Simulating High Energy Collisions

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DESY Theory Group

DESY Summer Student Programme 2013

Outline and Disclaimer

We'll discuss:

- Basics of event generators and Monte Carlo methods.
- Hard scattering, parton showers, and in between.
- Hadronization and underlying event models.

What should not be expected:

- An unbiased view.
- A complete reference on the subject.
- Any details of available codes.

If there are any questions, don't hesitate to interrupt me.

Resources

Recommended references:

- Buckley et al.:
  General-purpose event generators for LHC physics,
  `arXiv:1101.2599 [hep-ph]`
- Ellis, Stirling, Webber:
  QCD and Collider Physics,
  Cambridge Monographs on Particle Physics
- Dissertori, Knowles, Schmelling:
  Quantum Chromodynamics: HEP Theory and Experiment,
  Oxford University Press

The Task.

Calculate cross sections for typical LHC events:

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$$
\begin{aligned}
&\mathrm{d}\sigma\Big(pp \to e^+e^-p\pi^-\pi^+\pi^+\pi^-\pi^+\pi^+K^+\bar{n}p\bar{n}\pi^-pn\bar{n}nK^+\\
&\pi^-\pi^+K^-\pi^-\pi^-\pi^-K^+\pi^-K^-\pi^+\pi^+K^+K^+\bar{p}\pi^-\pi^+\pi^+\bar{p}n\pi^+\pi^-\\
&\pi^+\pi^-K^+K^-\pi^+\pi^-\pi^-\pi^+\pi^-K^-\bar{n}K^+K^-K^-\pi^+\pi^-\pi^+\pi^-\\
&\pi^+\pi^-\pi^+n\bar{n}\pi^+n\pi^-\pi^+\pi^+\bar{n}\pi^-\pi^+\pi^-K^+\pi^+\pi^-\pi^+\bar{p}\bar{n}p\pi^-\pi^+\pi^+\pi^-\\
&nK^-K^+\pi^-\pi^+p\bar{p}\pi^-\pi^+\pi^-\bar{n}p\pi^-n\pi^+\pi^+\pi^-\bar{n}\pi^+\pi^-\pi^+\pi^-\pi^-\gamma\gamma\pi^+\\
&\pi^-\gamma\gamma\gamma\gamma\bar{n}\pi^-\gamma\gamma\pi^-\pi^+\gamma\gamma\gamma\gamma\pi^+\pi^-\gamma\gamma\gamma\gamma\pi^-\gamma\gamma\pi^-\pi^+\gamma\gamma\pi^+\pi^-\pi^-\\
&\pi^+\gamma\gamma\pi^-\pi^+\gamma\gamma\gamma\gamma\pi^-\gamma\gamma\gamma\gamma\gamma\gamma\pi^+\gamma\gamma n\gamma\gamma K^+\pi^-\pi^+K_Ln\pi^+\pi^+\\
&\pi^+\pi^-\gamma\gamma n\pi^-\pi^-\pi^+\gamma\gamma\gamma\bar{p}\pi^+\pi^+\pi^-\pi^+\pi^-\gamma\gamma\gamma\gamma\pi^-\gamma e^-e^+\pi^+\gamma\\
&\gamma\pi^+\pi^-\gamma\gamma K^-\pi^+\pi^+\bar{n}\pi^-\gamma\gamma\pi^+\gamma\gamma K^-\gamma\gamma\ldots\Big) = ???
\end{aligned}
$$

[Sequence of first few outgoing particles from a simulated LHC event.]

The Task, Simplified.

$$
\begin{aligned}
\mathrm{d}\sigma(pp \to ...) = & P(i \text{ in proton } 1)P(j \text{ in proton } 2) \\
& \times \mathrm{d}\sigma(\text{hard scattering of } i, j) \\
& \times P(\text{secondary scatterings}) \\
& \times P(\text{QCD radiation}) \\
& \times P(\text{partons} \to \text{hadrons}) \\
& \times P(\text{unstable hadrons} \to \text{stable hadrons})
\end{aligned}
$$

Still quite complex, but dealing with probabilities $\to$ Monte Carlo methods.

The Task, in Practice.

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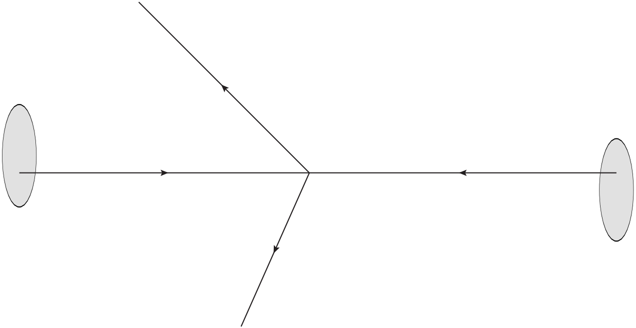
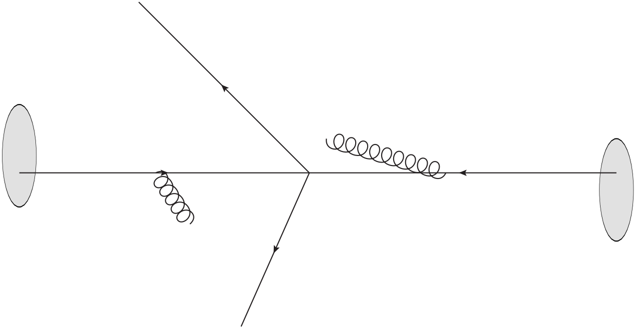


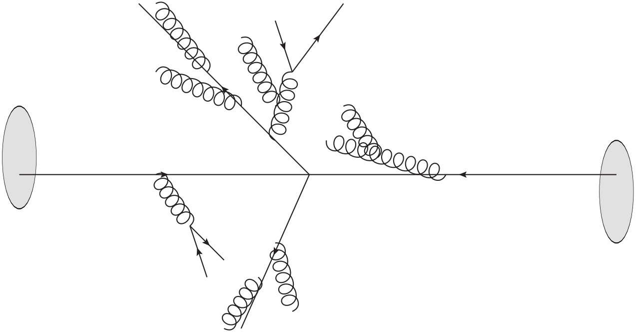
Hard partonic scattering.

The Task, in Practice.



Initial state parton shower.

The Task, in Practice.



Final state parton shower.

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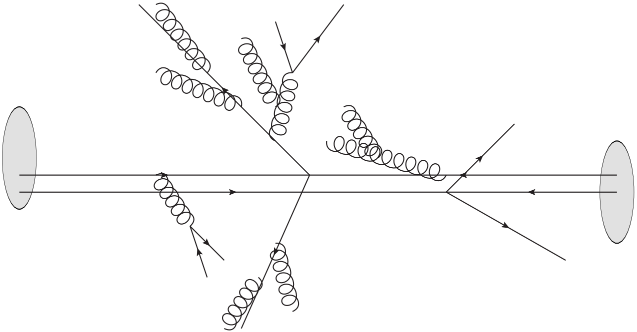

Multiple interactions.

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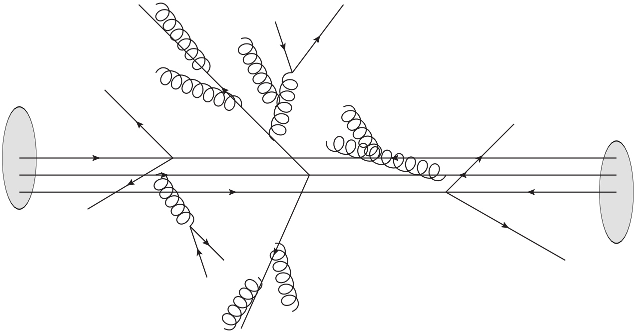

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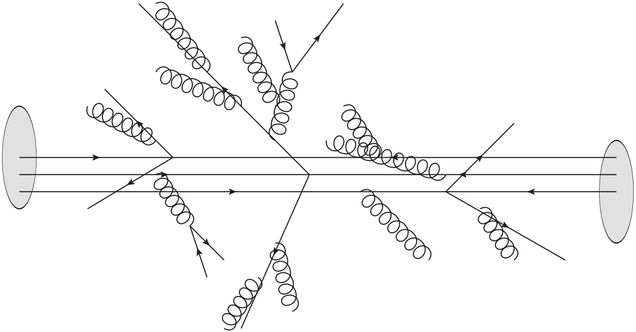
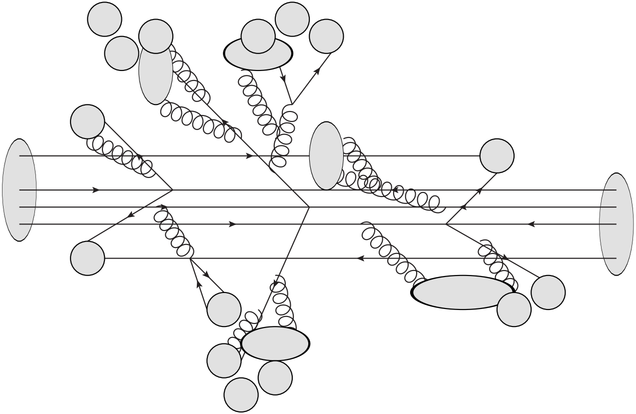


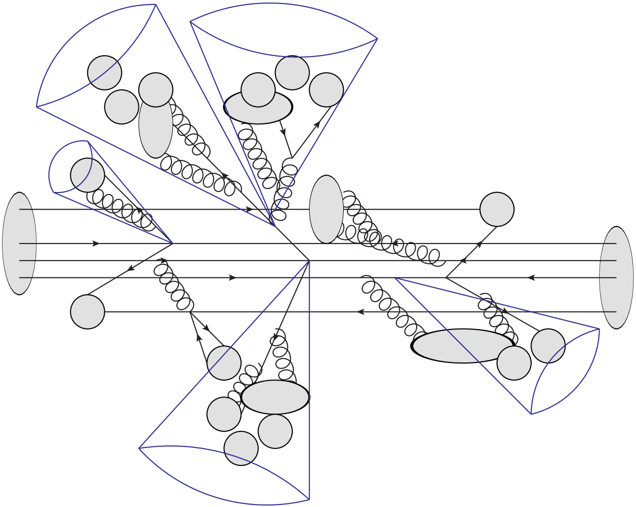
Parton showers off secondary scatterings.

The Task, in Practice.



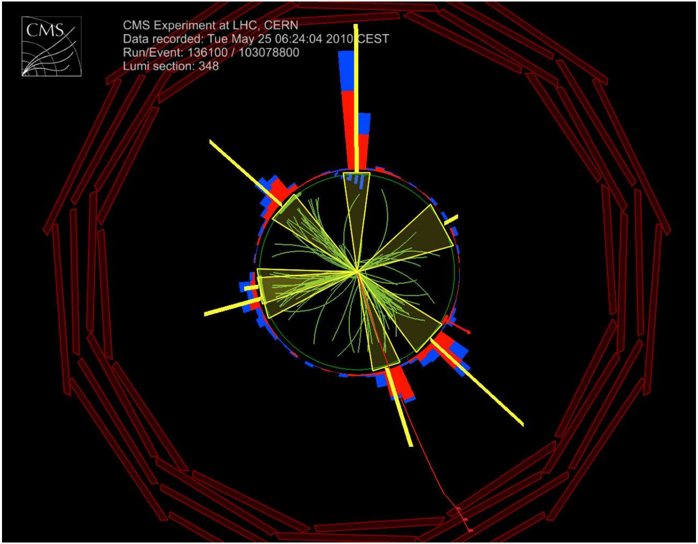
Hadronization.

The Task, in Practice.



Observed final state.

The Task, in Practice.



Plan.

- General structure of event generators. ✓
- Basic Monte Carlo methods.
- Partonic cross sections.
- Parton showers.
- Dipoles and soft gluons. ~
- Higher orders (and parton showers). ✗
- Multiple interactions.
- Hadronization models.
- Decays of unstable hadrons. ✗
- Some highlight results. ~

Basic Monte Carlo Methods.

– Monte Carlo methods: What and how.
– Sampling from a probability density.
– Monte Carlo integration.
– Importance sampling. ~
– From weights to events.
– The art of (pseudo-) random number generation. ✗
– MC methods outside HEP. ✗

The Task, Physics Wise.

Given some cross section differential in all momentum components ...

- Calculate the total cross section σ.
- With arbitrary acceptance criteria (‘cuts’).
- Produce a sample of events $(p_1, ..., p_n)$ with probability density

$$\frac{1}{\sigma} \mathrm{d}\sigma(p_1, ..., p_n) \ .$$

- Book histograms for arbitrary observables, and compare to data

The Task, Technically.

Given a function $p(x_1, ..., x_n)$,
and a volume $V = \{(x_1, ..., x_n) \in \mathbb{R}^n | v(x_1, ..., x_n) = 1\}$ ...

- Calculate
$$N(p, V) = \int_V p(x_1, ..., x_n) \mathrm{d}^n x \ .$$
- Produce a sample of events $(x_1, ..., x_n) \in V$ with probability density
$$\frac{1}{N(p, V)} p(x_1, ..., x_n) \mathrm{d}^n x \ .$$
- Book histograms for arbitrary functions $O(x_1, ..., x_n)$.

Why go to Monte Carlo at all?

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Well, we could just do numerical integrations,

$$N(p, V) = \int_V^{\text{Gauss},\ldots} p(x_1, \ldots, x_n)\mathrm{d}^n x \ .$$

Don't even need 'events', nor histograms, but calculate

$$\frac{\mathrm{d}p}{\mathrm{d}O} = \int_V p(x_1, \ldots, x_n)\delta(O - O(x_1, \ldots, x_n))\mathrm{d}^n x$$

the same way, and just plot into measured histograms.

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So what?

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MC methods are the only feasible way to achieve the task.

We'll see how ...

Getting into Touch with MC Methods.

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We'll start off with drawing random variates in a single variable.

May seem unrelated to the problems we want to solve, yet:

- Gives a first feeling for what is going on.
- Often needed as helper for more efficient algorithms.

How to do things with some probability?

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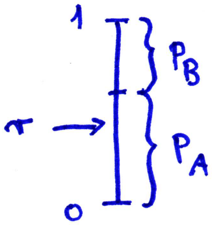
Choose between two outcomes A and B with probabilities $P_{A,B}$.
How to implement an algorithm selecting either one according to $P_{A,B}$?

Have **rnd**() to return equally distributed random numbers $r \in [0, 1]$.

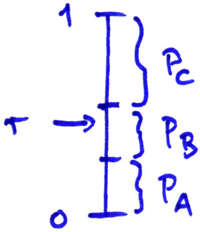
$r \leftarrow$ **rnd**()
if $r < P_A$ **then**
  return A
else
  return B
end if



How to do things with some probability?

Choose between three outcomes A, B and C with probabilities $P_{A,B,C}$.

$r \leftarrow$ **rnd**()
if $r < P_A$ **then**
  return A
else if $r < P_A + P_B$ **then**
  return B
else
  return C
end if



Etc.

Basic Monte Carlo Methods.

- Monte Carlo methods: What and how. ✓
- Sampling from a probability density.
- Monte Carlo integration.
- Importance sampling. ~
- From weights to events.
- The art of (pseudo-) random number generation. ✗
- MC methods outside HEP. ✗

Sampling by Inversion.

Suppose we got $p(x) \geq 0$ and $V = [a, b]$ to define a probability density

$$P(x)\mathrm{d}x = \theta(b - x)\theta(x - a)\frac{p(x)\mathrm{d}x}{\int_a^b p(z)\mathrm{d}z}$$

from which we are to draw random variates.

We'll assume that p is sufficiently simple such that

- we can calculate the integral of p, and
- we can solve

  $$\int_a^x p(z)\mathrm{d}z = r \int_a^b p(z)\mathrm{d}z$$

  for x as a function of $r \in [0, 1]$.

Sampling by Inversion.

The algorithm generating events according to $P(x)$ is simple:

$r \leftarrow$ **rnd**()
$x \leftarrow$ solution of
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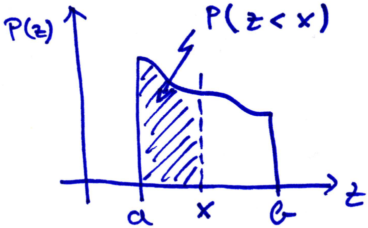

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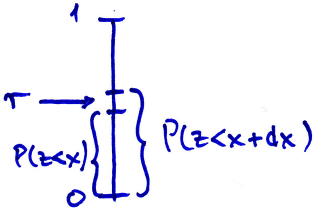

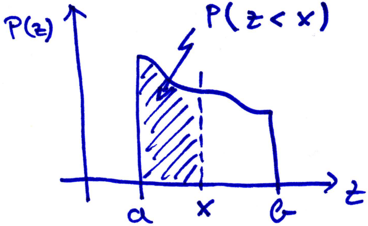

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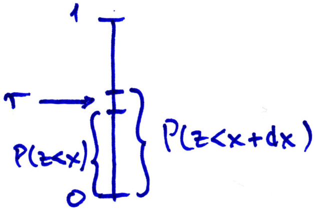

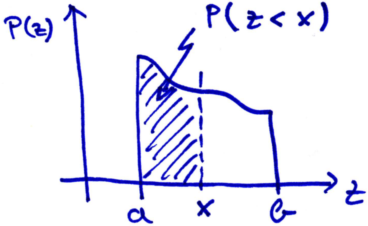

$$\mathrm{d}r \quad = \quad P(z)\mathrm{d}z$$

We only solved a change of variables.

Sampling by Inversion: Example.

Suppose we have $p(x) = x$ on $[0, 1]$. Then solve $\frac{x^2}{2} = r \ \frac{1}{2}$.

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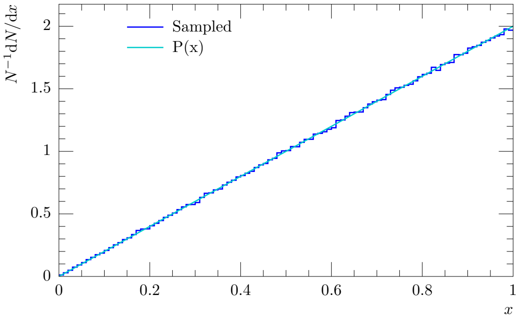

Dealing with Many Variables: Hit-and-Miss.

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Before trying many variables: What if we cannot invert the integral?
Suppose we know $c \geq p(x)$.

loop
  $r \leftarrow$ **rnd**()
  $x \leftarrow a + r(b - a)$
  $r' \leftarrow$ **rnd**()
  if $r' < p(x)/c$ **then**
    return x
  end if
end loop

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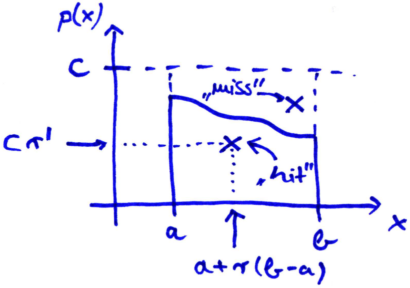

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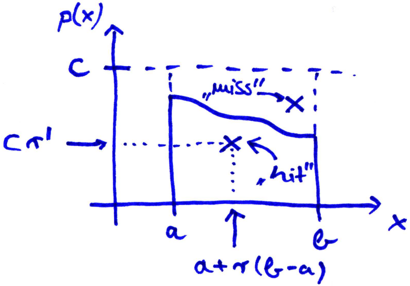
```
loop
  r ← rnd()
  x ← a + r(b − a)
  r′ ← rnd()
  if r′ < p(x)/c then
    return x
  end if
end loop
```



The frequency of hits in $[x, x + \mathrm{d}x]$ is directly proportional to $p(x)$.

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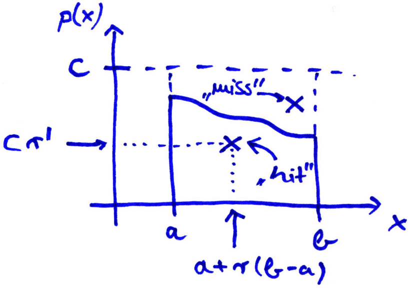

Note that we did not have to know the normalization!

Dealing with Many Variables: Hit-and-Miss.

Given a function $p(x_1, ..., x_n)$,
and a volume $V = \{(x_1, ..., x_n) \in \mathbb{R}^n | v(x_1, ..., x_n) = 1\}$ ...

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And a hypercube $I = [a_1, b_1] \times \cdots \times [a_n, b_n]$ with $V \subset I$.

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Define

$$p_V(x_1, ..., x_n) = \begin{cases} p(x_1, ..., x_n) & : \quad v(x_1, ..., x_n) = 1 \\ 0 & : \quad \text{otherwise} \end{cases} .$$

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```
loop
  for i = 1..n do
    r ← rnd()
    x_i ← a_i + r(b_i − a_i)
  end for
  r' ← rnd()
  if r' < p_V(x_1, ..., x_n)/c then
    return (x_1, ..., x_n)
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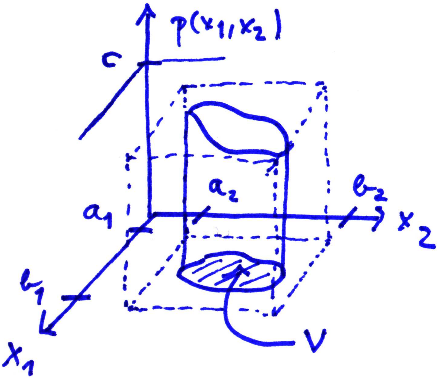

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- MC methods outside HEP. ✗

Remark.

Unless stated otherwise: Back to one variable.

Generalizations should be obvious now.

If not: Please ask!

From Hits to Weights and Integrals.

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Before trying many variables: What if we cannot invert the integral?
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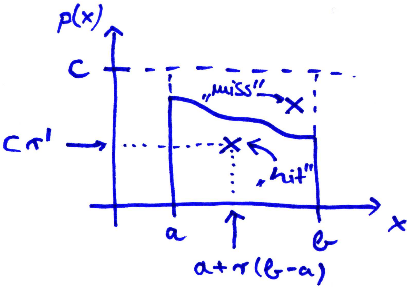
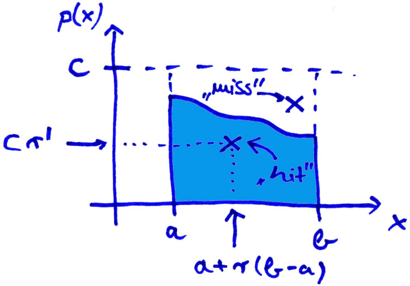


Note that we did not have to know the normalization!

From Hits to Weights and Integrals.

We actually *estimated* the normalization, if we were counting hits:

$$\int_a^b p(x)\mathrm{d}x \approx \frac{\#\text{hits}}{\#\text{hits} + \#\text{misses}} \times c(b-a) \; .$$



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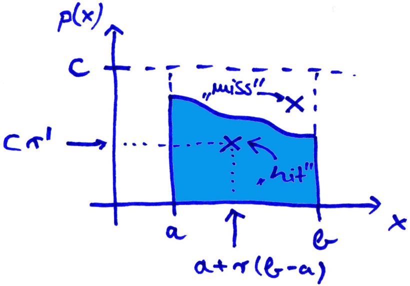

In other words: We have just (approximately) calculated an integral!

Weights, Averages, and Variances.

Let's put the estimate onto a more waterproof ground.

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Averaging p over $[a, b]$ is connected to its integral,

$$\langle p \rangle = \frac{1}{b-a} \int_a^b p(x) \, \mathrm{d}x \; .$$

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$$\langle p \rangle = \frac{1}{b-a} \int_a^b p(x) \, \mathrm{d}x \ .$$

Now estimate the average by

- recording p's value at random points x_i,
- for a total of N points:

$$\langle p \rangle_{\text{estimate}} = \frac{1}{N} \sum_{i=1}^{N} p(x_i) \ .$$

Weights, Averages, and Variances.

We'll call $w_i = p(x_i)$ the *weight* of an event x_i.
w_i is a measure of how many hits we should expect in $[x_i, x_i + \mathrm{d}x]$.

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The variance of $\langle p \rangle_{\text{estimate}}$ is

$$\sigma^2 \left[\langle p \rangle_{\text{estimate}} \right] = \frac{1}{N} \left(\frac{1}{N} \sum_{i=1}^{N} w_i^2 - \left(\frac{1}{N} \sum_{i=1}^{N} w_i \right)^2 \right) .$$

Monte Carlo Integrals.

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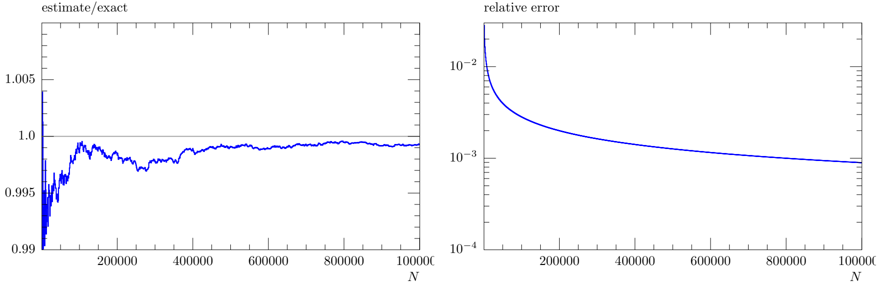
Monte Carlo Integrals.

By recording p's value at random points x_i, $i = 1, ..., N$ we can approximately calculate its integral:

$$\boxed{\int_a^b p(x)\ \mathrm{d}x = (b-a)\langle p\rangle_{\text{estimate}} \pm (b-a)\sigma\left[\langle p\rangle_{\text{estimate}}\right]}$$

Monte Carlo Integrals: Example.

Integrate $p(x) = x^2$ on $[0, 1]$.



Uncertainty drops as $1/\sqrt{N}$. Mind the independent measurements.

Monte Carlo Integrals: Example.

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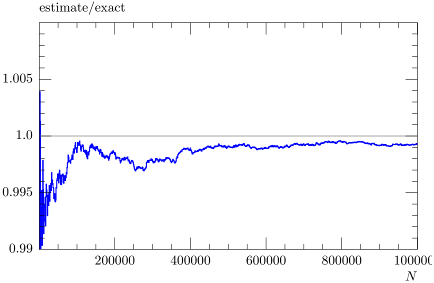
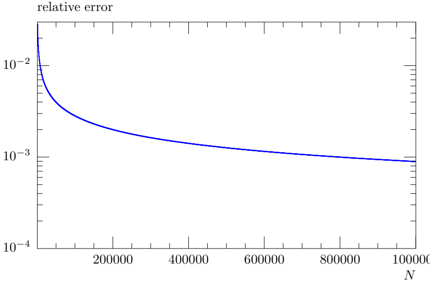
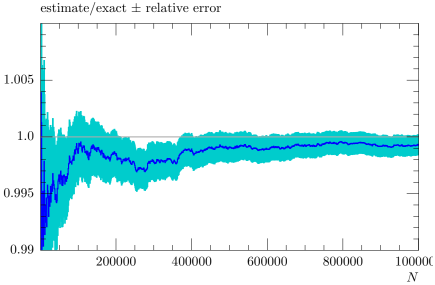




Doesn't really converge to the true value, right?

Monte Carlo Integrals: Example.

Integrate $p(x) = x^2$.



Even worse: Error band just scratches true value for large N.

Monte Carlo Integrals.

Mind the choice of your random number generator!

Monte Carlo Integrals.

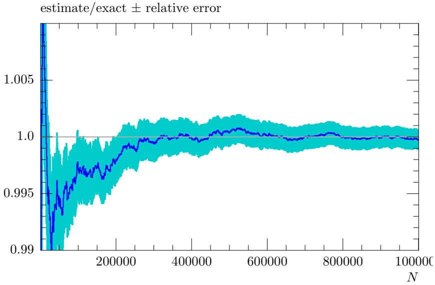
Mind the choice of your random number generator!

Never, ever use things like:

`rnd(), drand48() ...`

Monte Carlo Integrals: Example.

Integrate $p(x) = x^2$.



Same thing, better random number generator.

How do weights connect to hits and misses?

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Assign weights

- $w_i = c$ to any 'hit' x_i, and
- $w_j = 0$ to any 'miss' x_j.

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Then

$$\int_a^b p(x)\mathrm{d}x \approx \frac{\#\text{hits}}{\#\text{hits} + \#\text{misses}} \times c(b-a)$$

as conjectured.

But now we know how accurate this estimate is.

How do weights connect to hits and misses?

We still miss an explanation for $w_{\text{hit}} = c$.

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We have actually 'measured' $p(x)$ in units of c...

- by accepting $N \times p(x)/c$ hits in $[x, x + \mathrm{d}x]$, thus
- recording the value of $p(x)/c$ by the number of hits.

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For any hit we therefore need to multiply by the unit c we've chosen.

Just a scaling of variables:

$$\int_a^b p(x)\mathrm{d}x = c \int_a^b \frac{p(x)}{c}\mathrm{d}x$$

True *changes* of variables when trying to cheat in the casino ...

Basic Monte Carlo Methods.

- Monte Carlo methods: What and how. ✓
- Sampling from a probability density. ✓
- Monte Carlo integration. ✓
- Importance sampling. ~
- From weights to events.
- The art of (pseudo-) random number generation. ✗
- MC methods outside HEP. ✗

Uncertainties, continued.

Mind the integral's uncertainty,

$$\sigma^2 \left[\langle p \rangle_{\text{estimate}} \right] = \left\langle \sigma^2 \left[p \right] \right\rangle_{\text{estimate}} \ .$$

If p has large variance, need a very large N for a reasonable uncertainty.

Uncertainties, continued.

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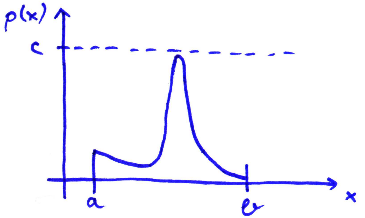
If p has large variance, need a very large N for a reasonable uncertainty.

Connected to this is an unacceptable efficiency of hit-and-miss,

$$\epsilon = \frac{\#\text{hits}}{\#\text{hits} + \#\text{misses}} \ll 1\ .$$



Loading the Dice: Variance Reduction.

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We've been honest gamblers, using equally distributed random numbers.

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Now we'll start to cheat.

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Now we'll start to cheat.

First set some notation,

$$\langle p \rangle \to \langle p \rangle_1 \ ,$$

where in general

$$\langle p \rangle_r = \int_a^b p(x) r(x) \mathrm{d}x \ .$$

Loading the Dice: Variance Reduction.

The basic ingredients to variance reduction:

- A constant function has zero variance.
- And we always have

$$\langle p \rangle_1 = \left\langle \frac{p}{r} \right\rangle_r \quad .$$

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- A constant function has zero variance.
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$$\langle p \rangle_1 = \left\langle \frac{p}{r} \right\rangle_r \ .$$

So, ideally

$$\langle p \rangle_1 = \langle 1 \rangle_p$$

with zero variance ???

Loading the Dice: Variance Reduction.

What does $\left\langle \frac{p}{r} \right\rangle_r$ actually mean?

Loading the Dice: Variance Reduction.

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– A change of variables,

$$p(x)\mathrm{d}x = p(x(R))\frac{\mathrm{d}x(R)}{\mathrm{d}R}\mathrm{d}R$$

with $r(x)\mathrm{d}x = \mathrm{d}R$.
Record p/r at points inside the transformed volume.

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- If $r(x)$ is normalized to define a probability density:
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– If $r(x)$ is normalized to define a probability density:
Record p/r at points distributed with density r.

To arrive at $\langle 1 \rangle_p$ we would actually have to know the integral.
Then the uncertainty is – of course – zero.

Loading the Dice: Variance Reduction.

The best we can hope for is finding a r, which is very similar to p,

$$\frac{p(x)}{r(x)} \approx \text{constant} \ .$$

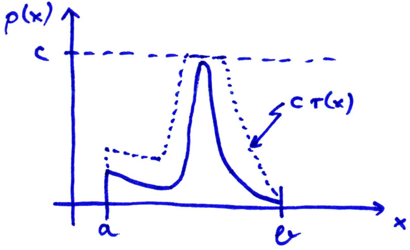
And sufficiently simple, such that we can distribute points with a probability density defined by r.

Loading the Dice: Variance Reduction.

This also helps with the hit-and-miss efficiency:

If we know c such that $c\ r(x) \geq p(x)$, we can

- Propose points with density defined by r, and
- accept a hit x with probability $\frac{p(x)}{c\ r(x)}$.



Loading the Dice: Variance Reduction.

Bottom line:

- Generate more points where p has large fluctuations.
- Generate less points where p is essentially constant.
- Divide out the bias introduced thereby.

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A bit of terminology:

What we got to know here is known as **'importance sampling'**.

Loading the Dice: Variance Reduction.

A bit of terminology:

There is also **'stratified sampling'**.

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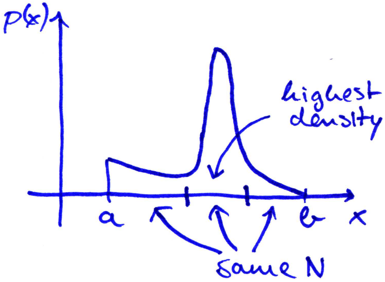
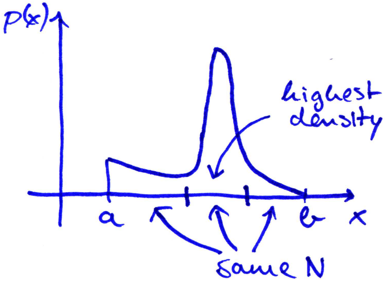


Loading the Dice: Variance Reduction.

A bit of terminology:

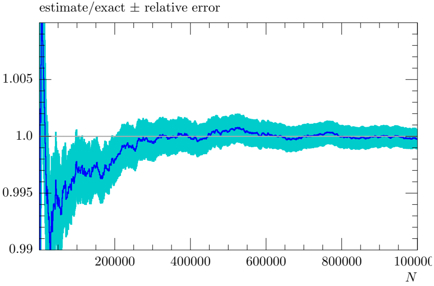
There is also **'stratified sampling'**.



This is just another way of implementing a $r(x)$ made up of step functions.

Variance Reduction: Example.

Integrate $p(x) = x^2$ on $[0, 1]$. Importance sampling with $r(x) = x$.



Basic Monte Carlo Methods.

- Monte Carlo methods: What and how. ✓
- Sampling from a probability density. ✓
- Monte Carlo integration. ✓
- Importance sampling. ✓
- From weights to events.
- The art of (pseudo-) random number generation. ✗
- MC methods outside HEP. ✗

From Integrals to Hits.

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From MC integration we obtain a sample of *weighted* events, (x_i, w_i).

For event generation, we are interested in *unweighted* events, (x_i, c).
Recap that w_i is a measure of the frequency of events in $[x_i, x_i + \mathrm{d}x]$.

From Integrals to Hits.

From MC integration we obtain a sample of *weighted* events, (x_i, w_i).

For event generation, we are interested in *unweighted* events, (x_i, c).
Recap that w_i is a measure of the frequency of events in $[x_i, x_i + \mathrm{d}x]$.

To get to unweighted events,

- find the maximum weight $w_{\max}$,
- keep each weighted event (x_i, w_i) with probability $w_i/w_{\max}$, and
- assign common weight $c = N(p, V)/N_{\rm uw}$ to $N_{\rm uw}$ accepted events.

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Basic Monte Carlo Methods: Things to Take Home.

- Pseudo-random numbers $\rightarrow$ events with given probability density.
- Can calculate integrals with complicated boundaries and functions.
- The only choice of method when it comes to many variables.
- Apply to differential cross sections:
  $\rightarrow$ **Simulate** collider events with frequency as occurring in nature.

Plan.

- General structure of event generators. ✓
- Basic Monte Carlo methods. ✓
- Partonic cross sections.
- Parton showers.
- Dipoles and soft gluons. ~
- Higher orders (and parton showers). ✗
- Multiple interactions.
- Hadronization models.
- Decays of unstable hadrons. ✗
- Some highlight results. ~

Partonic Cross Sections.

What did we start with?

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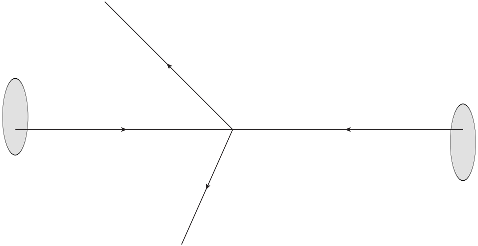
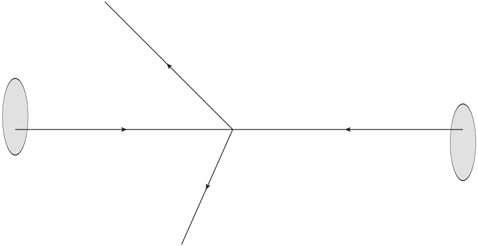


Scatter $q\bar{q} \to e^+e^-$, $gq \to gq$, ...

Partonic Cross Sections.

What did we start with?



What we observe is $pp \to e^+e^- + X$, $pp \to 2$ jets $+ X$, ...

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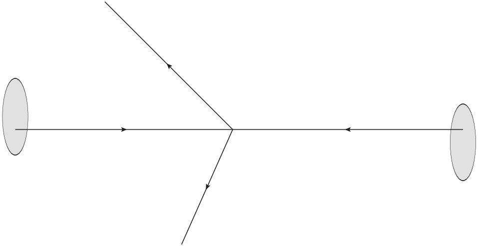

What we observe is $pp \to e^+e^- + X$, $pp \to 2$ jets$+X$, ...

Partonic Cross Sections.

The factorization theorem.

$$
\mathrm{d}\sigma\left(pp \to F_{\text{hadronic}} + X\right) = \sum_{i,j,k} \int \mathrm{d}x_i \mathrm{d}x_j f_{P\leftarrow i}(x_i, \mu_F^2) f_{P\leftarrow j}(x_j, \mu_F^2) \mathrm{d}\sigma\left(ij \to F_{\text{partonic},k}, \mu_F^2\right)
$$

Formula for an *inclusive* cross section.

Works, as long as:

- We do not ask about any detail of X, and
- F has the same definition in terms of hadrons or partons.

Partonic Cross Sections.

A fully *exclusive* cross section:

$$\mathrm{d}\sigma\left(pp \to e^+e^-p\pi^-\pi^+\pi^+\pi^-\pi^+\pi^+K^+\bar{n}p\bar{n}\pi^-pn\bar{n}nK^+\ldots\right)$$
$$\equiv \mathrm{d}\sigma\left(F_{\text{hadronic},l} + \bar{n}\pi^-\pi^+\pi^-K^+\ldots\right) \equiv \mathrm{d}\sigma\left(F_{\text{hadronic},l} + X_l\right)$$

Then:

$$\mathrm{d}\sigma\left(pp \to F_{\text{hadronic}} + X\right) = \sum_l \mathrm{d}\sigma\left(F_{\text{hadronic},l} + X_l\right) =$$
$$\sum_{i,j,k,l} \int \mathrm{d}x_i \mathrm{d}x_j f_{P\leftarrow i}(x_i, \mu_F^2) f_{P\leftarrow j}(x_j, \mu_F^2) \mathrm{d}\sigma\left(ij \to F_{\text{partonic},k}, \mu_F^2\right)$$
$$\otimes P(F_{\text{partonic},k} \to F_{\text{hadronic},l} + X_l)$$

Mind that this is only a heuristic argument. We can't (always) prove this from first principles, but up to now it worked remarkably well.

Partonic Cross Sections.

A well defined recipe:

- $\mathrm{d}\sigma\,(ij \to F_{\text{partonic},k})$ can be calculated in perturbation theory.
- The μ_F^2 dependence of the parton distributions f as well.
- The boundary condition for the f's can be fitted to data.

At leading order, all the ingredients are positive definite and well behaved.

Use Monte Carlo methods to generate events at *parton level*
$\to$ the first cornerstone of our simulation.

Partonic Cross Sections: To take home.

Inclusive quantities at *hadron level* can be calculated in perturbation theory at *parton level*, as long as:

- We ask for a final state definition F which equally applies to partons and hadrons.
- We do not ask about any details of stuff not entering the final state definition.

Plan.

- General structure of event generators. ✓
- Basic Monte Carlo methods. ✓
- Partonic cross sections. ✓
- Parton showers.
- Dipoles and soft gluons. ~
- Higher orders (and parton showers). ✗
- Multiple interactions.
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Parton showers.

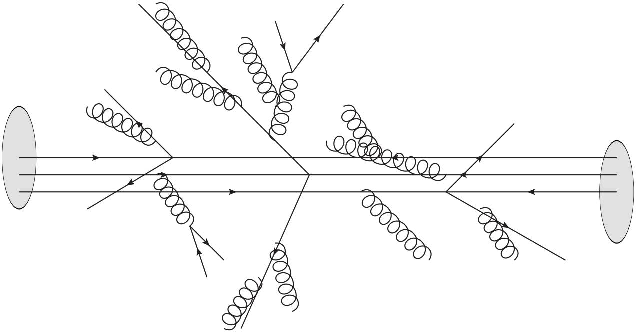
Troubles with the parton level:

- Only sufficiently inclusive observables.
- Not really a realistic final state.
- Quarks and gluons carry colour charge. Don't they radiate???

We don't observe a small number of hadrons comparable to the number of partons in a partonic cross section, but jets made out of hundreds of hadrons.

$\rightarrow$ Add radiation as a first step towards a more realistic final state.

Parton showers.

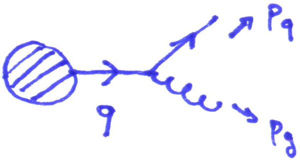


Parton showers.

- Shortcomings of the parton level. ✓
- Radiation, and the breakdown of naive perturbation theory.
- Factorization and parton showers.

Radiation, and the breakdown of naive perturbation theory.

Let's add one more parton from radiation.



Propagator factor (massless quarks for the time being):

$$\frac{1}{q^2} = \frac{1}{(p_q + p_g)^2} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

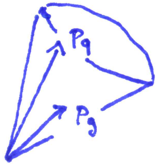
Diverges, whenever $E_g \to 0$ or $\theta_{qg} \to 0$.

Radiation, and the breakdown of naive perturbation theory.

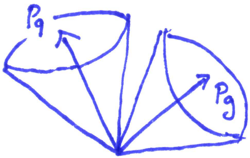
What to do about it?

Introduce a cutoff μ.

- If $p_\perp(q,g) < \mu$ we call it one jet.
- If $p_\perp(q,g) > \mu$, we call it two jets.



No divergence for $p_\perp > \mu$.
Divergence below will cancel with loop correction, when integrating over emissions with $p_\perp < \mu$ (see Jürgen Reuter's lecture).

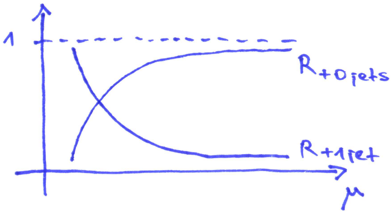


Radiation, and the breakdown of naive perturbation theory.

So, we're fine: Can calculate jet rates, which are finite.

$$R_{+0 \text{ jets}} = \frac{\sigma_{+0 \text{ jets}}}{\sigma_{\text{total}}}$$

$$R_{+1 \text{ jet}} = \frac{\sigma_{+1 \text{ jet}}}{\sigma_{\text{total}}}$$



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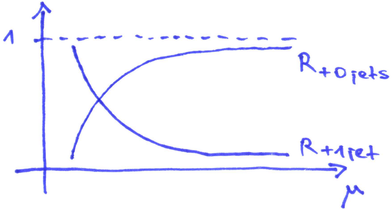


Is this making sense?

Radiation, and the breakdown of naive perturbation theory.

What happened?

Radiation, and the breakdown of naive perturbation theory.

What happened?

Large logarithms, overcoming the smallness of α_s

$$R_{+1\text{ jet}} \sim c_2\ \alpha_s \log^2 \frac{Q}{\mu} + c_1\ \alpha_s \log \frac{Q}{\mu}$$

These large contributions appear at *every* order in α_s.

We'll have to take into account all of them, or:
take into account any number of additional emissions.

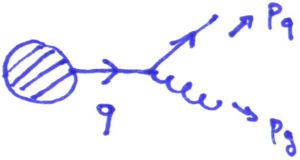
Fixed order is only valid, if jets are well separated.

Parton showers.

- Shortcomings of the parton level. ✓
- Radiation, and the breakdown of naive perturbation theory. ✓
- Factorization and parton showers.

Factorization and parton showers.

Let's add one more parton from radiation.



Whenever q^2 is much smaller than any other scale in the process, the cross section factorizes:

$$\mathrm{d}\sigma_{+1 \text{ gluon}} \approx \mathrm{d}\sigma_{+0 \text{ gluons}} \times \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2} P_{q\to qg}(z)\mathrm{d}z$$

where

$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$

and z is the energy fraction of the quark with respect to its parent.

Factorization and parton showers.

Similar results for $g \to gg$ and $g \to q\bar{q}$:

$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$

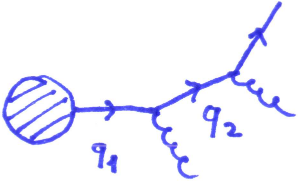
$$P_{g\to gg}(z) = 2C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$

$$P_{g\to q\bar{q}} = T_R \left(1 - 2z(1-z)\right)$$

These are the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting functions.

Factorization and parton showers.

Let's add two more partons from radiation.



Whenever $q_2^2 \ll q_1^2$ and q_1^2 is much smaller than any other scale in the process, the cross section factorizes again,

$$\mathrm{d}\sigma_{+2\ \text{gluon}} \approx \mathrm{d}\sigma_{+0\ \text{gluons}} \times \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q_1^2}{q_1^2} P_{q\to qg}(z_1)\mathrm{d}z_1 \times \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q_2^2}{q_2^2} P_{q\to qg}(z_2)\mathrm{d}z_2$$

(We have to include the second gluon emitted off the first one as well).

Factorization and parton showers.

Bottom line:

For **soft-collinear** emissions which are **strongly ordered** in a measure of hardness (i.e. from larger to smaller q^2), we can iteratively build up a parton cascade by **independent emissions**.

The cascade will start at a q^2 typical to the hard scattering.

It will terminate if we reach q^2 for which perturbation theory ceases to make sense, i.e. $q \sim 1$ GeV.

We say that partons 'evolve' from larger to smaller scales.

Factorization and parton showers.

We have transitions (=emissions), which are independent of each other and ordered in some measure, and occur with some probability dependent on this measure.

Factorization and parton showers.

We have transitions (=emissions), which are independent of each other and ordered in some measure, and occur with some probability dependent on this measure.

This is a stochastic process, more precisely, a Markovian process.

Detour: Markovian processes.

Suppose we have a set of states $\{i, j, k, ...\}$ and a time variable t.
We change state instantaneously at times $t_i, t_j, t_k, ...$
For a Markovian process, the transition only depends on the state before:

$$\text{Prob}(i, t_i | j, t_j | k, t_k | ...) = \text{Prob}(i, t_i | j, t_j) \equiv \Delta(i, t_i | j, t_j)$$

Detour: Markovian processes.

Transition probabilities determined by instantaneous transition rates:

$$\Delta(j, t + \delta t | i, t) = \delta_{ij} + \mathcal{P}(j|i,t)\delta t + \mathcal{O}(\delta t^2)$$

Unitarity ($\rightarrow$ shower does not change total inclusive cross sections)

$$\sum_j \Delta(j, t' | i, t) = 1 \qquad \sum_j \mathcal{P}(j|i,t) = 0$$

'$+$' regularization of transition rates:

$$\mathcal{P}(j|i,t) = [P(j|i,t)]_+ \equiv P(j|i,t) - \delta_{ij} \sum_k P(k|i,t)$$

Detour: Markovian processes.

Suppose we have a set of states $\{i, j, k, ...\}$ and a time variable t.

- Given: transition rates $P(j|i,t)$ to make a transition from state i to state j at time t.
- Want: probability to make a transition to state j at time t', given state i has been occupied at time t: $\Delta(j,t'|i,t)$

One can derive a differential equation for $\Delta(j,t'|i,t)$:

$$\frac{\partial}{\partial t}\Delta(j,t'|i,t) = \sum_k \Delta(j,t'|k,t)P(k|i,t) - \Delta(j,t'|i,t)\sum_k P(k|i,t)$$

Detour: Markovian processes.

$$\frac{\partial}{\partial t}\Delta(j,t'|i,t) = \sum_k \Delta(j,t'|k,t)P(k|i,t) - \Delta(j,t'|i,t)\sum_k P(k|i,t)$$

Let us derive the probability of keeping a state, provided we can visit each state only once:

$$\frac{\partial}{\partial t}\Delta(i,t'|i,t) = -\Delta(i,t'|i,t)\sum_k P(k|i,t)$$

$$\Rightarrow \Delta(i,t'|i,t) = \exp\left(-\int_{t'}^{t}\sum_k P(k|i,\tau)\mathrm{d}\tau\right)$$

Detour: Markovian processes.

How to implement in a Monte Carlo simulation?

Detour: Markovian processes.

How to implement in a Monte Carlo simulation?

Want the probability density of the time variable for a transition at time q, given no transition between Q and q.
Terminate the evolution at $\mu \leftrightarrow$ jet resolution: no radiation below μ.

Draw events from a probability density

$$\frac{\mathrm{d}S_p(\mu, q|Q)}{\mathrm{d}q} = \Delta_p(\mu|Q)\delta(q-\mu) + p(q)\Delta_p(q|Q)\theta(Q-q)\theta(q-\mu)$$

where

$$\Delta_p(q|Q) = \exp\left(-\int_q^Q P(t)\mathrm{d}t\right) \; .$$

Detour: Markovian processes.

How to achieve this?

Detour: Markovian processes.

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First note, that we're truly facing a probability density,

$$\int_{\mu}^{Q} \frac{\mathrm{d}S_p(\mu, q|Q)}{\mathrm{d}q} \mathrm{d}q = 1 \ .$$

Detour: Markovian processes.

How to achieve this?

First note, that we're truly facing a probability density,

$$\int_{\mu}^{Q} \frac{\mathrm{d}S_p(\mu, q|Q)}{\mathrm{d}q}\mathrm{d}q = 1 \ .$$

We just use sampling by inversion, solving for q in

$$\int_{\mu}^{q} \frac{\mathrm{d}S_p(\mu, t|Q)}{\mathrm{d}t}\mathrm{d}t = \Delta_p(q|Q)\theta(q-\mu) = \mathbf{rnd}() \ .$$

Detour: Markovian processes.

There's a caveat:

$$\Delta_p(q|Q)\theta(q-\mu) = \mathbf{rnd}()$$

has no solution if **rnd**() returned a value smaller than $\Delta_p(\mu|Q)$.

Detour: Markovian processes.

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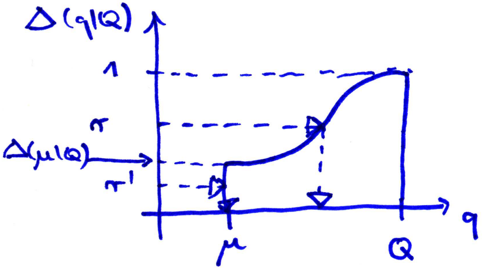
has no solution if **rnd**() returned a value smaller than $\Delta_p(\mu|Q)$.

This is precisely giving us the contribution multiplying the δ-function.



Detour: Markovian processes.

What if we can't solve $\Delta_p(q|Q) = \mathbf{rnd}()$ for q?

Detour: Markovian processes.

What if we can't solve $\Delta_p(q|Q) = \mathbf{rnd}()$ for q?

There's something like a hit-and-miss algorithm, known as the 'Sudakov veto algorithm'.

Factorization and parton showers.

Ingredients to the Monte Carlo simulation:

– The probability for parton i to evolve from Q^2 to q^2 without emission:

$$\Delta_i(q^2|Q^2) = \exp\left(-\int_{q^2}^{Q^2} \frac{\alpha_s(q^2)}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int_{z_-(q^2,Q^2)}^{z_+(q^2,Q^2)} \mathrm{d}z \sum_{j,k} P_{i\to jk}(z)\right)$$

This is called the 'Sudakov form factor'.

– The q^2, z-density for emission $i \to j, k$ when starting from Q^2:

$$\frac{\alpha_s(q^2)}{2\pi}\frac{\mathrm{d}q^2}{q^2}\mathrm{d}z P_{i\to jk}(z)\Delta_i(q^2|Q^2)$$

Note that there is a large family of possible ordering measures q^2.

Factorization and parton showers.

What does this have to do with the large logarithms?

Factorization and parton showers.

What does this have to do with the large logarithms?

Choose $q^2 = p_\perp^2$ and calculate the two- and three-jet rates for $e^+e^- \to q\bar{q}$:

$$R_2 = \Delta_q^2(\mu^2|Q^2) \qquad R_3 = 2\Delta_q^2(\mu^2|Q^2)\int_{\mu^2}^{Q^2} \Gamma_q(q^2|Q^2)\Delta_g(\mu^2|q^2)\frac{\mathrm{d}q^2}{q^2}$$

Here,

$$\Gamma_q(q^2|Q^2) = \frac{\alpha_s}{2\pi}C_F\left(\ln\frac{Q^2}{q^2} - \frac{3}{2}\right) \approx \frac{\alpha_s}{2\pi}\int_{z_-(q^2,Q^2)}^{z_+(q^2,Q^2)} P_{q\to qg}(z)\mathrm{d}z$$

Factorization and parton showers.

Let's check the first order in α_s:

$$R_2 = 1 - 2\int_{\mu^2}^{Q^2} \Gamma_q(q^2|Q^2)\frac{\mathrm{d}q^2}{q^2} + \mathcal{O}(\alpha_s^2)$$

$$R_3 = 2\int_{\mu^2}^{Q^2} \Gamma_q(q^2|Q^2)\frac{\mathrm{d}q^2}{q^2} + \mathcal{O}(\alpha_s^2)$$

and

$$\int_{\mu^2}^{Q^2} \Gamma_q(q^2|Q^2)\frac{\mathrm{d}q^2}{q^2} = \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{Q^2}{\mu^2} - \frac{3}{2}\ln\frac{Q^2}{\mu^2}\right)$$

Exactly what we expected.

Factorization and parton showers.

But how did it cure the problem?

Factorization and parton showers.

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$$R_2 = \Delta_q^2(\mu^2, Q^2) \qquad R_3 = 2\Delta_q^2(\mu^2, Q^2) \int_{\mu^2}^{Q^2} \Gamma_q(q^2|Q^2)\Delta_g(\mu^2|q^2)\frac{\mathrm{d}q^2}{q^2}$$

All powers of $\alpha_s^n \ln^{2n}$ and $\alpha_s^n \ln^{2n-1}$

Sudakov form factors suppress the logarithmic behaviour in Γ,
$\Delta(\mu^2|Q^2) \to 0$ as $\mu^2 \to 0$.

Factorization and parton showers.

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Sudakov form factors suppress the logarithmic behaviour in Γ,
$\Delta(\mu^2|Q^2) \to 0$ as $\mu^2 \to 0$.

But now $R_2 + R_3 \neq 1$?
We took into account *any number of emissions*
$\to$ for small enough μ^2 we start seeing 4,5,6,... jets.

Factorization and parton showers.

So far dealt with final state radiation. What about initial state radiation?

Going back to the factorization property, we actually now have:

$$f_{P\leftarrow q}(x,q^2)\mathrm{d}\sigma_{+1\text{ gluon}}(x) \approx$$

$$f_{P\leftarrow q}\left(\frac{x}{z},q^2\right)\mathrm{d}\sigma_{+0\text{ gluons}}(x)\times\frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}P_{q\to qg}(z)\frac{\mathrm{d}z}{z} =$$

$$f_{P\leftarrow q}(x,q^2)\mathrm{d}\sigma_{+0\text{ gluons}}(x)\times\frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}P_{q\to qg}(z)\mathrm{d}z\times\frac{f_{P\leftarrow q}\left(\frac{x}{z},q^2\right)}{z\ f_{P\leftarrow q}(x,q^2)}$$

This generates the DGLAP evolution of parton densities
(see Jürgen Reuter's lecture).

Parton showers.

- Shortcomings of the parton level. ✓
- Radiation, and the breakdown of naive perturbation theory. ✓
- Factorization and parton showers. ✓

Parton showers: Things to take home.

- Simulate the bulk of radiation, i.e. soft and collinear.
- Approximation only reliable in this region of phase space.
- Takes care of large logarithms in all orders.
- Cascade of subsequent, independent emissions
  $\to$ Markovian process.
- Infrared cutoff is free parameter $\to$ fit from data.

Plan.

- General structure of event generators. ✓
- Basic Monte Carlo methods. ✓
- Partonic cross sections. ✓
- Parton showers. ✓
- Dipoles and soft gluons. ~
- Higher orders (and parton showers). ✗
- Multiple interactions.
- Hadronization models.
- Decays of unstable hadrons. ✗
- Some highlight results. ~

Dipoles and soft gluons.

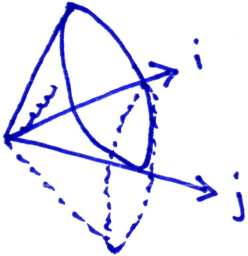
Cross sections factor in the soft-collinear limit.

This is what led us to parton showering.

In the soft limit, the amplitude factorizes.

→ Dipoles of partons of opposite colour charge radiate.

Important interference effect:

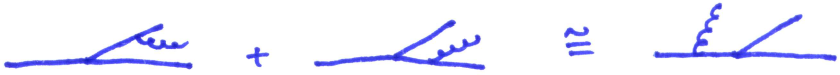


Dipoles and soft gluons.

Two approaches to get soft gluon emissions right:

- Improve dipole functions to include the collinear behaviour
  $\rightarrow$ build up a dipole cascade.
- Improve soft-collinear parton shower to account for the interference.

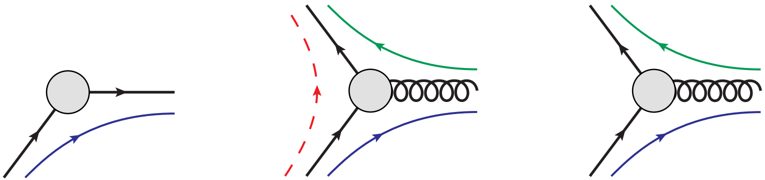
Coherent emission and angular ordering:



Dipoles and soft gluons.

Dipole cascades:

Instead of $1 \to 2$ splittings have $2 \to 3$ splittings:
$1 \to 2$ dipole splittings.



Use approximation where the number of colour charges is considered large.

Plan.

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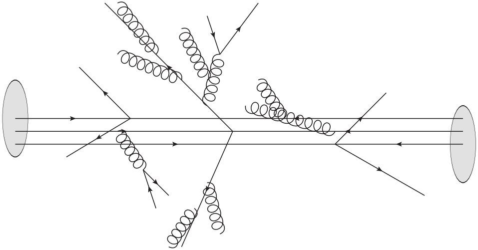
Multiple interactions.



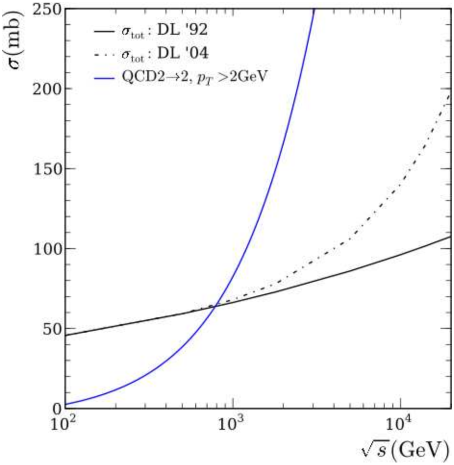
Appearance is intuitively clear $\rightarrow$ proton is an extended object.

Multiple interactions.

Intuitively clear, but what are the details of the physics behind?

Multiple interactions.

Intuitively clear, but what are the details of the physics behind?
Cross section for $pp \to 2$ jets, with a cut on the $p_\perp$ of the jets.



Multiple interactions.

Eventually,

$$\sigma_{pp\to 2\text{ jets}}(p_\perp > p_{\perp,\text{min}}) > \sigma_{pp,\text{total}}$$

for small $p_\perp$-cutoff.

What is going on here?

Multiple interactions.

Eventually,

$$\sigma_{pp\to 2\text{ jets}}(p_\perp > p_{\perp,\text{min}}) > \sigma_{pp,\text{total}}$$

for small $p_\perp$-cutoff.

What is going on here?

On average,

$$\left\langle \frac{\sigma_{pp\to 2\text{ jets}}}{\sigma_{pp,\text{total}}} \right\rangle \geq 1$$

$\to$ more than one hard scattering in a single collision!

Mind that $\sigma_{pp\to 2\text{ jets}}$ is an *inclusive* cross section, counting all scatterings. Ask for *any* two jets $\to$ gets large just by combinatorics.

Multiple interactions.

Assume independent scatters: Poisson distribution for the number of additional scatters m

$$P_m(b,s) = \frac{\langle n(b,s)\rangle^m}{m!} e^{-\langle n(b,s)\rangle}$$

Mean depends on CM energy and impact parameter
$\rightarrow$ modelled by parton luminosity:

$$\langle n(b,s)\rangle = \sum_{i,j,k,l} \int \mathcal{L}(x_i, x_j, b)\sigma_{ij\to kl}(x_i, x_j, s)$$

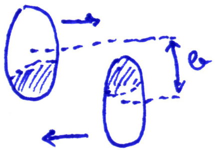
Multiple interactions.

Inelastic cross section:

$$\sigma_{\text{inelastic}} = \sum_{m=1}^{\infty} \int \mathrm{d}^2 b\; P_m(b,s) = \int \mathrm{d}^2 b \left(1 - e^{-\langle n(b,s)\rangle}\right)$$

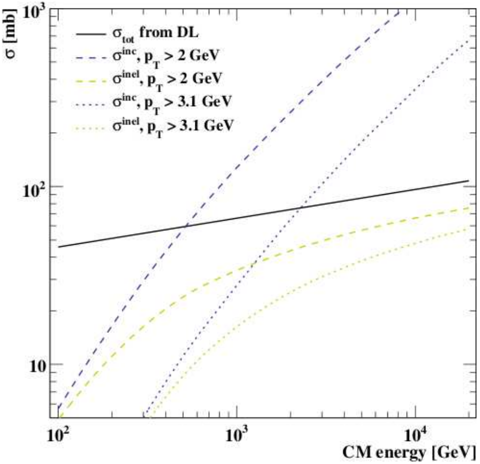
Simplest models of $\langle n(b,s)\rangle$ by geometrical arguments:

$$\langle n(b,s)\rangle = A(b)\sigma_{pp\to 2\text{ jets}}(s, p_{\perp,\min})$$



Multiple interactions.

Unitarized cross sections:



Multiple interactions: To take home.

- On average, there is no single hard scattering in a single *pp* collision.
- Independent scatterings and unitarized inelastic cross sections.
- $p_{\perp,\mathrm{min}}$ and the modelling of $\langle n \rangle$ distinguish different models.
- $p_{\perp,\mathrm{min}}$ and other parameters from fit to data
- Below $p_{\perp,\mathrm{min}}$ we have no clue $\rightarrow$ pure modelling.

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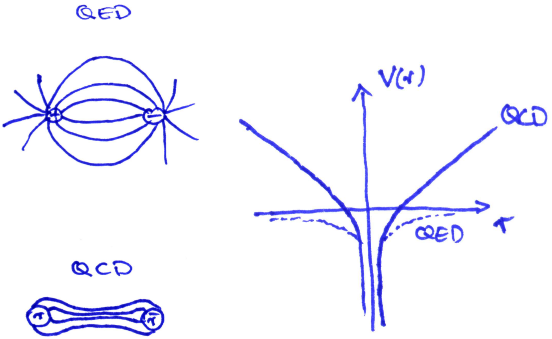
Hadronization models.

Confinement:

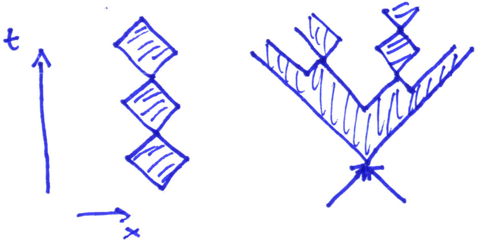
- QCD strongly coupled at small energy scales.
- Colour charges are bound by flux tubes.



Hadronization models.

String models.

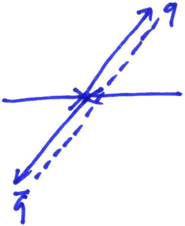
- Meson = $q\bar{q}$ stretching oscillating string
- Colour dipole in hard scattering acts as point like source of strings.
- String breaks up by creation of $q\bar{q}$ if field gets too intense.

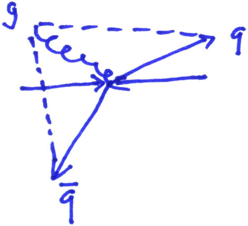


Hadronization models.

String models and gluons.

Gluons produce kinks on strings.

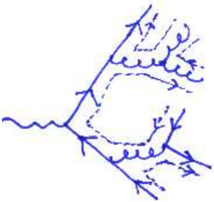


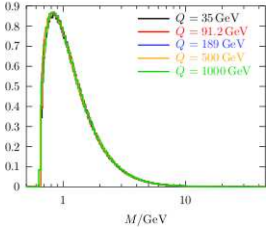


Hadronization models.

Cluster models:

End of a parton shower: colour singlet ‘$q\bar{q}$’ pairs close in phase space
$\rightarrow$ colour neutral clusters with masses peaked at low values.





Spectrum is universal
$\rightarrow$ hadronization does not care about the hard process.

Hadronization models.

Cluster models.

- Split gluons into $q\bar{q}$ pairs.
- Find clusters.
- Decay clusters with mass above some threshold.
- Pop $q\bar{q}$ pair from vacuum and turn cluster into two mesons, or
- pop $q\bar{q}Q\bar{Q}$ pair from vacuum and turn into two baryons.

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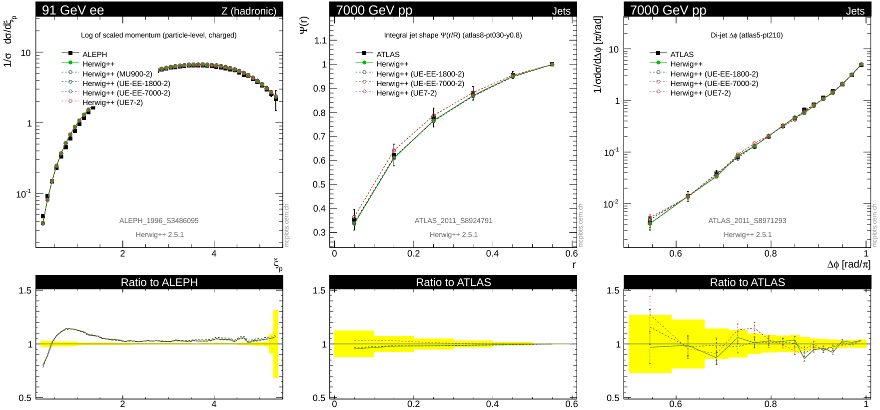
Some Highlight Results.



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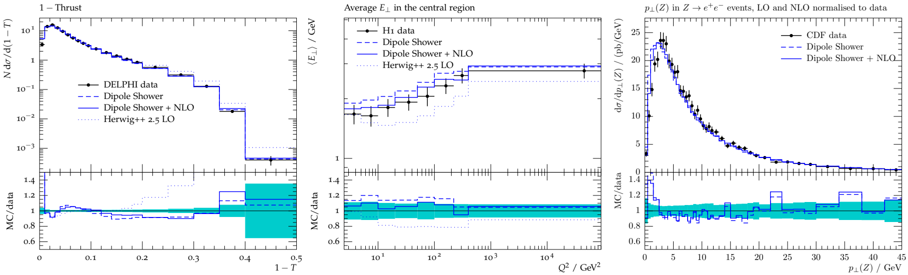

Thinks to keep in mind.

We certainly do have a very good understanding of
what is going on in high energy collisions.

We can really simulate them, as seen in experiment.

However, no one has an ultimate understanding of the subject.

Take care to compare and validate different approaches.

Try the programs. We're happy if they are used.

Some of the major event generators:

- Sherpa
  http://sherpa.hepforge.org
- Pythia
  http://home.thep.lu.se/~ torbjorn/Pythia.html
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Now: Open for discussion!