Stefano Frixione

QCD made easy

DESY, 22/5/2013

Special thanks to V. Hirschi and A. Papanastasiou

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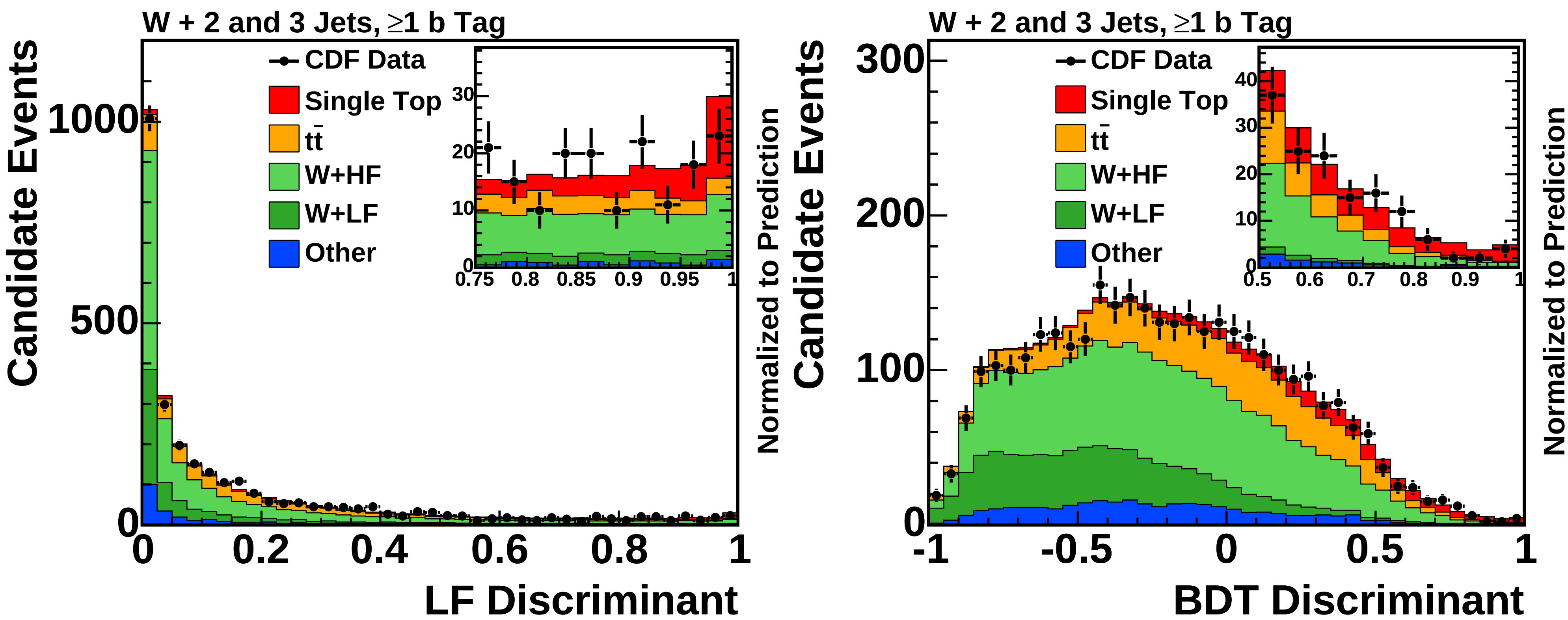
How far is the real world from the ideal one?

Rather far (Nature does not seem willing to cooperate)

Let's consider an example in some sense extreme

Single top at CDF (1004.1181)



Cut and count hopeless: uncertainty much larger than signal

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Precision Higgs physics will not be done by cutting and counting

BSM (barring sheer luck) will be presumably be worse

Single-top discovery at the Tevatron is a paradigm for (many) LHC analyses

- An extremely large number of background processes, which swamp the small-cross-section signal
- If predictions for signal are wrong, there is no safety net
- For backgrounds: one must not overstretch predictions when tuning, since this may “hide” a signal. In other words: shapes must be trustable
- It is important to use fully-exclusive theoretical results, that can go through detector simulation

So what are the lessons to be learned?

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3: "Exclusive" parton-level predictions are mandatory. Experimenters will always prefer to have them attached to event generators (parton showers)

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YES

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Does this mean NLO in (QCD) perturbation theory?

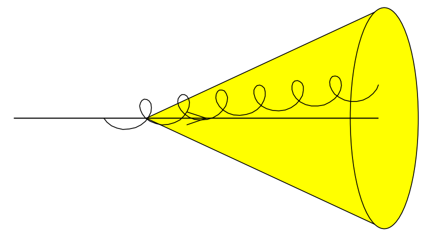
YES

But beware: the usual motivations given by many theorists (e.g.: better description of jet structure; extra contributions from initial-state partons; "NLO" effects on distributions) are actually motivations for *tree-level* calculations (*beyond LO*)

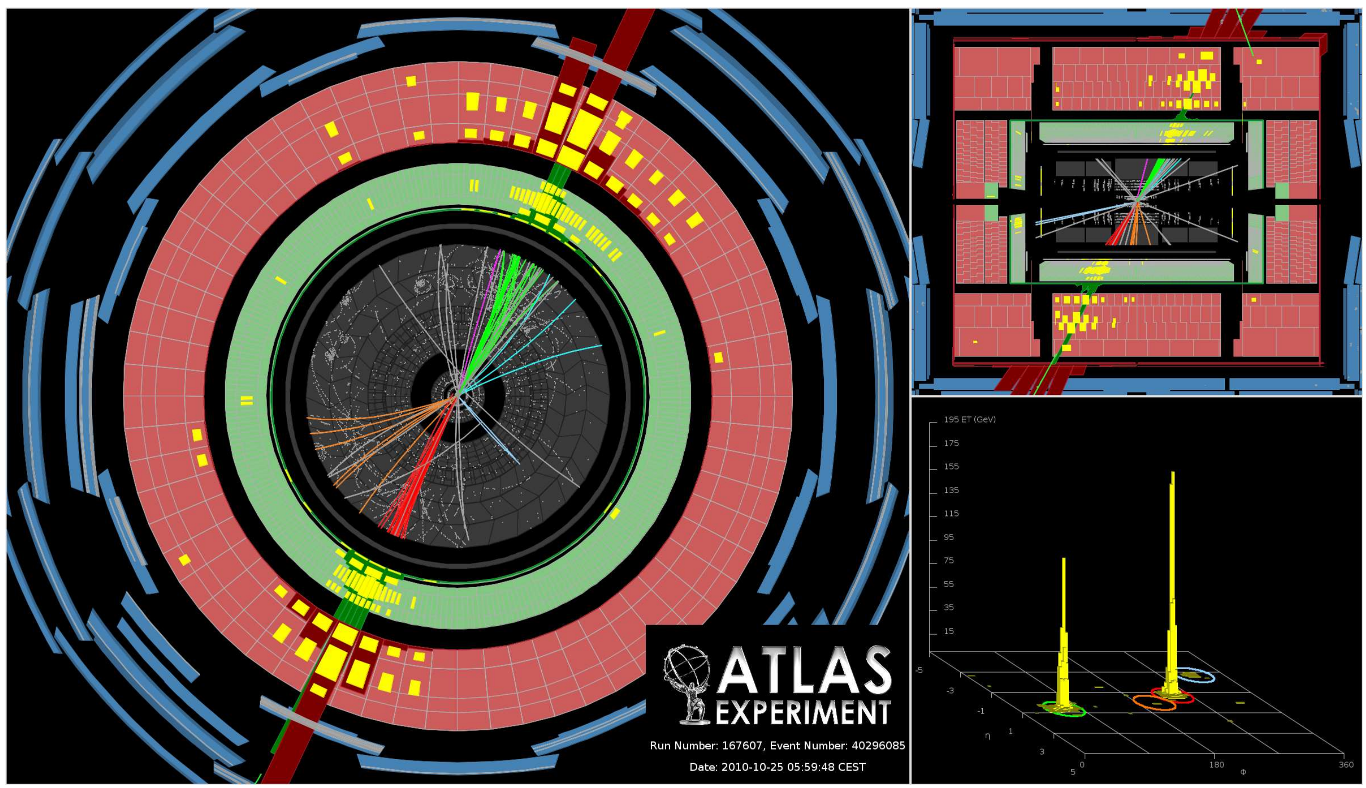
A: Better description of jet structure

(...in the sense that doing worse is difficult, perhaps?)

A jet in an NLO computation:



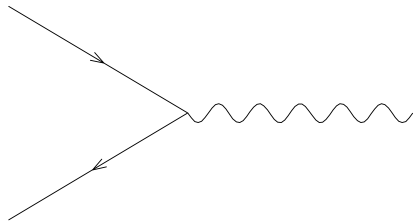
Actual jets:

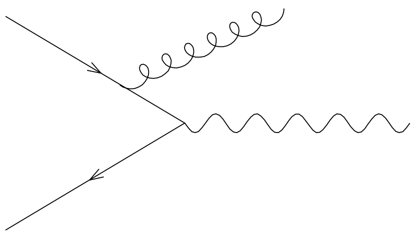
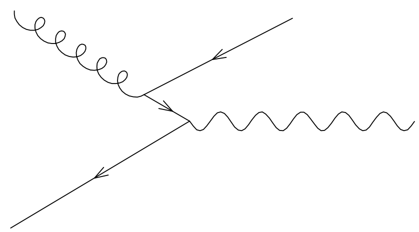


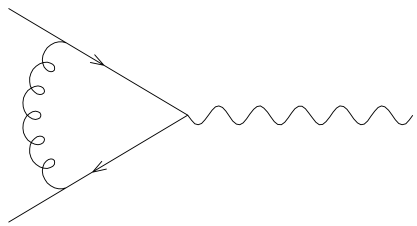
- One or two partons vs $\mathcal{O}(50)$ hadrons

B: More combinations of initial-state partons

True, but misses the point. Consider Z production:

Born: 

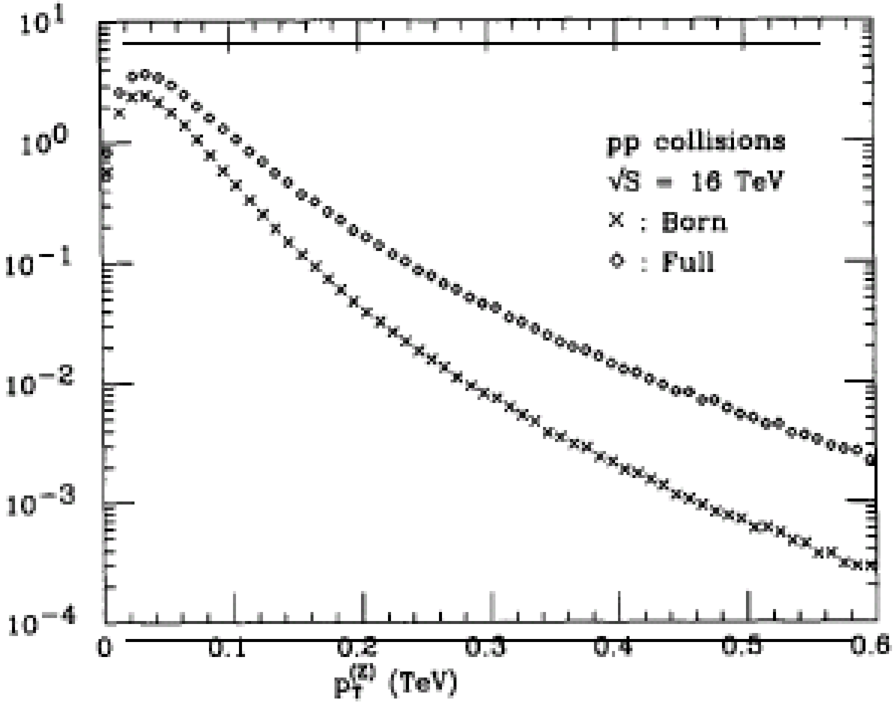
Real corrections:  

Virtual corrections: 

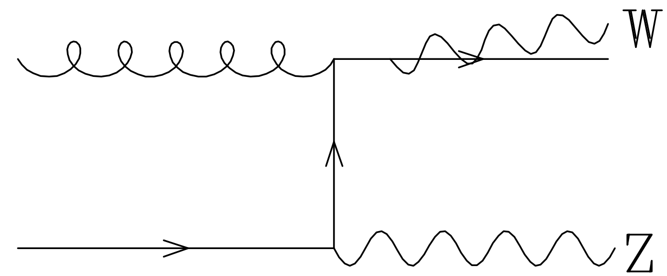
- This is a feature of tree-level corrections

C: NLO effects on distributions

Again, misses the point. Consider $p_T^{(Z)}$ in WZ production:



A K factor of about 6 at $p_T^{(Z)} = 600$ GeV. But:



is what dominates (double Sudakov log)

- Again, a feature of tree-level corrections

As far as I'm concerned, the really crucial thing is:

- Precision: NLO is the first order at which the assessment of theoretical uncertainties is reliable (for both postdictions and *predictions* – predictivity gives immense benefits, and is key)

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This is even more true for N^{k}LO, $k \geq 2$, so what does the trick is:

- We can perform an NLO calculation (almost) as straightforwardly as an LO one
- Same for its matching with parton showers

Note: these two items are very recent advances, which enlarged the scope of NLO results far beyond what was previously thought possible

Let me put it in a slightly different way

NLO is now what LO used to be 10 years ago

Difficulties are irrelevant, since NLO calculations are carried out by a computer

This is most definitely not the case for beyond-NLO results

Timeline for NLO

1979 Passarino-Veltman tensor reduction

1980 Ellis-Ross-Terrano: subtraction method (observable dependent)

1980-circa 1995 Pain, pain, and more pain

1995-1997 Discovery of process-independent subtraction procedures

1997-circa 2008 Less pain, but pain still

$\sim$ 2000 Tree-level calculations fully automated

2002-2005 Discovery of NLO-MC matching techniques

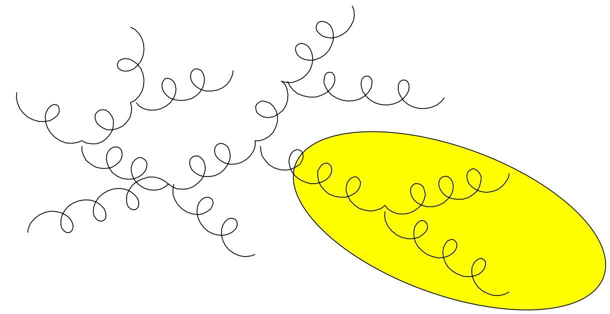
2005-2008 (Re)-discovery of process-independent one-loop techniques

2009-present Automation – pain is over

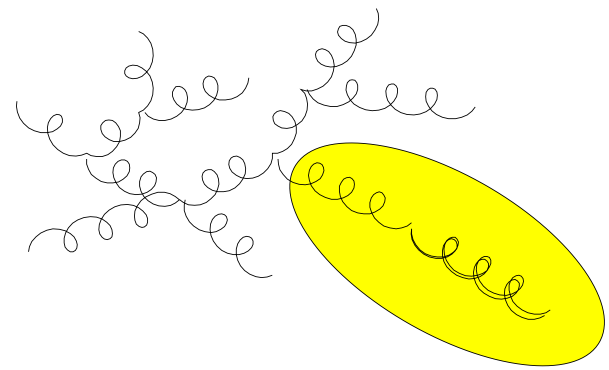
Fixed order

Anatomy of NLO computations

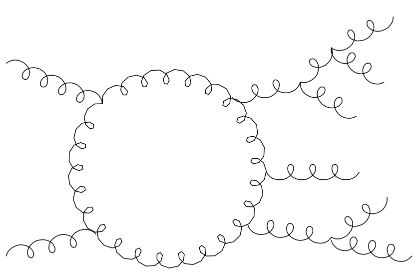
$$\frac{d\sigma}{dO} = \int d\phi_{n+1} \Big(\mathcal{M}^{(r)}(\phi_{n+1})\delta(O - O_{n+1}(\phi_{n+1}))$$

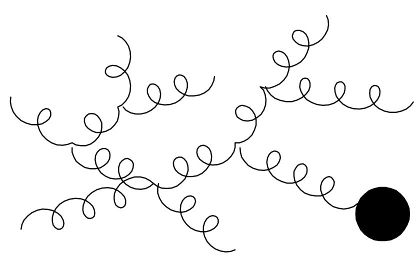


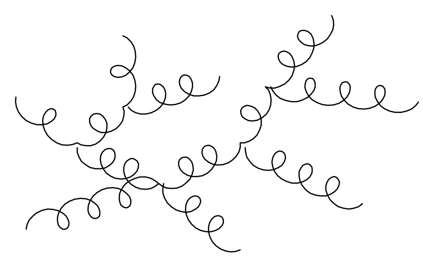
$$-\mathcal{M}^{(c.t.)}(\mathcal{P}\phi_{n+1})\delta(O - O_n(\mathcal{P}\phi_{n+1}))\Big)$$



$$+\int d\phi_n \Big(\mathcal{M}^{(v)}(\phi_n) + \mathcal{M}^{(rem)}(\phi_n) + \mathcal{M}^{(b)}(\phi_n)\Big)\, \delta(O - O_n(\phi_n))$$







Things to do:

- Compute the real, Born, and one-loop matrix elements
- Subtract the singularities of the real matrix elements, thus cancelling the one-loop ones
- Parametrize the phase space, and integrate the (finite) results of the previous step

Major bottlenecks were the subtraction and the one-loop computations

Subtraction

Consider a $2 \to n$ pure-gluon process. There are

- $(n^2 + 3n)/2$ collinear singularities (two-body correlations)
- n soft singularities (three-body correlations)

Their systematic subtractions for any n in a process-independent manner is a solved problem

- FKS (Frixione, Kunszt, Signer, hep-ph/9512328 + ...)
- Dipole (Catani, Seymour, hep-ph/9605323 + ...)
- Antenna (Kosower, hep-ph/9720213)

An alternative technique is slicing (Owens, Harris; Laenen, Keller; ...), not suited to large n

FKS

- Use collinear singularities to organize subtractions $\Longrightarrow$ two-body kernels
- Define subtractions on-shell
- Use arbitrary functions (whose sum is equal to one) to damp all singularities except one collinear and one soft. These newly-constructed quantities are treated independently from each other

Dipole

- Use soft singularities to organize subtractions $\Longrightarrow$ three-body kernels
- Define subtractions off-shell. The recoil is distributed using a mapping defined by the three partons of each kernel. Hence, each subtraction term has a different kinematics
- All singularities are subtracted simultaneously

Implications:

- Number of subtraction terms scales as n^2 in FKS, and as n^3 in dipoles. By exploiting symmetries, FKS reduce this to a constant

  Variants of dipoles (Chung, Krämer, Robens, 2010) achieve n^2

- Numerics in dipoles become intractable for large n without the use of α-dependent subtractions (Nagy). This is not the case in FKS

- Importance sampling must be done dynamically in dipoles, not so in FKS

- FKS has a "collinear" structure. It is therefore the method of choice for NLO-parton shower matching formalisms (MC@NLO and POWHEG)

  [except for dipole-based showers (SHERPA, both MC@NLO and POWHEG matchings)]

- Dipole is manifestly Lorentz invariant, FKS is not

Automation

- MadFKS (Frederix, Frixione, Maltoni, Stelzer 0908.4272)
- HELAC (Czakon, Papadopoulos, Worek 0905.0883)
- MadDipole (Frederix, Gehrmann, Greiner 1004.2905, 0808.2128)
- SHERPA (Gleisberg, Krauss 0709.2881)
- Other less systematic attempts (Seymour, Tevlin 0803.2231; Hasegawa, Moch and Uwer 0911.4371)

Level and scope of automation differ, and at the moment it is difficult to assess the capabilities of these codes. I suppose dust will settle soon

One-loop computations

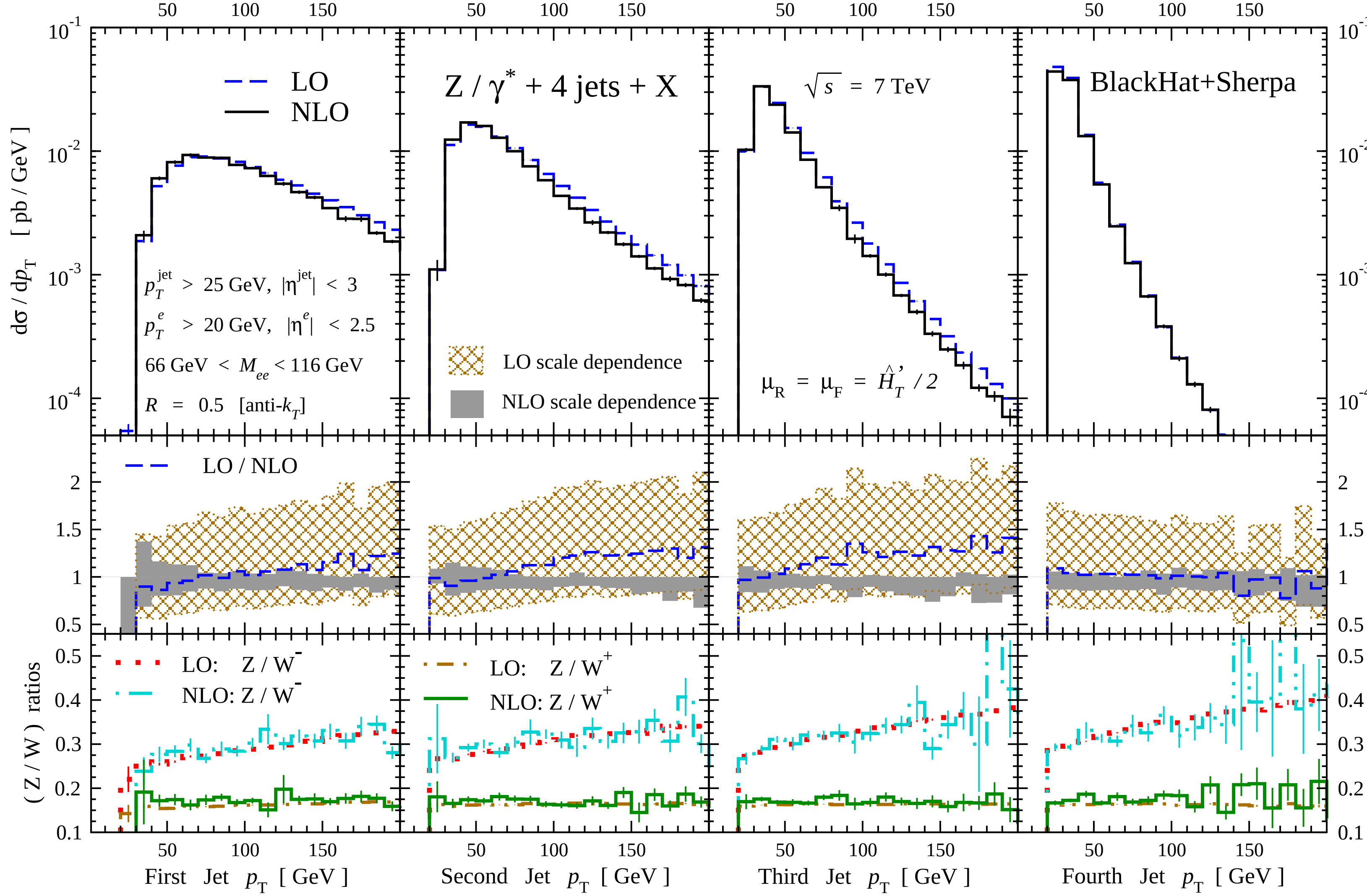
Several methods are now established:

- Generalized Unitarity (Bern, Dixon, Dunbar, Kosower hep-ph/9403226 + ...; Ellis, Giele, Kunszt 0708.2398, +Melnikov 0806.3467)
- Integrand Reduction (Ossola, Papadopoulos, Pittau hep-ph/0609007; del Aguila, Pittau hep-ph/0404120; Mastrolia, Ossola, Reiter, Tramontano 1006.0710)
- Tensor Reduction (Passarino, Veltman 1979; Denner, Dittmaier hep-ph/0509141; Binoth, Guillet, Heinrich, Pilon, Reiter 0810.0992)

...and have been put to use and automated $\longrightarrow$

- GU $\longleftarrow$ BlackHat (Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre 1009.2338 + ...)
- GU $\longleftarrow$ Rocket (Ellis, Giele, Kunszt, Melnikov, Zanderighi 0810.2762 + ...)
- IR(+TR) $\longleftarrow$ MadLoop (Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau, 1103.0621), HELAC-NLO (Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek, 1110.1499 + ...), GoSam (Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano, 1111.2034), OpenLoops (Cascioli, Maierhofer, Pozzorini, 1111.5206)

So far, GU applied mostly to large-multiplicity, massless final states (e.g. W+5 jets by BlackHat), IR to lower-multiplicity, massive final states (e.g. HELAC $t\bar{t}b\bar{b}$)



$Z + 4$ jets, BlackHat+SHERPA, 1108.2229

Simply unthinkable just a few years ago

Matching to showers (NLOwPS)

This problem has attracted a lot of attention in the theory community, being quite challenging (and very relevant to phenomenology)

Main issue: MC's and NLO's "generate" identical classes of Feynman diagrams, that must not be counted twice

Why it is tricky: NLO computations are inclusive by nature; MC's are fully exclusive. Opposite requirements!

One can take a pragmatic attitude (make do with existing MCs), or a more long-term one (improve MCs, endowing them with NLO-like features such as interferences, subleading effects, ...)

The former is presently the standard choice for phenomenology, with two methods (MC@NLO, POWHEG) which have made it to mass-production in experiments

MC@NLO

- Compute what the MC does at the first non trivial order, and subtract it from the matrix elements. The resulting short-distance cross sections can be unweighted, and the hard events thus obtained are used as initial conditions for parton showers

- One set of analytical computations per MC
- Negative weights
- Strictly identical to MC in soft/collinear regions
- Strictly identical to NLO in hard emission regions;
  all $\mathcal{O}(\alpha_S^{2+b})$ terms not logarithmically enhanced are zero
- Inclusive cross sections identical to total cross section @NLO

POWHEG

- Replace the first MC emission with one generated with a p_T-ordered Sudakov, constructed by exponentiating the *full real matrix element*. Requires a truncated shower to restore the correct pattern of soft emissions for angular-ordered showers

- Short-distance computations independent of MCs
- No negative weights
- Differs from MC in soft/collinear regions if MC is not p_T-ordered. For angular-ordered showers, agreement with MC is restored by truncated showers (only up to subleading terms)
- Differs from NLO in hard emission regions by $\mathcal{O}(\alpha_S^{2+b})$ terms; no piece of information on NNLO is used

MC@NLO vs POWHEG

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- In MC@NLO the MC generates all non-hard emissions. This is not the case in POWHEG. Technically, this implies an ordering in p_T; thus, double-log accuracy is spoiled if an MC is used that is not ordered in p_T (such as HERWIG). It can be restored by adding a "soft" shower

MC@NLO basics

The generating functional is:

$$\mathcal{F}_{\text{MC@NLO}} = \mathcal{F}^{(2\to n+1)}\, d\sigma^{(\mathbb{H})}_{\text{MC@NLO}} + \mathcal{F}^{(2\to n)}\, d\sigma^{(\mathbb{S})}_{\text{MC@NLO}}$$

with the two *finite* short-distance cross sections

$$d\sigma^{(\mathbb{H})}_{\text{MC@NLO}} = d\phi_{n+1}\left(\mathcal{M}^{(r)}(\phi_{n+1}) - \mathcal{M}^{(\text{MC})}(\phi_{n+1})\right)$$

$$d\sigma^{(\mathbb{S})}_{\text{MC@NLO}} = \int_{+1} d\phi_{n+1}\left(\mathcal{M}^{(b+v+rem)}(\phi_n) - \mathcal{M}^{(c.t.)}(\phi_{n+1}) + \mathcal{M}^{(\text{MC})}(\phi_{n+1})\right)$$

- Black terms: pure NLO, same as before
- Red terms: MC subtraction terms, with a factorized form

$$\mathcal{M}^{(\text{MC})} = \mathcal{K}^{(\text{MC})}\mathcal{M}^{(b)}$$

Automation of MC@NLO $\equiv$ MadFKS+MadLoop+automation of $\mathcal{K}^{(\text{MC})}$
$\equiv$ aMC@NLO

MC@NLO $\longrightarrow$ aMC@NLO

Each Monte Carlo corresponds to a finite set of $\mathcal{K}^{(\mathrm{MC})}$, which need be computed analytically

- Done for Herwig6, Pythia6 (ISR-only for p_T-ordered), Herwig$++$, Pythia8 (being validated)
- The automatic implementation of $\mathcal{K}^{(\mathrm{MC})}$ also relies on finding pairs associated with collinear singularities – inherited from MadFKS
- Need to figure out colour connections

The first version (spring 2011) was not public, and not user-friendly.
The now-public code (Nov 8^{th} 2012) is *very* user-friendly

The simplest run

```
your_shell> ./bin/mg5

MG5> generate p p > t t~ w+ [QCD]

aMC@NLO> output MYDIR

aMC@NLO> launch
```

This prints out some information on the screen (very much MG5-like: model, multiparticles, ...), and the following prompt

$\longrightarrow$

```
Which programs do you want to run?
0 / auto :  NLO event generation and -if cards exist- shower and madspin
1 / NLO : Fixed order NLO calculation (no event generation).
2 / aMC@NLO : NLO event generation (include running the shower).
3 / noshower :  NLO event generation (without running the shower).
4 / LO : Fixed order LO calculation (no event generation).
5 / aMC@LO : LO event generation (include running the shower).
6 / noshowerLO : LO event generation (without running the shower).
+10 / +madspin :  Add decays with MadSpin (before the shower).
[0, auto, 1, NLO, 2, aMC@NLO, 12, aMC@NLO+madspin, 3, ...  ][60s to answ
```

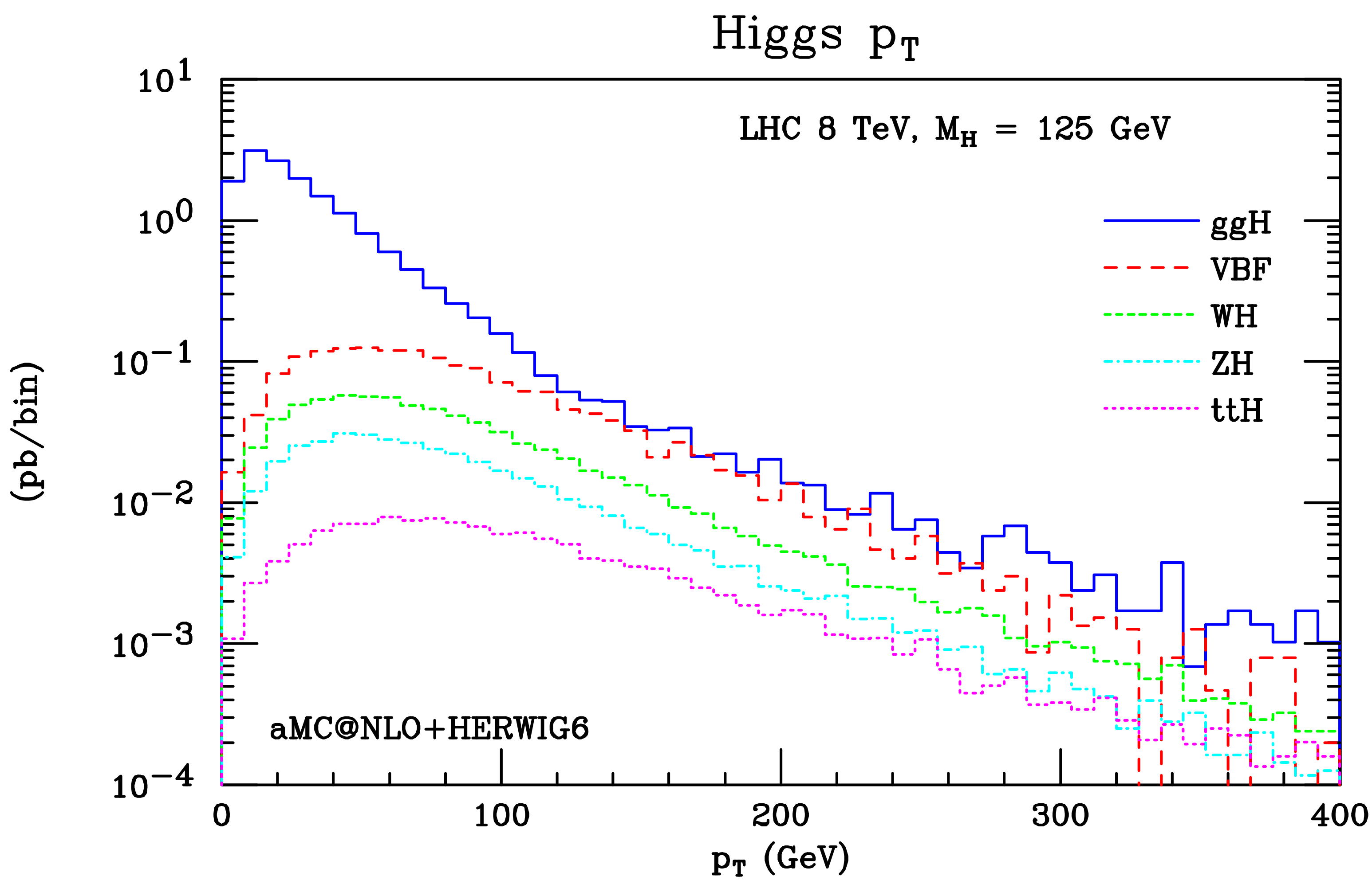
If one enters:

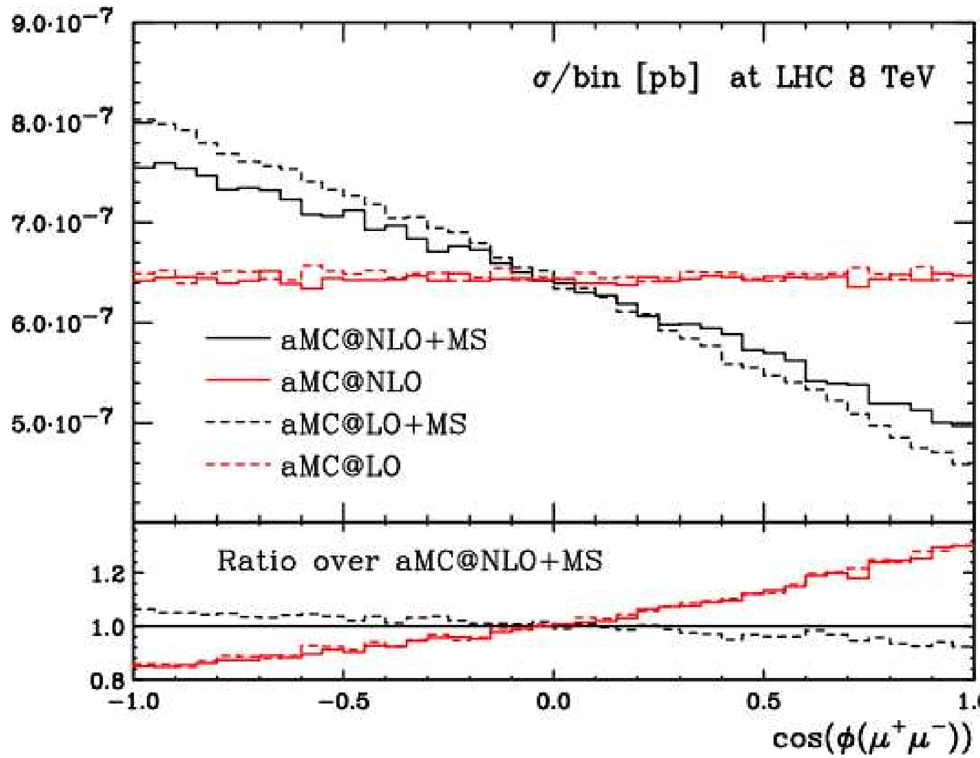
\>2

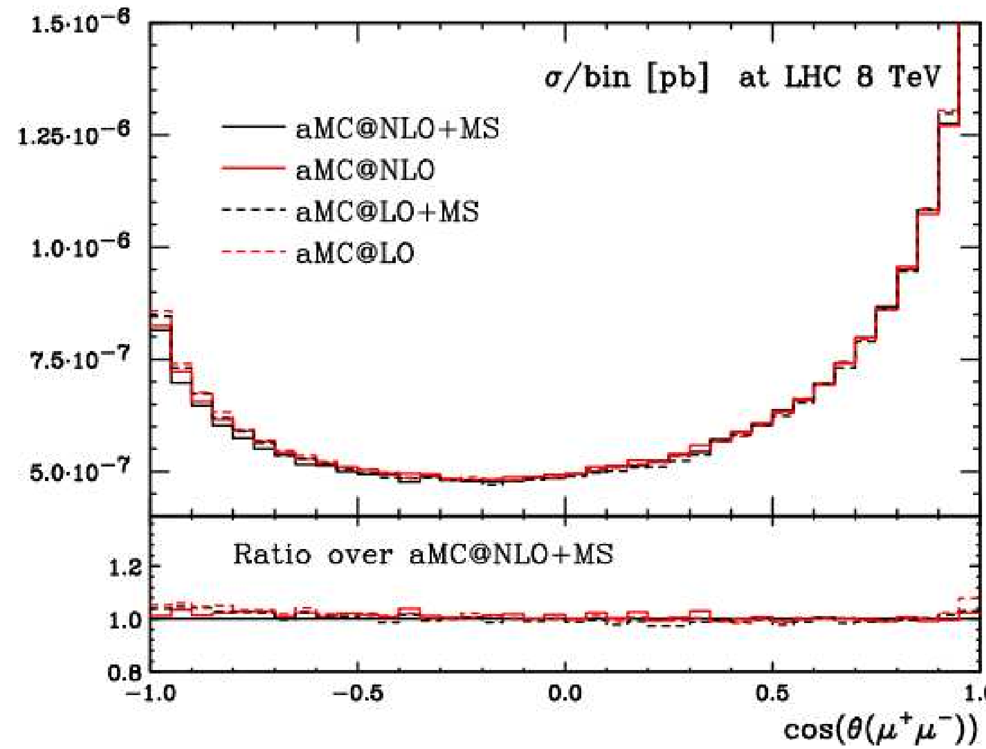
after a while the hard-event file, and an StdHEP or HepMC file that contains the complete event record *after shower*, will be found in:

./MYDIR/Events/run_01/

A typical example...

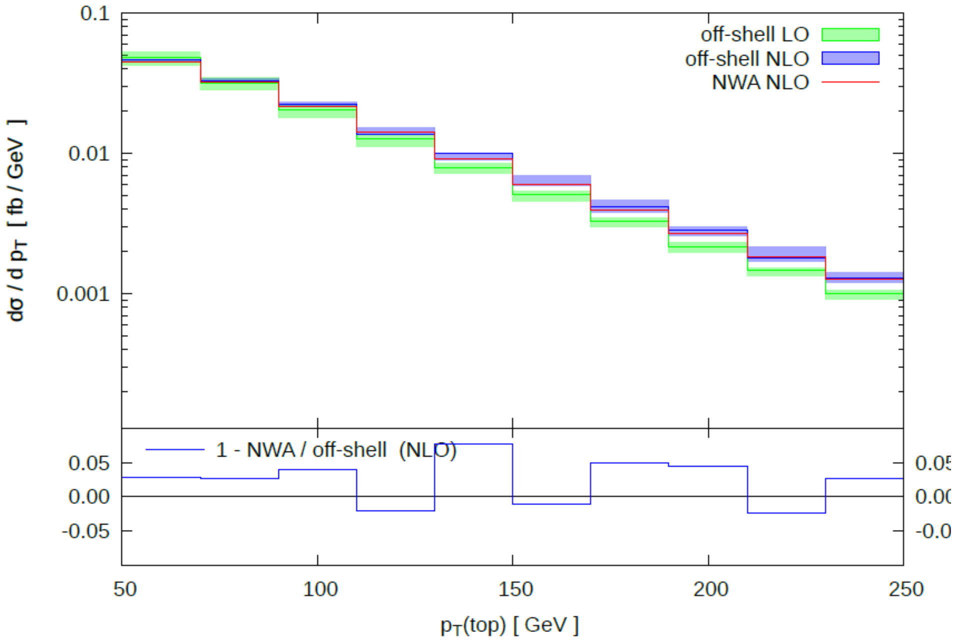


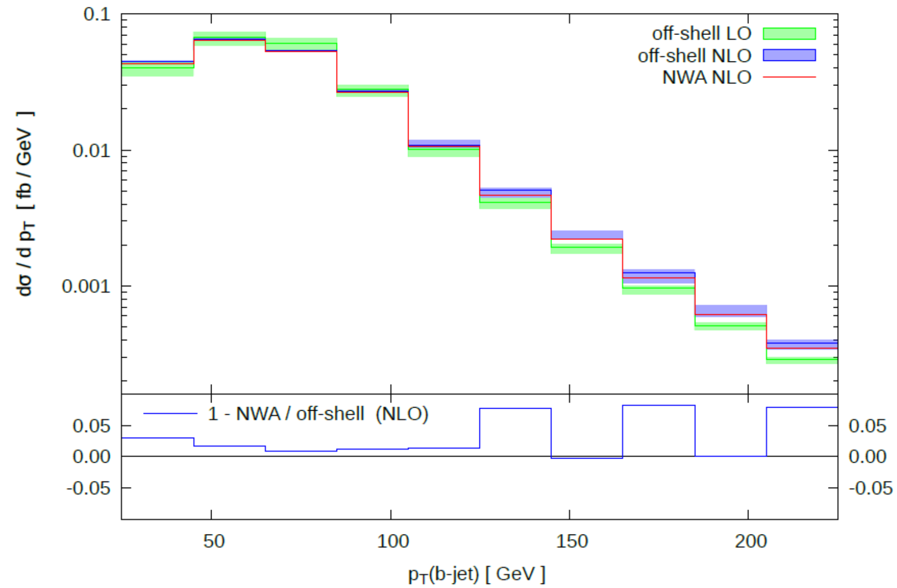




Angular correlation variables in $W^+(\to \mu^+\nu_\mu)W^-(\to \mu^-\bar{\nu}_\mu)Z(\to e^+e^-)$ production

From V. Hirschi's thesis – preliminary



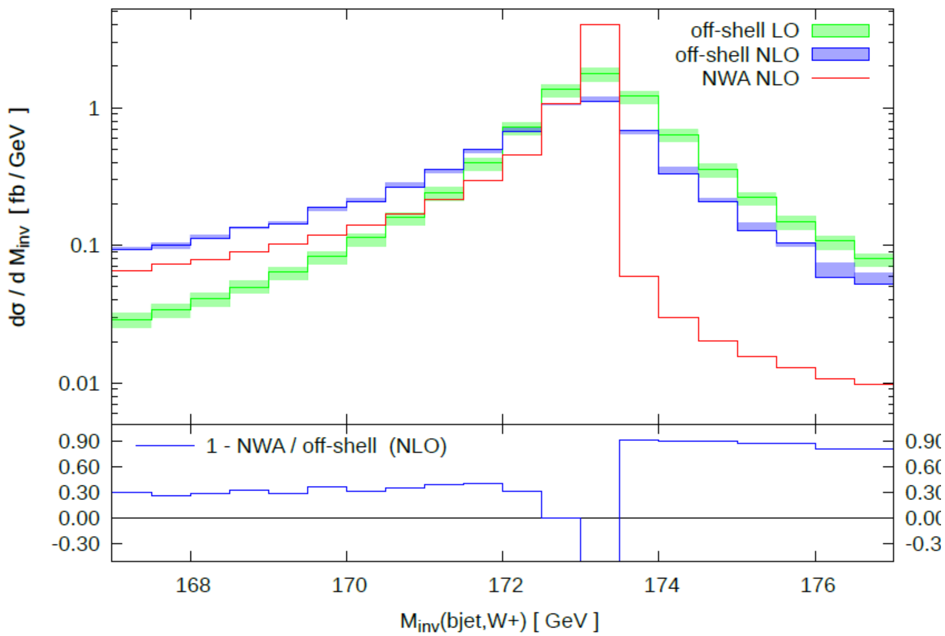


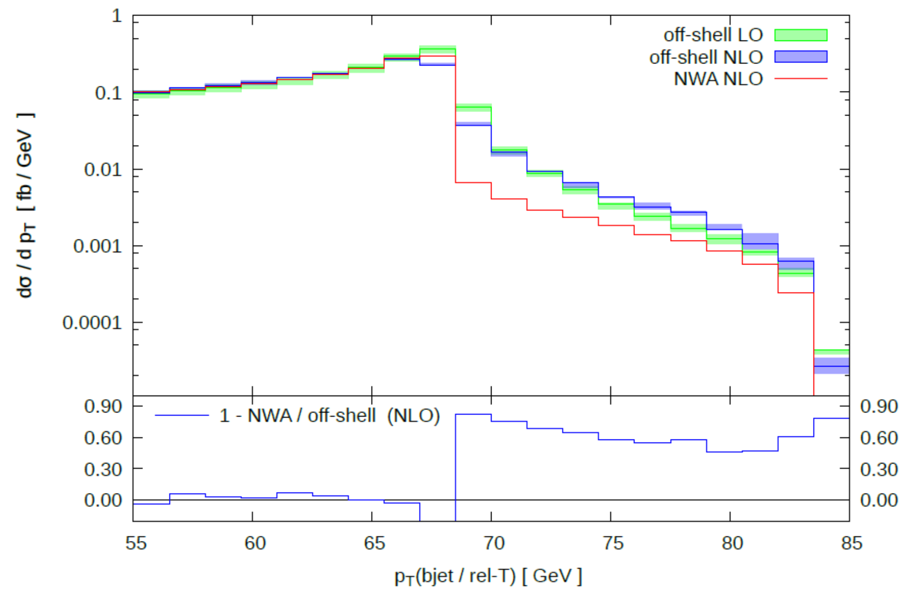
$$pp \longrightarrow W^+ J_b J_{light}$$

$$p_T(j_b) \geq 25 \text{ GeV} \qquad p_T(j) \geq 25 \text{ GeV}$$

$$|j_b| < 4.5 \qquad |j| < 4.5 \qquad 140 \leq M(W^+ j_b) \leq 200 \text{ GeV}$$

Γ_t-insensitive observables





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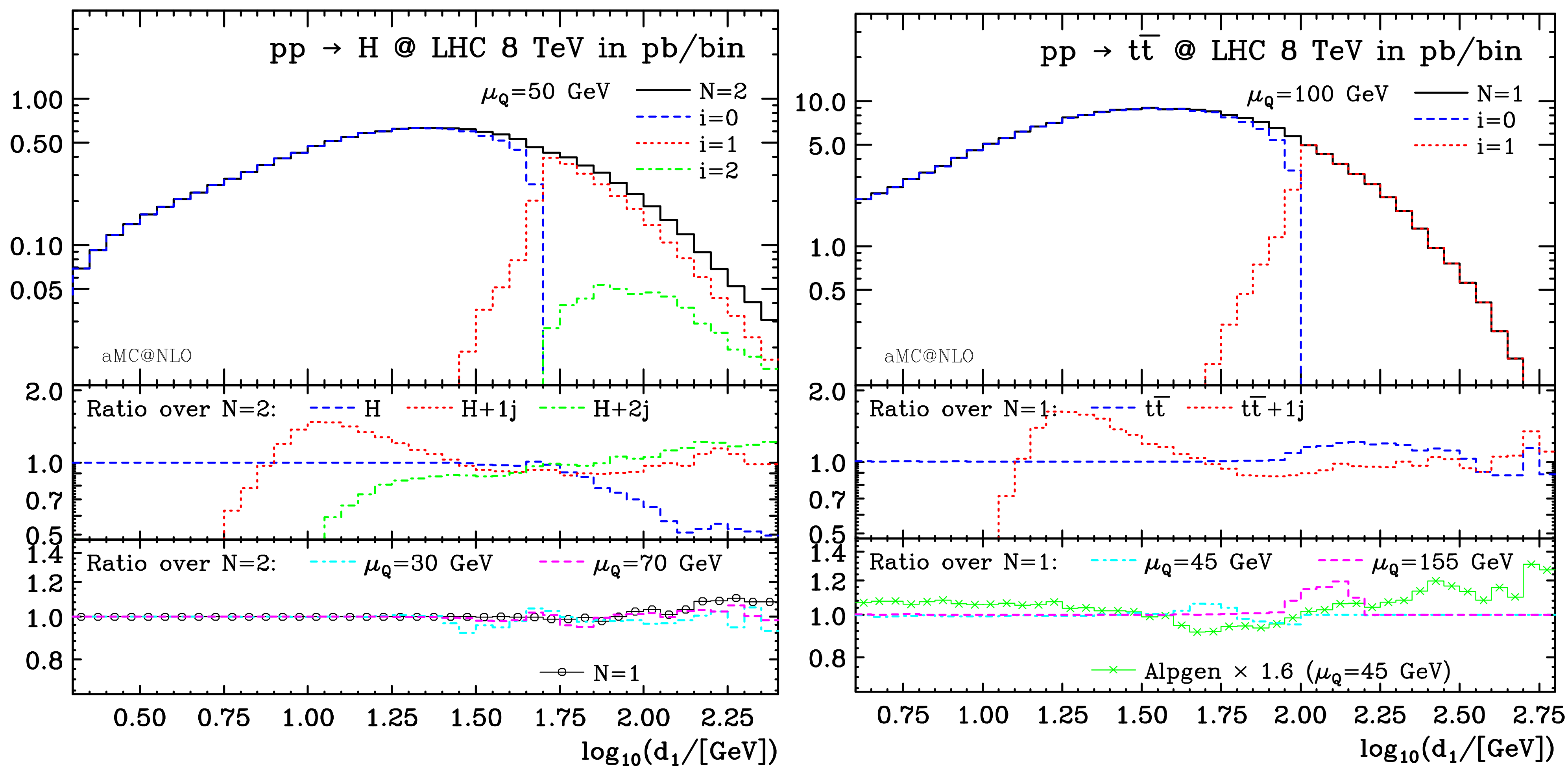

$0 \longrightarrow 1$ rates in H^0 and $t\bar{t}$ production

Merging (FxFx, 1209.6215) at the NLO

Outlook

A significant effort was that of moving to MadGraph5. Now the code is stable, user-friendly, and ready for the next round of development

- EW loop corrections and QCD/EW double perturbative expansion (we are almost there)
- FeynRules can generate Feynman rules from the Lagrangian. It is now understood that it is also possible to generate UV and R_2 counterterms
- Hence, we shall be able to compute NLO corrections *to and in* arbitrary, user-defined theories (e.g. MSSM), thus strictly putting LO and NLO in MadGraph on the same footing
- Use faster trees (recursion). Obviously doable in FKS. Less obvious with OPP, but true (the key information are topologies and colour structures)
- Consistent use of $1/N_c$ expansion

Conclusions

The significant progress made in the past few years by several groups has not only led to remarkable physics results, but also to two (unintended) sociological consequences:

- NLO computations will not require any expertise
- Hiring PhD's or young postdocs as (highly-skilled) human computers is not justified any longer

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So while it is not true that QCD has become "easy" by magic, it is true that one entire class of difficult problems has been fully solved, thus paving the way for precision hadron phenomenology and, for theorists, putting back the emphasis on more conceptual problems

Extras

FKS #1: Simplify the problem

Find parton pairs (i, j) that can give collinear singularities. Then:

$$\mathcal{M}^{(r)} = \sum_{ij} \mathcal{M}_{ij}^{(r)} \qquad \mathcal{M}_{ij}^{(r)} = \mathcal{S}_{ij} \mathcal{M}^{(r)}$$

with

$$\begin{aligned}
&\sum_{ij} \mathcal{S}_{ij} = 1 && \\
&\sum_{j} \mathcal{S}_{ij} \longrightarrow 1 && k_i \longrightarrow 0 \\
&\mathcal{S}_{ij} \longrightarrow 1 && k_i \parallel k_j \\
&\mathcal{S}_{ij} \longrightarrow 0 && \text{all other singularities}
\end{aligned}$$

- $\mathcal{M}_{ij}^{(r)}$ has one soft and one collinear singularity at most
- Soft singularities are "split" into underlying collinear structures
- The $\mathcal{M}_{ij}^{(r)}$'s are independent from each other

FKS #2: Subtract

For a given $\mathcal{M}_{ij}^{(r)}$, the choice of variables associated with singularities is natural and essentially unique

$$\mathcal{M}_{ij}^{(r)} d\phi_{n+1} \longrightarrow \left(\frac{1}{E_i}\right)_+ \left(\frac{1}{1-\cos\theta_{ij}}\right)_+ E_i^2 \left(1-\cos\theta_{ij}\right) \mathcal{M}_{ij}^{(r)} \frac{d\phi_{n+1}}{E_i}$$

- Plus distributions understand the projections $\mathcal{P}\phi_{n+1}$. FKS defines subtraction terms *exactly* on shell, and thus eliminates the problem of recoil altogether (NO approximation is required)
- Soft and collinear counterterms can be defined so as they have the *same* kinematics $\Longrightarrow$ the subtraction term is unique

By-product: important sampling (thanks to E_i and θ_{ij}) is straightforward

FKS #3: Exploit symmetries

Collinear structure and definition of projections imply that the total number of subtraction terms scales as n^2. However for any observable O

$$O(\mathcal{M}_{ij}^{(r)}) = O(\mathcal{M}_{kl}^{(r)})$$

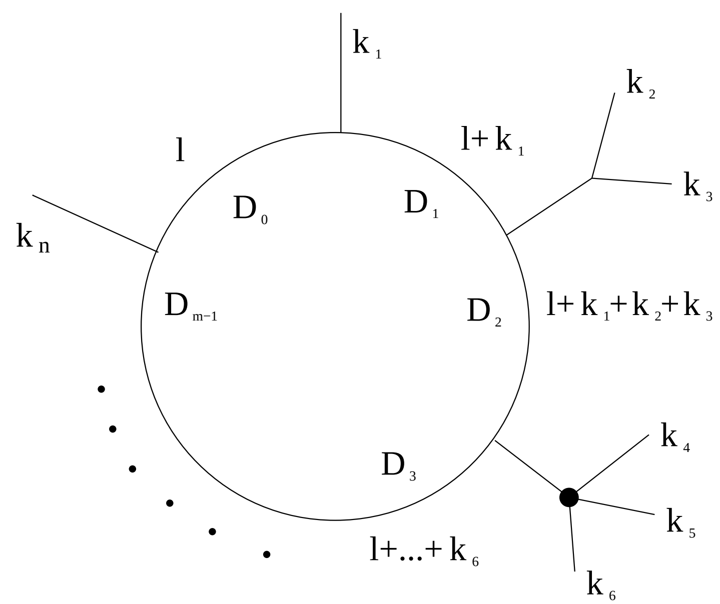
when flavour$_i$=flavour$_k$ and flavour$_j$=flavour$_l$, the key being the independence of $\mathcal{M}_{ij}^{(r)}$ and $\mathcal{M}_{kl}^{(r)}$

$\Longrightarrow$ The majority of contributions can be taken into account simply with an overall symmetry factor

Thus: for a $2 \to n$ gluon process, the total number of subtractions is 3 ($\forall n$). The presence of quarks complicates the counting, but the subtractions are never more than a few

OPP #1: Remember Passarino-Veltman

$$
\begin{aligned}
\mathcal{C} \quad = \quad & \sum_{0\le i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
& + \sum_{0\le i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
& + \sum_{0\le i_0<i_1}^{m-1} b(i_0 i_1) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
& + \sum_{i_0=0}^{m-1} a(i_0) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}} \\
& + R
\end{aligned}
$$



$$\bar{\ell} = \ell + \tilde{\ell}\,, \qquad \ell\cdot\tilde{\ell} = 0$$

$$\bar{D}_i = (\ell + p_i)^2 - m_i^2 + \tilde{\ell}^2$$

OPP #2: Move to the integra*nd* level

$$\mathcal{C} = \int d^d\bar\ell\, \bar C(\bar\ell)\,, \qquad \bar C(\bar\ell) = \frac{\bar N(\bar\ell)}{\prod_{i=0}^{m-1} \bar D_i}$$

Separate 4- and ϵ-dimensional part in the numerator

$$\bar N(\bar\ell) = N(\ell) + \widetilde N(\ell, \tilde\ell) \quad \Longrightarrow \quad \bar C(\bar\ell) = \frac{N(\ell)}{\prod_{i=0}^{m-1} \bar D_i} + \frac{\widetilde N(\ell, \tilde\ell)}{\prod_{i=0}^{m-1} \bar D_i}$$

This defines a cut-constructible-plus-rational part, and a pure-rational part:

$$\begin{aligned}
\mathcal{C} &= \mathcal{C}_{cc+R_1} + R_2 \\
\mathcal{C}_{cc+R_1} &= \int d^d\bar\ell\, \frac{N(\ell)}{\prod_{i=0}^{m-1} \bar D_i} \\
R_2 &= \int d^d\bar\ell\, \frac{\widetilde N(\ell, \tilde\ell)}{\prod_{i=0}^{m-1} \bar D_i}
\end{aligned}$$

OPP #3: Expand the numerator

$$
\begin{aligned}
N(\ell) \quad &= \sum_{0\le i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0i_1i_2i_3) + \hat{d}(\ell; i_0i_1i_2i_3)\right] \prod_{\substack{i=0 \\ i\notin\{i_0,i_1,i_2,i_3\}}}^{m-1} D_i \\
&+ \sum_{0\le i_0<i_1<i_2}^{m-1} \left[c(i_0i_1i_2) + \hat{c}(\ell; i_0i_1i_2)\right] \prod_{\substack{i=0 \\ i\notin\{i_0,i_1,i_2\}}}^{m-1} D_i \\
&+ \sum_{0\le i_0<i_1}^{m-1} \left[b(i_0i_1) + \hat{b}(\ell; i_0i_1)\right] \prod_{\substack{i=0 \\ i\notin\{i_0,i_1\}}}^{m-1} D_i \\
&+ \sum_{0\le i_0}^{m-1} \left[a(i_0) + \hat{a}(\ell; i_0)\right] \prod_{\substack{i=0 \\ i\neq i_0}}^{m-1} D_i
\end{aligned}
$$

- $d \ldots a$ are the *same as at the integral* level
- Spurious terms $\hat{d} \ldots \hat{a}$ vanish upon integration
- Mismatch between D_i and $1/\bar{D}_i$ is the origin of R_1

OPP #4: Enter CutTools

The system

$$N(\ell) = f(\ell; d, c, b, a)$$

is solved for $d \ldots a$ by the recursive applications of unitarity-cut-like conditions (4, 3, 2, or 1 denominators are imposed to vanish):

$$D_{i_0}(\ell^{\pm}) = D_{i_1}(\ell^{\pm}) = D_{i_2}(\ell^{\pm}) = D_{i_3}(\ell^{\pm}) = 0$$

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Bottom line:

- CutTools solves a system of linear equations. One can see this (owing to the values of ℓ) as unitarity-cutting, but it is not essential
- Given the function $N(\ell)$, momenta, and masses, CutTools returns the cut-constructible part and R_1, using pre-tabulated results for scalar one-loop integrals
- R_2 must be computed independently

Construction of POWHEG

Start with an exact phase-space factorization $d\phi_{n+1} = d\phi_n d\phi_r$, and construct

$$\overline{\mathcal{M}}^{(b)}(\phi_n) = \mathcal{M}^{(b+v+rem)}(\phi_n) + \int d\phi_r \left[\mathcal{M}^{(r)}(\phi_{n+1}) - \mathcal{M}^{(c.t.)}(\phi_{n+1})\right]$$

For a *given* p_T, define the *vetoed* *process-dependent* Sudakov

$$\Delta_R(t_I, t_0; p_T) = \exp\left[-\int_{t_0}^{t_I} d\phi'_r \frac{\mathcal{M}^{(r)}}{\mathcal{M}^{(b)}} \Theta(k_T(\phi'_r) - p_T)\right]$$

Obtain hard configurations (to be given to shower as initial conditions) from the short-distance cross section

$$d\sigma_{\text{POWHEG}} = d\phi_n \overline{\mathcal{M}}^{(b)}(\phi_n)\left[\Delta_R(t_I, t_0; 0) + \Delta_R(t_I, t_0; k_T(\phi_r)) \frac{\mathcal{M}^{(r)}(\phi_{n+1})}{\mathcal{M}^{(b)}(\phi_n)} d\phi_r\right]$$

which includes Sudakov suppression at $p_T \to 0$

- $k_T(\phi_r)$ will play the role of hardest emission
- The full real matrix element is exponentiated

Attaching (angular-ordered) showers

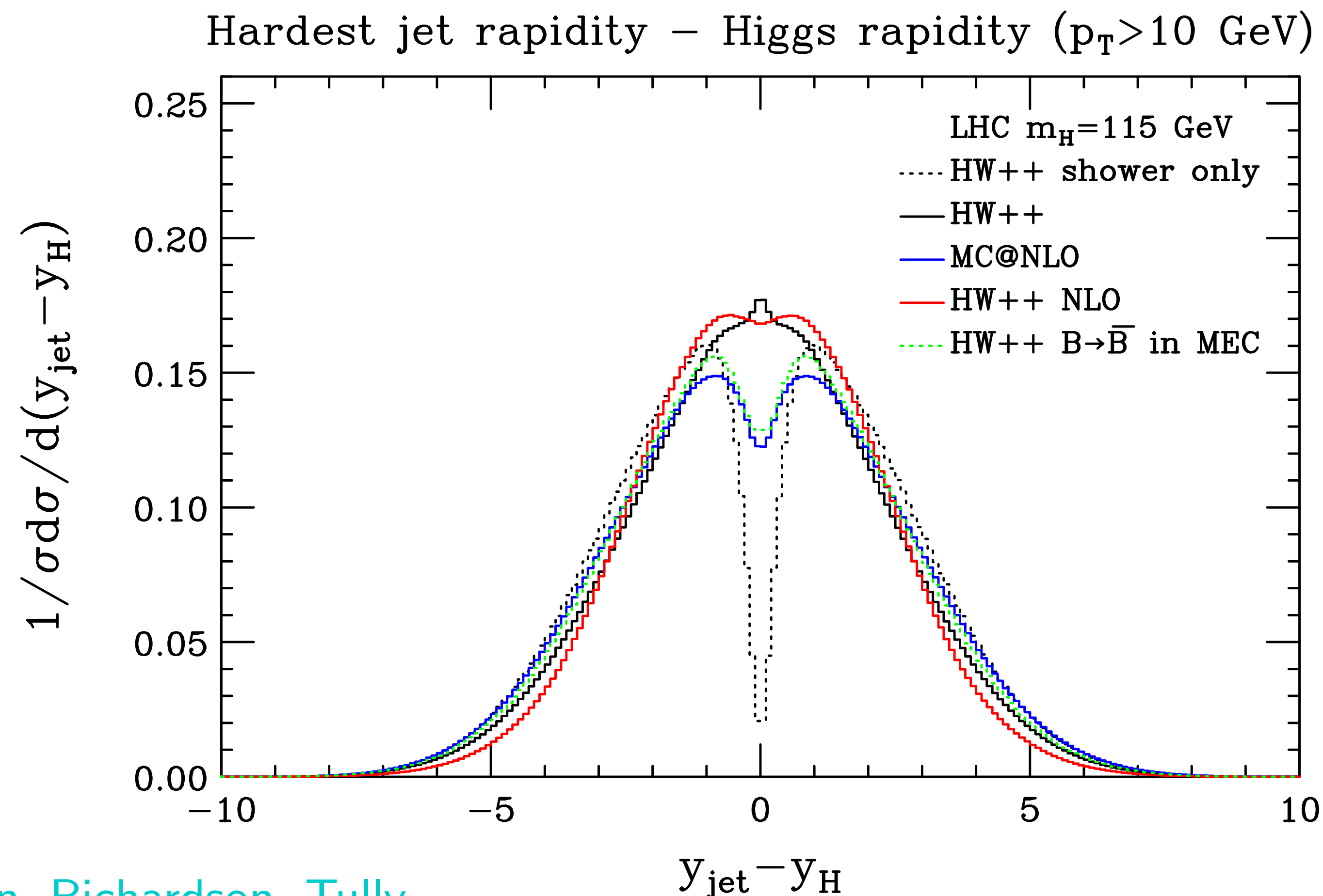
- One wants the matrix-element-generated p_T to be the hardest
  $\Longrightarrow$ veto emissions harder than p_T during shower
- But this screws up colour coherence

Colour coherence can be restored at the price of a more involved structure

$$\begin{aligned}\mathcal{F}_{\text{POWHEG}}[t_I;p_T] &= \Delta(t_I,t_0)+\int_{t_0}^{t_I}\frac{dt}{t}\int dz\Delta_R(t_I,t;p_T)\frac{\alpha_S}{2\pi}P(z)\\ &\quad\times\mathcal{F}_{\text{V}}((1-z)^2t;p_T)\,\mathcal{F}_{\text{V}}(z^2t;p_T)\,\mathcal{F}_{\text{VT}}(t_I,t;p_T)\end{aligned}$$

- $\mathcal{F}_{\text{V}}(t;p_T)$ are *vetoed* showers. Evolve down to t_0, with all emissions constrained to have a transverse momentum smaller than p_T
- $\mathcal{F}_{\text{VT}}(t_I,t;p_T)$ are *vetoed-truncated* showers. Evolve from t_I down to t (i.e., *not* t_0) along the hardest line. On top of that, they are vetoed

MC@NLO vs POWHEG: discrepancies



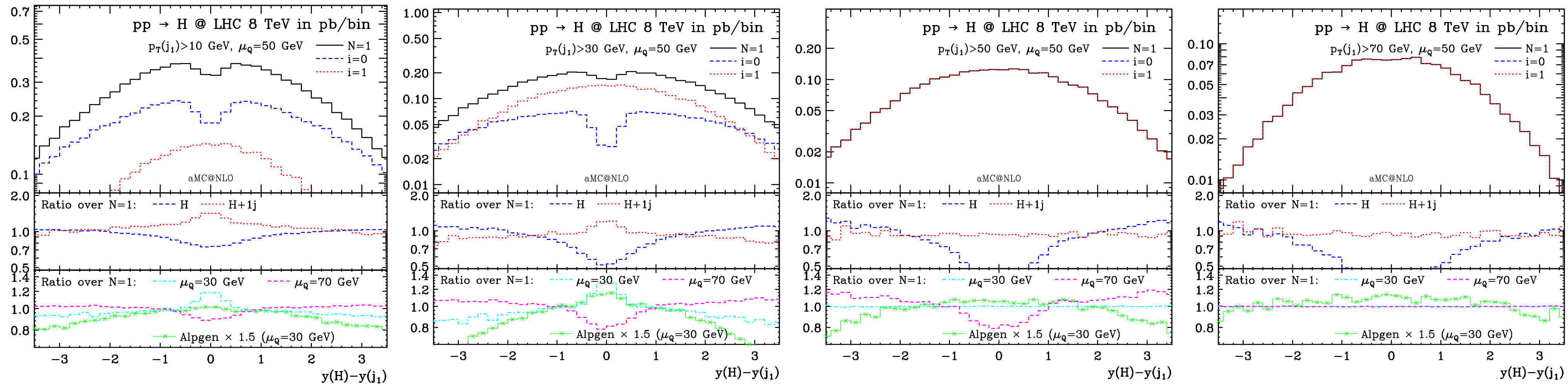
Hamilton, Richardson, Tully

HW/HW++ have dips at $\Delta y = 0$. Likely an artifact of dead zones

MC@NLO fills that dip, via hard radiation

POWHEG fills it much more, mainly owing to own Sudakov

aMC@NLO



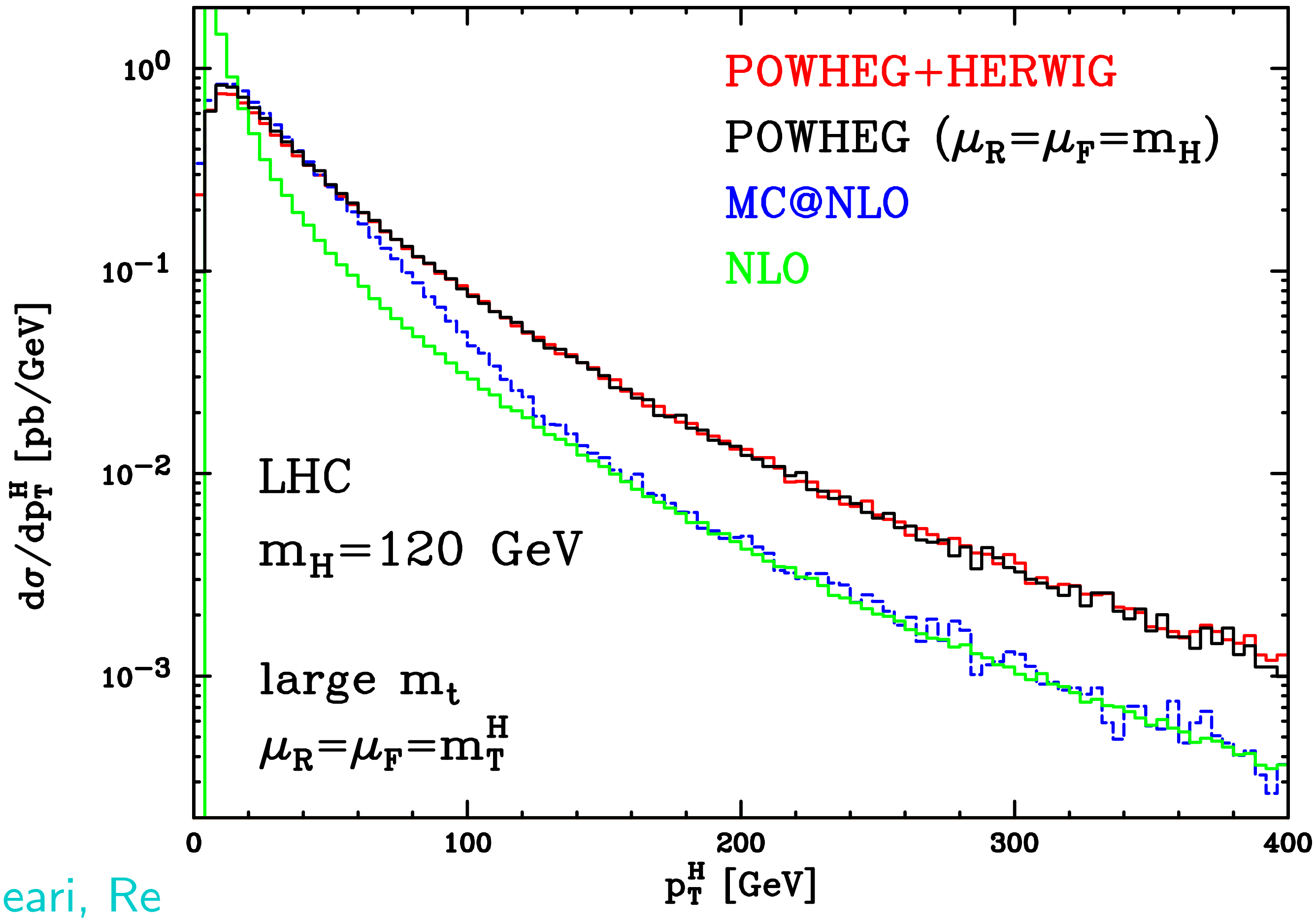
This is a rather interesting case, somewhat involved

In short: this is a 1-jet-dominated observables, so $H + 0j$ (even at the NLO) may not be ideal

Furthermore, there is a significant MC dependence

Merging (here, FxFx, 1209.6215) solves it

MC@NLO vs POWHEG: discrepancies



Alioli, Nason, Oleari, Re

POWHEG a factor ~ 3 larger than MC@NLO$\equiv$ NLO in the tail

POWHEG result can be decreased by removing part of the real contribution from the exponent. Predictive power?

Note: MC@NLO and POWHEG use the *same matrix elements*