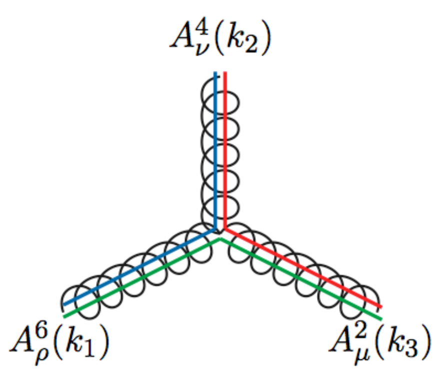
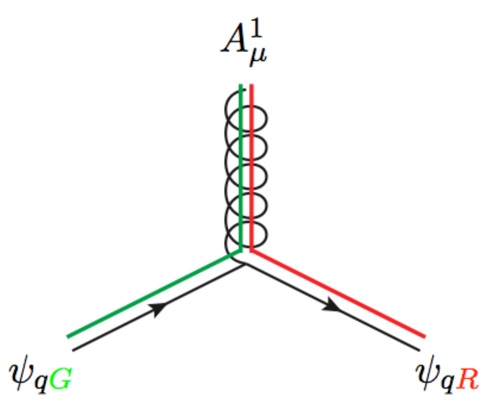
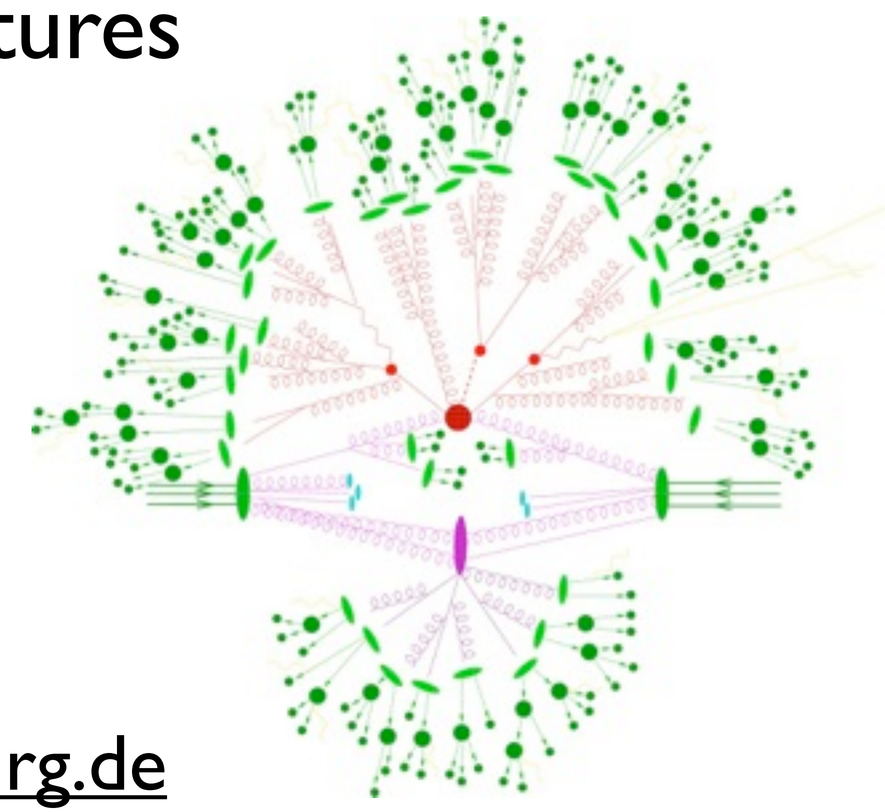
Quantum Chromodynamics





DESY Summer Student Lectures

August 5 and 9, 2013



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Overview

Part 1 - Setting the Stage

The Static Quark Model

Deep-Inelastic Scattering

Discovery of quarks and colour

The QCD Lagrangian

Discovery of gluons

Part 2 - Working with QCD

Renormalisation

Perturbative QCD

Jets

Factorisation and Parton Distribution Functions

Part 1

Note On Units

Natural Units

In particle physics, it is customary and convenient to set $\hbar = c = 1$

Implications:

Energy (mc^2), momentum (mc) and mass (m): units of GeV

Length (l) and time (t): units of GeV^{-1}

Conversion:

using $\hbar = 6.582119 \cdot 10^{-16}$ eV·s

Energy: 1 GeV = $1.609 \cdot 10^{-10}$ J

Momentum: 1 GeV/c = $5.36 \cdot 10^{-19}$ kg·m/s

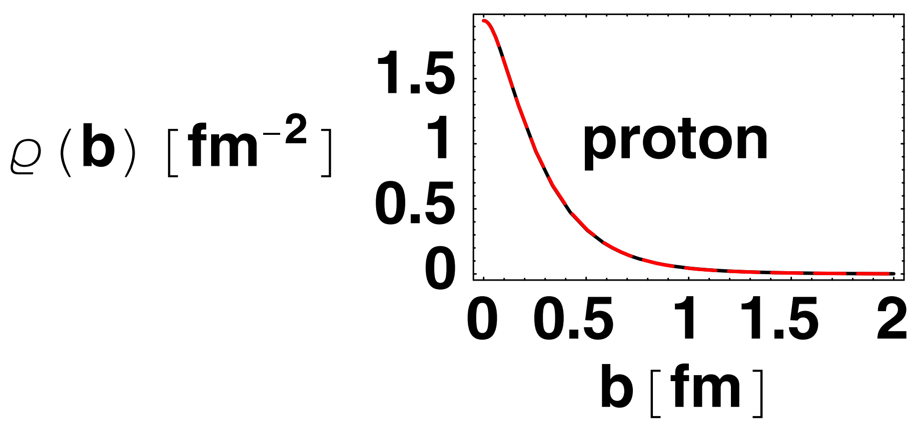
Mass: 1 GeV/c² = $1.79 \cdot 10^{-27}$ kg

Length: 1 GeV^{-1} = $1.97 \cdot 10^{-16}$ m = 0.197 fm

The Proton:

Mass: ~1GeV/c²

Size: ~1 fm

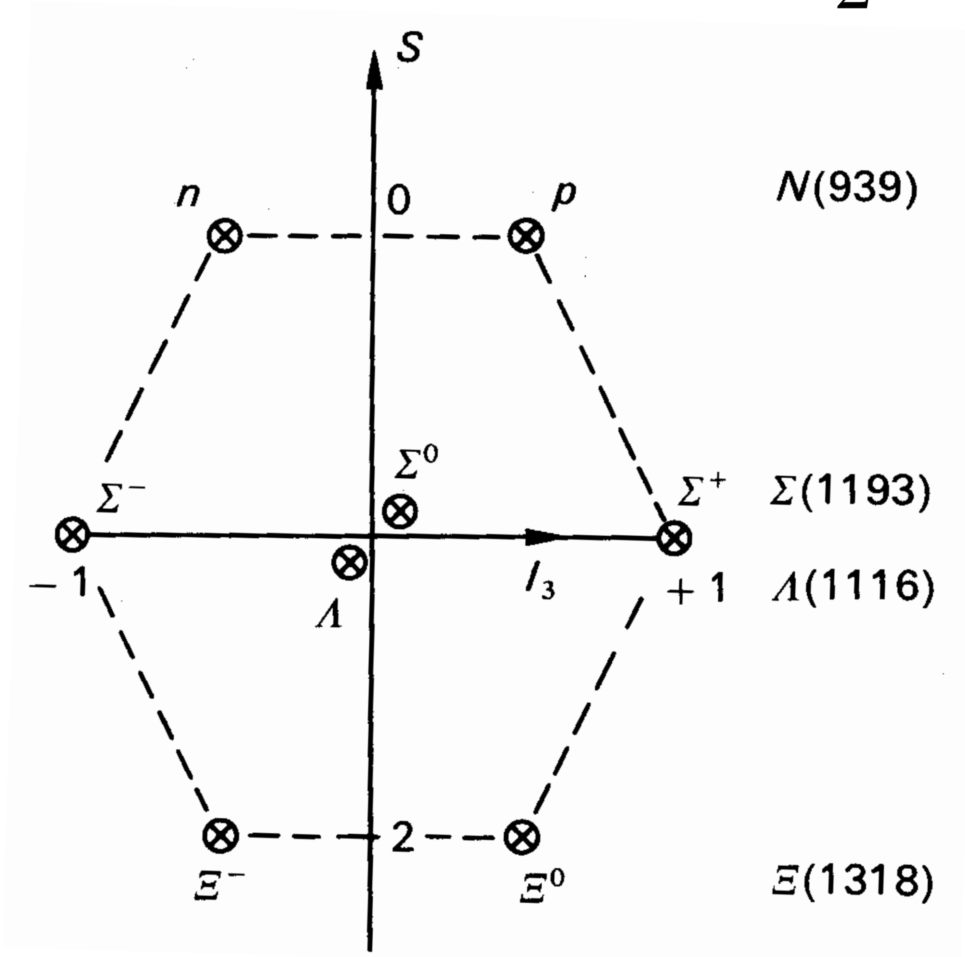


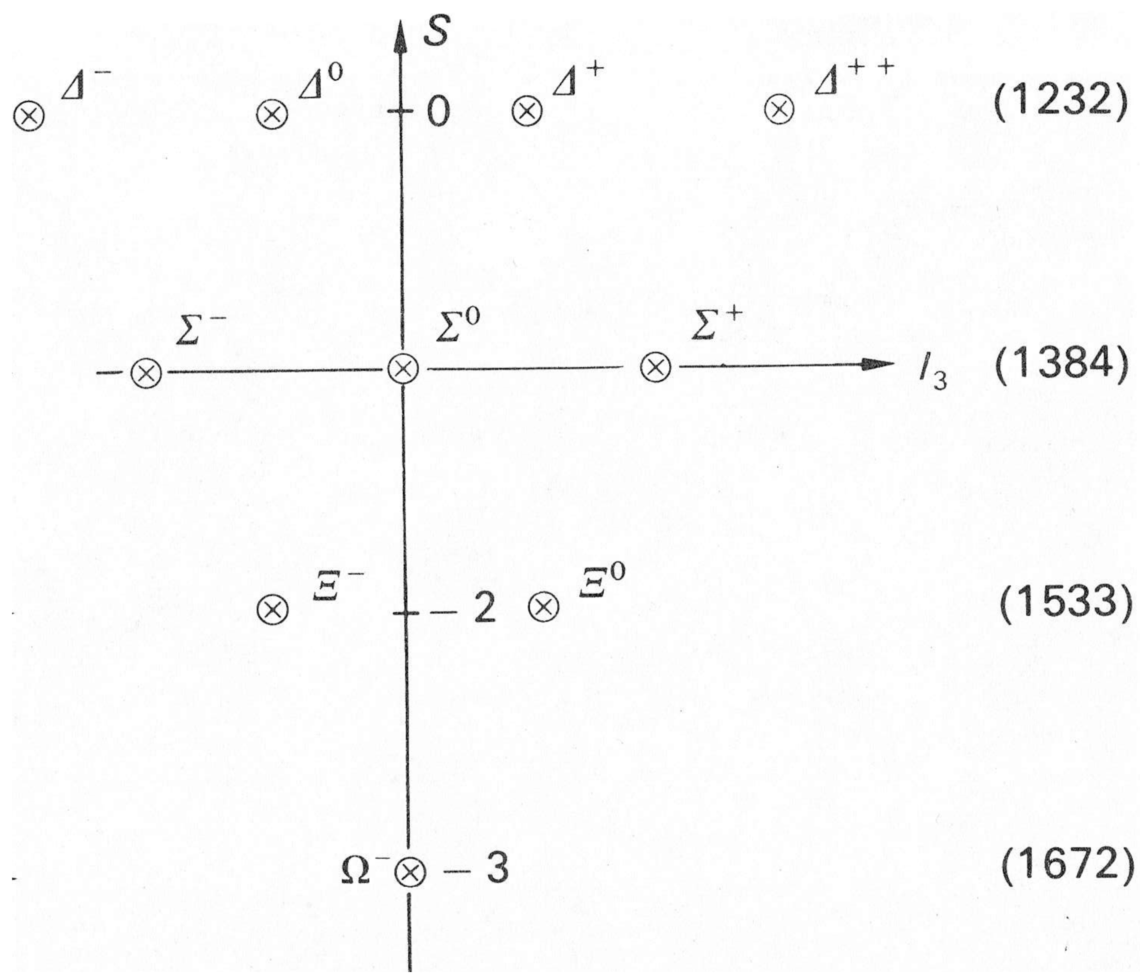
The Static Quark Model

The Particle Zoo

In 1960s accumulation of data from many new baryon and meson resonances: sort them by their strangeness S and isospin I_3

Spin-Parity $J^P = \frac{1}{2}^+$ $J^P = \frac{3}{2}^+$





The Static Quark Model

Postulating constituents

The flavour-states build up a symmetry group: $SU(3)_{flavour}$

Physical particles: reduce the products 3 x 3 x 3 (baryons) and 3 x $\overline{3}$ (mesons) and combine with $SU(2)_{spin}$

Flavour	B	J	I	I_3	S	Q
up	⅓	½	½	+½	0	+⅔
down	⅓	½	½	−½	0	−⅓
strange	⅓	½	0	0	-1	−⅓

The quantum numbers are related through

$$Q = \frac{1}{2}(B + S) + I_3$$

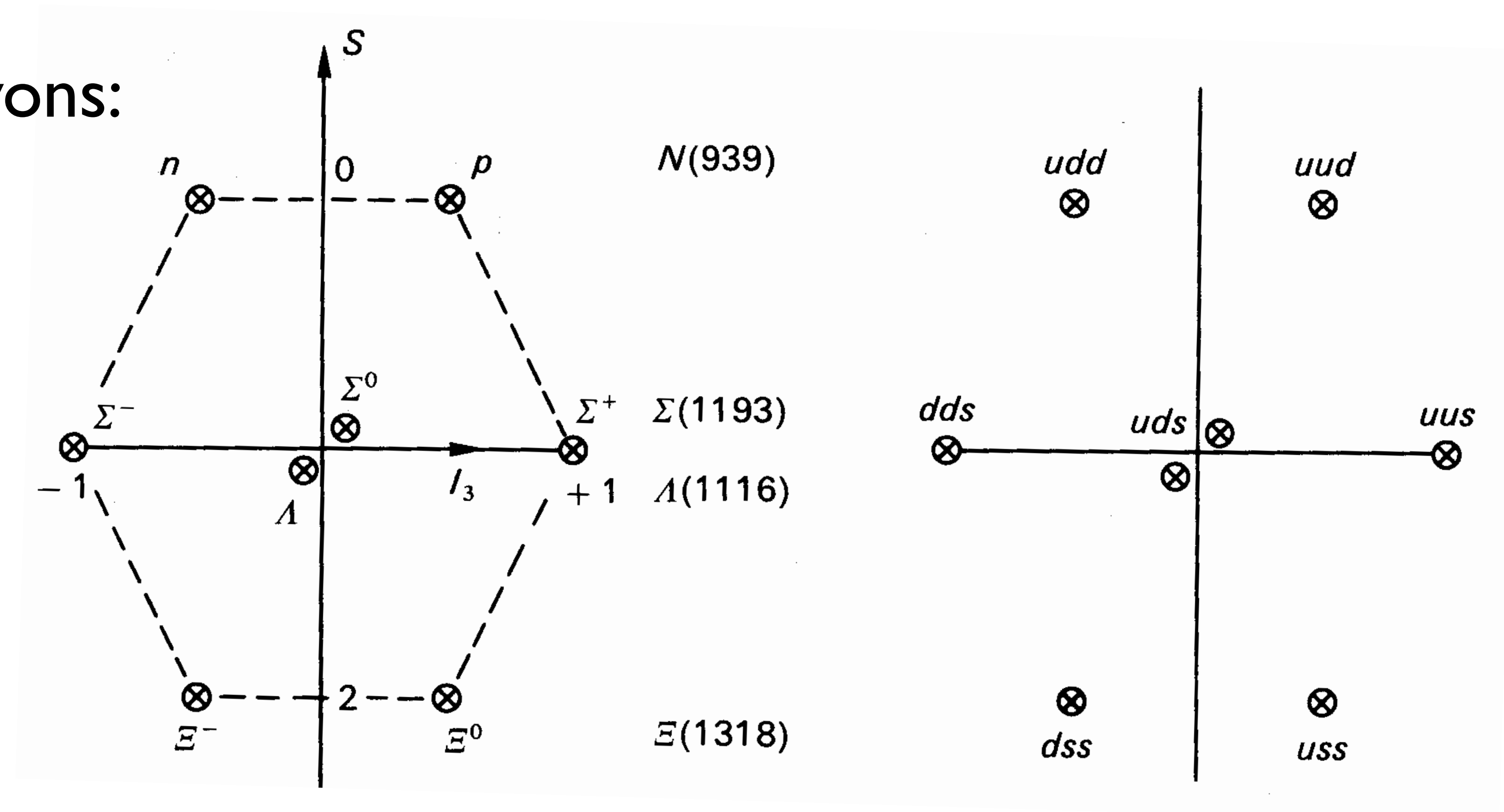
where the hypercharge $Y = B + S$

The anti-particles have the signs of B, I_3, S and Q reversed

Masses: $m_d - m_u \approx$ 4 MeV and $m_s - m_d \approx$ 150 MeV

The Static Quark Model

Spin 1/2 Baryons:



Light Mesons:

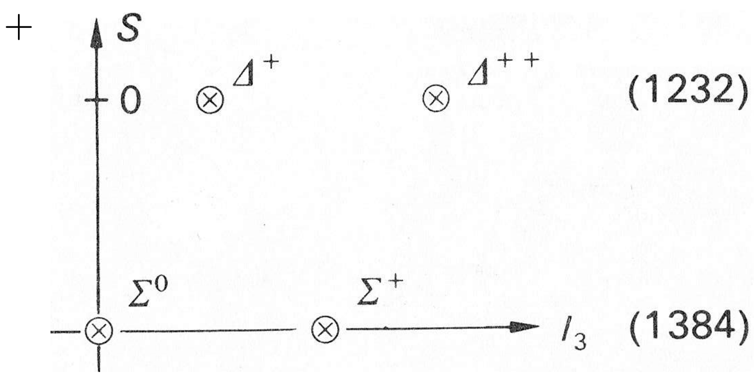
(pseudoscalars

with $J^P = 0^-$)

I	I_3	Wavefunction	Q
1	+1	$\lvert\pi^+\rangle = \lvert u,\bar{d}\rangle$	+1
1	-1	$\lvert\pi^-\rangle = -\lvert\bar{u},d\rangle$	-1
1	0	$\lvert\pi^0\rangle = 1/\sqrt{2}(\lvert d,\bar{d}\rangle - \lvert u,\bar{u}\rangle)$	0
0	0	$\lvert\eta^0\rangle = 1/\sqrt{2}(\lvert d,\bar{d}\rangle + \lvert u,\bar{u}\rangle)$	0

Colour

$J^P = \frac{3}{2}^+$



Wave function of Δ^{++}: $|\Delta^{++}\rangle = |u,\uparrow\rangle + |u,\uparrow\rangle + |u,\uparrow\rangle$

Symmetric in flavour, spin and space (quarks are in ground state: s-wave)

Violates the Pauli Principle!

Solution: one more internal degree of freedom - colour!

$$|\Delta^{++}\rangle = |u,\uparrow,g\rangle + |u,\uparrow,r\rangle + |u,\uparrow,b\rangle$$

Antisymmetric: $|\Delta^{++}\rangle = \Sigma\, \varepsilon_{ijk}\, |u,\uparrow,i\rangle + |u,\uparrow,j\rangle + |u,\uparrow,k\rangle$

- With the arguments given, are you convinced that quarks have physical reality? Why?
- Do colour charges exist?
- How many colours are there?
- How can we test these assumptions?

Deep-Inelastic Scattering

Scattering Experiments

Used to probe the structure of matter and forces involved

Different high-energy scattering experiments: e^+e^-, pp, $e^{\pm}p$

Deep-Inelastic Scattering (DIS)

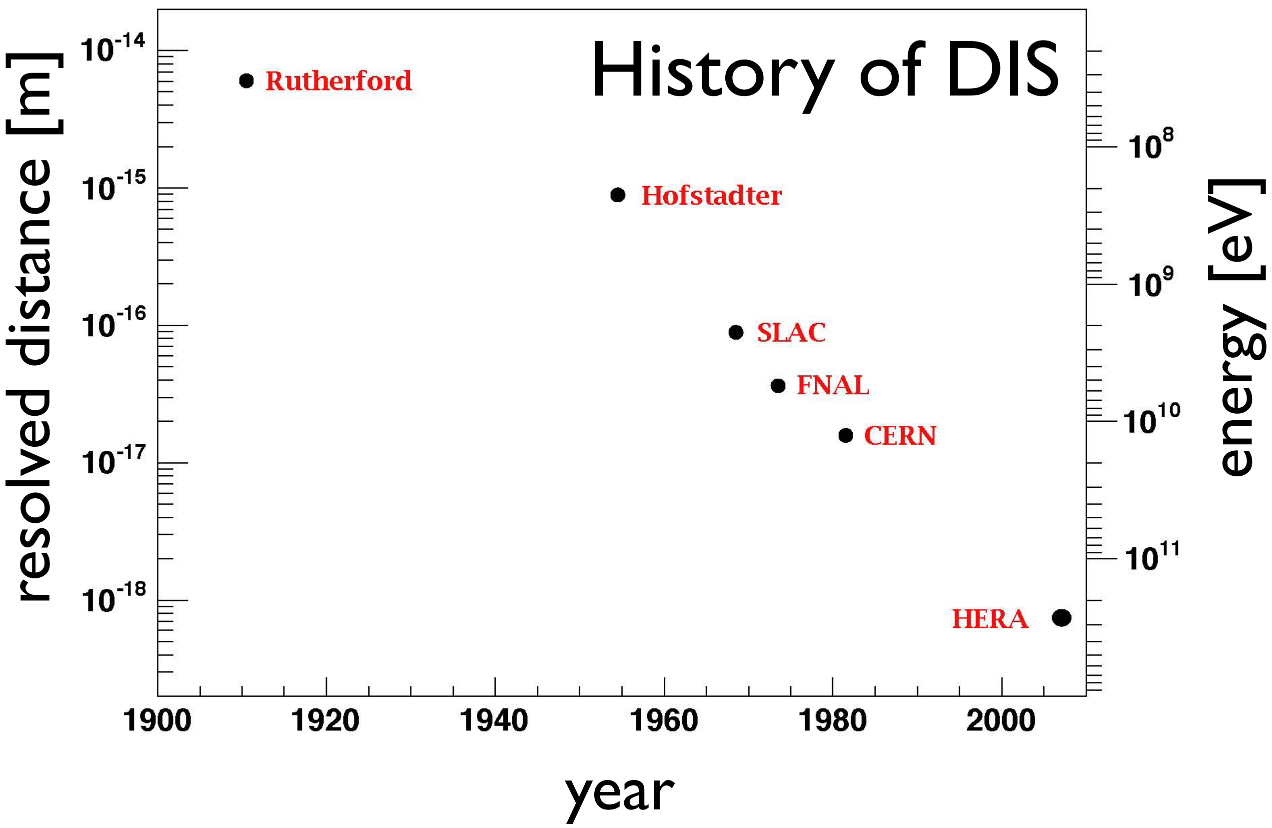
Lepton-nucleus scattering at high energies.

Use the lepton as clean probe to explore the structure of matter - initial state well known.

Distance scale in DIS:

$$r\ [\mathrm{fm}] \approx \frac{\hbar c}{Q} \approx \frac{0.2}{Q\ [\mathrm{GeV}]}$$

where Q is related to the transferred four-momentum



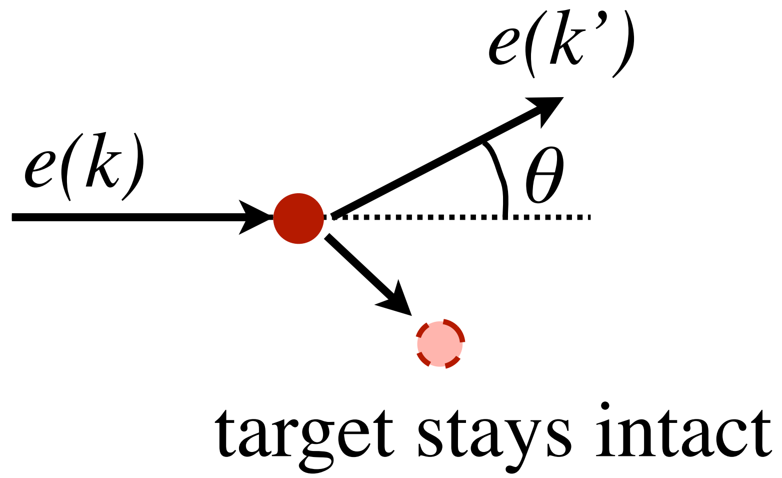
Elastic Electron Scattering

Electron (charge 1) scatters on a nucleon N with charge Z, mass M.
Take recoil into account (assume point-like particles).
Outgoing electron: $k' = (E', 0, E' \sin\theta, E' \cos\theta)$

Variables

$$Q^2 = -q^2 = 4EE' \sin^2(\theta/2)$$

$$E - E' = \frac{Q^2}{2M} \Rightarrow E' = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)}$$



Only one independent variable!

Mott Scattering: $$\left.\frac{d\sigma}{dQ^2}\right|_{\mathrm{Mott}} = \frac{4\pi\alpha^2 Z^2}{Q^4}\frac{E'}{E}\cos^2\left(\frac{\theta}{2}\right)$$

Assume a Dirac particle with spin 1/2:

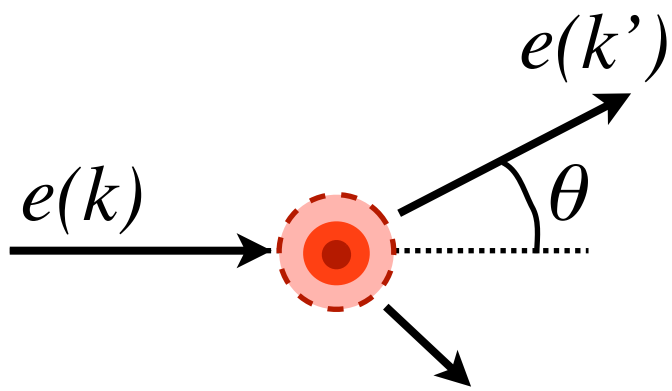
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Z^2}{Q^4}\frac{E'}{E}\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{Q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

Proton Form Factors

So far we have assumed a point-like nucleon with spin 1/2.
Now take a proton with $Z=1$ and allow for a charge distribution $\varrho(r)$.

Potential becomes: $V(\boldsymbol{r}) = \int \frac{\rho(\boldsymbol{r'})}{4\pi|\boldsymbol{r}-\boldsymbol{r'}|}\mathrm{d}\boldsymbol{r'}$ with $\int \rho(\boldsymbol{r'})\mathrm{d}\boldsymbol{r'} = 1$

This introduces two (a priori unknown) form factors in the cross section: $G_E(Q^2),\ G_M(Q^2)$



Rosenbluth Formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 Z^2}{Q^4}\frac{E'}{E}\left[\cos^2\left(\frac{\theta}{2}\right)\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2 \sin^2\left(\frac{\theta}{2}\right)\right]$$

with $\tau = Q^2/(4M^2) > 0$ (space-like)

Using the Mott cross section:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}\right)_{\mathrm{ela}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}\right)_{\mathrm{Mott}}\left[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right]$$

Proton Form Factors

At small $\tau \ll 1$, and hence at small Q^2, the electric and magnetic form factors are just the Fourier-transforms of the charge and magnetic moment distributions of the proton:

$$G_E(Q^2) \approx \int e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r}) \mathrm{d}\boldsymbol{r} \quad \text{and} \quad G_M(Q^2) \approx \int e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mu(\boldsymbol{r}) \mathrm{d}\boldsymbol{r}$$

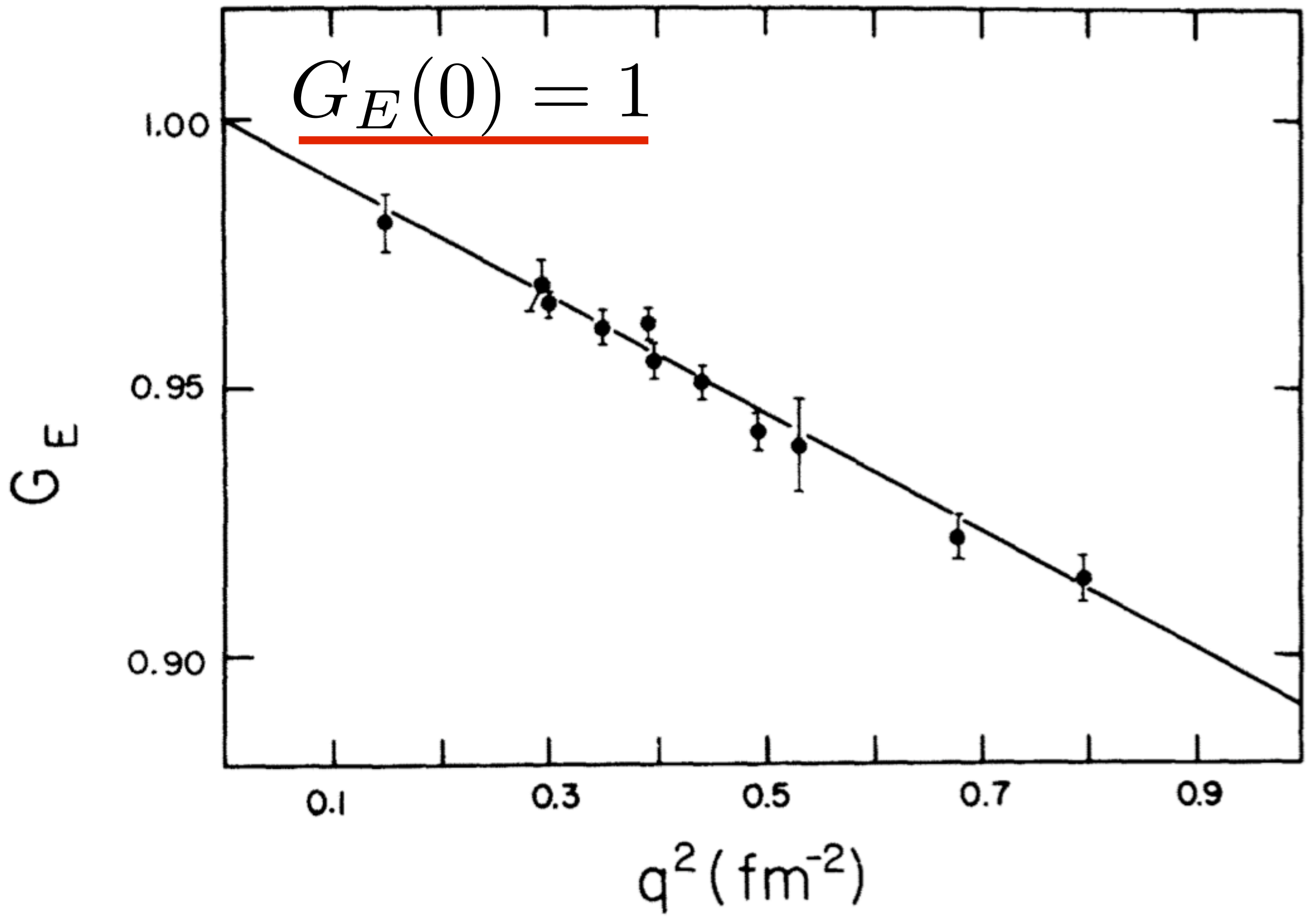
This means that at $Q^2 \approx 0$ we expect

$$\underline{G_E(0)} = \int \rho(\boldsymbol{r}') \mathrm{d}\boldsymbol{r}' = \underline{1} \quad \text{and} \quad G_M(0) = \int \mu(\boldsymbol{r}') \mathrm{d}\boldsymbol{r}' = \vec{\mu}$$

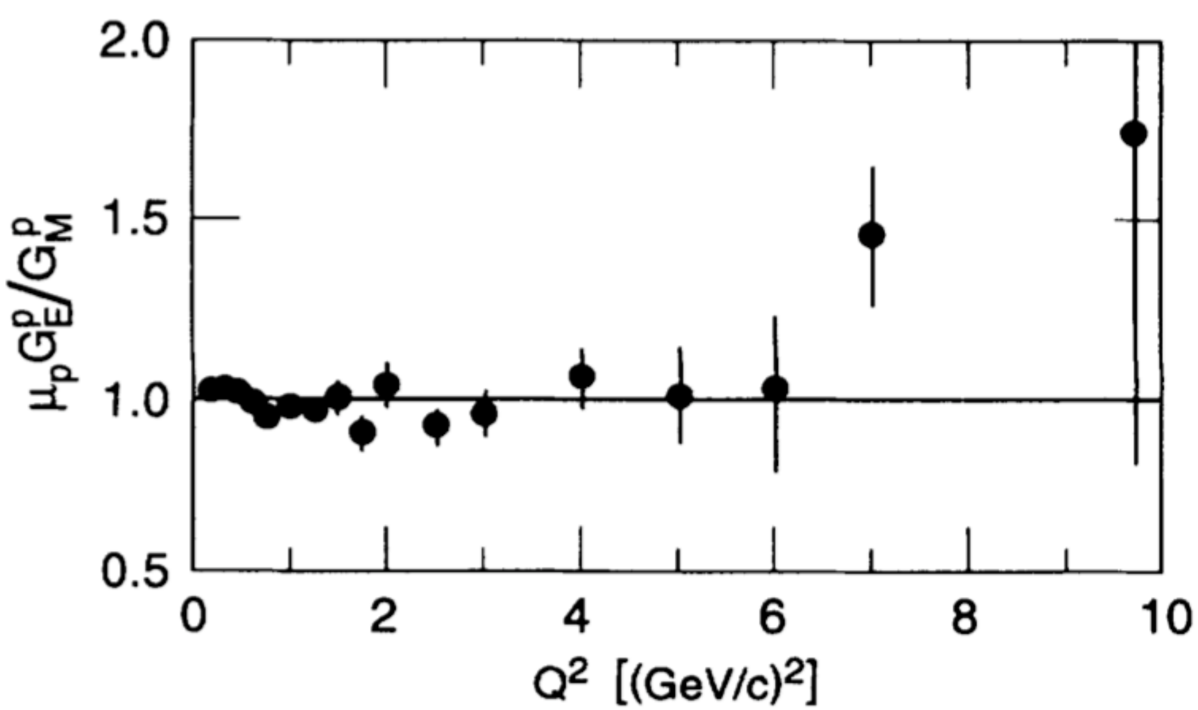
The experimental value of the anomalous magnetic moment of the proton $\mu_p = 2.79$, so we expect $\underline{G_M(0) = 2.79}$

Proton Form Factors

J.J.Murphy *et al.*, Phys. Rev. C9, 2125 (1974)



R.C.Walker *et al.*, Phys. Rev. D49, 5671 (1994)



In fact, we find that $\mu_p G_E(Q^2) = G_M(Q^2) \Rightarrow G_M(0) = \mu_p$

This means that the charge distribution is the same as the current spatial distribution in the proton.

Inelastic Electron Scattering

Transferred momentum:

$$q = k - k'$$

Virtuality of exchanged boson:

$$Q^2 = -q^2 > 0$$

Squared centre-of-mass energy:

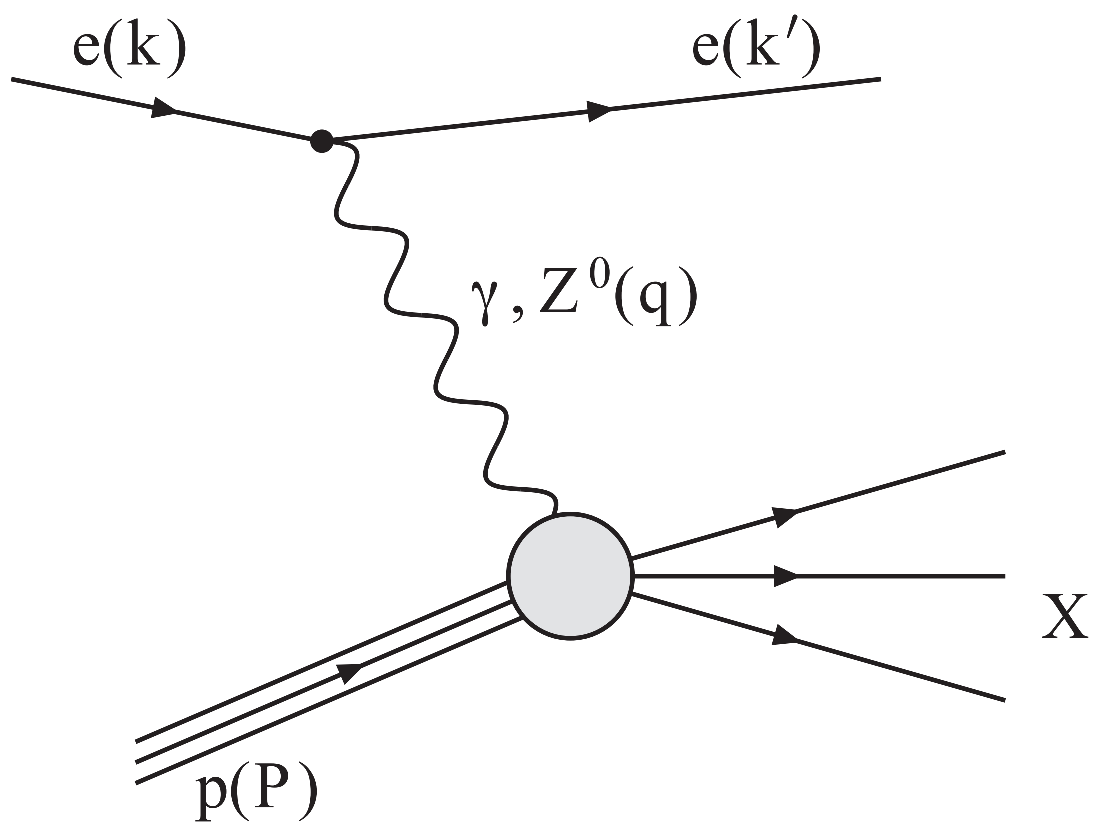
$$s = (P + k)^2$$

Squared mass of the hadronic final state:

$$W^2 = (P + q)^2 = M^2 + 2q \cdot P - Q^2$$

Inelasticity: $y = \dfrac{q \cdot P}{k \cdot P}$ with $0 \leq y \leq 1$

Scaling variable: $x = \dfrac{Q^2}{2q \cdot P}$ with $0 \leq x \leq 1$



Deep: $Q^2 \gg M^2$

Inelastic: $W > M$

Deep-Inelastic Scattering (DIS)

Deep: $Q^2 \gg M^2$

Inelastic: $W > M$

Neglect rest masses whenever

$W \gg m_e, \; W \gg M$

$$Q^2 = -(k - k')^2 \qquad W^2 = (P + q)^2$$

$$x = \frac{Q^2}{2q \cdot P} \qquad y = \frac{q \cdot P}{k \cdot P}$$

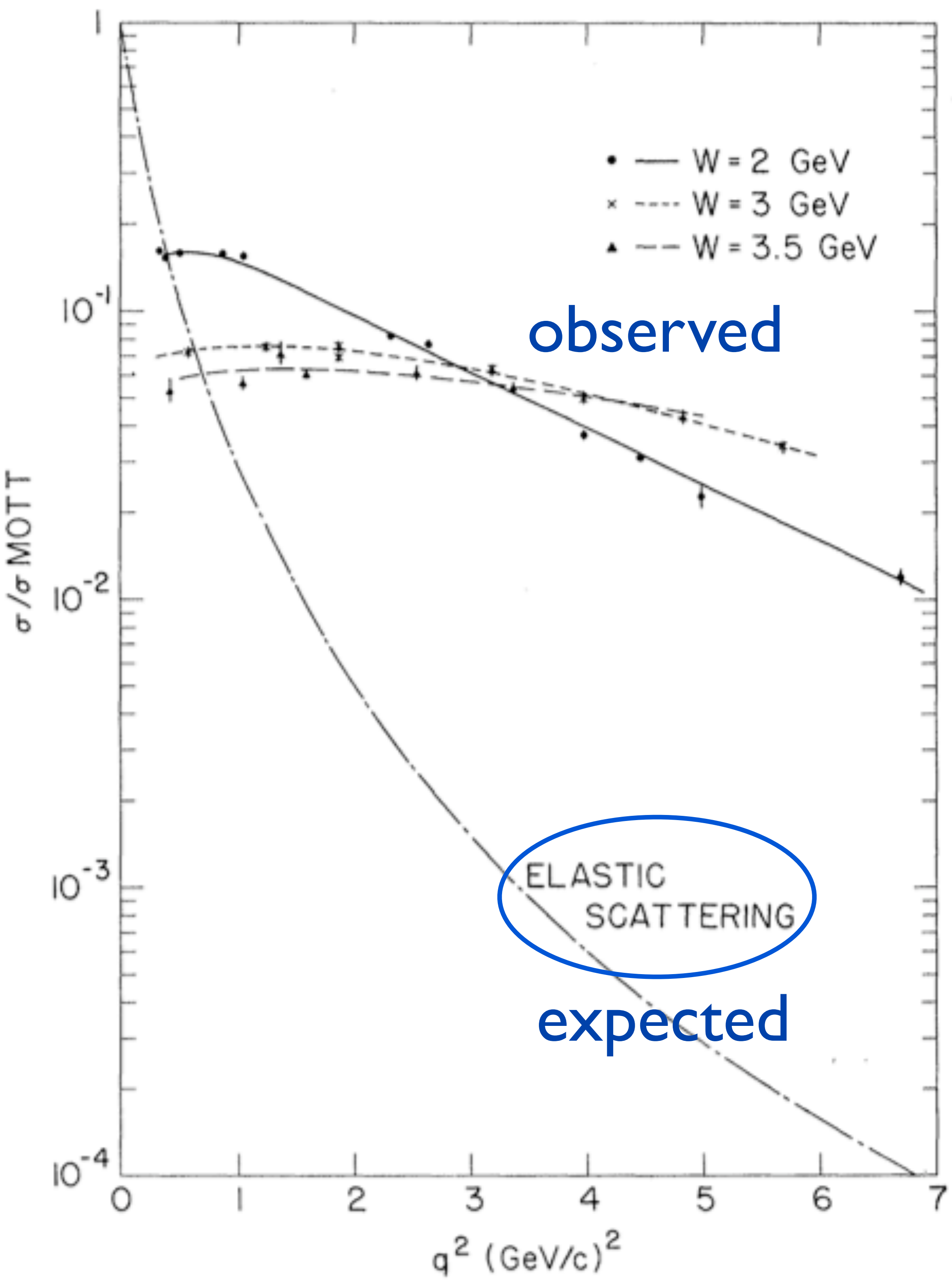
Squared centre-of-mass energy: $s = 4\, E_e\, E_p$

Out of Q^2, x, y, W only two are independent at fixed centre-of-mass energy, since they are related through $Q^2 = s\,x\,y$ and $W^2 = M^2 + 2\,q{\cdot}P - Q^2$

Thus, pairs of these variables fully determine the kinematics of the scattering.

Often used: (Q^2, x) and (Q^2, W^2)

Deep-Inelastic Scattering Results



Early results from SLAC (1969):

$E = 7 - 17.7$ GeV

$\theta = 10°$

Elastic cross section falls off rapidly due to the proton not being point-like

Inelastic: $W > M$

Ratio to Mott cross section nearly flat in Q^2

Q^2 dependence becomes weaker for increasing W

Proton a composite particle!

M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969)

Partons

Assume that proton consists of partons, then the electron scatters off a parton with momentum xP:

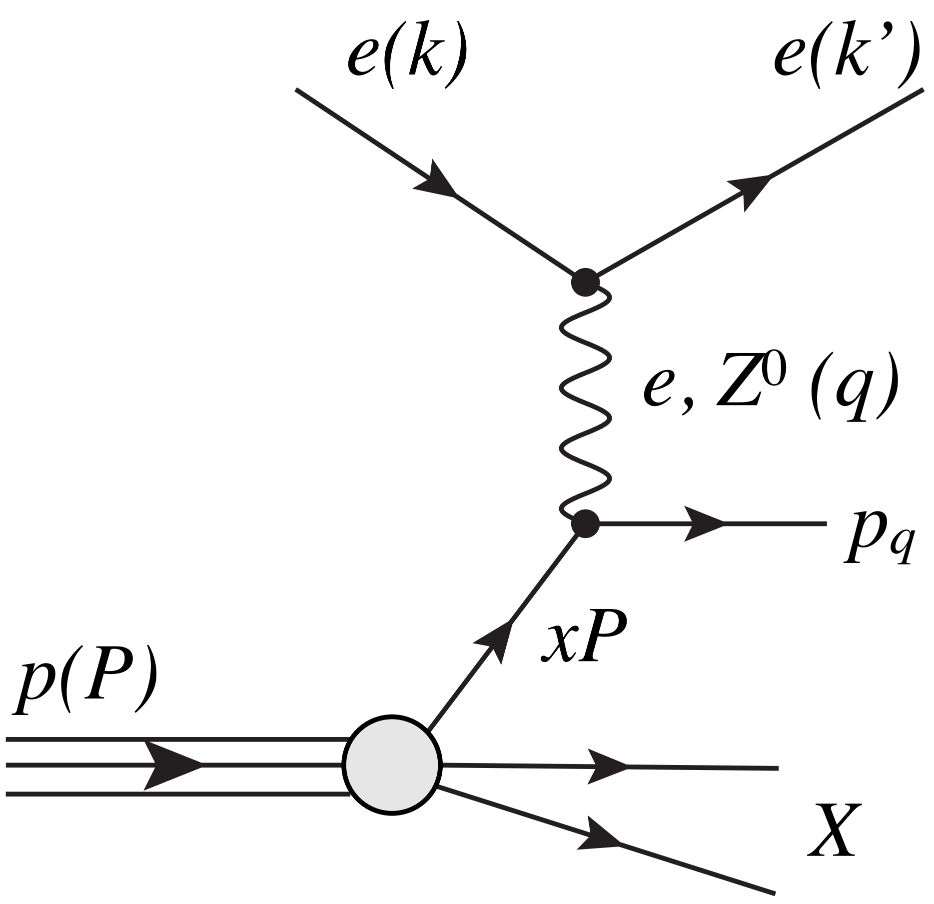
$$p_q = q + xP$$

$$p_q^2 = (xP)^2 = m_q^2 = 0$$

$$= (q + xP)^2$$

$$= -Q^2 + 2xq \cdot P + (xP)^2$$

so we get $x = \frac{Q^2}{2q \cdot P}$

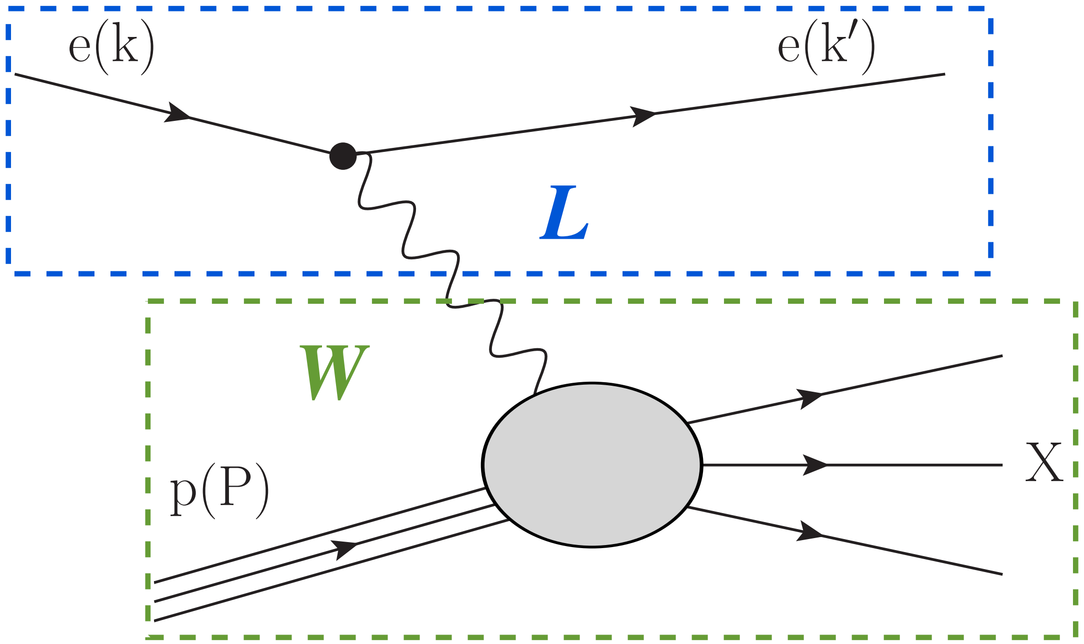


The variable x can be interpreted as the momentum fraction of the proton carried by the struck parton

DIS Cross Section

The amplitude for the deep inelastic scattering diagram is given by

$$\mathcal{A} = \underline{e\bar{u}(k')\gamma^\alpha u(k)}\frac{1}{q^2}\underline{\langle X|j_\alpha(0)|P\rangle}$$



The cross section is proportional to $|\mathcal{A}|^2$ (Fermi's golden rule)

$$\frac{d^2\sigma}{dx dQ^2} \propto |\mathcal{A}|^2 = \frac{\alpha^2}{Q^4}\underline{L_{\alpha\beta}}\ \underline{W^{\alpha\beta}}$$

The leptonic tensor $L_{\alpha\beta}$ is fully determined by QED:

$$\underline{L_{\alpha\beta}} = 2\left(k_\alpha k'_\beta + k_\beta k'_\alpha - g_{\alpha\beta} k \cdot k'\right)$$

The hadronic tensor $W_{\alpha\beta}(P,q)$ is unknown, since it involves all the structure of the proton

$$\underline{W_{\alpha\beta}(P,q)} = \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\langle P,S|\left[j^\dagger_\alpha(z), j_\beta(0)\right]|P,S\rangle$$

⇒ Absorb our ignorance in structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$

DIS Cross Section

The DIS cross section then becomes

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{xQ^4}\left[(1-y)\underline{F_2(x,Q^2)} + \frac{y^2}{2}2x\underline{F_1(x,Q^2)}\right]$$

Rewrite the Rosenbluth formula in terms of Q^2 and y

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left[\cos^2\left(\frac{\theta}{2}\right)\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2\sin^2\left(\frac{\theta}{2}\right)\right]$$

with $y = 1 - \frac{E'}{E}\sin^2(\theta/2)$

we get (elastic scattering):

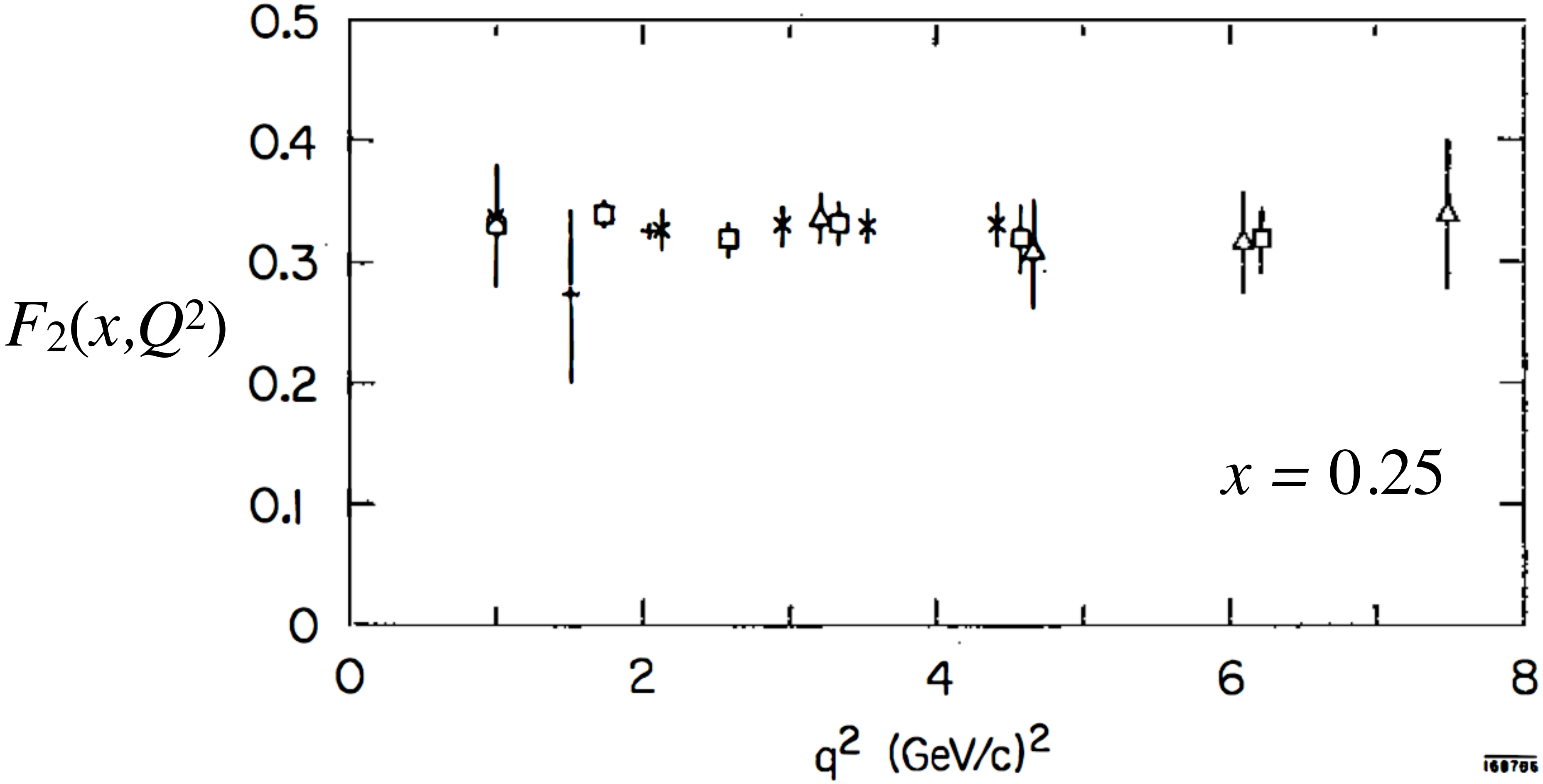
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\underline{\frac{G_E^2+\tau G_M^2}{1+\tau}} + \frac{1}{2}y^2\underline{G_M^2}\right]$$

$F_2(x,Q^2)$ corresponds to the electromagnetic field of the parton

$F_1(x,Q^2)$ corresponds to the spin of the parton

Early F_2 Data

J.T. Friedman, H.W. Kendall, Ann. Rev. Nucl. Sci. 22, 203 (1972)



Scaling!

Independence of the structure functions of Q^2: $F_i(x,Q^2) = F_i(x)$

J.D. Bjørken predicted scaling for $Q^2 \to \infty$ as x stays fixed.
Scaling is obtained using Gell-Mann's current algebra in the quark model.

Scattering from point-like constituents of the proton!

Callan-Gross Relationship

F_1 and F_2 are not independent, but satisfy the Callan-Gross relationship:

$$2xF_1 = F_2$$

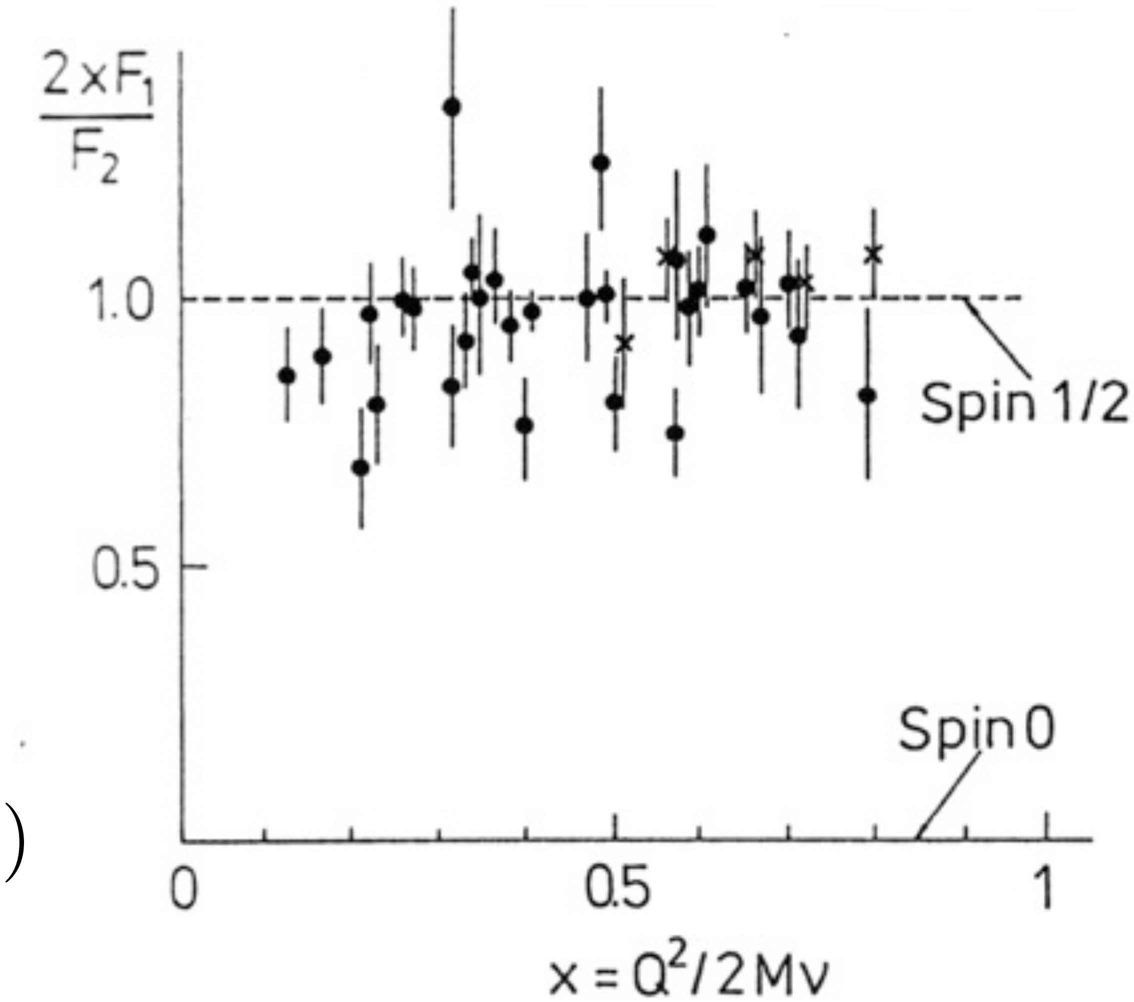
This means that partons are spin 1/2 particles! (spin 0, would mean $2xF_1 = 0$)

The cross section now becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{xQ^4}\left(1 - y + \frac{y^2}{2}\right) F_2(x, Q^2)$$

The electric charge and magnetic moment are fixed with respect to each other

→ scattering from point-like Dirac particles



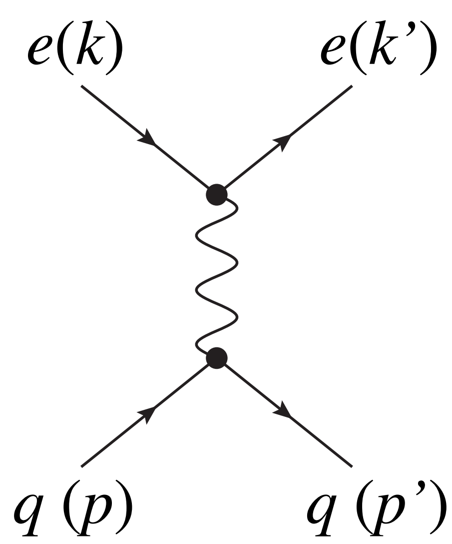
P. Schmüser, *Feynman-Graphen und Eichtheorien für Experimentalphysiker*, Springer Verlag (1988)

Clear evidence for Quarks!

Quark Parton Model (QPM)

Elastic electron-quark scattering:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left(1 - y + \frac{y^2}{2}\right)$$



same as e-μ scattering with charge e_q

<u>Assumptions</u>

- Single photon exchange
- incoherent scattering of quarks from the proton
- take $q_i(x)\mathrm{d}x$ to be the probability to find quark of type i inside the proton with momentum fraction between x and $x+\mathrm{d}x$

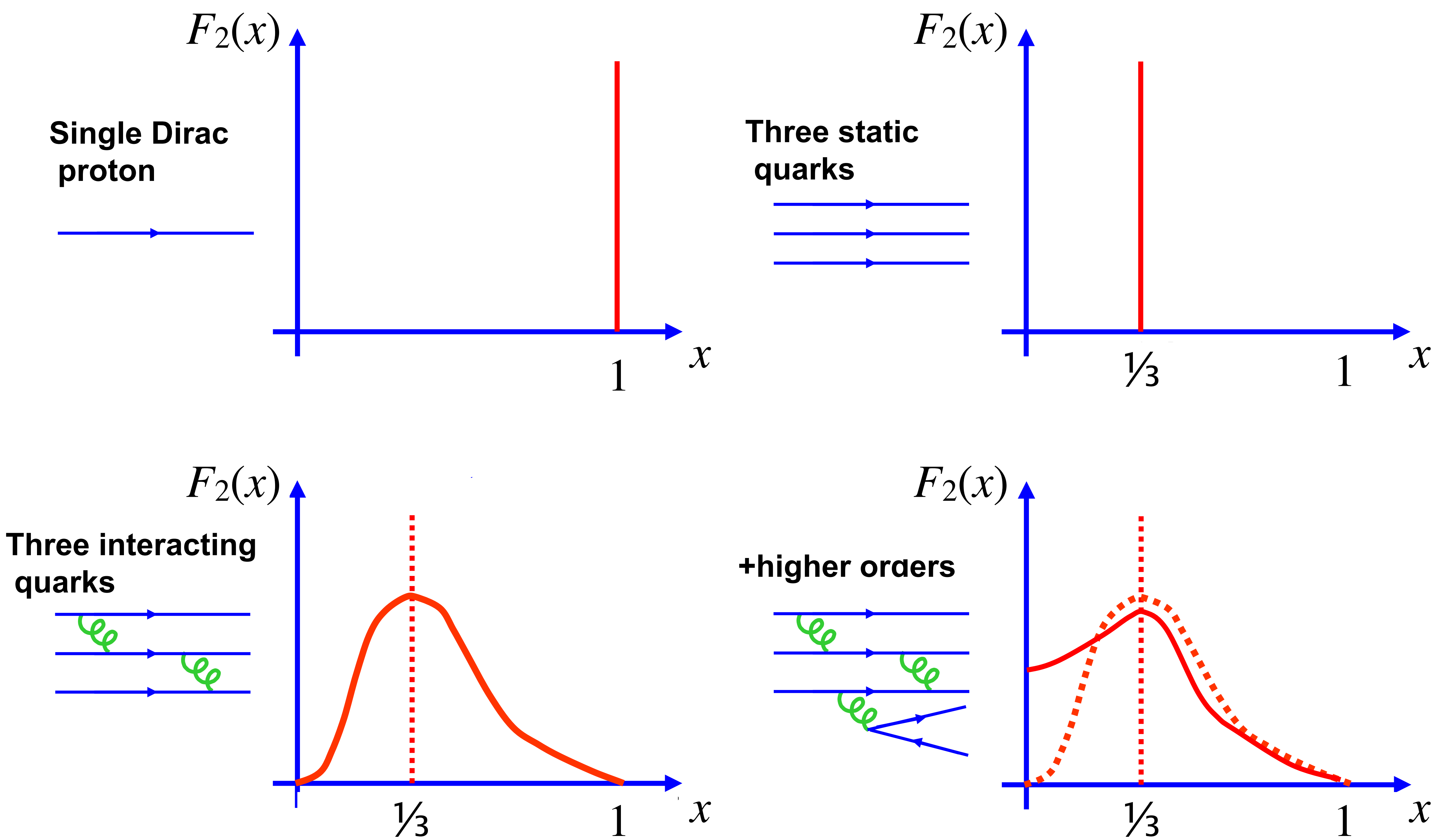
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4}\left(1 - y + \frac{y^2}{2}\right)\sum_i e_i^2 q_i(x)$$

Compare with:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{xQ^4}\left(1 - y + \frac{y^2}{2}\right) F_2(x, Q^2)$$

$$F_2(x) = x\sum_i e_i^2 q_i(x)$$

How Does $F_2(x)$ Look?



Early Experimental Results on F_2

A. Bodek, *et al.*, Phys. Rev. D20, 1471 (1974)

$$F_2(x, Q^2) =$$
$$= \frac{d\sigma}{dQ^2 dx} \frac{xQ^4}{4\pi\alpha^2} \frac{1}{(1 - y + y^2/2)}$$

Experimentally accessible!

In the QPM $F_2(x)$ is directly proportional to the quark distributions $q_i(x)$

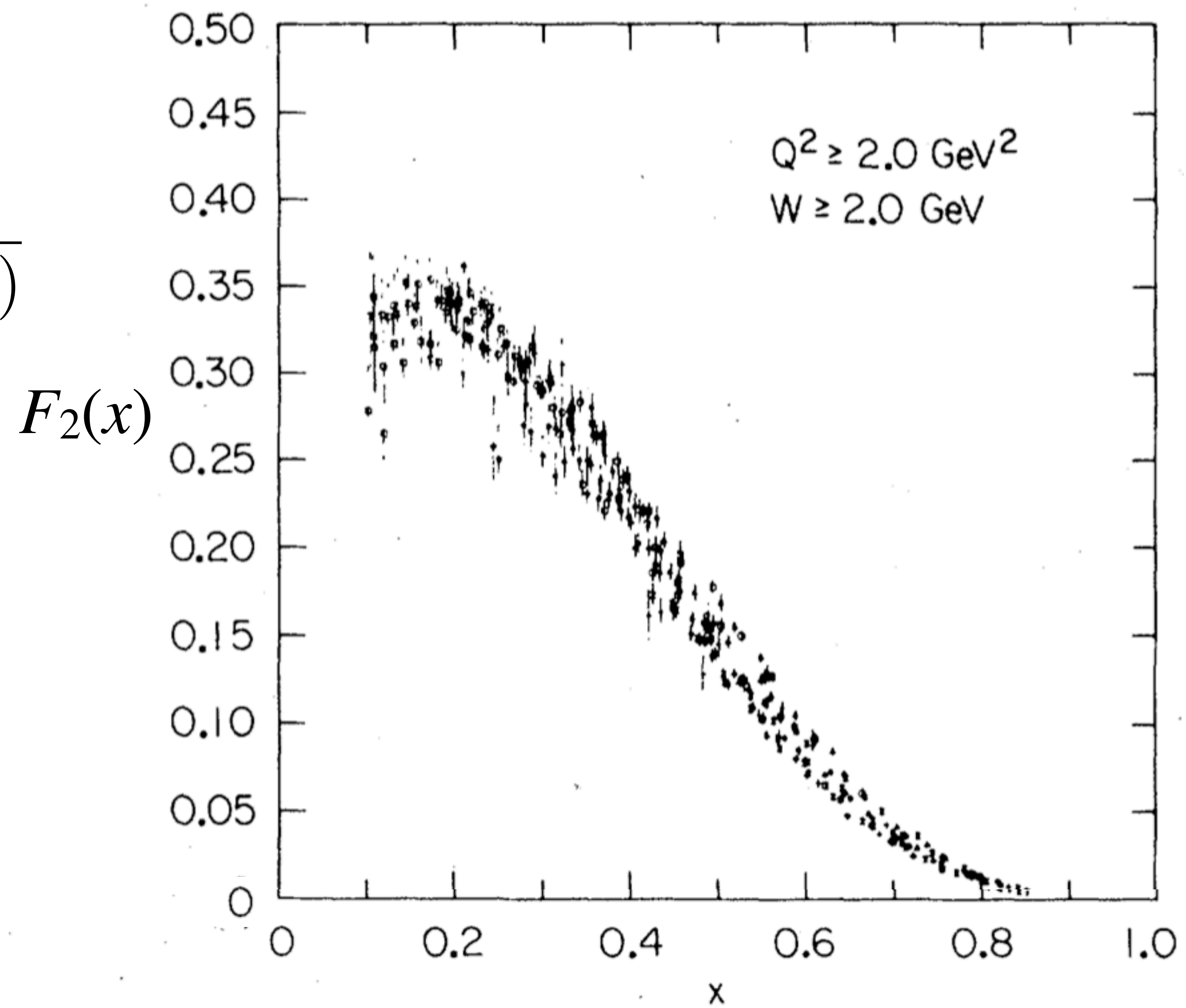


FIG. 38. Values of νW_2^p, ~~νW_2^n, and νW_2^d~~ plotted against x. The errors shown are purely random.

The QPM - Mini Summary

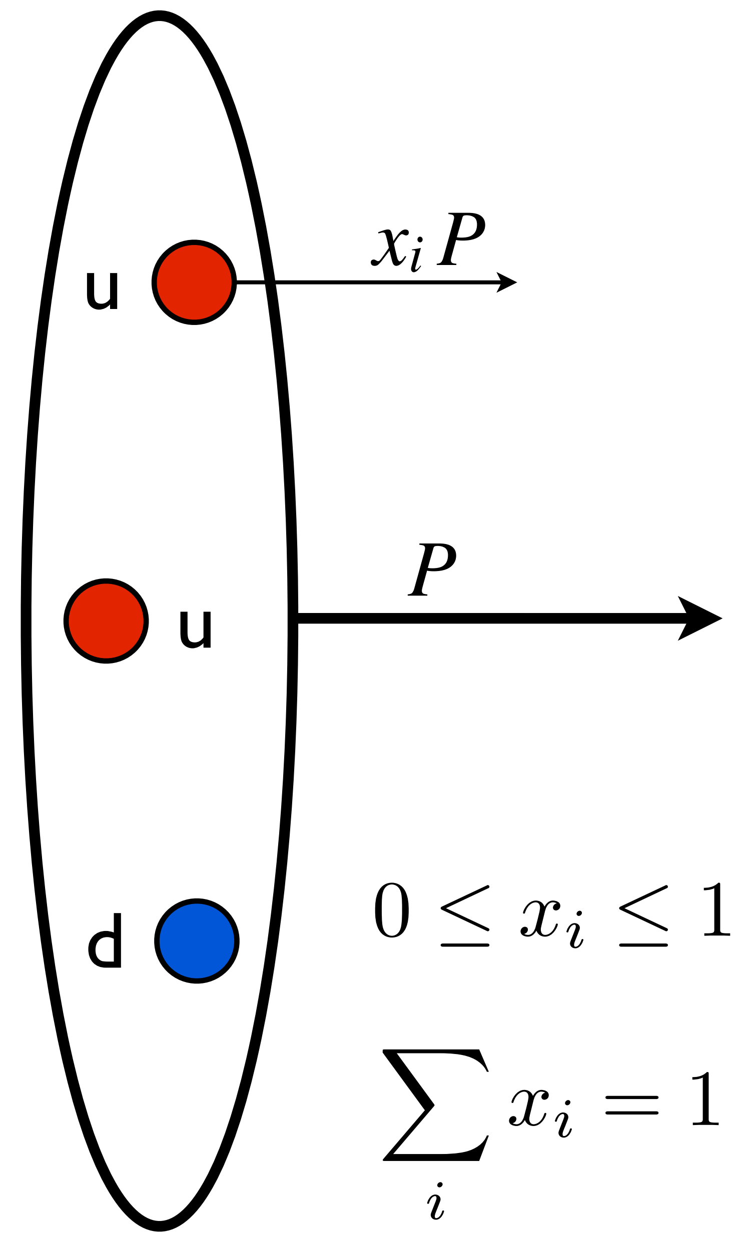
Proton consists of 3 partons, which can be identified with spin-1/2 quarks

Electron-proton scattering is then a sum of incoherent electron-quark scatterings with single photon exchange

Proton structure is defined by parton distributions $q_i(x)$

The Structure function is directly proportional to the quark content of the proton

$$F_2(x) = x \sum_i e_i^2 q_i(x)$$



$$0 \leq x_i \leq 1$$

$$\sum_i x_i = 1$$

What's Missing?

$$\int_0^1 F_2^p(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)\mathrm{d}x$$

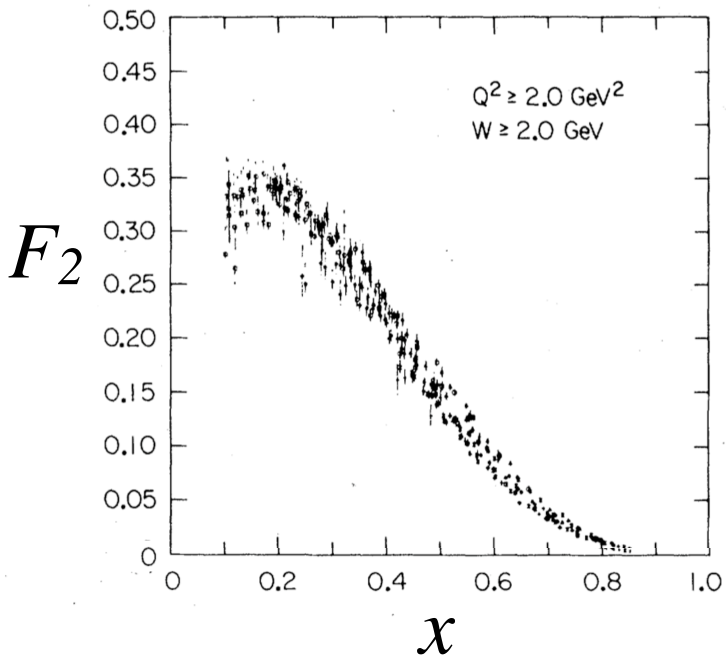
proton

$$= \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{with} \quad f_u = \int_0^1 xu(x)\mathrm{d}x$$

$$\int_0^1 F_2^n(x)\mathrm{d}x = \frac{1}{9}f_u + \frac{4}{9}f_d \quad \text{(assume isospin symmetry)}$$

neutron

f_u and f_d are the fractions of the proton or neutron momenta carried by the up or down quarks



Exp.: $\int_0^1 F_2^p(x)\mathrm{d}x \approx 0.18$ and $\int_0^1 F_2^n(x)\mathrm{d}x \approx 0.12$

$$\Rightarrow \boxed{f_u = 0.36 \text{ and } f_d = 0.18}$$

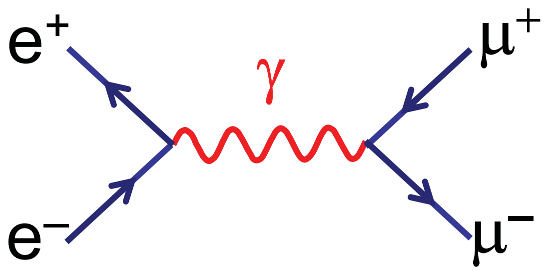
In the proton up-quark carry twice as much momentum as down-quarks ✓

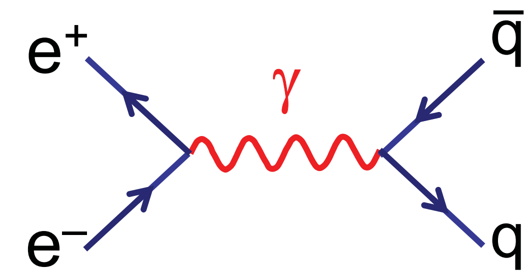
What about colour ?

Where are 50% of the proton momentum ?

Discovering Colour

Rate for $e^+e^- \to$ hadrons





$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = \frac{e^2}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\mu^+)\gamma^\mu \bar{u}(\mu^-)]$$

$$\mathcal{M}(e^+e^- \to \mathrm{q}\bar{\mathrm{q}}) = \frac{e\, e_q}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\bar{\mathrm{q}})\gamma^\mu \bar{u}(\mathrm{q})]$$

- Ignoring differences in the phase space,

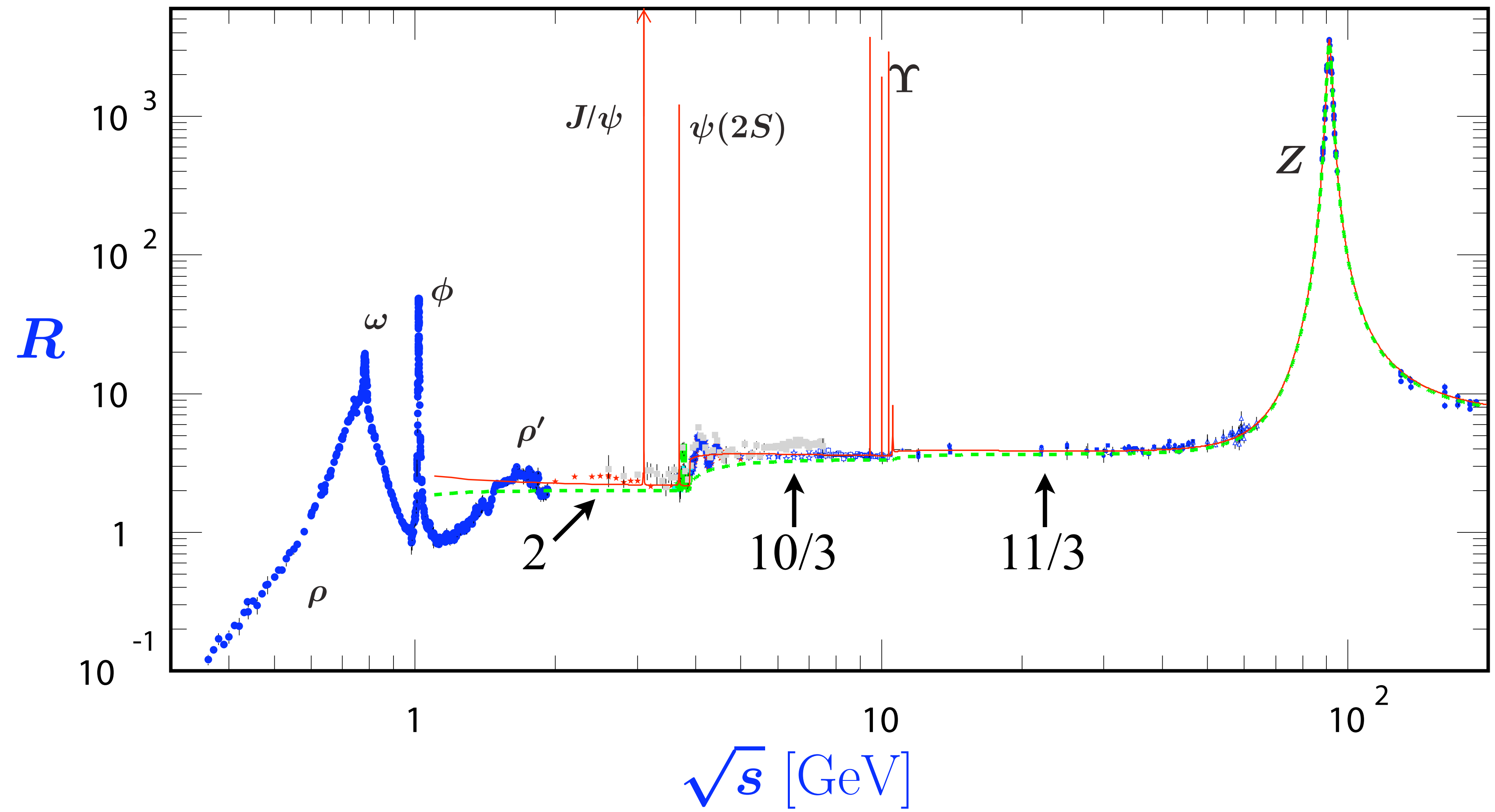
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_C \frac{e_q^2}{e^2}$$

- R directly sensitive to N_C
- Number of available flavours depends on $s = q^2$, with $\sqrt{s} > 2m_q$ for a quark of flavour q to be produced

CM energy [GeV]	available quark pairs	R with $N_C = 3$
$1 < \sqrt{s} < 3$	u, d, s	2
$4 < \sqrt{s} < 9$	u, d, s, c	10/3
$\sqrt{s} > 10$	u, d, s, c, b	11/3

Discovering Colour

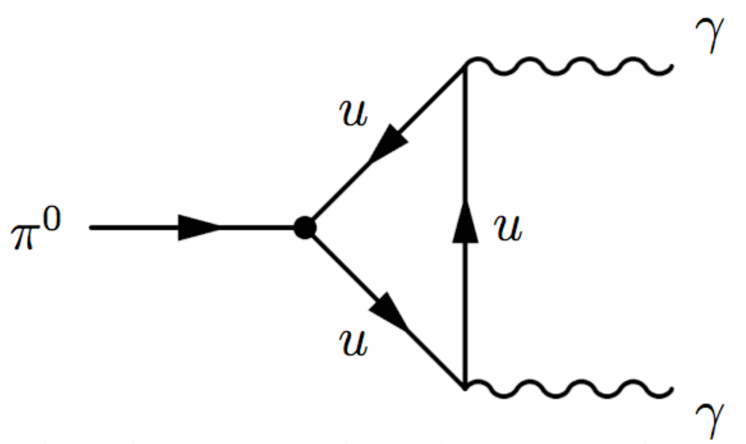
J. Beringer, *et al.*(Particle Data Group), Phys. Rev. D86, 10001 (2012)



- Compendium of many measurements from e^+e^- colliders
- Consistent with $N_C = 3$
- Resonances at quark production thresholds: $q\bar{q}$-bound states
- At $\sqrt{s} > 100$ GeV contributions from Z-exchange

More Evidence

Also $\pi^0 \to 2\gamma$ is sensitive to number of colours N_C



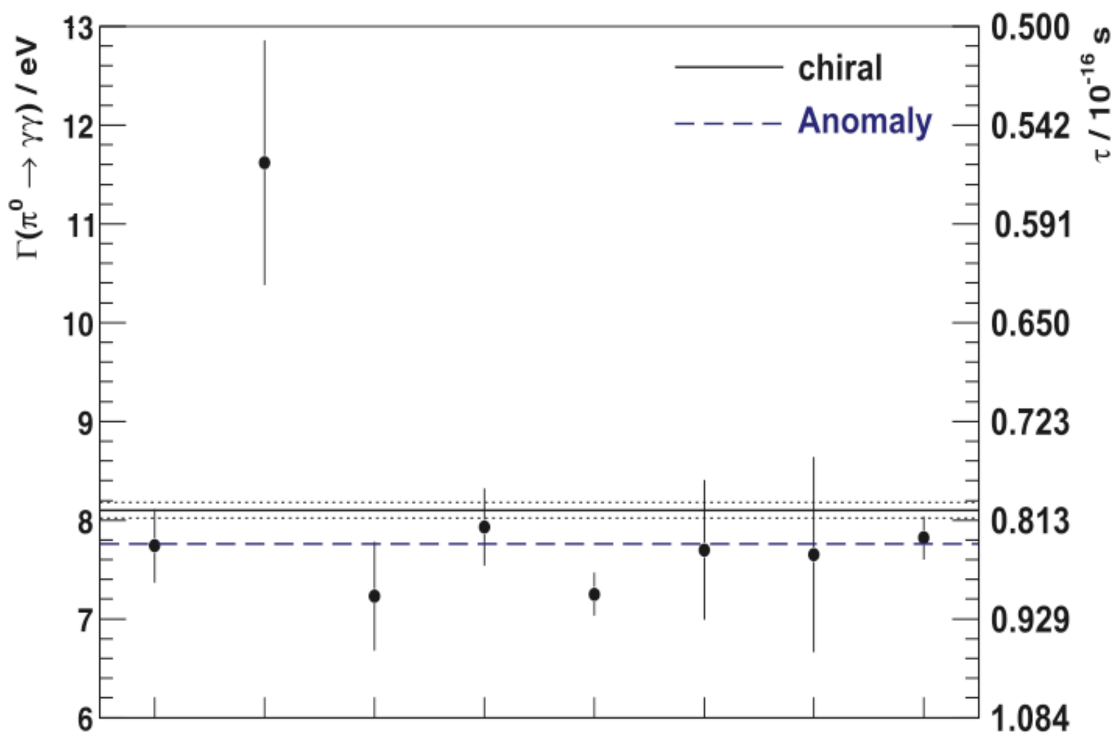
Every quark colour contributes in the loop

$$\Gamma(\pi^0 \to 2\gamma) = \left(\frac{N_C}{3}\right)^2 \frac{\alpha^2 m_{\pi^0}^3}{64\pi^3 f_\pi^2} = 7.76 \left(\frac{N_C}{3}\right)^2 \text{eV}$$

Experimentally, average of several measurements:

$\Gamma(\pi^0 \to 2\gamma) = 7.74 \pm 0.37$ eV

$\Rightarrow N_C = 2.99 \pm 0.11$



A. Bernstein, A. Holstein, arXiv:1112.4809 (2011)

Constructing the QCD Lagrangian

Instead of $SU(3)_{\text{flavour}}$ we use $SU(3)_{\text{colour}}$, theory must be invariant under gauge transformations in colour space

Theory is fully determined by the Lagrangian

$$S = \int dt L = \int dt d^3x \mathcal{L} = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi) \quad \Rightarrow \quad \boxed{\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}}$$

Euler-Lagrange equations: lead to equations of motion

Quark fields: three colours $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Construct the Lagrangian analogous to QED:

Free Lagrangian: $\mathcal{L}_0 = \bar{\psi} \left[i\gamma_\mu \partial^\mu - m \right] \psi$

Constructing the QCD Lagrangian

Require invariance under SU(3)$_c$ gauge transformations:

$$\psi(x) \longrightarrow \tilde{\psi}(x) = U\,\psi(x)$$

With $U = \exp\left[\; ig\sum_a \theta_a(x)\frac{\lambda_a}{2}\right]$ and $\theta_a(x)$ a real function and $a = 1,..,8$

Eight generators $t_a = \lambda_a/2$ of SU(3), with the Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Lie algebra:

$$\mathrm{tr}\{\lambda_a\} = 0$$

$$\mathrm{tr}\{\lambda_a\,\lambda_b\} = 2\,\delta_{ab}$$

$$\left[\lambda_a\,,\,\lambda_b\right] = 2\,i\,f_{abc}\,\lambda_c$$

structure constants:

$$f_{123} = 1$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = \\ = f_{345} = -f_{367} = \frac{1}{2}$$

$$f_{458} = f_{678} = \sqrt{\frac{3}{2}}$$

Constructing the QCD Lagrangian

Lagrangian has to stay invariant under transformations

Infinitessimal gauge transformation:

$$U = \exp\left[ig \sum_a \theta_a(x) t_a \right] = 1 + ig \sum_a \theta_a(x) t_a + \dots$$

$$\widetilde{\psi}_a(x) = \psi_a(x) + ig \sum_b \theta_b(x) t^b \psi_a(x)$$

Derivative of the transformed field:

$$\partial^\mu \widetilde{\psi}_a(x) = \partial^\mu \psi_a(x) + ig \sum_b t^b \theta_b(x) \partial^\mu \psi_a(x) - ig \sum_b t^b \left(\partial^\mu \theta_b(x) \right) \psi_a(x)$$

transforms differently than the field → $\mathcal{L}_0$ is not gauge-invariant

Solution: Introduction of eight gauge fields: $A_a(x)$

Gauge transformation $\widetilde{A}^\mu = U A^\mu U^{-1} + \frac{i}{g} (\partial^\mu U) U^{-1}$

$$\widetilde{A}^\mu_a(x) = A^\mu_a(x) + ig \sum_{b,c} f_{abc} \theta_b(x) A^\mu_c(x) - \partial^\mu \theta_a(x)$$

Constructing the QCD Lagrangian

Similarly to QED construct the covariant derivative

$$D^\mu = \partial^\mu + ig \sum_a A_a^\mu(x) t^a$$

Gauge invariance given by $\widetilde{D^\mu \psi_a} = \partial^\mu \widetilde{\psi}_a + ig \sum_b \widetilde{A}_b^\mu(x) t^b \widetilde{\psi}_a$

We find that $\widetilde{D^\mu \psi_a} = D^\mu \psi_a + ig \sum_a \theta_a(x) t^a D^\mu \psi_a = U\left(D^\mu \psi_a\right)$

⇒ the derivative transforms similar to the quark fields

We arrive at the Lagrangian $\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m_j)\psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$

with $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu$

BUT this Lagrangian is again not invariant under $SU(3)_c$!

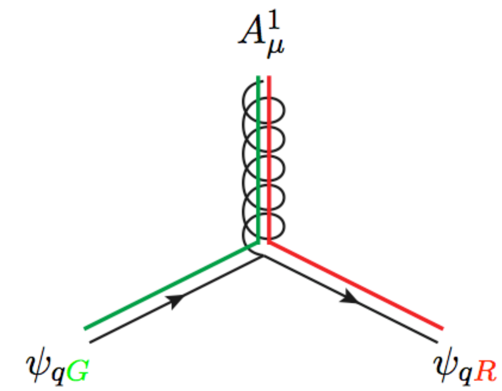
Reason: transformation of field strength tensor

$$\widetilde{A}_a^\mu(x) = A_a^\mu(x) + ig \sum_{b,c} \underline{f_{abc} \theta_b(x) A_c^\mu(x)} - \partial^\mu \theta_a(x)$$

Constructing the QCD Lagrangian

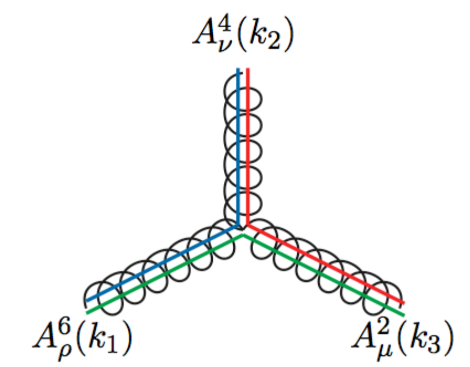
Add an additional term:

$$G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g \sum f_{abc} A_b^\mu A_c^\nu$$



And finally we have the full Lagrangian of QCD

$$\mathcal{L} = \sum_f \bar{\psi}_f (i\gamma^\mu \mathcal{D}_\mu - m_f)\psi_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$



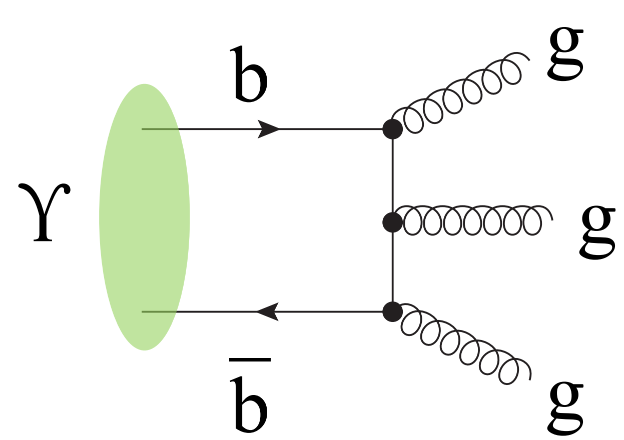
Gauge invariance lead to

- introduction of 8 gluon fields: $\{3\} \times \{3\} = 8 \oplus 1$ possible fields, but only the octet fields carry colour
- gluons are massless, since a term $m_g A_a^\mu A_\mu^a$ would violate gauge invariance
- same coupling strength g for quark-gluon and gluon-gluon interaction
- renormalisability - a non-Abelian gauge theory is renormalisable if it is gauge invariant (t'Hooft)

The Missing Piece: Gluons

Decay of the $\Upsilon(9.46)$

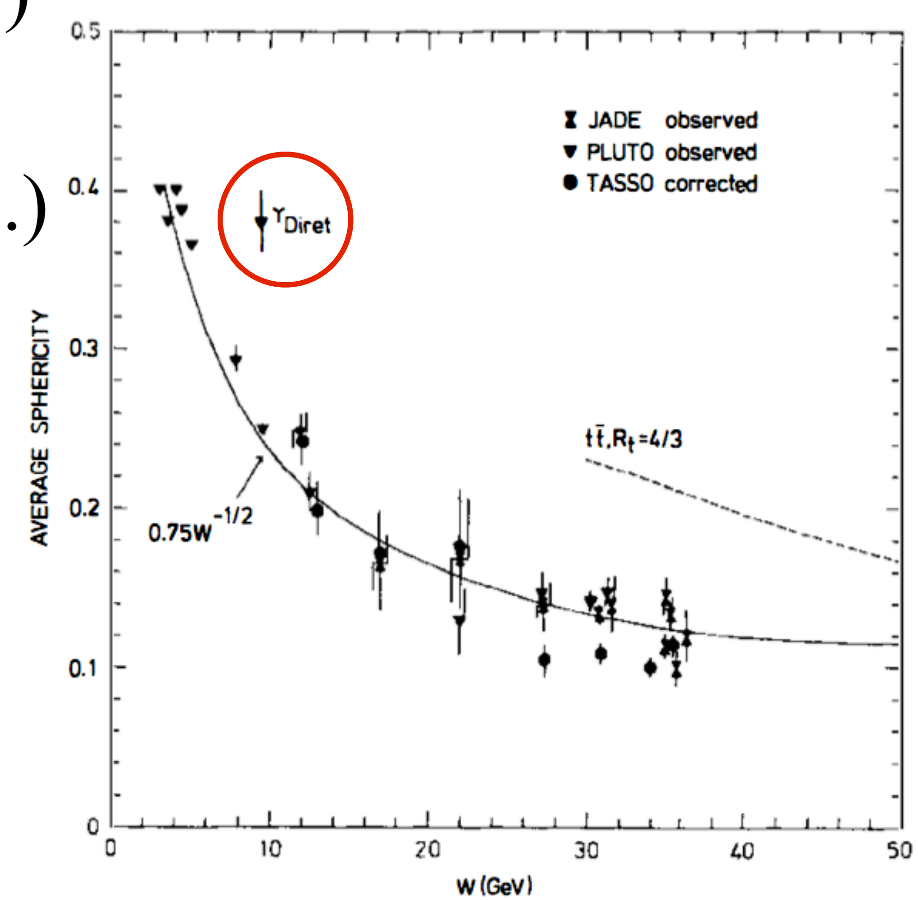
If gluons existed, the primary decay would be through three gluons



Decay width (from DORIS):

$$\frac{\Gamma(\Upsilon \to ggg \to \text{hadrons})}{\Gamma(\Upsilon \to e^+e^-)} = \frac{(10\pi^2 - 90)\alpha_s^3}{81\pi e_q^2 \alpha^2} \approx 34 \text{ (the.)}$$

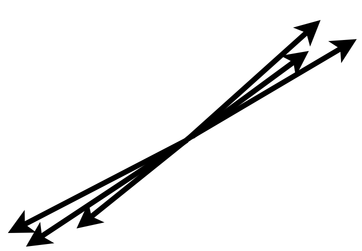
$$\frac{\Gamma(\Upsilon \to ggg \to \text{hadrons})}{\Gamma(\Upsilon \to e^+e^-)} = \frac{39^{+19}_{-10}}{1.29 \pm 0.14} = 30^{+20}_{-10} \text{ (exp.)}$$



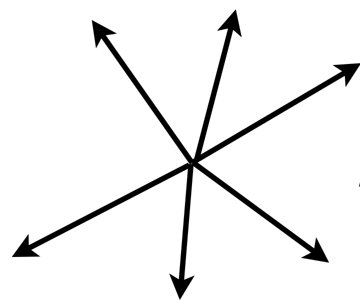
P. Söding, G.Wolf, Ann.Rev.Nucl.Part.Sci.31,231 (1981)

Event shape: Sphericity

$$S = \frac{3\sum P_{T,i}^2}{2\sum P_i^2}, 0 < S < 1$$



$S = 0$

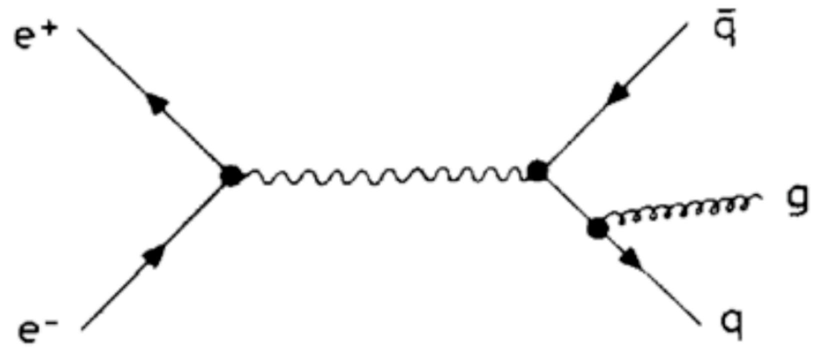


$S = 1$

The Missing Piece: Gluons

Three-Jet Events in e^+e^-

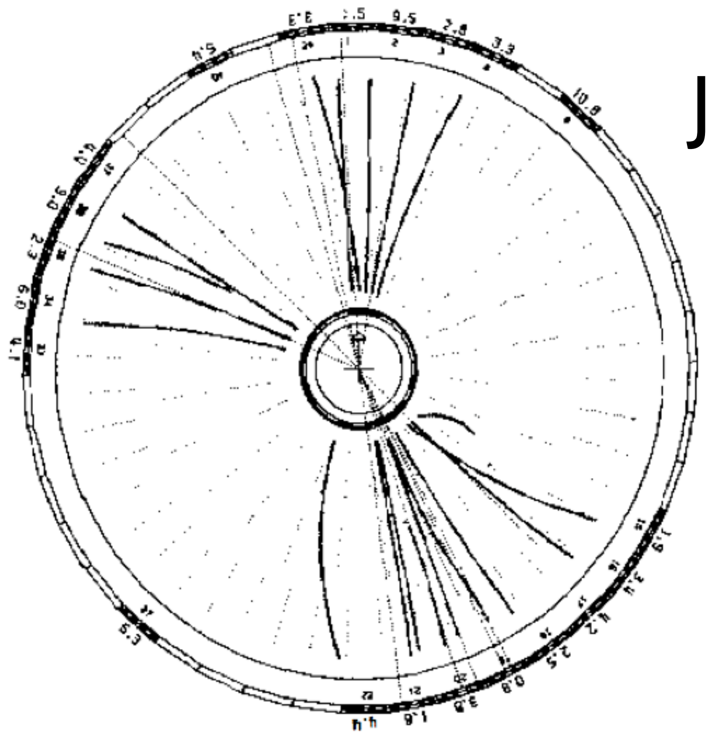
Radiation of a gluon leads to 3-jet structure



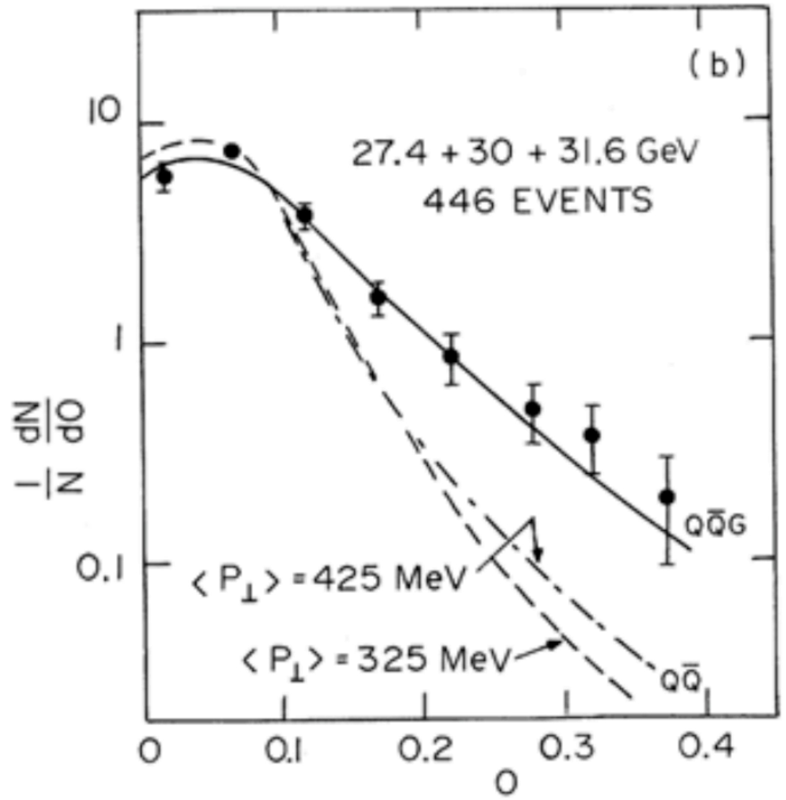
First observed at PETRA (higher CMS energy than at DORIS)

Oblateness: $O = F_{\mathrm{major}} - F_{\mathrm{minor}}$

O is small for 2-jet events and becomes larger for 3-jet events, proportional to the P_T of the radiated gluon



JADE



D. P. Barber (Mark-J), Phys.Lett.B89, 139(1979)

Summary of Part I

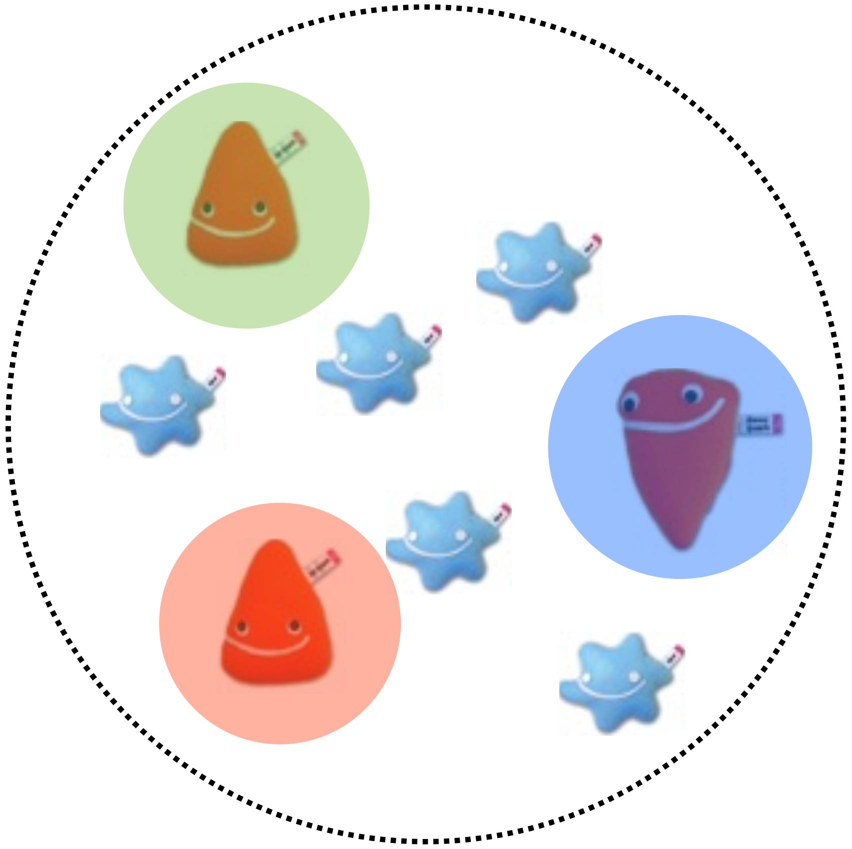
We saw that...

Quarks - 6 flavours, massive spin-1/2 particles ✓

Gluons - massless spin 1 particles ✓

3 colours ✓

Hadrons: composite particles made of quarks and gluons ✓



... and found QCD

Beautiful field theory with local gauge invariance, but can it explain

- quasi-free partons observed in DIS ?
- non-observation of free quarks and gluons ?
- formation of jets and production of hadrons in particle collisions ?