Beyond Inflation

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We present a model that realizes an Open Inflationary Universe and study the possible observable signatures.

Outline:

(I) Bubble formation

(II) The Universe after the tunneling

(III) Signatures

i- Value of Ω_k today

ii- CMB Spectrum

iii- Bubble collisio

References:

[1] Phys. Rev D 21 (1980), Coleman, de Luccio
"Gravitational effects on and of vacuum decay"

[2] "Tunneling and gravity" (Workshop seminar, Vacuum transition)

[3] "Observational Consequences of a Landscape"
hep-th/0505232 v2 Susskind et al.

[4] "CMB in open Inflation"
astro-ph/9901135, Linde, Sasaki and Tanaka

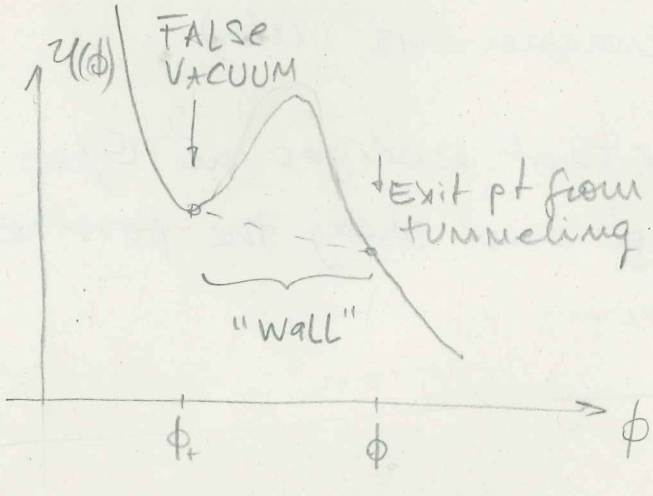
[5] "A toy model for Open Inflation"
hep-ph/9807493, Linde.

[6] "Open Inflation in the Landscape"
hep-th/1103.2674, Sasaki et al.

[7] "Watching worlds collide"
hep-th/0810.512 Kleban et al

[8] "Towards observable signatures of other bubble universes"
hep-th/041347 Aguirre et al

(I) Bubble formation



A possible way to realize an open Universe is through a tunneling process. This is called "bubble nucleation" and the solutions are described by the well known "CDL" (Coleman-de Luccia) instantons.

The solution is constructed using WKB approx., in which the probability density is given by:

$$\Gamma \propto e^{-B/\hbar} \quad / \quad B = S_E(\phi) - \underbrace{S_E(\phi_+)}_{\text{value of action at false vacuum}}$$

$$t \to i\tau \qquad U \to -U$$

We take as solution of ϕ the one that minimizes B i.e. the one that maximises the probability density

$\Rightarrow$ ϕ has $O(4)$ symmetry (and so does the metric)

$$\phi = \phi(\tau) \quad ; \quad ds^2 = d\tau_E^2 + a^2(\tau_E)\, d\Omega_3 \to g_{\mu\nu}$$

(τ: radius of 4-sphere)

$$d\Omega_3 = (d\chi_E^2 + \sin^2\chi_E\, d\Omega_2^2)$$

The Euclidean action in the presence of gravity is given by:

$$S_E = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu \phi + U(\phi) - \frac{8\pi G}{3} R \right]$$

Replacing with $g_{\mu\nu}$:

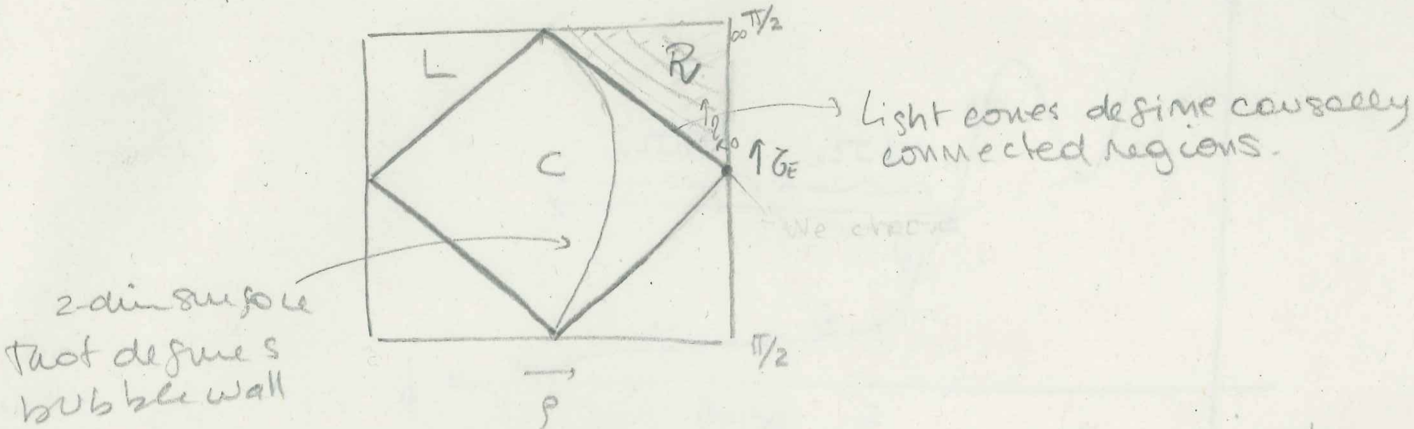
$$S_E = 2\pi^2 \int d\tau_E \left[a^3 \left(\tfrac{1}{2}(\dot{\phi})^2 + \mathcal{U}(\phi) + \frac{3}{8\pi G}(a^2\ddot{a} + a\dot{a}^2 - a) \right) \right] \quad (\dot{\ }) = \frac{d}{d\tau_E}$$

The euclidean background equations are given by:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{d\mathcal{U}}{d\phi} = 0$$ → ≡ KG field eq. in an expanding Universe with potential $(-\mathcal{U})$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{a^2} + \frac{8\pi G}{3}\left(\tfrac{1}{2}(\dot{\phi})^2 - \mathcal{U}\right)$$ → ≡ FRW equation with $k = -1$

We can draw the Penrose diagram ~~for the euclidean sol~~ in the $O(4)$ symm. ~~background~~ in which global coordinates can be defined



We choose $\tau = 0$ as the moment the bubble nucleates
⇒ R corresponds to the region inside the bubble.
L " outside

It can be shown that bubble wall moves at c ⇒ an observer from the inside can never reach the wall

The background geometry and the field configuration are obtained by the analytical continuation into the Lorentzian regime.

The coordinates in the Lorentzian regime are given by:

$$\begin{cases} \tau_E = \tau_C = -\tau_R - \frac{\pi}{2}i = \tau_L + \frac{\pi}{2}i \\ \chi_E = -i\chi_C + \pi/2 = -i\chi_R = -i\chi_L \\ a_E = a_C = ia_R = ia_L \end{cases} \quad \Rightarrow$$

The coordinate τ defined by: $dt = a(\tau)\,d\tau$

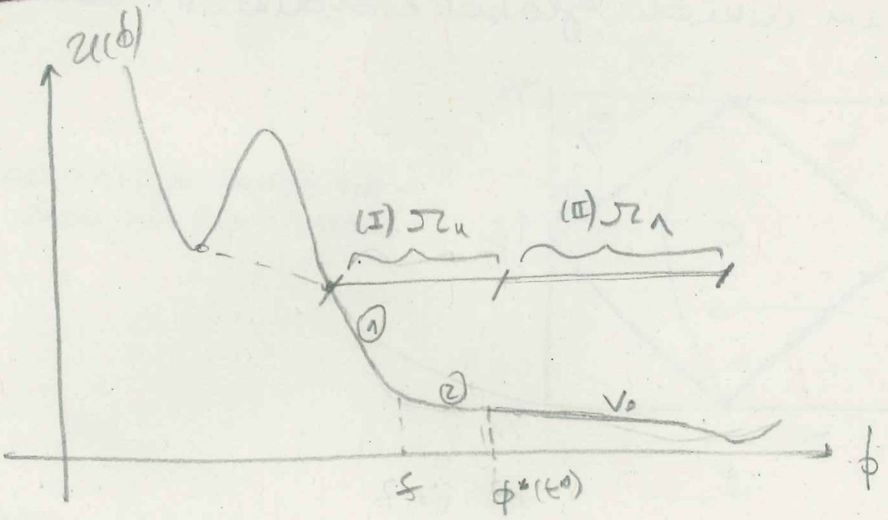
⇒ On the interior of the bubble:

$$ds^2 = -dt^2 + a^2(t)\,(d\chi^2 + \sinh^2\chi\, d\Omega_2^2)$$

$$\Rightarrow \boxed{ds^2 = a^2(\eta)\,(-d\eta^2 + d\chi^2 + \sinh^2\chi\, d\Omega_2^2)} \rightarrow \text{Open Universe!}$$

⇒ Our starting point is a bubble, whose interior looks like a open Universe and whose evolution will depend on the ~~form~~ behaviour of the potential at the exit point of the tunneling process.

(II) The Universe after the tunneling



Toy model:

① $\ddot{\phi} + 3H\dot{\phi} - V/f = 0$; $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\dot{\phi}^2 + V\right) + \frac{-k}{a^2}$ $(k=-1)$

② $\ddot{\phi} + 3H\dot{\phi} = 0$

"Λ" term "curvature term"

⇒

(I) Curvature domination

① Fast Roll – Tunneling brings us to a point in which $H^2 < V''$

② Slow Roll – ϕ arrives to the plateau, but curvature term is still significant

At small times $a(t) \sim t \Rightarrow H \sim \frac{1}{t}$ $\Rightarrow$ $\boxed{r_{hubble} \cong 1}$

Eom for ϕ: $\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{friction term} \to \infty} + V = 0 \Rightarrow \dot{\phi}$ small

(II) Λ domination

Eventually the curvature term will fall below the potential energy and the eqs will become dominated by effective Λ on the plateau. This happens at a time we call t^*:

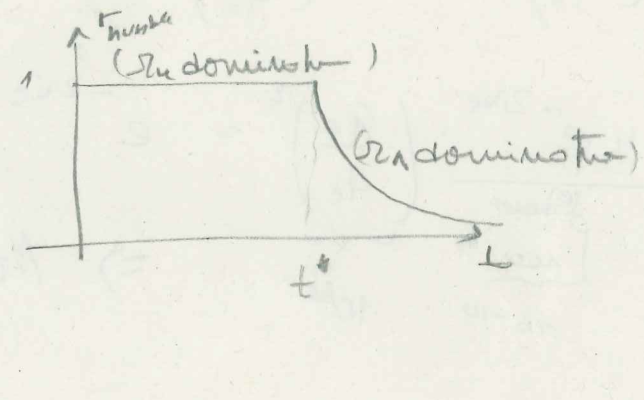
$$t^* \approx \sqrt{\frac{3}{8\pi G V_0}}$$

$$a(t) \sim t^* e^{H(t-t^*)}$$

$$H^2 = \frac{8\pi G}{3} V_0$$

$$\Rightarrow \boxed{r_{hubble} = e^{H(t^*-t)}}$$

Putting together (I) + (II) the horizon evolution is give by



(III) Signatures

i- Value of Ω_k today

Could we "see" Ω_k?

To answer this question we relate the suppression of the value of Ω_k with the # e-folds the Universe has undergone during inflation

$$a_e = e^{N_e} \quad \text{(value of } a \text{ at the end of inflation)}$$

For $a > a_e$ the major contribution to the energy density comes from radiation term.

$$\rho(a_e) = \underbrace{\rho_\Lambda (-k) e^{-2N_e}}_{\rho_k} + \underbrace{\rho_{rad}(a_e)}_{\lesssim V(\phi) >> \rho_k} \quad , \quad \rho_\Lambda \simeq \rho^{(e)}_{rad.} \equiv \rho_{rad.}(a_e)$$

$$H^2 = \rho_{crit} \sim \rho_{rad}$$

$$\Rightarrow \text{normalizing} \quad H^2 = \rho^e_{rad}\left(\frac{a_e}{a}\right)^4$$

$$\Omega_k(a) \equiv \frac{\rho_k(a)}{\rho_{crit}} \sim \frac{\rho_k(a)}{\rho_{rad}} = \frac{-k\, e^{-2N_e} \rho^e_{rad.}\left(\frac{a_e}{a}\right)^2}{\rho^e_{rad}\left(\frac{a_e}{a}\right)^4} = \frac{-k e^{-2N_e}\left(\frac{a_e}{a}\right)^2}{\left(\frac{a_e}{a}\right)^4}$$

$$= -k e^{-2N_e}\left(\frac{a}{a_e}\right)^2$$

$\Lambda_{GUT} \sim 10^{-3} M_{Pl}$

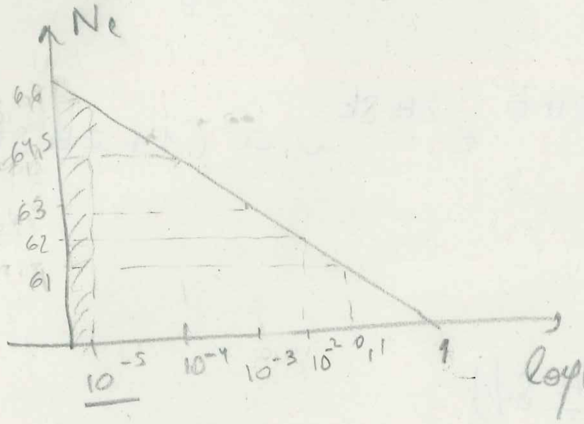
$$\rho^e_{rad} \sim 10^{-10} \quad ; \quad \rho^{(0)}_{rad} \sim 10^{-120}$$

$$\rho^0_{rad} = \rho^e_{rad}\left(\frac{a_e}{a_0}\right)^4 \sim 10^{-10}\left(\frac{a_e}{a_0}\right)^4 \sim 10^{-120} \Rightarrow \left(\frac{a_0}{a_e}\right) \sim 3 \times 10^{-28}$$

$$\therefore \quad \Omega_k = -k e^{-2N_e} \underbrace{\left(\frac{a_0}{a_e}\right)^2}_{10^{55}} \sim e^{-2N_e + 126}$$

$$\Rightarrow N_e \gtrsim 63$$

$$\Rightarrow \left\{ N_e = -\frac{1}{2}\ln(\Omega_u) + 60 \right\}$$



With only a few e-folds more that the lowest bound we couldn't detect Ω_u!

(iii) CMB Spectrum

Anisotropies in CMB spectrum originate from fluctuations on the energy-momentum tensor, ~~in~~ related to fluctuations of the inflaton field. In order to study these perturbations one expands the field in mode functions and compute the fluctuations around the vacuum. Of course the modes are defined in all scales, but if we recall the Penrose diagram of our original $O(4)$-space we see that the region our coordinates don't describe the whole space.

⇒ We compute the scalar perturbations in the (c) region and then analytically continue the solution to get the expression in our region (R)

Scalar fluctuations (motivation for a good expression)

We write the perturbations around the background metric (Newton-gauge) as:

$$g_{\mu\nu} = \begin{pmatrix} (1+2\Phi) & \\ & a^2(1+2\Phi)\underset{\sim}{g_{ij}} \end{pmatrix}$$

$\mathcal{R}$ ☆

During inflation a perturbation on the scale factor can be written as:

$$a^2 + \delta a^2 = e^{2H(t+\Delta t)} = e^{2Ht}\, e^{2H\delta t} \sim a^2(1 + 2H\delta t)$$

$$\delta t \sim \frac{\delta\phi}{\dot\phi}$$

$$\Rightarrow \; a^2 + \delta a^2 \sim a^2\left(1 + 2\frac{H}{\dot\phi}\delta\phi\right)$$

Comparing with the expression in the metric

$$(*) \quad \mathcal{R} \sim \frac{H}{\dot\phi}\delta\phi \;, \quad \Delta_{\mathcal{R}} = |\mathcal{R}|^2 \frac{p^3}{2\pi^3}$$

↑ power spectrum of scalar pert.

Using the expression (*) and the expression of the metric in the region (c), i.e

$$ds^2 = dt_c^2 + \sin^2 t_c\,(-dx_c^2 + \cosh^2 x_c\, d\Omega_2)$$

we are ready to compute the scalar perturbation modes:

$$q = \sum \hat a_{plm}\, q^p(\eta)\, f^{pl}(x_c)\, Y_{lm}(\Omega_2) + h.c.$$

The spatial eigenfunctions q^p satisfy

$$\left[-\frac{d^2}{d\eta_c^2} + U(\eta_c)\right] q^p = p^2 q^p \;\Big/\; U_s = 4\pi G\phi'^2 + \phi'\left(\frac{1}{\phi'}\right)'' - 4$$

and the temporal f^{pl}:

$$\left[\frac{-1}{\cosh^2 x_c}\frac{\partial}{\partial x_c}\cosh^2 x_c \frac{\partial}{\partial x_c} - \frac{l(l+1)}{\cosh^2 x_c}\right] f^{pl} = (p^2+1) f^{pl}$$

$Y_{lm}(\Omega_2)$ = spherical harmonics

We finally analytically continue q to the region inside the light cone emanating from the center of the bubble (R) ⇒ f^{pl} become radial function on a unit spatial 3 hyperboloid, and the q^p is the temporal mode fc.

⇒

We can analyze the behaviour of the scalar spectrum by studying the following analytic expression [6]

"effect of the wall"

$$|R|^2 \frac{p^3}{2\pi^2} = \left(\frac{H^2}{2\pi\dot\phi}\right)^2 \frac{\cosh \pi p + \cos \delta_p}{\sinh \pi p} \frac{p^2}{(1+p^2)}$$

In the limit:

$p \ll 1$ (i.e. low ℓ's)

$$\cosh p = \frac{e^{\pi p} + e^{-\pi p}}{2} \sim \frac{2}{2}(1 + (\pi p)^2)$$

$$\sinh p = \frac{e^{\pi p} - e^{-\pi p}}{2} \sim \frac{1}{2}(2\pi p + O(p^3))$$

$$\therefore \frac{\cosh \pi p + \overbrace{\cos \delta_p}^{\sim -1}}{\sinh \pi p} \frac{p^2}{(1+p^2)} \sim \frac{1 + (\pi p)^2 + (-1)}{(\pi p)} \frac{p^2}{(1+p^2)}$$

$$\Rightarrow |R|^2 \frac{p^3}{2\pi^2} \propto p^3$$ for low p's! (Not scale invariant as in flat inflationary case)

low p's correspond to the modes that leave the horizon earlier, i.e. are those who are most influenced by a non vanishing value of a_n. Indeed, we get a different behaviour for the scalar power spectrum.

$p \gg 1$

$$\cosh(\pi p) \sim \frac{e^{\pi p}}{2} \; ; \; \sinh(\pi p) \sim \frac{e^{\pi p}}{2}$$

$$\therefore \frac{\cosh \pi p + \overbrace{\cos \delta_p}^{\leq 1}}{\sinh \pi p} \frac{p^2}{(1+p^2)} \sim \frac{e^{\pi p}/2}{e^{\pi p}/2} (1 + \overbrace{2 e^{-\pi p} \cos \delta_p}^{\to 1}) \overbrace{\frac{p^2}{(1+p^2)}}^{\to 1} \equiv \text{const}$$

For high p's we recuperate the behaviour of flat inflationary background, i.e. <u>scale invariant</u>

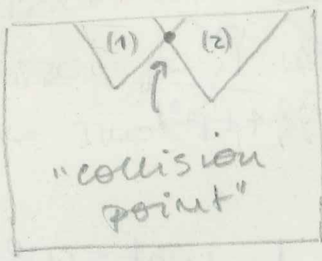
<u>Note</u>: the behaviour of tensor power spectra is identical [6]

Comments:

- One should recall that the expression does <u>not</u> consider the effect of fast roll condition which would additionally suppressed low p modes.
- Due to the fact that fast roll also has low p suppression ~~this effect would~~ the observation of this signature on CMB spectrum is <u>not enough</u> to claim in favour of a Open Universe. We also need to observe $\Omega_k > 0$.
- We don't expect supercurvature modes because m_{eff} is big! ⇒ the inflaton at the false vacuum does not fluctuate

(iii) <u>Bubble Collisions</u>

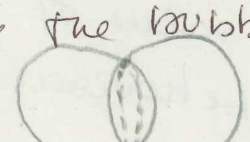
Penrose diagram



If two bubbles (or more) were formed such that they are causally connected they would eventually collide!

(1), (2) interior of the bubbles.

The point (•) corresponds to the event of the collision of the bubbles which would be seen as



a ring of density perturbations in the sky!