar{U} = \bar{u}\,(+\,\bar{c}), \quad \bar{D} = \bar{d} + \bar{s}\,(+\bar{b})$$

Then the valence quark distributions are

$$u_v = U - \bar{U}, \quad d_v = D - \bar{D}$$

The total sea distribution is often expressed as

$$S = 2(\bar{U} + \bar{D})$$

and the momentum sum rule has to be fulfilled

$$\int_0^1 \Big[\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)\Big] x\mathrm{d}x = 1$$

From F_2 To PDFs

QCD (DGLAP) predicts scale dependence of quark and gluon densities

x-dependence can not be calculated in perturbative QCD
(reminder: renormalisation of the bare quark and gluon densities - soft, long-range effects are absorbed in the PDF)

Need to obtain the x-dependence from experiment!

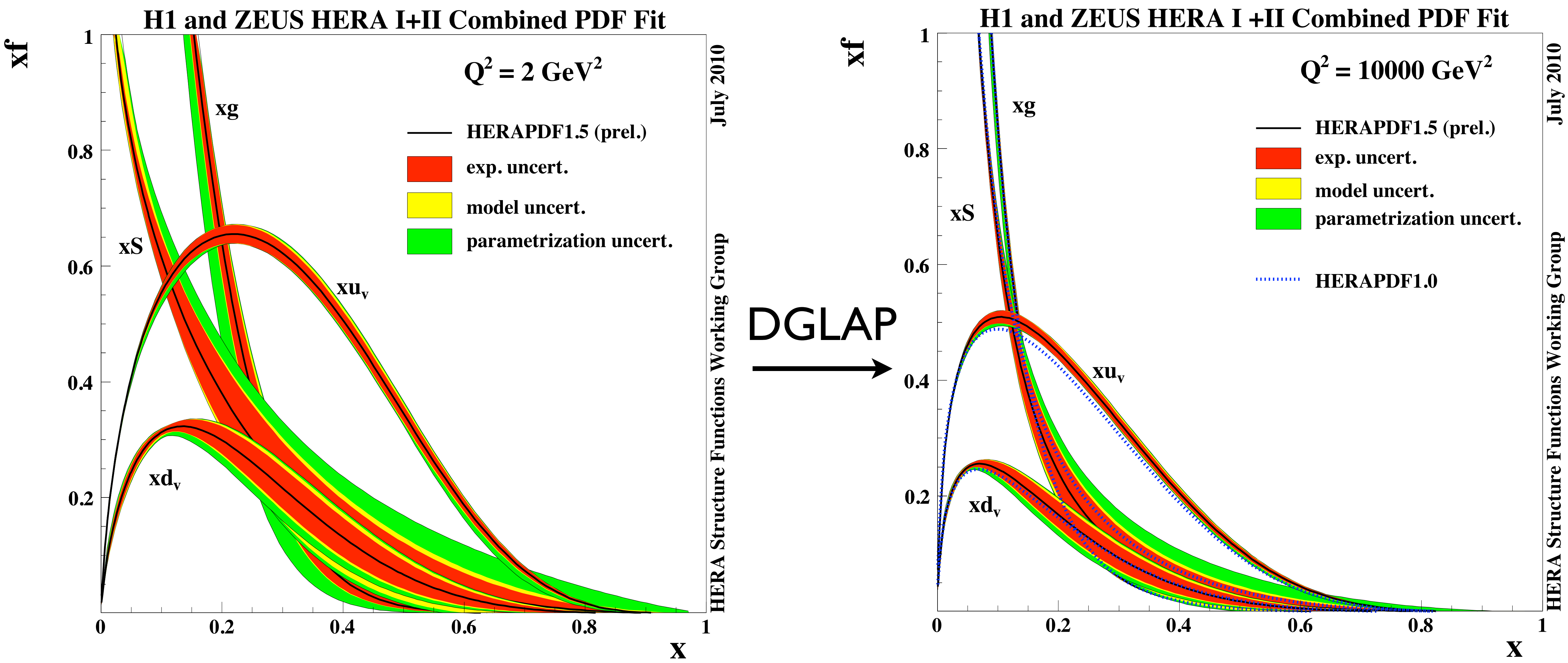
- Parametrise $q_i(x)$, $g(x)$ at a starting scale Q_0
- Use DGLAP to evolve F_2 to a higher scale (and calculate $\sigma_r(x,Q^2)$)
- Determine the parameters from a fit to data

Note: Q_0^2 has to be smaller than the lowest value of Q^2 in the data

Only limited number of free parameters possible

Use physical constraints for PDFs

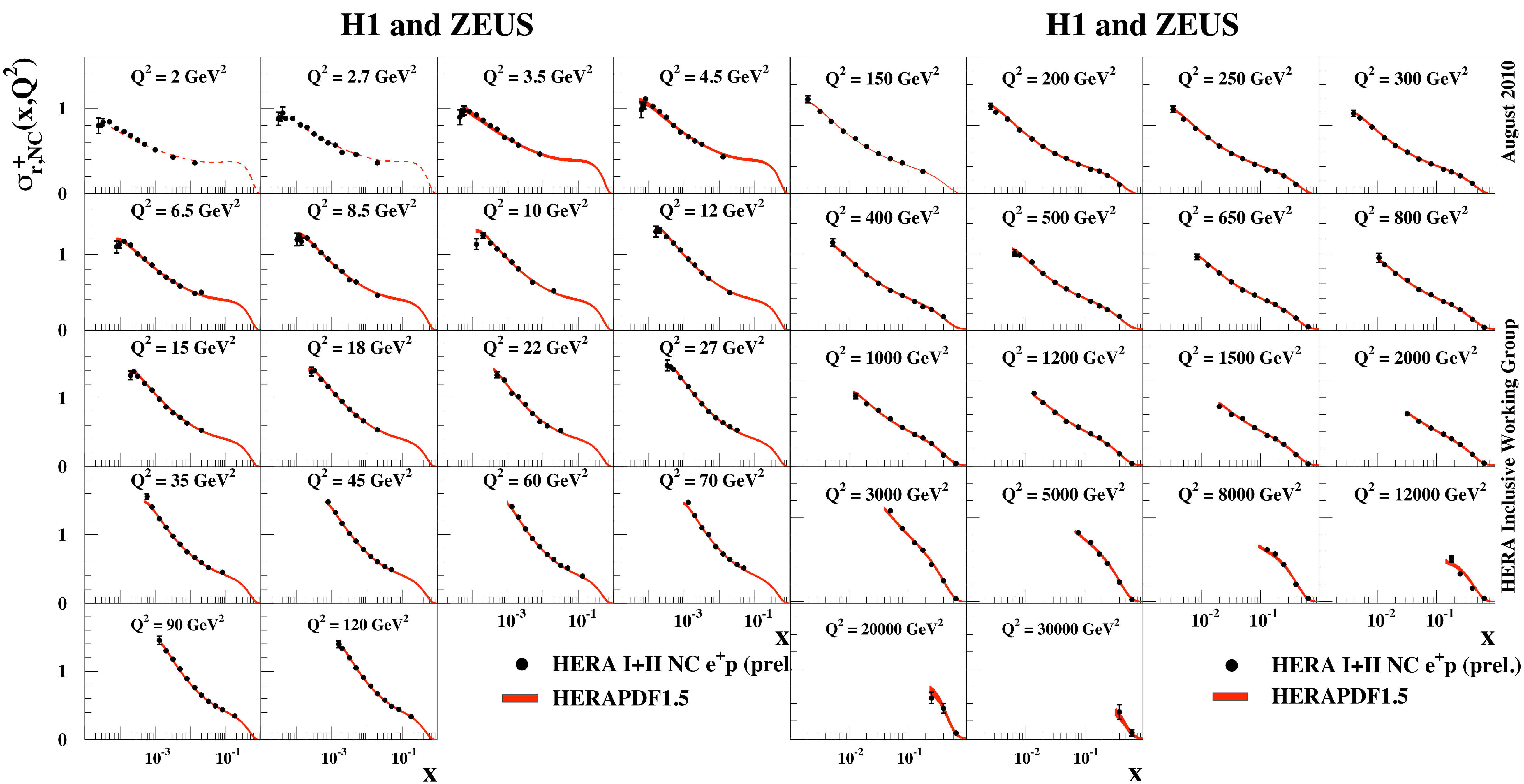
Parton Distribution Functions



10 free parameters, about 1000 data points entered the fit, $\chi^2/\mathrm{n.d.f} \approx 0.94$

$u_v \approx 2d_v$, gluon starts to dominate around $x \sim 0.2$

Strong Rise Of F_2 Versus x



Strong rise of F_2 towards small x, becoming steeper with increasing Q^2

Impressive agreement between calculations (using DGLAP) and data

Formation Of Hadrons

Last missing piece before we can calculate real-life cross sections

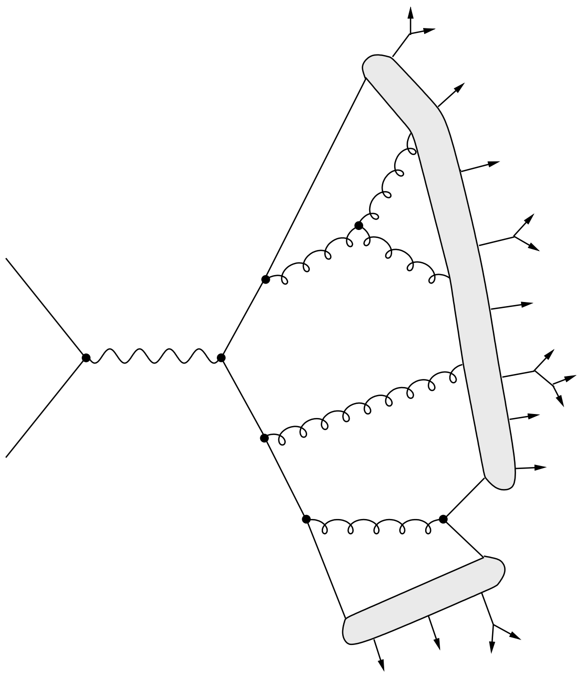
Full-scale event generators generate QCD branching according to emission probabilities - the parton shower approach

Once the scale of the emitted partons becomes small, perturbative QCD is not applicable anymore

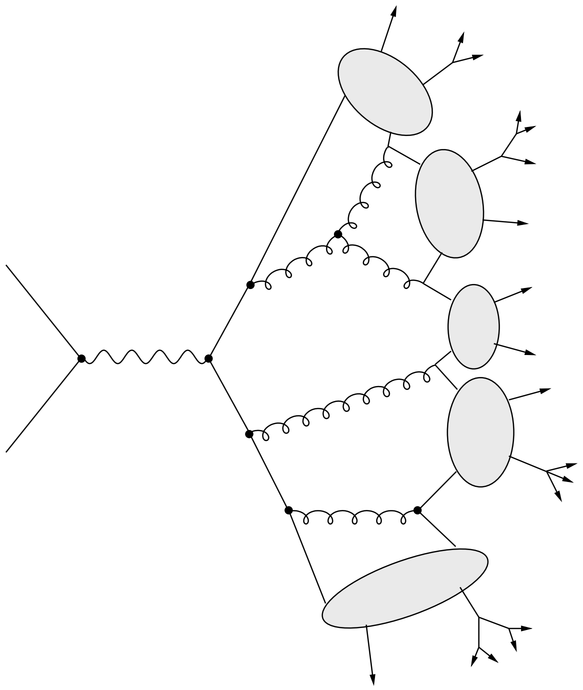
Model the formation of hadrons with phenomenological approaches

Based on the idea of the QCD potential

$$V(r) \propto k \cdot r$$



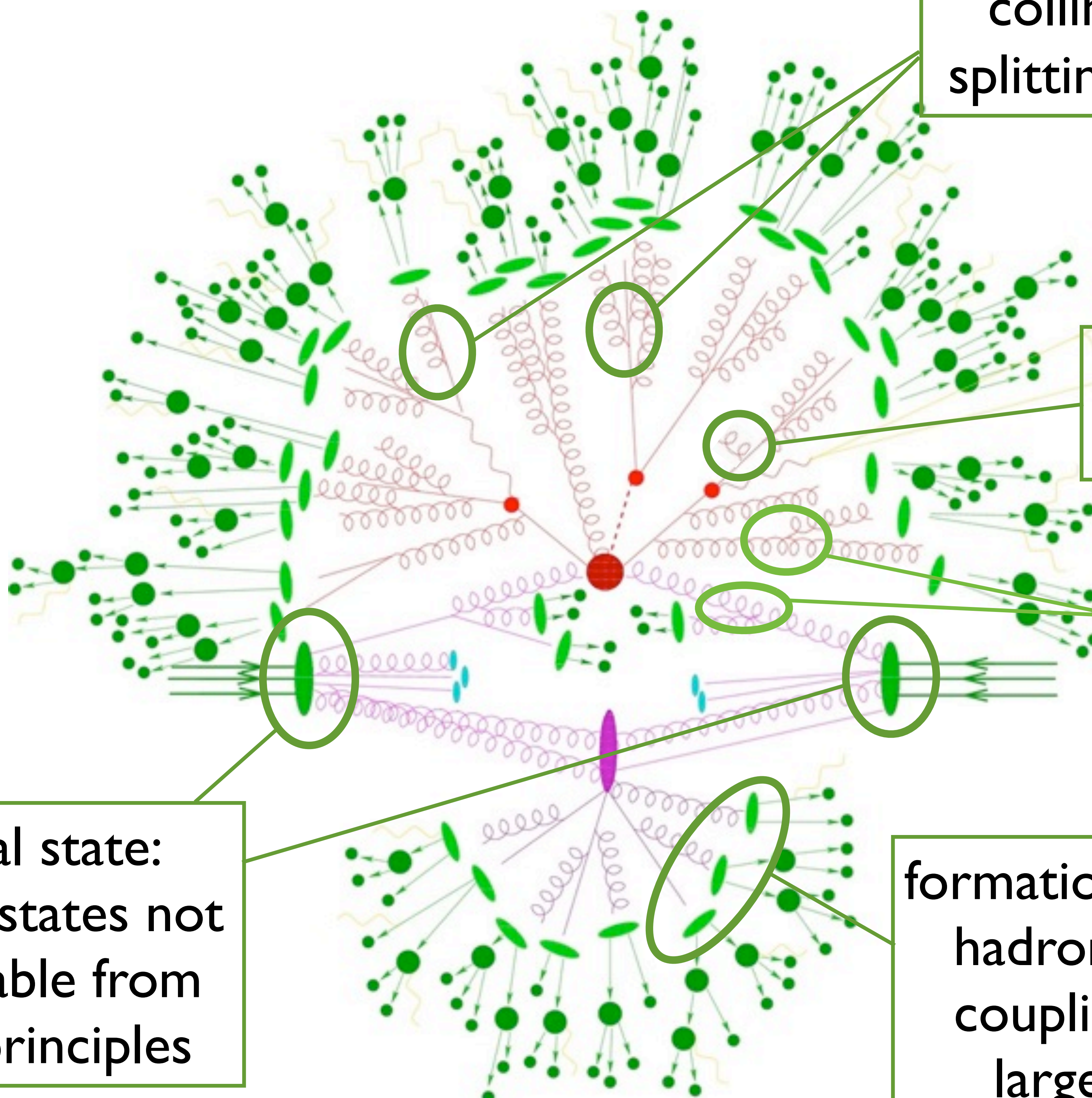
String Fragmentation
(Pythia and friends)



Cluster Fragmentation
(Herwig)

→ don't forget to model particle decays

Putting The Pieces Together



collinear splittings: ∞ ✓

use IRC safe observables

soft radiation: ∞ ✓

renormalisation

propagators and qg vertex: ∞ ✓

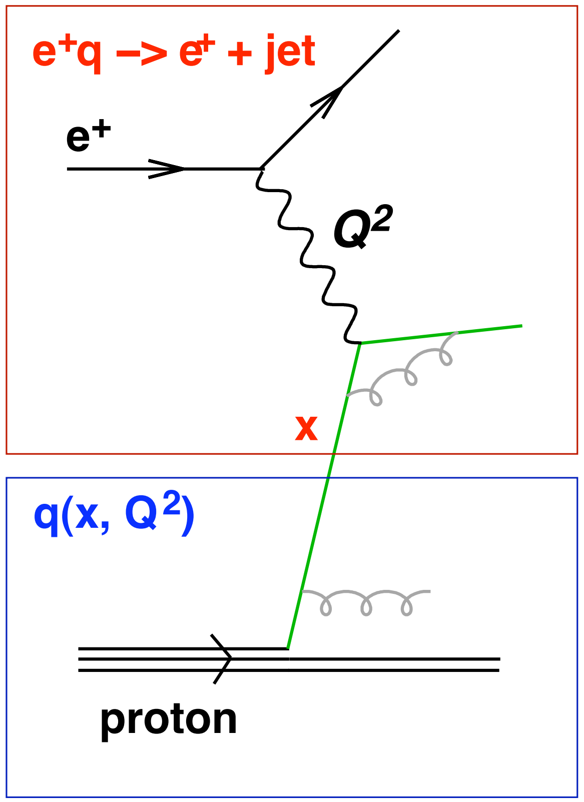
Parton Distribution Functions

initial state: bound states not calculable from first principles ✓

formation of hadrons: coupling large ✓

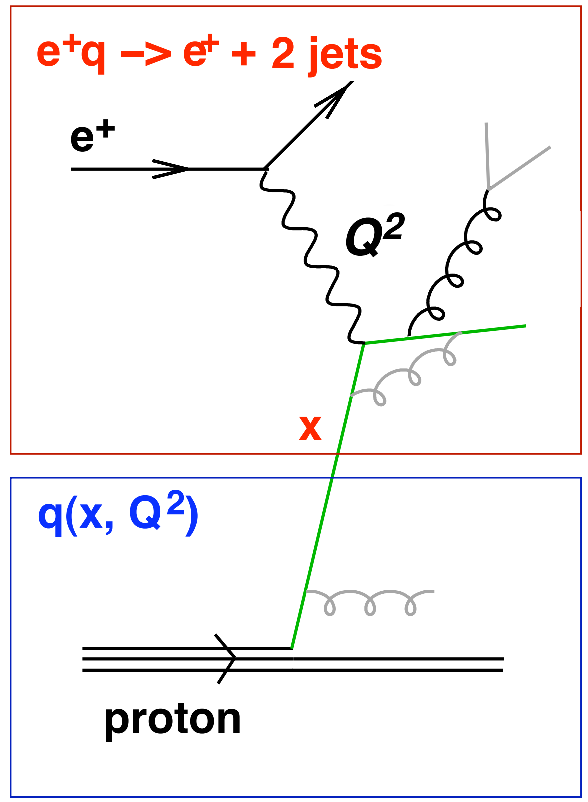
fragmentation models

Testing QCD: Jet Production



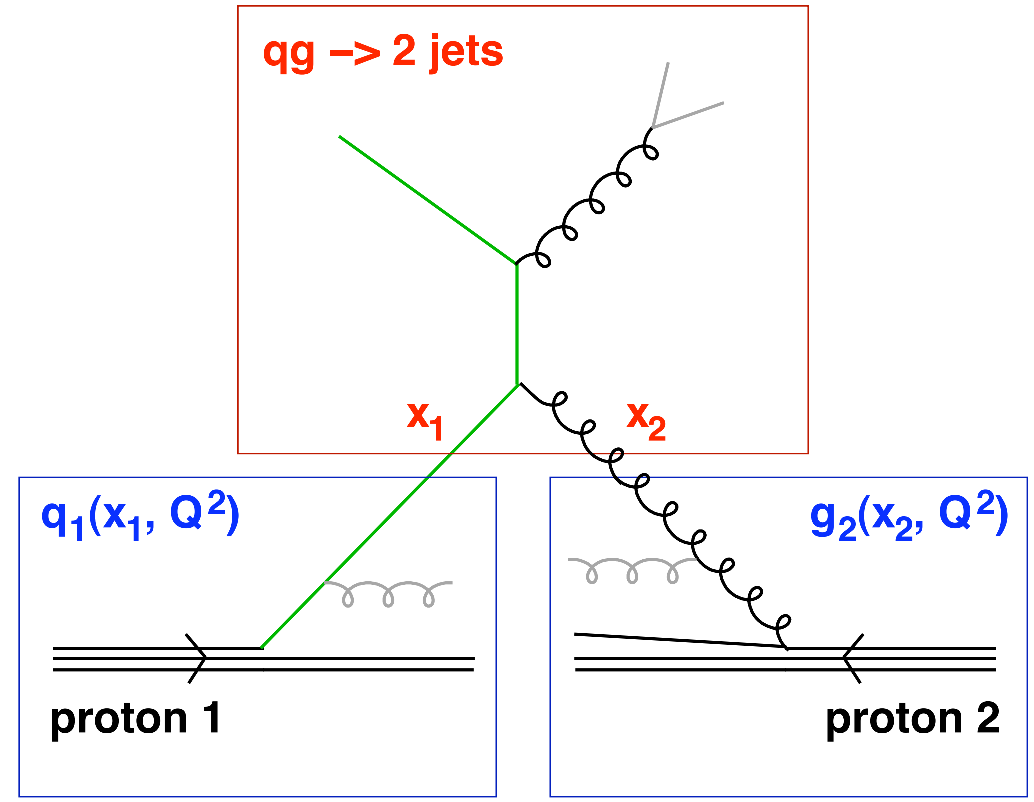
$$\sigma_{ep} = \sigma_{eq} \otimes q$$

Inclusive DIS, this we used for extracting PDFs



$$\sigma_{ep\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q$$

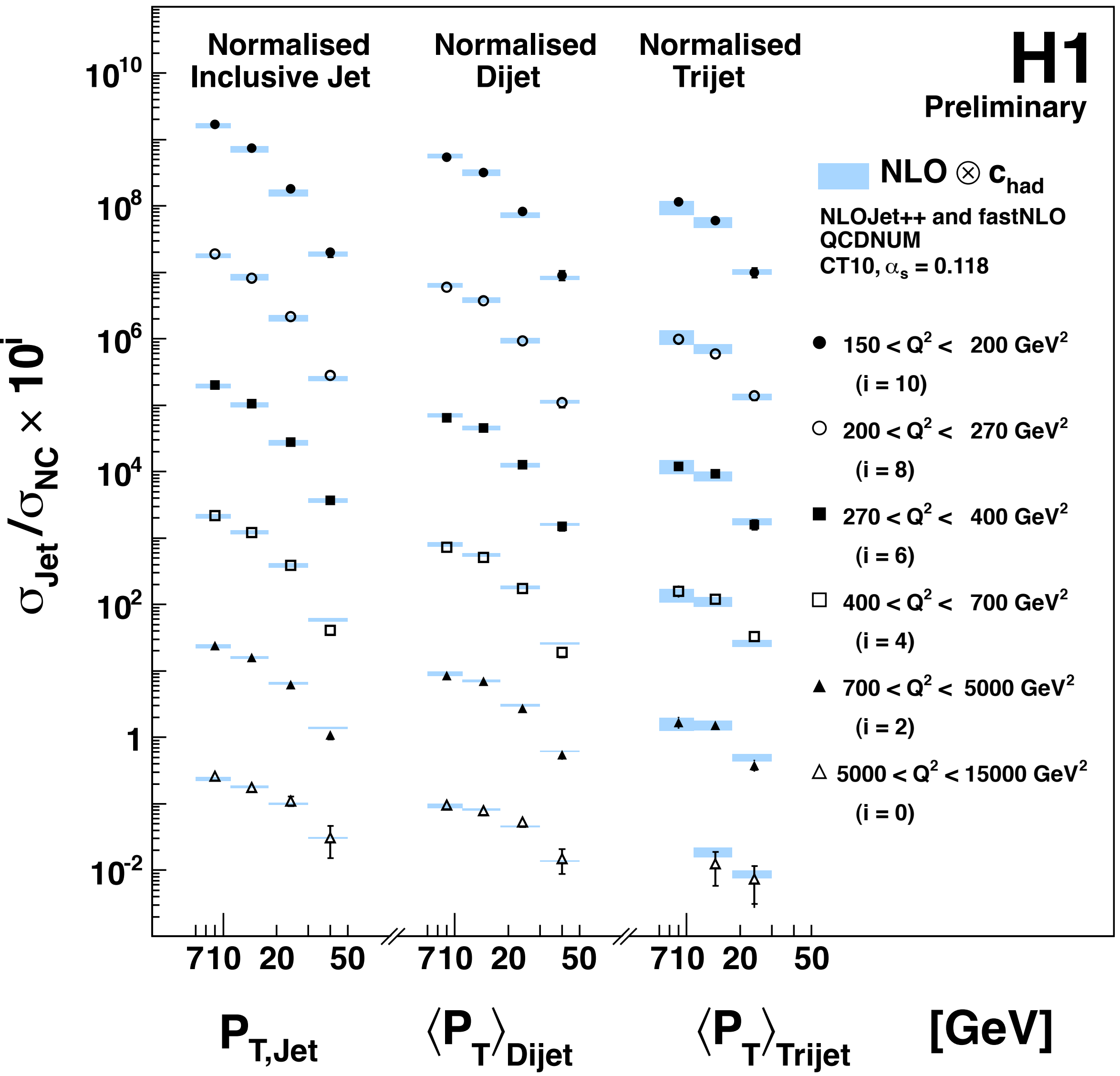
Test calculation of exclusive observables, PDFs in different processes, ...



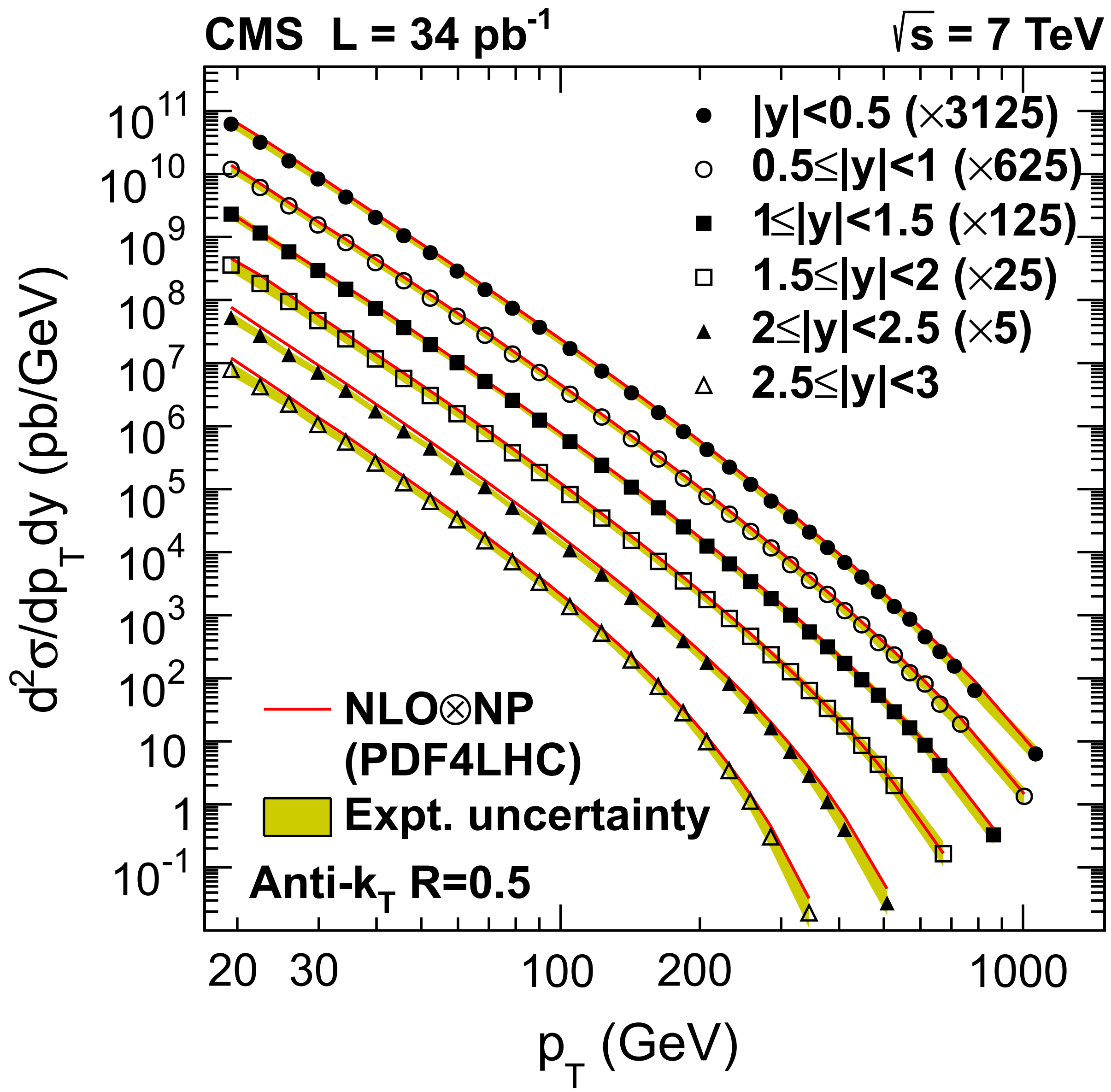
$$\sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Test universality of PDFs, how well do we understand QCD at the LHC energies?

Jet Production



Inclusive Jet, Dijet and Trijet Production in DIS at HERA



Inclusive Jet Production at the LHC

Very good agreement between NLO calculations and data - huge success!

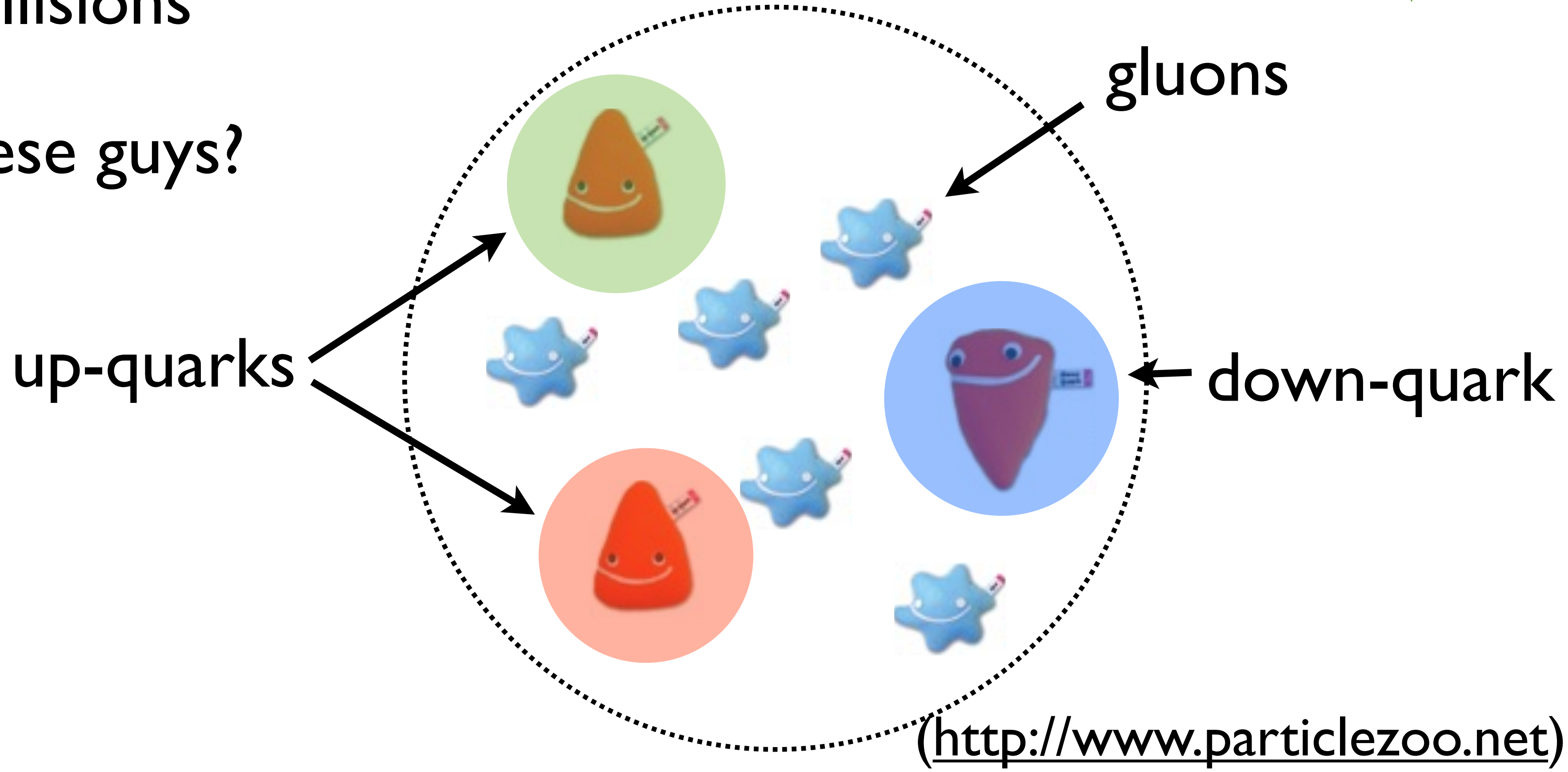
Summary

QCD

Beautiful field theory with local gauge invariance, but can it explain:

- quasi-free partons observed in DIS ⇒ asymptotic freedom ✓
- non-observation of free quarks and gluons ⇒ confinement ✓
- scaling violations in DIS ⇒ evolution equations ✓
- formation of jets and production of hadrons in particle collisions ⇒ success of perturbative QCD ✓

And finally: Who are these guys?



(http://www.particlezoo.net)