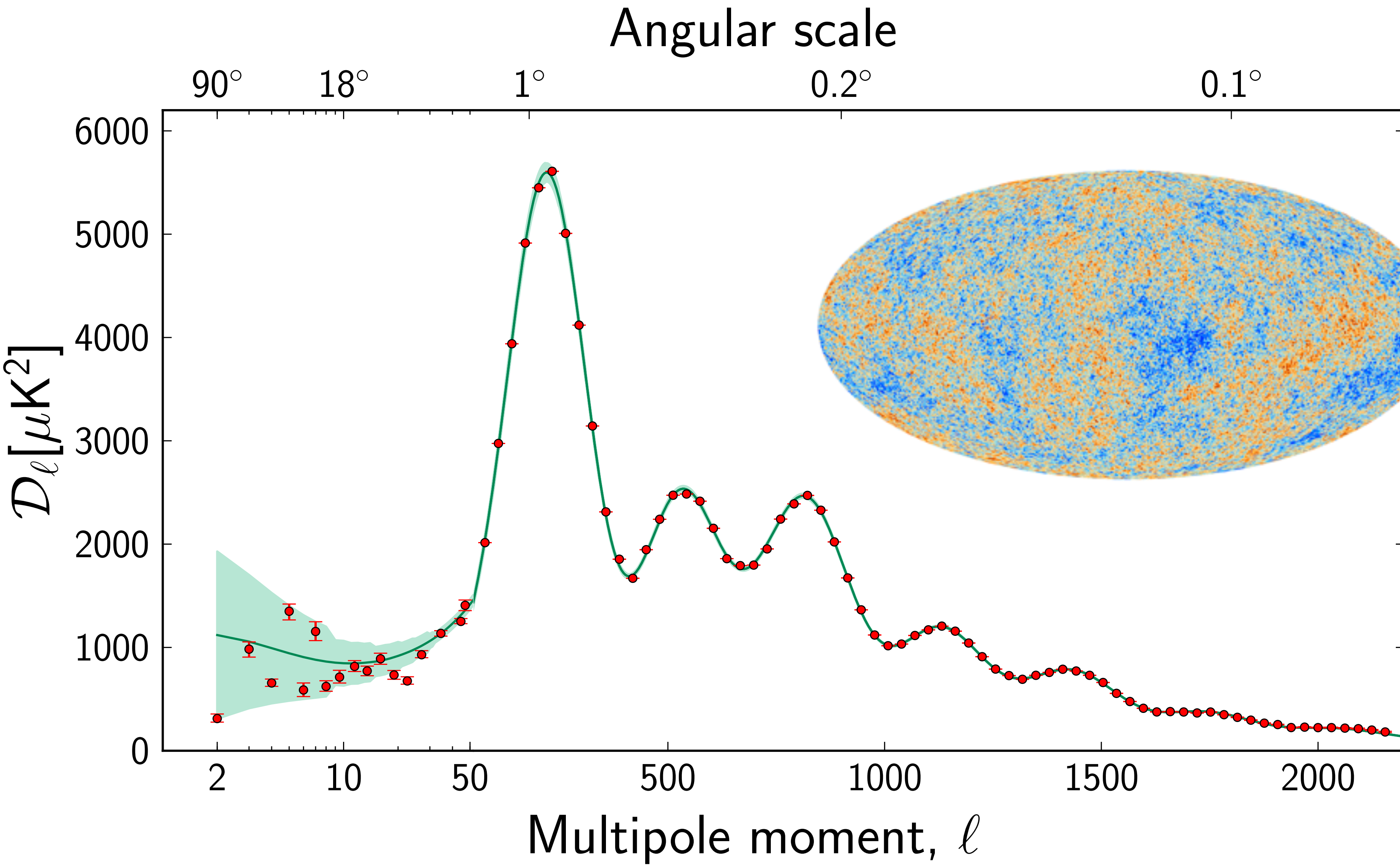
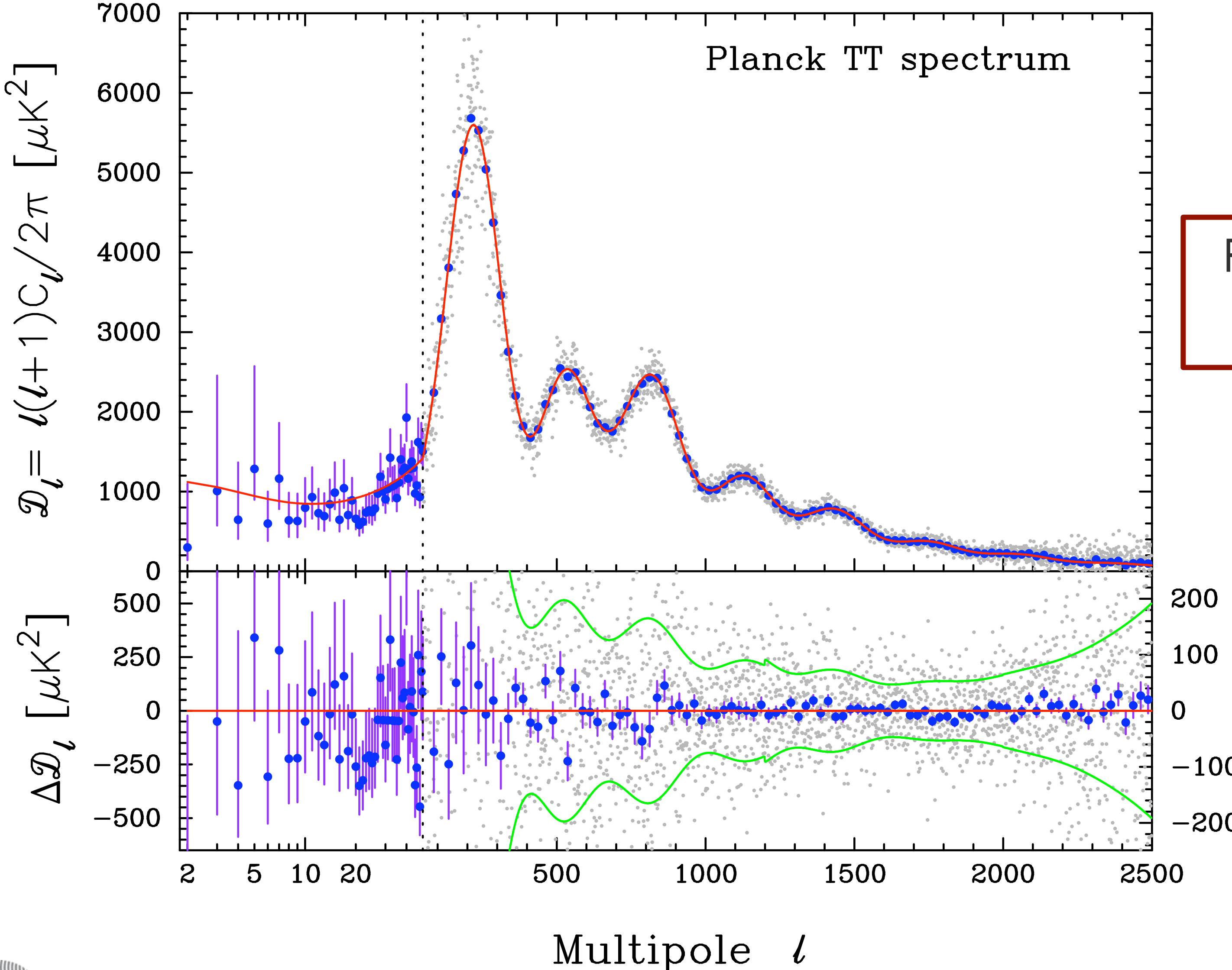

Angular scale
90°
18°
1°
0.2°
0.1°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell [\mu K^2]$
2
10
50
500
1000
1500
2000
Multipole moment, ℓ

ΛCDM is a very good fit



Reduced χ^2 =1.09
for ~ 7500 dof

Table 1. Cosmological parameters used in our analysis. For each, we give the symbol, prior range, value taken in the base ΛCDM cosmology (where appropriate), and summary definition (see text for details). The top block contains parameters with uniform priors that are varied in the MCMC chains. The ranges of these priors are listed in square brackets. The lower blocks define various derived parameters.

Parameter	Prior range	Baseline	Definition
$\omega_b \equiv \Omega_b h^2$	[0.005, 0.1]	...	Baryon density today
$\omega_c \equiv \Omega_c h^2$	[0.001, 0.99]	...	Cold dark matter density today
$100\theta_{MC}$	[0.5, 10.0]	...	$100 \times$ approximation to r_*/D_A (CosmoMC)
τ	[0.01, 0.8]	...	Thomson scattering optical depth due to reionization
Ω_K	[−0.3, 0.3]	0	Curvature parameter today with $\Omega_{tot} = 1 - \Omega_K$
$\sum m_\nu$	[0, 5]	0.06	The sum of neutrino masses in eV
$m^{eff}_{\nu,\,sterile}$	[0, 3]	0	Effective mass of sterile neutrino in eV
w_0	[−3.0, −0.3]	−1	Dark energy equation of state[a], $w(a) = w_0 + (1-a)w_a$
w_a	[−2, 2]	0	As above (perturbations modelled using PPF)
N_{eff}	[0.05, 10.0]	3.046	Effective number of neutrino-like relativistic degrees of freedom (see text)
Y_P	[0.1, 0.5]	BBN	Fraction of baryonic mass in helium
A_L	[0, 10]	1	Amplitude of the lensing power relative to the physical value
n_s	[0.9, 1.1]	...	Scalar spectrum power-law index ($k_0 = 0.05\mathrm{Mpc}^{-1}$)
n_t	$n_t = -r_{0.05}/8$	Inflation	Tensor spectrum power-law index ($k_0 = 0.05\mathrm{Mpc}^{-1}$)
$dn_s/d\ln k$	[−1, 1]	0	Running of the spectral index
$\ln(10^{10}A_s)$	[2.7, 4.0]	...	Log power of the primordial curvature perturbations ($k_0 = 0.05\,\mathrm{Mpc}^{-1}$)
$r_{0.05}$	[0, 2]	0	Ratio of tensor primordial power to curvature power at $k_0 = 0.05\,\mathrm{Mpc}^{-1}$
Ω_Λ		...	Dark energy density divided by the critical density today
t_0		...	Age of the Universe today (in Gyr)
Ω_m		...	Matter density (inc. massive neutrinos) today divided by the critical density
σ_8		...	RMS matter fluctuations today in linear theory
z_{re}		...	Redshift at which Universe is half reionized
H_0	[20,100]	...	Current expansion rate in $\mathrm{km\,s^{-1}Mpc^{-1}}$
$r_{0.002}$		0	Ratio of tensor primordial power to curvature power at $k_0 = 0.002\,\mathrm{Mpc}^{-1}$
$10^9 A_s$		...	$10^9 \times$ dimensionless curvature power spectrum at $k_0 = 0.05\,\mathrm{Mpc}^{-1}$
$\omega_m \equiv \Omega_m h^2$		...	Total matter density today (inc. massive neutrinos)
z_*		...	Redshift for which the optical depth equals unity (see text)
$r_* = r_s(z_*)$		...	Comoving size of the sound horizon at $z = z_*$
$100\theta_*$		...	$100 \times$ angular size of sound horizon at $z = z_*$ (r_*/D_A)
z_{drag}		...	Redshift at which baryon-drag optical depth equals unity (see text)
$r_{drag} = r_s(z_{drag})$		...	Comoving size of the sound horizon at $z = z_{drag}$
k_D		...	Characteristic damping comoving wavenumber (Mpc^{-1})
$100\theta_D$		...	$100 \times$ angular extent of photon diffusion at last scattering (see text)
z_{eq}		...	Redshift of matter-radiation equality (massless neutrinos)
$100\theta_{eq}$		...	$100 \times$ angular size of the comoving horizon at matter-radiation equality
$r_{drag}/D_V(0.57)$		...	BAO distance ratio at $z = 0.57$ (see Sect. 5.2)

[a] For dynamical dark energy models with constant equation of state, we denote the equation of state by w and adopt the same prior as for w_0.

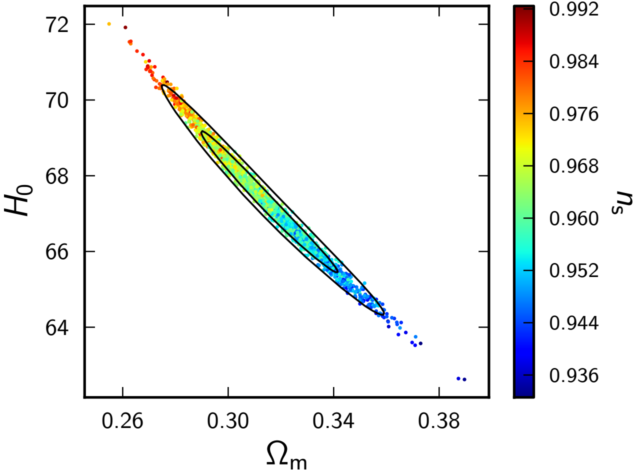


Fig. 3. Constraints in the Ω_m–H_0 plane. Points show samples from the *Planck*-only posterior, coloured by the corresponding value of the spectral index n_s. The contours (68% and 95%) show the improved constraint from *Planck*+lensing+WP. The degeneracy direction is significantly shortened by including WP, but the well-constrained direction of constant $\Omega_m h^3$ (set by the acoustic scale), is determined almost equally accurately from *Planck* alone.

ΛCDM

Planck +WP

$\Omega_b h^2$ = 0.02205 ± 0.00028

$\Omega_c h^2$ = 0.1199 ± 0.0027

n_s = 0.960 ± 0.007

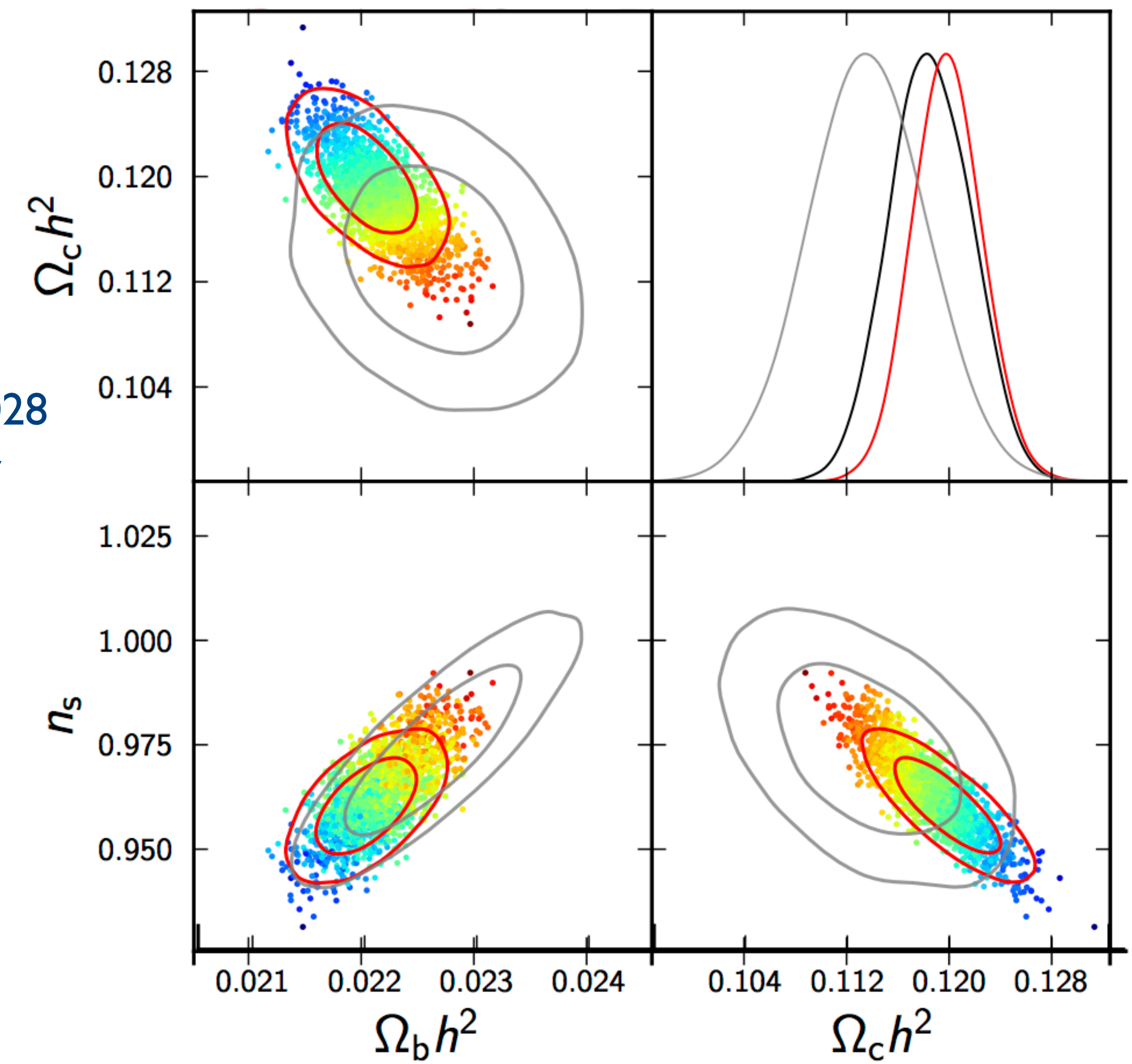
τ = 0.089 ± 0.014

$10^9 A_s$ = 2.20 ± 0.06

H_0 = 67.3 ± 1.2

Ω_Λ = 0.685 ± 0.017

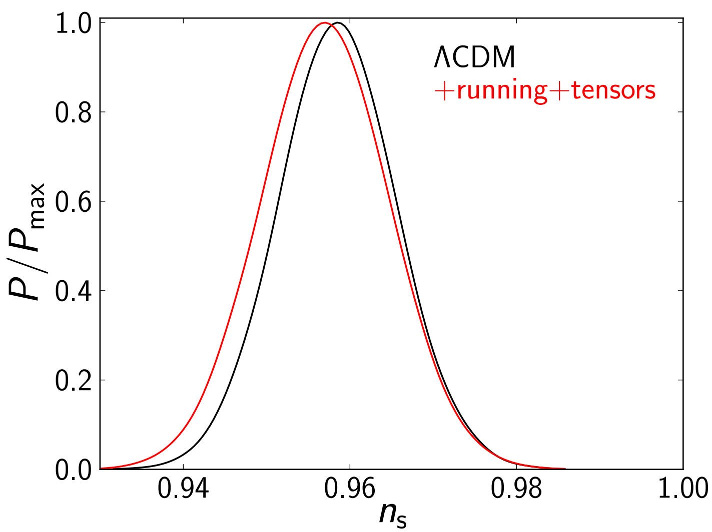
σ_8 = 0.829 ± 0.012

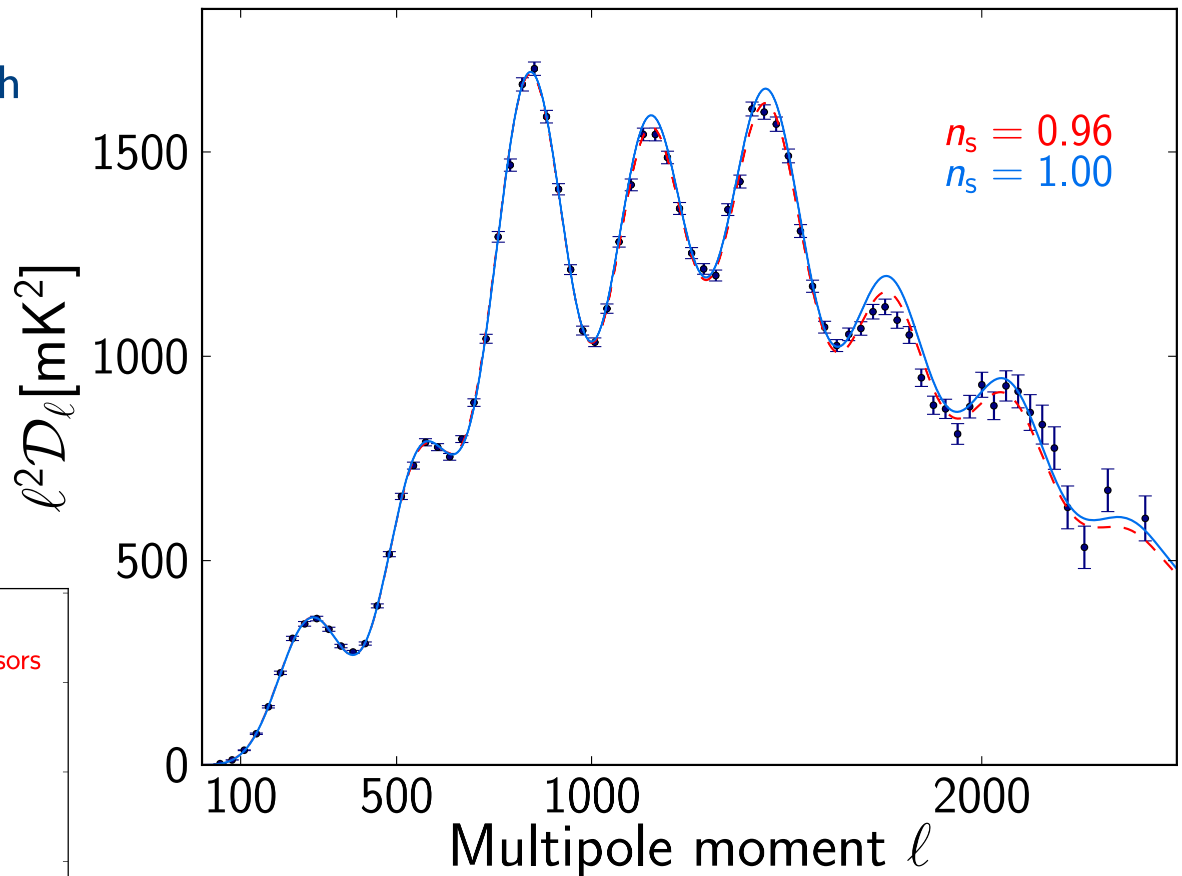


Scalar spectral index: n<1

Harrison-Zel'dovich: too much power on small scales
Ruled out at >5σ

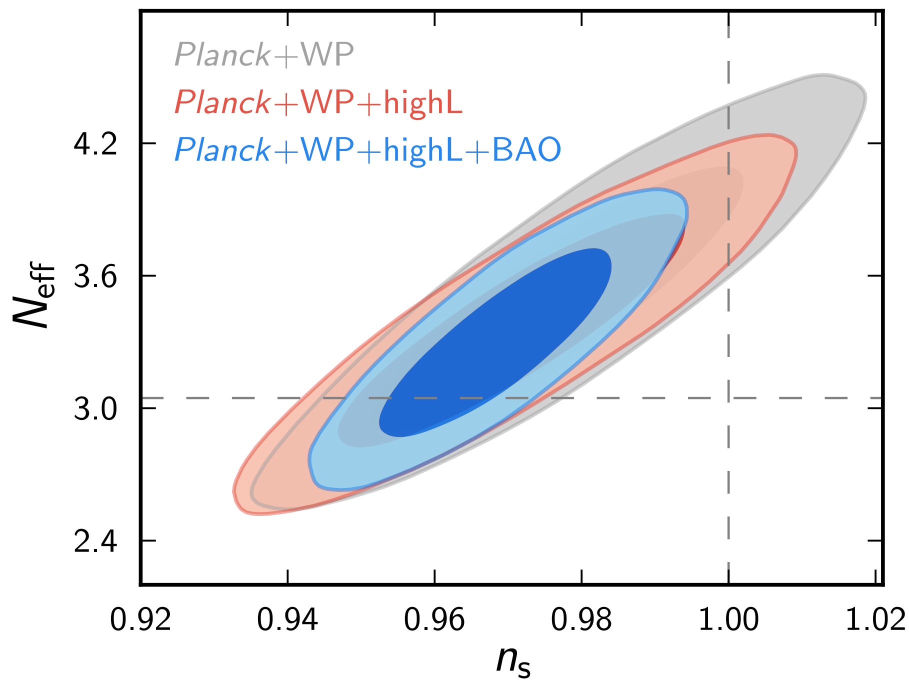
n_s = 0.960 ± 0.007

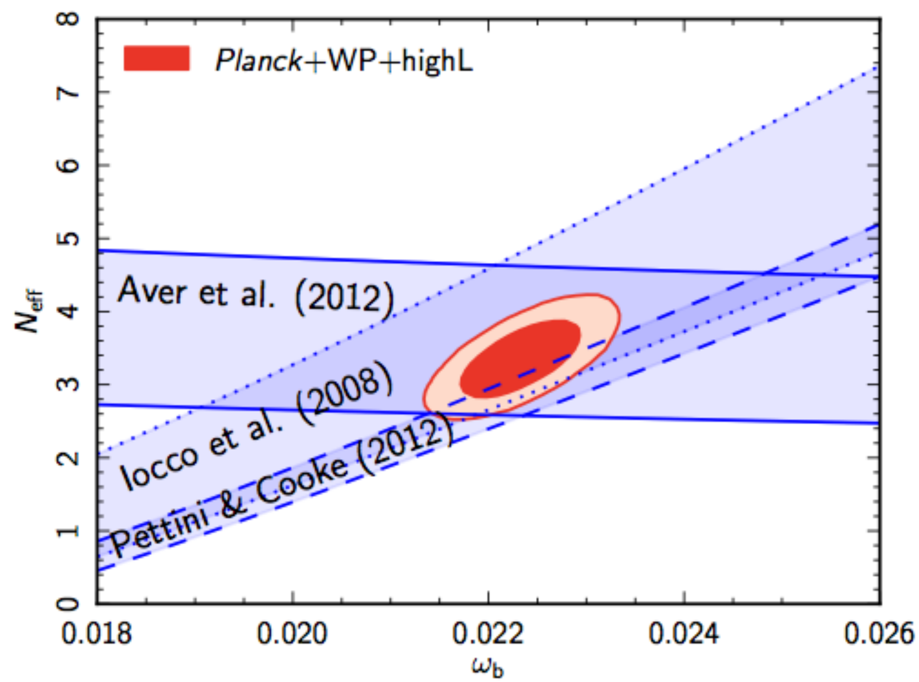




Relativistic species

$$\rho_{rel} = \left[\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma$$





More species, longer radiation domination; suppress early acoustic oscillations in primary CMB; have anisotropic stress

N_{eff} = 3.36 ± 0.34 (68%, Planck+WP+highL)
N_{eff} = 3.30 ± 0.27 (+BAO)

Spatial curvature

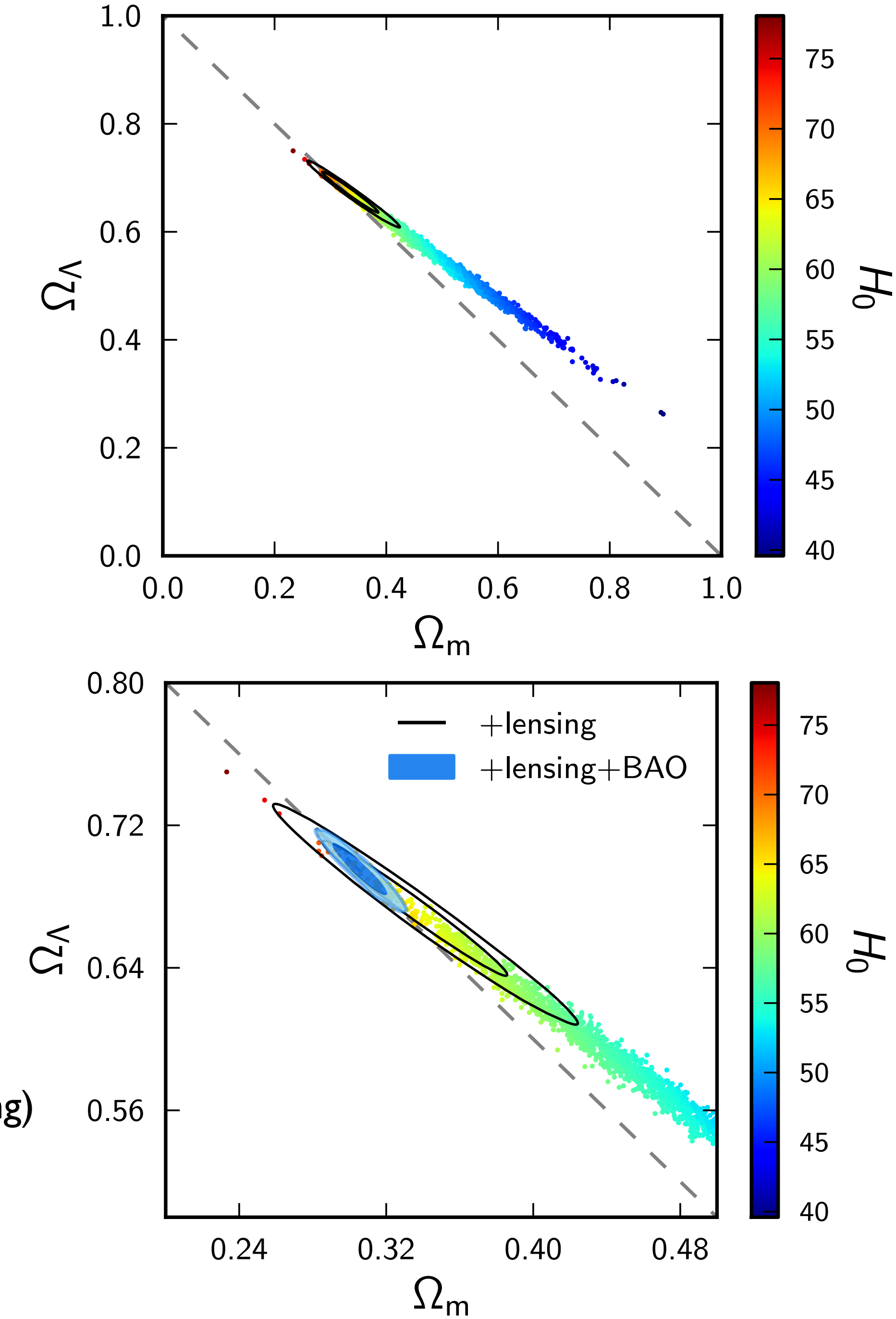
With primary CMB, cannot measure curvature.

Planck measures curvature through lensing
(more closed, less dark energy → more lensing)

(i) smears out the small-scale TT peaks and moves power to small scales

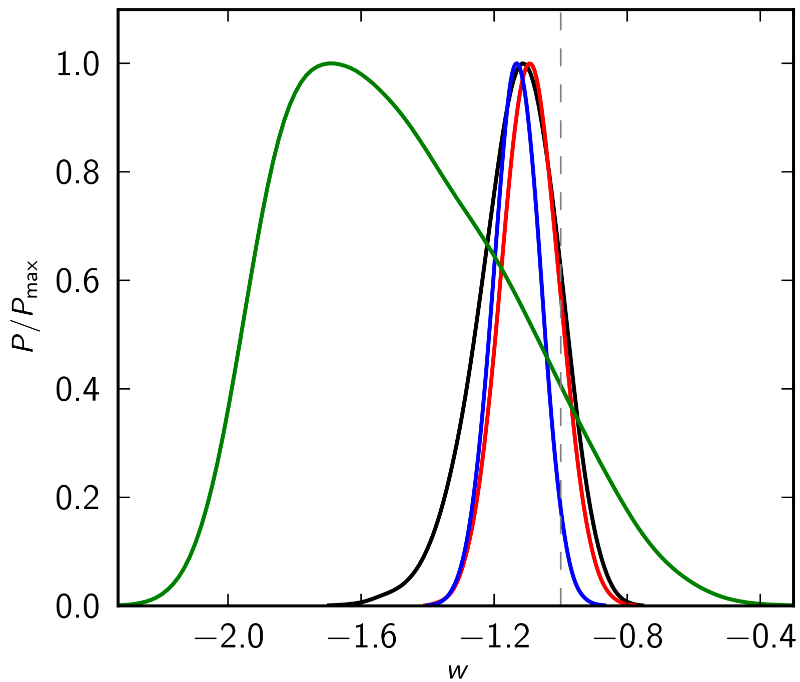
(ii) boosts the deflection power spectrum (about double at $\Omega_\Lambda=0$)

Ω_k = -0.01 ± 0.009 (68%, Planck+WP+highL+lensing)
Ω_k = -0.001 ± 0.0032 (+BAO)



Dark energy

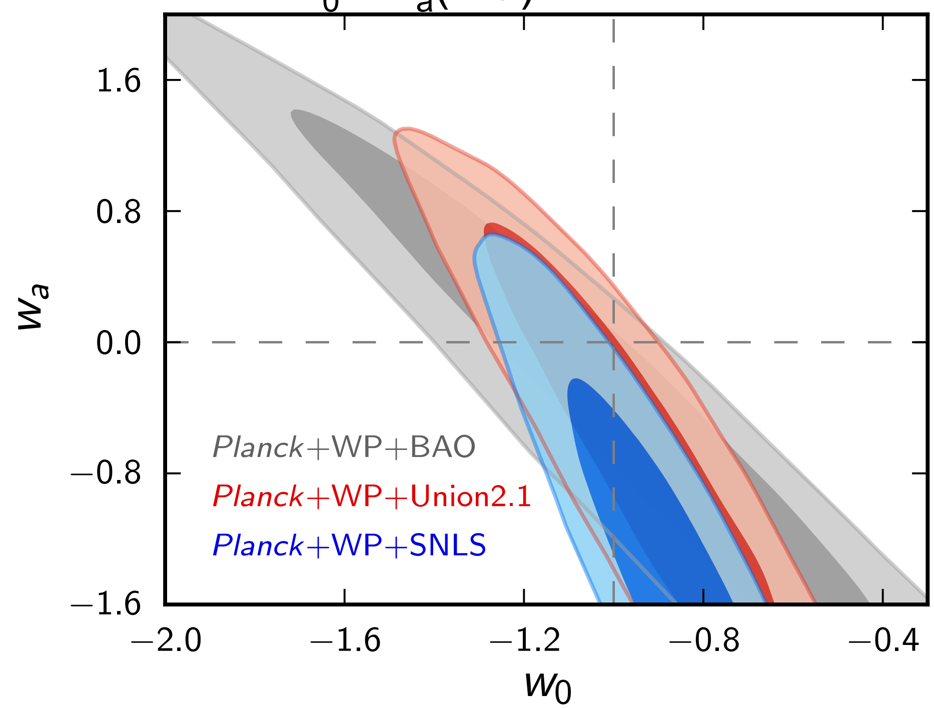
1: w=const



w = -1.13 ± 0.12 (68%, Planck+WP+BAO)

SNLS (blue) favours phantom dark energy, w<-1

2: w=w$_0$ +w$_a$(1-a)

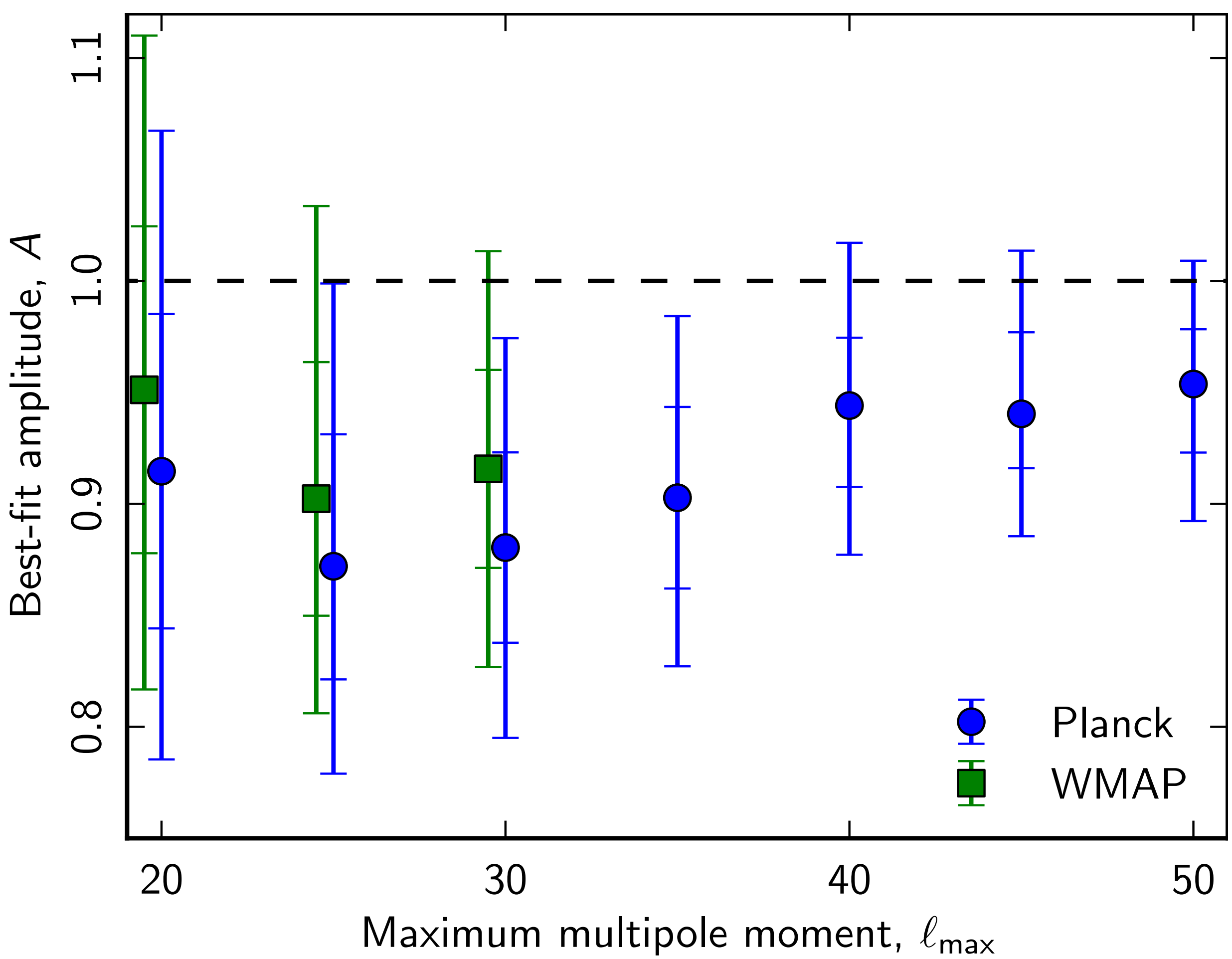


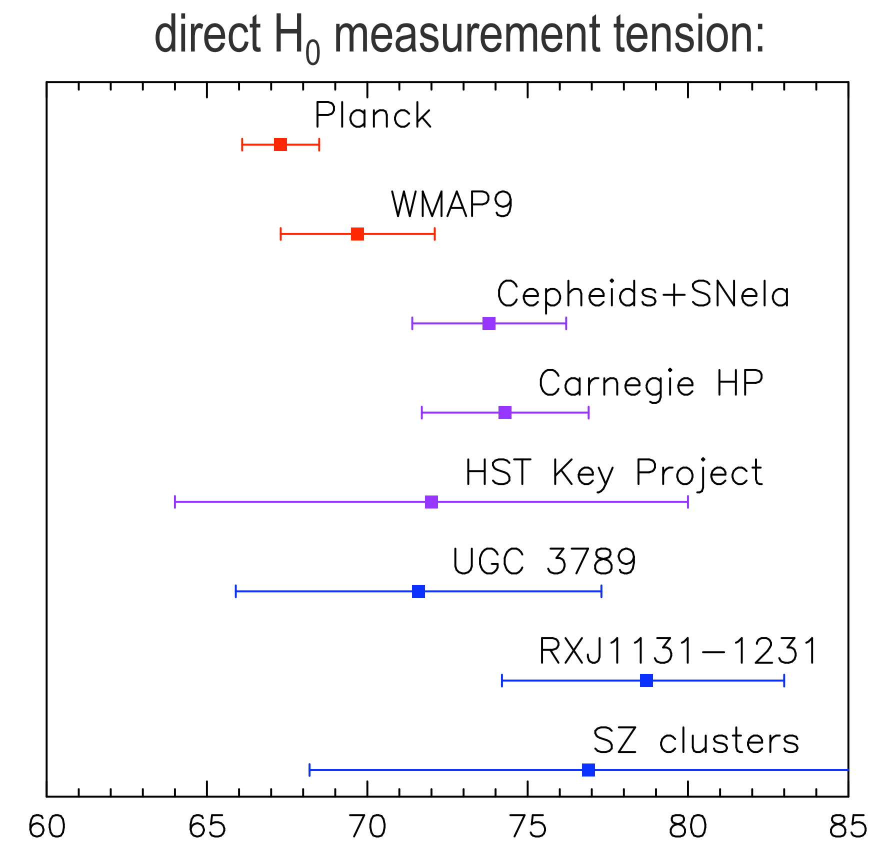
3: Early dark energy

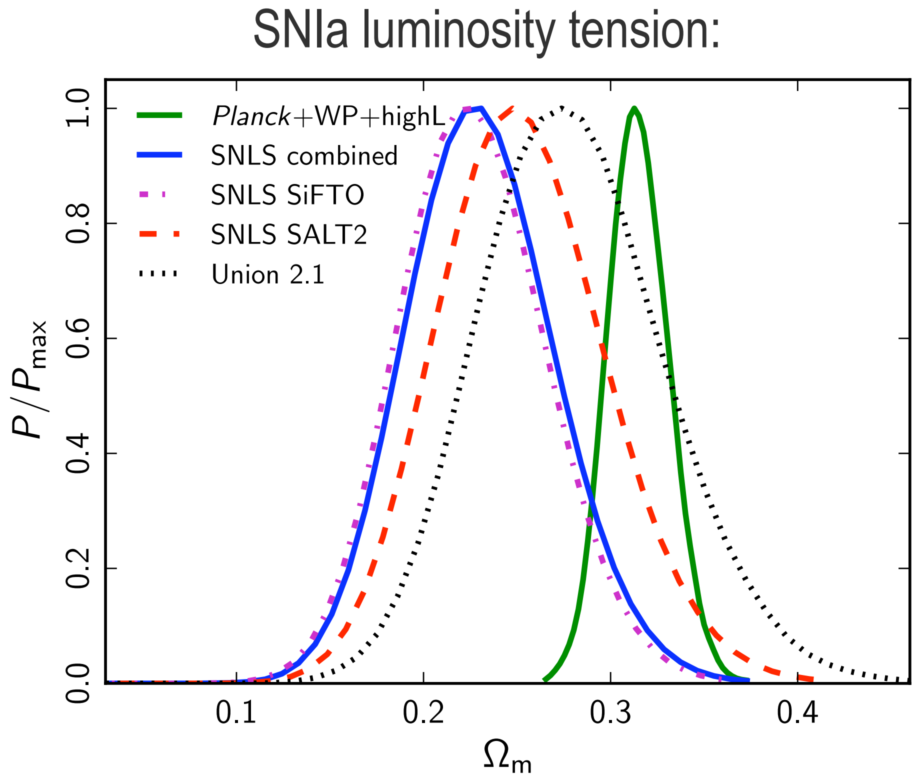
$$\Omega_{\rm de}(a) = \frac{\Omega^0_{\rm de} - \Omega_{\rm e}(1 - a^{-3w_0})}{\Omega^0_{\rm de} + \Omega^0_{\rm m} a^{3w_0}} + \Omega_{\rm e}(1 - a^{-3w_0}) \ .$$

$$\Omega_{\rm e} < 0.009 \quad (95\%; \textit{Planck}\text{+WP+highL}).$$

Low-ell ‘anomaly’: 2-3σ low




direct H_0 measurement tension:
Planck
WMAP9
Cepheids+SNeIa
Carnegie HP
HST Key Project
UGC 3789
RXJ1131−1231
SZ clusters
60
65
70
75
80
85



SNIa luminosity tension:
Planck+WP+highL
SNLS combined
SNLS SiFTO
SNLS SALT2
Union 2.1
P / P_max
Ω_m
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.2
0.3
0.4

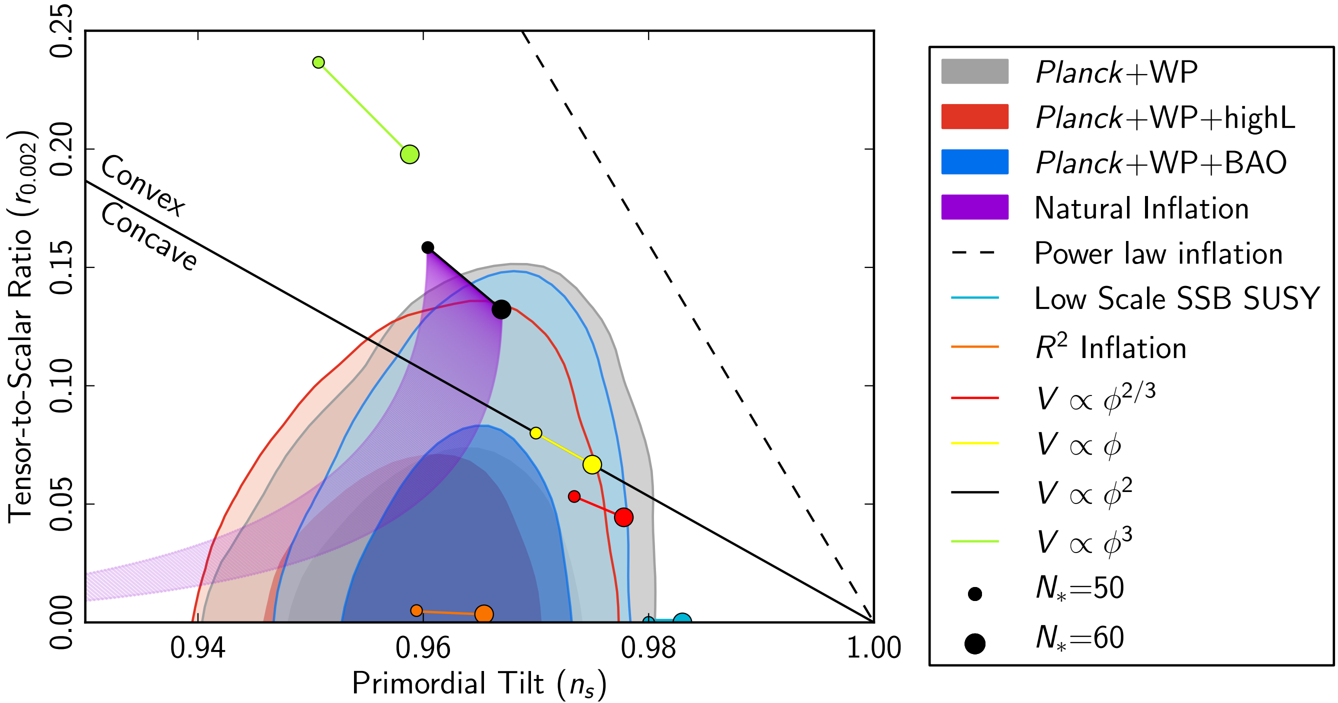


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

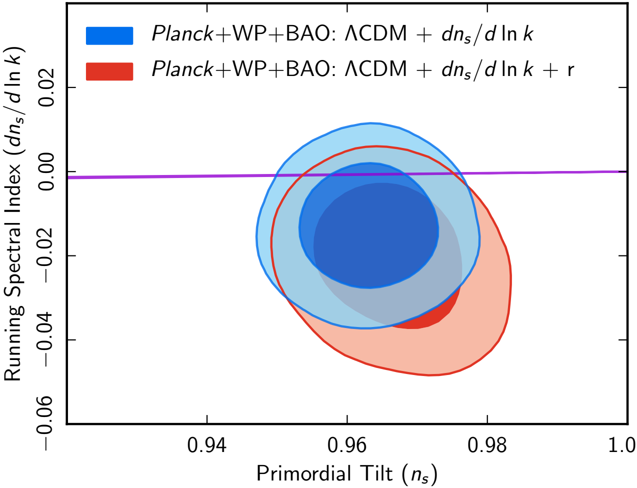


Fig. 2. Marginalized joint 68% and 95% CL for $(\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k\,, n_\mathrm{s})$ using *Planck*+WP+BAO, either marginalizing over r or fixing $r = 0$ at $k_* = 0.038\ \mathrm{Mpc}^{-1}$. The purple strip shows the prediction for single monomial chaotic inflationary models with $50 < N_* < 60$ for comparison.

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck*+WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + $\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	n_s	0.9561 ± 0.0080	0.9615 ± 0.0072	0.9548 ± 0.0073	0.9596 ± 0.0063
	$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	-0.0134 ± 0.0090	-0.0094 ± 0.0085	-0.0149 ± 0.0085	-0.0130 ± 0.0090
	$-2\Delta\ln\mathcal{L}_\mathrm{max}$	-1.50	-0.77	-2.95	-1.45
ΛCDM + $\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$ + $\mathrm{d}^2n_\mathrm{s}/\mathrm{d}\ln k^2$	n_s	$0.9514^{+0.087}_{-0.090}$	$0.9573^{+0.077}_{-0.079}$	$0.9476^{+0.086}_{-0.088}$	$0.9568^{+0.068}_{-0.063}$
	$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	$0.001^{+0.016}_{-0.014}$	$0.006^{+0.015}_{-0.014}$	$0.001^{+0.013}_{-0.014}$	$0.000^{+0.016}_{-0.013}$
	$\mathrm{d}^2n_\mathrm{s}/\mathrm{d}\ln k^2$	$0.020^{+0.016}_{-0.015}$	$0.019^{+0.018}_{-0.014}$	$0.022^{+0.016}_{-0.013}$	$0.017^{+0.016}_{-0.014}$
	$-2\Delta\ln\mathcal{L}_\mathrm{max}$	-2.65	-2.14	-5.42	-2.40
ΛCDM + r + $\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	n_s	0.9583 ± 0.0081	0.9633 ± 0.0072	0.9570 ± 0.0075	0.9607 ± 0.0063
	r	< 0.25	< 0.26	< 0.23	< 0.25
	$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$	0.021 ± 0.012	0.017 ± 0.012	$-0.022^{+0.011}_{-0.010}$	$-0.021^{+0.012}_{+0.010}$
	$-2\Delta\ln\mathcal{L}_\mathrm{max}$	-1.53	-0.26	-3.25	-1.5

Table 5. Constraints on the primordial perturbation parameters for ΛCDM+$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$, ΛCDM+$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$+$r$ and ΛCDM+$\mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k$+$\mathrm{d}^2n_\mathrm{s}/\mathrm{d}\ln k^2$ models from *Planck* combined with other data sets. Constraints on the spectral index and its dependence on the wavelength are given at the pivot scale of $k_* = 0.05\ \mathrm{Mpc}^{-1}$.

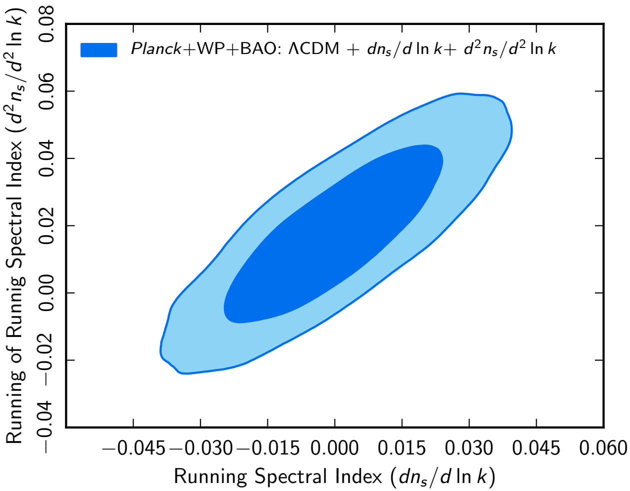


Fig. 3. Marginalized joint 68% and 95% CL regions for $(\mathrm{d}^2n_\mathrm{s}/\mathrm{d}\ln k^2\,, \mathrm{d}n_\mathrm{s}/\mathrm{d}\ln k)$ using *Planck*+WP+BAO.

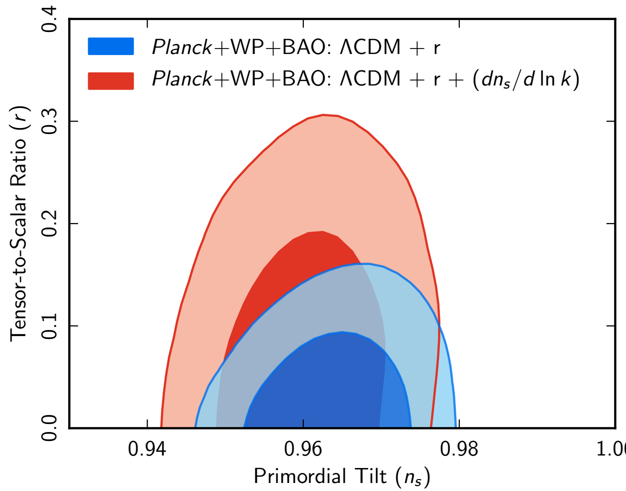


Fig. 4. Marginalized joint 68% and 95% CL regions for $(r\,, n_\mathrm{s})$, using *Planck*+WP+BAO with and without a running spectral index.