Brane Inflation

Martin Sprenger

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This talk mostly follows chapter 2 of [1] and [6].

1 Motivation

Beyond any doubt, inflation is an essential ingredient in modern cosmology. In the simplest realisation, a scalar field φ, called the inflaton, is **postulated** and is governed by

$$\mathcal{L} = \sqrt{-\mathrm{det} g_{\mu\nu}} \left(-\frac{1}{2} \left(\partial_\mu \varphi\right)^2 - V(\varphi) \right). \tag{1}$$

However, not all potentials $V(\varphi)$ are allowed, but we need the slow-roll parameters

$$\epsilon := \frac{1}{2}\left(M_{Pl}\frac{V'}{V}\right)^2, \quad \eta := M_{Pl}^2\frac{V''}{V}, \tag{2}$$

where M_{Pl} is the four-dimensional Planck mass, to be very small, $\epsilon, \eta \ll 1$, to get the correct number of e-folds.
Why should we try to embed this framework into string theory? First of all, the nature of the particle φ remains elusive when just postulating its existence. Furthermore, string theory is sometimes claimed to be a *theory of everything*. Since inflation is *something*, it should somehow be included in string theory. While these are aesthetical reasons, there is a more severe problem and that is the UV-sensitivity of inflation. Indeed, consider a Planck-suppressed operator contributing to the inflaton potential,

$$\Delta V = \frac{\varphi^2}{M_{Pl}^2}\mathcal{O}_4. \tag{3}$$

If the vev of $\mathcal{O}_4$ is of the order of the inflaton potential, $\langle\mathcal{O}_4\rangle \sim V$, this gives a contribution of order one to η, which potentially spoils the slow-roll behaviour. String theory, being the most promising candidate for a UV-complete theory at the Planck scale, is therefore the most natural candidate to study inflation.

2 Inflation in String Theory

The aim of embedding inflation in string theory can be stated as follows: Start from a full ten-dimensional string theory and specify the background data (compactification, fluxes,...) in such a way that the four-dimensional low-energy effective action gives rise to slow-roll inflation. At first sight, this seems easy enough - string compactifications naturally come with scalar fields, so-called moduli, which describe the parameters of the internal manifold (size, shape, brane positions,...). Therefore, we find natural inflaton candidates and there are several known possibilities to find flat enough potentials for one of the moduli. However, we face an embarrassment of riches - in general we have many moduli, all of which enter the scalar potential,

$$V_{\text{inf}} = V_{\text{inf}}(\varphi, \varphi^{\perp}), \tag{4}$$

where $\varphi^{\perp}$ denotes all moduli which are not our inflaton candidate. The problem now is that while we may have a flat potential in φ, the potential in $\varphi^{\perp}$ can be very steep. Then the theory would of course follow the steep $\varphi^{\perp}$-direction, completely spoiling slow-roll inflation. Therefore, we need to stabilise the $\varphi^{\perp}$-directions, i.e. find a mechanism that introduces a stabilising potential V_{stab} such that

$$m_{\varphi^{\perp}}^2 > 0. \tag{5}$$

The total potential would then be given by

$$V_{\text{tot}} = V_{\text{inf}} + V_{\text{stab}}. \tag{6}$$

However, in general V_{stab} depends on φ, as well. There is no reason why this additional potential should be compatible with slow-roll inflation. Therefore, we face a strong interplay between inflation and moduli stabilisation and neglecting the latter would lead to questionable results.
The aim of this talk is to study one specific setup ($D3/\overline{D3}$-inflation), highlight the features and problems mentioned above and find **one** explicit realisation of inflation in string theory.

3 Lightning review: IIB and its moduli

3.1 Particle content and action

The low-energy dynamics of type IIB string theory is given by type IIB SUGRA with the bosonic spectrum given by ten-dimensional metric g_{MN}, a NSNS 2-form $B_{(2)}$, a dilaton ϕ and the RR p-forms $C_{(p)}$ with $p = 0, 2, 4$, with field strengths

$$H_{(3)} := dB_{(2)},\ F_{(3)} := dC_{(2)}, \tilde{F}_{(5)} := dC_{(4)} + \frac{1}{2}B_{(2)} \wedge F_{(2)} - \frac{1}{2}C_{(2)} \wedge H_{(3)}. \tag{7}$$

With the combinations $\tau := C_{(0)} + ie^{-i\phi}$ ("axion-dilaton") and $G_{(3)} := F_{(3)} - \tau H_{(3)}$, the bosonic part of the IIB action is given by

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\left\{R - \frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\tau)^2} - \frac{G_{(3)}\cdot\bar{G}_{(3)}}{12\mathrm{Im}\tau} - \frac{\tilde{F}_{(5)}^2}{4\cdot 5!}\right\} + \frac{1}{8i\kappa_{10}^2}\int \frac{C_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)}}{\mathrm{Im}\tau}, \tag{8}$$

where κ_{10}^2 is the ten-dimensional gravitational coupling. In addition, in IIB we have stable p-branes when p is odd. Those are embedded in the ten-dimensional space-time by the action

$$\begin{aligned} S = S_{DBI} + S_{CS} = &- \mu_p \int d^{p+1}\xi e^{-P[\phi]}\sqrt{-\det\left(P[g + B_{(2)}] + 2\pi\alpha' F_{(2)}\right)} \\ &+ \mu_p \int_{\sigma_{p+1}} e^{P[B_{(2)}]+2\pi\alpha' F_{(2)}} \wedge \sum_q P[C_{(q)}]. \end{aligned} \tag{9}$$

We will compactify the additional six dimensions on a Calabi-Yau threefold, which breaks $\frac{3}{4}$ of the present SUSY, so that $\mathcal{N} = 2$ SUSY remains for IIB. To get the phenomenologically more interesting $\mathcal{N} = 1$ case, we have to do an additional orientifold projection. Then, the metric will be of the product form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \tilde{g}_{mn}dy^m dy^n. \tag{10}$$

3.2 Moduli

After compactifying IIB as described above, we find several scalar fields in the four-dimensional low-energy theory:

- Axion-dilaton τ
- Complex structure moduli $U^a \rightarrow$ describe deformations of complex structure of the CY_3 (i.e. deformations of Ω)
- Kähler moduli $\rightarrow$ describe size-deformations of CY_3 (e.g. sizes of cycles)

We will always find a Kähler modulus T, describing the overall volume of the CY_3 in the sense that $\mathrm{Im}T \sim (\mathrm{Vol}(\mathcal{M}_6))^{\frac{2}{3}}$. For simplicity, we will assume in the following that this is the only Kähler modulus. Each modulus lives in a $\mathcal{N}=1$ chiral supermultiplet with potentials of the general $\mathcal{N}=1$ SUGRA form

$$V = e^{K/M_{Pl}^2}\left[K^{I\bar{J}}\mathcal{D}_I W \mathcal{D}_{\bar{J}}\bar{W} - 3\frac{|W|^2}{M_{Pl}^2}\right], \tag{11}$$

with I, J labeling the moduli fields $z^I = \{U^a, T, \tau\}$. In Eq.(11), we introduced the following quantities:

- $K = K(z,\bar{z})$ is the Kähler potential
- $K_{I\bar{J}} := \partial_I\partial_{\bar{J}}K$, also enters in kinetic terms of z^I: $\mathcal{L}_{\text{kin}} = -\sqrt{-g}K_{I\bar{J}}\left(\partial_\mu z^I\right)\left(\partial^\mu \bar{z}^{\bar{J}}\right)$
- $W = W(z)$ is the superpotential
- $\mathcal{D}_I W := \partial_I W + (\partial_I K)\,\frac{W}{M_{\text{Pl}}^2}$

It is a known result that a critical point of V preserves $\mathcal{N}=1$ iff $\mathcal{D}_I W = 0$ for all I.

4 $D3/\overline{D3}$-inflation

4.1 Setup

For now, this is enough background to explain our model of inflation. We start with type IIB on a CY_3. We then add a pair of $D3/\overline{D3}$-branes which we choose to be space-time filling in the non-compact dimensions and localised in the CY_3. Their distance d in the internal manifold will be identified with the inflaton later on. It is known that in the backgrounds we are interested in the forces from gravity and the form field $C_{(4)}$ cancel for $D3$-branes to leading order in α'. This, however, is not true for $\overline{D3}$-branes, since their charges under the form fields is opposite to that of a $D3$-brane. Therefore, a $\overline{D3}$-brane is driven to special locations in the CY_3, where the forces cancel. The $D3$-brane is then only moving due to the "Coulomb"-attraction with the $\overline{D3}$-brane. For this reason, this is a very clean and simple setup. An additional virtue of this setup is that once the separation d between the branes is smaller than the string length ℓ_s, there is a tachyonic mode between the branes that leads to brane-antibrane annihilation, leading to an automatic endpoint of inflation.

In the six-dimensional space, the interaction potential between the branes is given by

$$V_{\text{Coulomb}} \sim 2T_3 \left(1 - \frac{1}{2\pi^2 T_3 d^4}\right), \tag{12}$$

where the first term describes the potential energy due to the presence of the $\overline{D3}$-brane, while the second term describes the Coulomb interaction (i.e. tree-level exchange of gravitons and $C_{(4)}$-quanta between the branes). To get a canonically-normalised scalar field, we define $\varphi := \sqrt{T_3}d$ and find for the potential Eq.(12)

$$V_{\text{Coulomb}} \sim 2T_3 \left(1 - \frac{T_3}{2\pi^2 \phi^4}\right). \tag{13}$$

Plugging

$$M_{\text{Pl}}^2 = \frac{T_3^2}{\pi} \text{Vol}(\mathcal{M}_6), \tag{14}$$

which follows from $M_{\text{Pl},10}^8 \text{Vol}(\mathcal{M}_6) = M_{\text{Pl},4}^2$, in Eq.(13) and using the definition of η, we find

$$\eta \cong -.3 \frac{\text{Vol}(\mathcal{M}_6)}{d^6}. \tag{15}$$

This shows that $\eta \ll 1$ is only possible if $d \gg \text{Vol}(\mathcal{M}_6)^{\frac{1}{6}}$, where the last quantity can be interpreted as the diameter of the CY_3, i. e. the branes would need to be separated larger than the diameter of the CY_3, which is impossible! Therefore, our setup is too simple and needs to be modified.

4.2 More on IIB: fluxes

To make sense of our setup described in the last section, we need an additional ingredient in the compactification. The three-form fields $H_{(3)}$, $F_{(3)}$ can lead to non-trivial behaviour if their integral along three-cycles in the CY_3 is non-vanishing,

$$\int\limits_{\Sigma_3} F_{(3)} \neq 0, \int\limits_{\Sigma_3} H_{(3)} \neq 0. \tag{16}$$

This is called a flux of the corresponding three-form. Since energy is stored in those fluxes, this leads to a back reaction of the geometry. In our case, this back reaction is rather mild: the metric attains a warping factor,

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n. \tag{17}$$

Then the internal manifold is still conformally equivalent to a CY_3. Changing the size of the cycle Σ_3 will change the energy stored in the flux, and should therefore lead to a modification of the scalar potential. Indeed, it can be shown that the fluxes lead to a contribution to the superpotential as

$$W_{\text{flux}} = W_{\text{flux}}(U^a, \tau) = \int\limits_{\mathcal{M}_6} G_{(3)} \wedge \Omega, \tag{18}$$

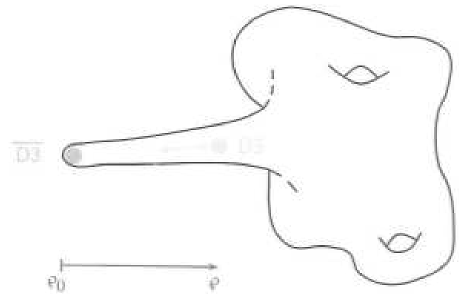

Figure 1: $\overline{D3}$-brane in a warped background. Taken from [1].

which is independent of the Kähler modulus T. Together with the tree-level Kähler potential

$$K(U^a, T, \tau) = K(U^a) + K(\tau) - 3\log(-i(T - \bar{T})) \tag{19}$$

this leads to the scalar potential

$$V = e^{K(z_i)+K(T)}\left[K^{i\bar{j}}\mathcal{D}_i W \mathcal{D}_{\bar{j}} W\right], \tag{20}$$

where $z^i = U^a, \tau$. This is manifestly non-negative and is minimised for $\mathcal{D}_i W = 0$. These are exactly as many equations as we have complex structure moduli plus the axion-dilaton. Therefore, these moduli will be stabilised at the minimum of the scalar potential! However, the volume modulus T remains unfixed.

4.3 $D3/\overline{D3}$-inflation in warped background

To study $D3/\overline{D3}$-inflation in a warped background, we choose a specific background called the Klebanov-Strassler background. This background, shown in figure 1, has a throat-like geometry. The tip of the throat is smoothed into a S^3, while the bottom is smoothly glued into a CY_3. The background fluxes are chosen such that

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_{(3)} =: M, \; \frac{1}{(2\pi)^2\alpha'}\int_B H_{(3)} := -K, \tag{21}$$

where A is the S^3 at the tip and B is the Poincaré-dual cycle to A. Inside the throat, the internal metric has the general form

$$ds^2_{\text{int}} = \sqrt{h(r)}\left(dr^2 + r^2 ds^2_{X^5}\right), \tag{22}$$

where r is cut off at both ends, $r_0 < r < r_{\max}$. The X^5 is a five-dimensional manifold, whose structure is irrelevant for our purposes, just note that it is parametrised by 5 angles. The exact metric in the throat is known. However, we will only need the form well inside the throat, which is given by

$$h(r) \cong \frac{R^4}{r^4}\log\left(\frac{r}{r_{\max}}\right). \tag{23}$$

Note that r_0 and $h_0 := h(r_0)$ depend exponentially on M and K.
In this background, the $\overline{D3}$-brane is moved to the tip of the throat. The $D3$-brane is then attracted by the $\overline{D3}$-brane and moves along the throat. We can then repeat the analysis of the Coulomb-potential, still neglecting volume stabilisation, and obtain

$$V_{\text{Coulomb}} = \frac{2h_0^{-1}T_3}{\mathcal{U}^2}\left(1 - \frac{27h_0^{-1}}{32\pi^2 T_3 r^4}\right), \tag{24}$$

where $\mathcal{U} \sim (\text{Vol}(\mathcal{M}_6))^{\frac{2}{3}}$. This shows that

$$\eta \sim h_0^{-1}, \tag{25}$$

which can be made arbitrarily small by the choice of fluxes. But V_{Coulomb} is much steeper in $\mathcal{U}$ than in r. Therefore, instead of the $D3$-brane moving along the throat $r \to r_0$, we would see that $\mathcal{U} \to \infty$, i.e. the internal manifold would decompactify! Wee see that there is no way around it - we need to stabilise the volume modulus, as well.

5 Volume stabilisation

5.1 General considerations

To stabilise the volume modulus T, we need to consider quantum corrections to the superpotential W. Due to a non-renormalisation theorem, the superpotential only receives non-perturbative corrections, for which two possible sources are known: Euclidean $D3$-branes or gaugino condensation on a stack of $D7$-branes wrapping a four-cycle Σ_4 in the CY_3. In both cases, the superpotential can be written as

$$W = W_0 + Ae^{iaT}, \tag{26}$$

where a depends on which non-perturbative corrections are considered and W_0 and A are constants up to a possible φ-dependence. Let us remark that up to now our approach is **very** similar to the KKLT setup [7]. This, of course, is no surprise, because we want to end up in a de Sitter-vacuum at the end of inflation. However, in [7], there is no mobile $D3$-brane. Therefore, A is a constant and the superpotential only depends on T. This is no longer true in our setup. In fact, including a mobile $D3$-brane the Kähler potential changes and now reads

$$K(T,\gamma) = -3\log(-i(T-\bar{T})) - k(\gamma,\bar{\gamma}), \tag{27}$$

where γ^a with $a = 1,2,3$ parametrises the position of the $D3$-brane in the internal manifold and where k is the Kähler potential of the original CY_3. Technically, it is this mixing of T and γ^a in the Kähler potential that leads to the strong interplay between inflation and moduli stabilisation. Of course, this introduces additional complications. In our case, however, this is a virtue, since assuming that $W = W(T)$ only, [2] show that the inflaton field acquires a mass

$$m_\varphi^2 \cong 2H^2, \tag{28}$$

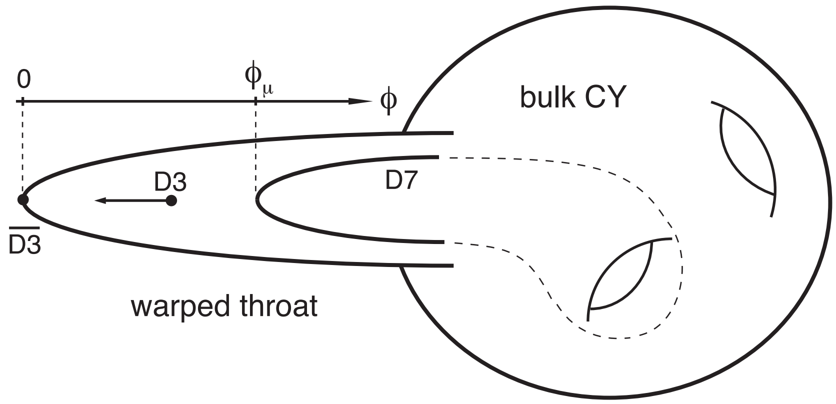

Figure 2: Internal manifold after introduction of the volume-stabilising $D7$-branes. Taken from [4].

where H is the Hubble parameter, which leads to

$$\eta \cong \frac{2}{3}, \tag{29}$$

which is much too large to get slow-roll inflation. An additional dependence of the superpotential on the $D3$-brane position introduces corrections and might cancel the $\frac{2}{3}$ in Eq.(29) to get $\eta \ll 1$. Indeed, [2] claim that with sufficient fine-tuning it should be possible to find vacua in which $\eta \ll 1$. However, it is conceivable that the corrections introduced by a position-dependence have the same sign as those from the volume stabilisation. Therefore, one needs to calculate the precise position-dependence of the superpotential to know for sure.

5.2 Explicit results

The explicit calculation of the superpotential in the case of gaugino condensation was carried out in [3]. Since the calculation is very technical, let us just motivate why this effect leads to a position-dependent superpotential and then just state the result.
We introduce a stack of n $D7$-branes that are space-time filling in the non-compact dimensions and that wrap a four-cycle Σ_4 in the CY_3 as shown in figure 2. On the worldvolume of those branes, we find a $SU(n)$ $\mathcal{N} = 1$ SYM theory. When settling to the energetic minimum, the gauginos of the SYM will condense. Note that this is a non-perturbative effect[1]. The introduction of the $D7$-branes also leads to additional vector fields which in the four-dimensional theory have kinetic terms

$$\mathcal{L}_{\text{kin}}^{\text{vec}} = -\frac{1}{4}\sqrt{-\text{det}g_{\mu\nu}}\text{Re}(f(z))F_{\mu\nu}F^{\mu\nu}. \tag{30}$$

The structure of this term is constrained by $\mathcal{N} = 1$ SUSY such that the prefactor of the canonical term has to be the real part of a holomorphic function of the moduli. $f(z)$ is

[1] Just compare this situation with the corresponding situation in QCD in which calculating $q\bar{q}$-condensation is only accessible via lattice calculations or holography.

called gauge kinetic function and a comparison with the canonical form for vector fields shows that we can identify

$$\mathrm{Re}(f(z)) = g^{-2}(z, \bar{z}), \tag{31}$$

where g is the moduli-dependent gauge coupling. In pure $SU(n)$ SYM gaugino condensation leads to a non-perturbative contribution to the superpotential of the form

$$W_{\mathrm{np}} = Ae^{-\frac{8\pi^2}{n}f(z)}, \tag{32}$$

in which the gauge kinetic function enters, as well. Consider now the embedding of the $D7$-branes, given by the DBI-action

$$\begin{aligned} S_{\mathrm{DBI}} &= -T_7 \int\limits_{\Sigma_4} d^4\sigma \sqrt{\det P[\tilde{g}_{mn} + \dots]} \int d^4x \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{(2)} + \dots)} \\ &= -T_7 \frac{(2\pi\alpha')^2}{4} \int\limits_{\Sigma_4} d^4\sigma h \sqrt{\det P[g_{mn}]} \int d^4x \sqrt{-\det g_{\mu\nu}} F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(\alpha'^2). \end{aligned} \tag{33}$$

The first integral in the second line of Eq.(33) is nothing but the volume of the four-cycle in the warped geometry. Therefore, we can identify

$$g^{-2} = T_7(2\pi\alpha')^2 \mathrm{Vol}(\Sigma_4). \tag{34}$$

The upshot of all this is that the presence of a mobile $D3$-brane leads to a deformation of the warpfactor h which enters in $\mathrm{Vol}(\Sigma_4)$. Therefore, the $D3$-brane leads to a modification of the gauge coupling g^{-2} and, via Eqs.(34),(32) to a modification of the superpotential, depending on the position of the $D3$. This is exactly what we wanted. The exact result depends on the analytic embedding equation $g(x_\alpha)$ of the $D7$-branes as

$$A(x_\alpha) = A_0 \left(\frac{g(x_\alpha)}{g(0)} \right)^{1/n}, \tag{35}$$

where x_α with $\alpha = 1, 2, 3$ are isotropic coordinates parametrising the internal manifold. Since $\sum |x_\alpha|^2 = r^2$, the x_α scale as $\varphi^{3/2}$. Therefore, Eq.(35) will generate powers of $\varphi^{3/2}$, but no purely quadratic term in φ and it is impossible to cancel the contribution Eq.(28) exactly, i.e. we cannot have slow-roll inflation over the whole range of φ.

6 One specific example

We can now finally consider a fully-explicit example. To do so, we choose the embedding equation $g(x_\alpha) = \mu$, where $\mu \in \mathbb{R}^+$, the so-called Kuperstein embedding. [4] then identify a radial trajectory of the $D3$-brane that is stable in the angular directions and which demands $x_1 = -\frac{1}{\sqrt{2}} r^{3/2}$. Plugging this into the general equation for the potential Eq.(20)

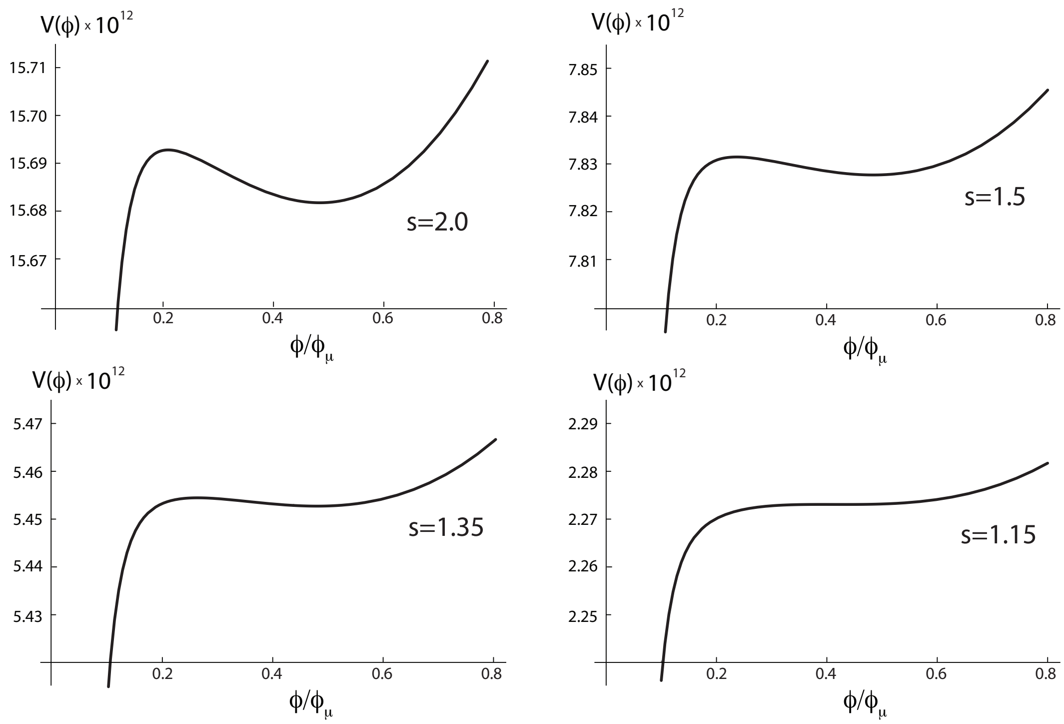

Figure 3: The single-field potential $\mathbb{V}$ after stabilising the volume modulus. It is always possible to tune the parameters such that we get an inflection point. Taken from [5].

and using $\sigma = \frac{1}{2}(T + \bar{T})$, we find

$$V(\varphi, \sigma) = \frac{a|A_0|^2}{3} \frac{e^{-2a\sigma}}{U^2(\varphi, \sigma)} g(\varphi)^{2/n} \left[2a\sigma + 6 - 6e^{a\sigma} \frac{|W_0|}{|A_0|} \frac{1}{g(\varphi)^1/n} + \frac{3c}{n} \frac{\varphi}{\varphi_0} \frac{1}{g(\varphi)^2} - \frac{3}{n} \frac{1}{g(\varphi)} \frac{\varphi^{3/2}}{\varphi_0^{3/2}} \right] + \frac{D}{U^2(\varphi, \sigma)}. \quad (36)$$

The only parameter of relevance in Eq.(36) is φ_μ, which describes the minimal value of the radius which the $D7$-branes reach in the throat. For more details on the other parameters see [5]. [4] then demand that at each value of φ the potential attains its minimum in σ. This gives rise to a function $\sigma_*(\varphi)$ such that $\frac{\partial V}{\partial \sigma}\big|_{\sigma_*(\varphi)} = 0$. This leads to a single-field potential $\mathbb{V}(\varphi) = V(\varphi, \sigma_*(\varphi))$ shown in figure 3. Generically, this potential has a metastable minimum. Indeed, it is possible to show that $\mathbb{V}$ has negative curvature near the tip and positive curvature far away. Therefore, η must vanish at one point in between. Furthermore, φ_μ can always be tuned such that the minimum becomes an inflection point. This is a big virtue, because at an inflection point both η and ϵ become small, leading to slow-roll inflation.

7 Conclusions

Let us conclude our findings. We see that it is possible to find an explicit realisation of slow-roll inflation in string theory. However, finding a working example turned out to be very difficult. For example, there are whole classes of embeddings other than the Kuperstein embedding that can never produce potentials that are flat enough for slow-roll inflation. Furthermore, the solution is very different than expected - no fine-tuning can cancel the quadratic mass term Eq.(28) and the non-perturbative contribution completely changes the character of the potential as compared to the naive Coulomb-potential Eq.(13).

References

[1] J. Erdmenger (Ed.), *String Cosmology*, Wiley (2009).

[2] S. Kachru et al., **JCAP 10**, 013 (2003).

[3] D. Baumann et al., **JHEP 11**, 031 (2006).

[4] D. Baumann et al., **Phys. Rev. Lett.** 99, 141601 (2007).

[5] D. Baumann et al., **JCAP 01**, 024 (2008).

[6] D. Baumann, *TASI Lecture Notes*, arXiv:0907.5424 (2009).

[7] S. Kachru et al., **Phys. Rev. D 68**, 046005 (2003).