Thermal conductivities in nonideal astrophysical plasmas

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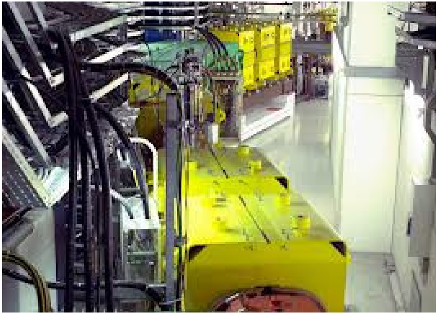
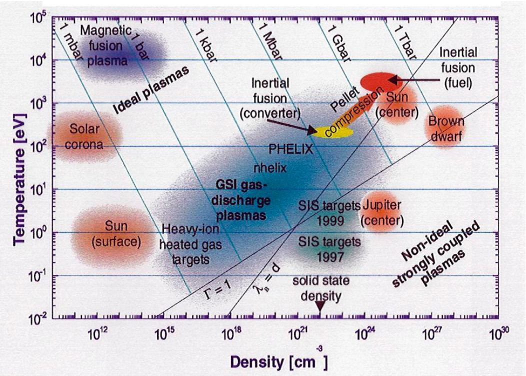


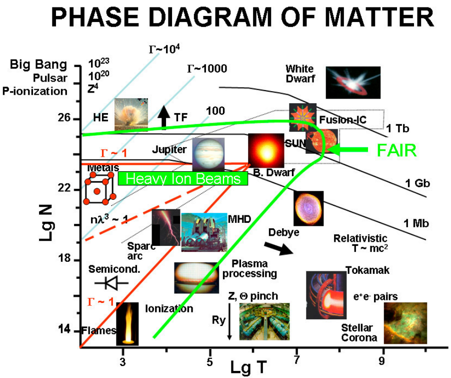
The Fragment Separator at GSI which will be replaced by the Super Fragment Separator of the Facility for Antiproton and Ion Research FAIR (Photo: A. Zschau/GSI)

Plasma classification



Parameters of nonideal plasmas (courtesy V. Fortov)

Plasma parameters of FAIR experiments



Phase diagram of plasmas (D.H.H. Hoffmann 2009)

Material destruction analysis related to future Super-Fragment-Separator experiments

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In the Super-FRS target and in the beam catchers at GSI stress waves are generated by intense, fast-extracted ion beams which deposit a high amount of energy within a very short time into target material.

- measurements of thermal parameters
- contactless electrical resistivity measurements
- registration of thermal parameters (specific heats, heat capacities, thermal conductivity

Inelastic thermal spike model for heat transport

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- electrons and lattice are two coupled subsystems
- kinetic energie of the projectiles deposited into electrons
- electron thermalization within about 10^{-15} s
- transfer of electron energy to the ions by electron-phonon coupling
- in lattice thermal equilibrium within 10^{-13} s
- The heat distribution in the electron and lattice subsystems described by classical heat transport equations

Inelastic thermal spike model for heat transport

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Heat transport equations for cylindrically symmetric penetration of projectiles into the target:

$$C_e(T_e)\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[rK_e(T_e)\frac{\partial T_e}{\partial r}\right] - g(T_e - T_a) + A(r,t),$$
$$C_a(T_a)\frac{\partial T_a}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[rK_a(T_a)\frac{\partial T_a}{\partial r}\right] + g(T_e - T_a).$$

- electron-ion energy exchange described by $g(T_e - T_a)$, g - coupling parameter
- T_j, C_j, K_j are temperature, specific heat coefficient, thermal conductivity of the electrons ($j = e$) and the lattice ($j = a$)
- $A(r,t)$ - spacio-temporal energy deposition rate. Most of the electrons deposit their energy close to the ion path within 10^{-15} s
- $\int dt \int d\vec{r}\, A(r,t)$ - total energy loss by the electrons.

Inelastic thermal spike model for heat transport

Recently used parameters:

- $K_e = C_e D_e$ (D_e - electron diffusivity)
- hot electrons in the conduction band of insulators: $C_e \approx 1.5 k_B n_e \approx 1$ J cm^{-3}K^{-1} ($n_e \approx 1$ - number of excited electrons per atoms)
- $D_e = v_F d \approx 2$cm^{2}s^{-1}, v_F - Fermi velocity, d - inter-atomic distance
- $\lambda_{ea}^2 = C_e D_e / g$, λ_{ea} - electron-phonon mean free path
- parameters of the lattice C_a, K_a, solid and liquid mass density, melting and vaporisation temperature (latent heat, sublimation energy) taken from experiments

Calculation of the equation of state and specific heats

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Virial expansion in density order 5/2 known of

$$F(V, n_e, n_i, T)$$

From there, equation of state (Meister et al., Astron. Nachr., 1999), inner energy

$$U(T, V, N) = F(T, V, N) + TS(T, V, N), \quad S = -\left(\frac{\partial F}{\partial T}\right)_{V,N=\mathrm{const}} .$$

and specific heat capacities as well as adiabatic coefficient found

$$c_v = \left(\frac{\partial U}{\partial T}\right)_{V,N=\mathrm{const}}, \quad c_p - c_V = -T\left(\frac{\partial p}{\partial T}\right)^2_{V,N=\mathrm{const}} \Big/ \left(\frac{\partial p}{\partial V}\right)_{T,N=\mathrm{const}},$$

$$\gamma_t = \frac{c_p}{c_v}, \quad \gamma_t = \frac{\rho}{p}\left(\frac{\partial p}{\partial \rho}\right)_{T=\mathrm{const}}, \quad \rho - \text{ mass density.}$$

Calculation of heat conductivity

Electrical currents j_E and heat currents j_Q are related to the corresponding generalized forces, the gradient of the temperature in the material ∇T and the gradient of the electrochemical potential $\nabla\zeta = -\nabla(\varphi + \mu/e)$, where $\varphi(r)$ is the external potential and $\mu(r)$ the chemical potential, by the linear relations (in isotropic systems)

$$j_E = e^2 L_{11} \nabla\zeta - eL_{12} \frac{\nabla T}{T}, \quad j_Q = eL_{21} \nabla\zeta - eL_{22} \frac{\nabla T}{T}.$$

Electrical conductivity σ, thermopower α, and thermal conductivity K defined by Onsager coefficients $L_{ik} = L_{ki}$ (Kraeft et al., Quantum statistics of charged particles, Akademie-Verlag 1986; Reinholz et al., Phys. Rev. E52, 5368-5386, 19959

$$\sigma = e^2 L_{11}, \quad \alpha = \frac{1}{eT} \frac{L_{12}}{L_{11}}, \quad K = \frac{1}{T} \left(L_{22} - \frac{L_{12} L_{21}}{L_{11}} \right).$$

Calculation of thermal conductivity

By non-equilibrium statistical mechAnics the transport coefficients are determined by the relevant scattering processes of the particles in the system. an interpolation between Spitzer and Ziman-Faber formula is found.

$$\sigma_S = 0.591 \frac{(4\pi\varepsilon_o)^2}{e^2} \frac{k_B T}{m_e}^{3/2} \frac{1}{\ln(3/\gamma)}$$

$$\sigma_Z = \frac{3}{4\sqrt{2\pi}} \frac{(4\pi\varepsilon_o)^2 (k_B T)^{3/2}}{e^2 \sqrt{m}_e \Phi}$$

$$\Phi = \frac{2V}{N_e} \left(\frac{m_e k_B T}{2\pi\hbar} \right) \int_0^\infty dE(k) \left(-\frac{df(E)}{dE} \right) \int_0^{2k_F} dq\, q^3 |V_{ei}|^2(q) \varepsilon_o / e^2.$$

Zubarev formalism for electrical conductivity

It is assumed to use the linear response theory to calculate the thermal conductivity. Therefore the Zubarev formalism will be used, as it takes also non-mechanical perturbations account.

Transport coefficients are expressed by force-force correlation functions, which are found in T-matrix approximation for the particle interactions. Numerical results are obtained for hydrogen and the alkali plasmas.

$$L = \frac{e^2}{k_B^2 T} \frac{K}{\sigma}.$$

Lorenz numbers L

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- non-degenerate plasma: $L = 4$ for electron-ion-interaction
- non-degenerate plasma: $L = 1.5966$ adding electron-electron interaction (Reinholz et al. 1995)
- degenerate plasmas with free electrons: $L = \pi^2/3$ ($L^* = 2.44 \cdot 10^{-8}$ WΩ/K^2).

metal	κ [W/m K]	$10^{-8} L^*$ [WΩ/T^2]	metal	κ [W/m K]	$10^{-8} L^*$ [WΩ/T^2]
Al	238	2.14	Na	138	2.12
Ag	418	2.31	Pb	38	2.47
Au	310	2.35	Pt	-	2.51
Cd	100	2.42	Sn	64	2.52
Cu	385	2.23	Nb	52	2.90
Fe	80	2.61	64	2.57	
In	88	2.58	W	-	3.04
Mo	-	2.61	Zn	1.13	2.31

Experimental values of thermal conductivity κ and Lorenz number $L^* = \kappa/(\sigma \cdot T)$ at $T = 272$ K (Kaye et al., 1966)

Electrical conductivity

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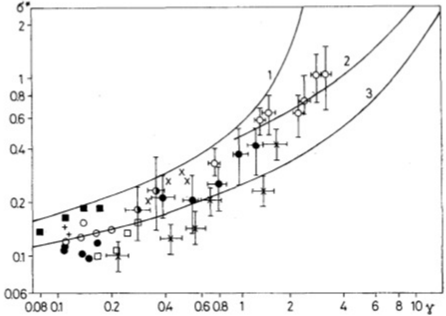
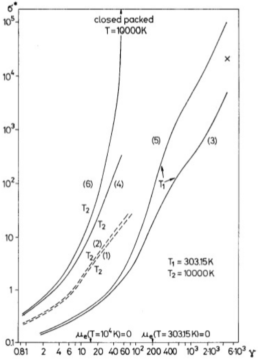


σ^* as function of γ. Comparison of theoretical results for Cs at $T = 10^4$ K with experimental values for Cs and inert gases. Theory at $T = 10^4$ K: (1) Spitzer thory, (2) statically screened T-matrix approximation, (3) statically screened Born approximation. Experiments: Black square - Ar at 11750 K $\leq T \leq$ 15920 K [Günther et al., 1981]; with error bares at 25000 K [Ivanov et al., 1976], black point - Ar, white point - Xe, black-white point - Ne; black point - Ar bei 12800 K $\leq T \leq$ 17400 K [Bakeev and Rovinskii, 1970]; white point - Xe bei 9000 K $\leq T \leq$ 13700 K [Bakeev and Rovinskii, 1970]; *x* with error bars - Cs at 4000 K $\leq T \leq$ 25000 K [Seshenov et al., 1975]; + - H at 15400 K $\leq T \leq$ 21500 K [Radtke et al., 1981]; white square - air at 13500 K $\leq T \leq$ 18800 K [Andreev and Gavrilova, 1975]; *x* - C_2H_2Cl at 37000 K $\leq T \leq$ 39000 K [Ogurzova et al., 1974]. (C.-V. Meister and G. Röpke, 1982)

Electrical conductivity



Influence of the structure factor $S(q)$ on the normalized electrical conductivity σ^*. Dotted line: Coulomb potential for (1) $S(q) = 1$ and (2) $S(q)$ in Debye approximation. Full line: Hellmann potential for (3,4) $S(q) = 1$ and (5,6) $S(q)$ in mean spherical approximation (MSA). x - experimental value for cesium at the melting point. The Hellmann potential is recently used to describe the transport coefficients of aluminum.

Outlook

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- A model for the electrical conductivity of warm dense matter is given. The thermal conductivities are yet estimated multiplying with the relevant Lorenz numers.
- The presented models for thermal and transport parameters have to be further developed for non-fully ionized plasmas of high density and lower temperatures.
- New models for the coupling coefficients between electrons and ion grids in solids have to be proposed.

THANK YOU VERY MUCH FOR YOUR ATTENTION

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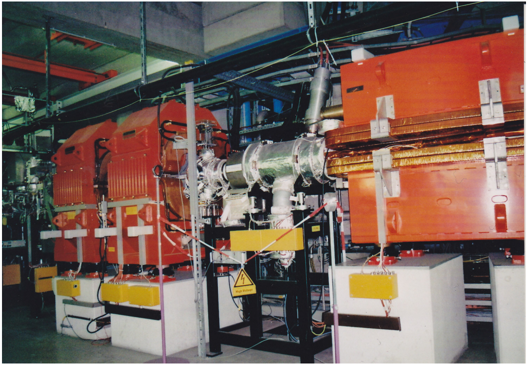

ESR at GSI. FRS is before ESR (photo: CVM 2010)