Constraining discrete leptonic flavour symmetries at long-baseline neutrino oscillation facilities

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Outline of talk

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- Long-baseline oscillation physics
  - Current knowledge
  - Next generation long-baseline experiments
- Phenomenological approach to discrete flavour symmetries
- Constraining atmospheric sum-rules
  - Discriminating between sum-rules
- Conclusions

Neutrino flavour and oscillations

- Contrary to the SM, neutrinos have mass and undergo flavour oscillations due to non-trivial mixing between the mass eigenstates and the flavour states.

- In the minimal scenario, the oscillation probability depends upon two mass squared splittings

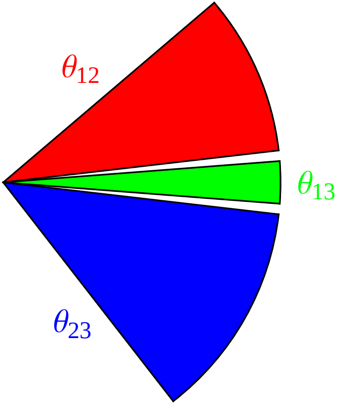
$$\Delta m^2_{21} \equiv m^2_2 - m^2_1 \qquad \text{and} \qquad \Delta m^2_{31} \equiv m^2_3 - m^2_1 .$$

- The remaining parameters describe the mapping between bases, expressed as a 3×3 unitary matrix, such that $\nu_\alpha = (U_{\mathrm{PMNS}})_{\alpha i}\, \nu_i$ where

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-\mathrm{i}\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{\mathrm{i}\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P.$$

What we know

- Thanks to decades of experimental work, including the discovery last year of θ_{13}, we now know all three of the angles which parameterize the PMNS matrix.



$$\sin^2\theta_{12} \approx 0.31,$$
$$\sin^2\theta_{23} \approx 0.52,$$
$$\sin^2\theta_{13} \approx 0.02.$$

- We also know the magnitudes of both mass squared differences and the sign of one.

$$\Delta m^2_{21} \approx 7.59 \times 10^{-5}$$
$$\left|\Delta m^2_{32}\right| \approx 2.50 \times 10^{-3}.$$

What we would like to know

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$$\Delta m_{13}^2 > 0?$$

What is the true hierarchy of neutrino masses?

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$$\delta_{CP} \in \{0, \pi\}?$$

Does the leptonic sector exhibit CP-violation?

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$\Delta m^2_{13} > 0?$

What is the true hierarchy of neutrino masses?

$\delta_{CP} \in \{0, \pi\}?$

Does the leptonic sector exhibit CP-violation?

$\theta_{12},\ \theta_{23},\ \theta_{13},$
$\Delta m^2_{12},\ \Delta m^2_{13},\ \delta_{CP}$

Is that all there is? Do we need to extend the 3ν-mixing paradigm?

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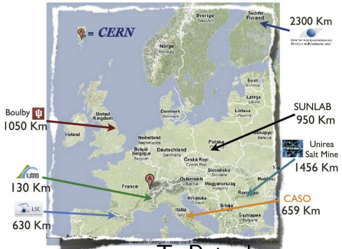
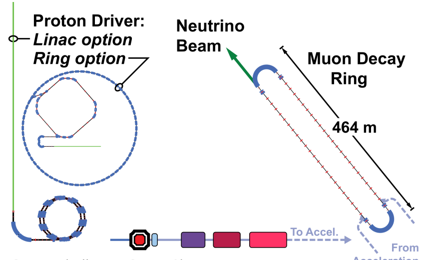
Candidate designs: superbeams, neutrino factories

- Superbeams are more powerful conventional neutrino beams. There are a number of proposed experiments: LBNE, LAGUNA-LBNO and T2HK (see e.g 1110.6249, SPSC-EOI-007, 1109.3262).



T. Patzak

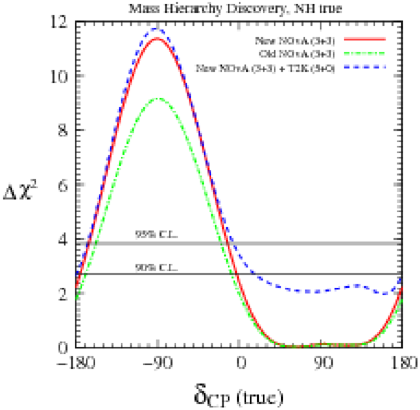


J.S. Berg (talk at IDS-NF 10)

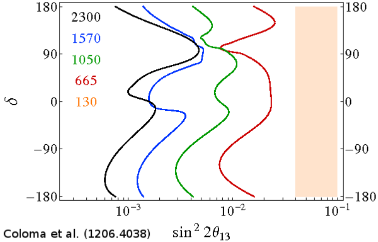
- A Neutrino Factory derives its beam from the decay of stored muons. This provides a very well understood and low background signal: wrong-sign muons. (see IDS-NF-020).

Prospects: mass hierarchy

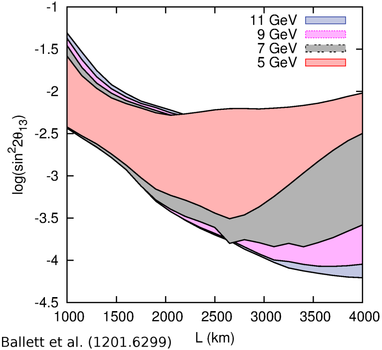
- Current generation offers reasonable reach: 40% at 2σ. Next generation: 100% at 3σ.



Agarwalla et al. (1208.3644)



Coloma et al. (1206.4038)

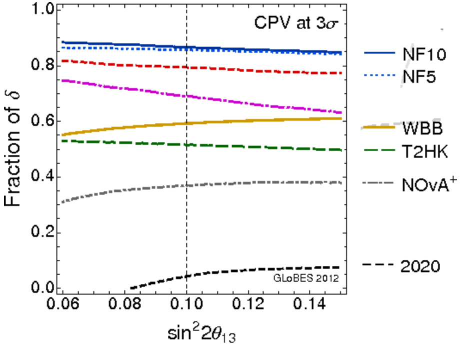


Ballett et al. (1201.6299)

Prospects: CP violation

- Trying to exclude $\delta_{\text{CP}} \in \{0, \pi\}$ is now the central focus of many next-generation experiments.

- Current generation has little sensitivity. Different proposed facilities offer varied chances to make the measurement. Potentially as high as 90% of parameter space.



Coloma et al. (1209.5973)

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Discrete leptonic flavour symmetries

- The distinctive mixing angles of the PMNS matrix have motivated many authors to look for models which use discrete symmetries in the leptonic sector. (for a recent review see 1301.1340)

$$e.g \qquad U_{\text{TBM}} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

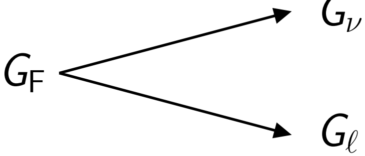
- Proposes some symmetry G_{F}, which is usually spontaneously broken by a set of flavons.

- The combination of the choice of particle representations, VEV alignment and the symmetry-compatable couplings shapes the resultant mass terms.

Discrete leptonic flavour symmetries

- Models of discrete flavour symmetries make predictive statements of correlations amongst the neutrino flavour parameters.
- With such a *large body of theoretical predictions*, constraining and excluding these correlations should be an aim of any next-generation facility.
- To focus our discussion, we have restricted our attention to a class of models based on a bottom-up approach due to Hernandez and Smirnov (see 1204.0445 and 1212.2149).

Hernandez-Smirnov approach

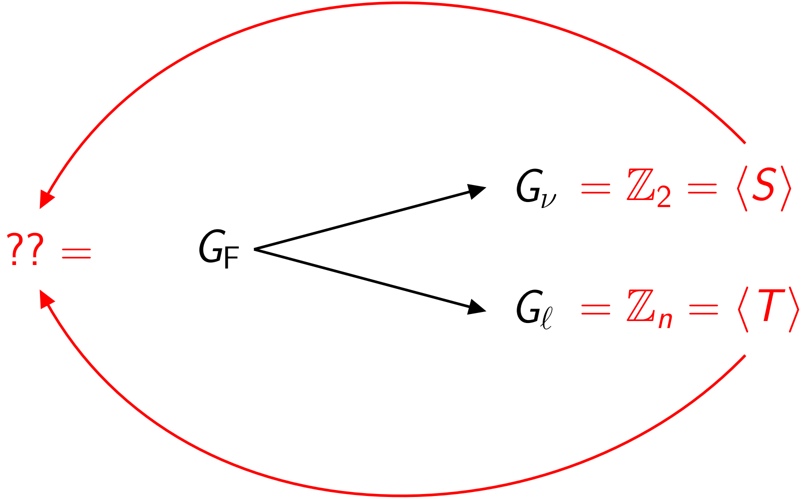


Hernandez-Smirnov approach

$$G_{\mathrm{F}} \to G_\nu = \mathbb{Z}_2 = \langle S \rangle$$

$$G_{\mathrm{F}} \to G_\ell = \mathbb{Z}_n = \langle T \rangle$$

Hernandez-Smirnov approach



Hernandez-Smirnov approach

$$D(2, n, m) = G_{\mathrm{F}}$$

$$G_{\nu} = \mathbb{Z}_2 = \langle S \rangle$$

$$G_{\ell} = \mathbb{Z}_n = \langle T \rangle$$

Hernandez-Smirnov approach (see 1204.0445 and 1212.2149)

- Attempts to constrain the PMNS from a bottom-up version of the symmetry breaking scenario. By specifying G_ν and G_ℓ, and making a few assumptions about G_{F}, we can derive constraints on U_{PMNS}.

- The subgroups G_ν and G_ℓ are chosen from the symmetries of the leptonic mass terms.

$$\mathcal{L}_\nu = \frac{1}{2}\overline{\nu^c}_L m_\nu \nu_L, \qquad \text{and} \qquad \mathcal{L}_\ell = \overline{E}_R m_\ell \ell_L.$$

- The symmetry of the neutrino mass term is $\mathbb{Z}_2 \times \mathbb{Z}_2$, whilst for the charged leptons it is $\mathrm{U}(1)^3$.

- It is *assumed* that the residual symmetries of these sectors are $G_\nu = \mathbb{Z}_2$ and $G_\ell = \mathbb{Z}_m$, and that the remaining symmetries are accidental.

Hernandez-Smirnov approach (cont.)

- Reversing the broken-symmetry scenario, these subgroups must be combined in some way to form the supergroup G_{F}. For any *finite* group we require the generators to obey $(g_\nu g_\ell)^p = 1$ for $p \in \mathbb{N}$.

- This assumption leads us to the von Dyck groups $D(2, m, p)$ given by the presentation

$$\langle S, T, W \mid S^2 = T^m = W^p = 1 \rangle.$$

- Assuming finiteness, the only permissible groups turn out to be small order groups already popular in the literature

$$\mathrm{D}(2,2,3) = \mathrm{S}_3, \qquad \mathrm{D}(2,3,3) = \mathrm{A}_4,$$
$$\mathrm{D}(2,3,4) = \mathrm{S}_4, \qquad \mathrm{D}(2,3,5) = \mathrm{A}_5.$$

Constraints and correlations

- In the framework that I've discussed, the symmetries can be shown to fix a column of the PMNS matrix. This leads to two constraints on the PMNS matrix parameters

$$e.g. \qquad \begin{pmatrix} |U_{e1}|^2 \\ |U_{\mu 1}|^2 \\ |U_{\tau 1}|^2 \end{pmatrix} = \begin{pmatrix} \frac{1-\eta}{2} \\ \frac{1-\eta}{2} \\ \eta \end{pmatrix}.$$

- For the models that we are interested in, these constraints can be expressed as a definition of θ_{12} in terms of θ_{13}, called a *solar sum-rule*, and a correlation between θ_{23}, θ_{13} and $\cos\delta$, which is called the atmospheric sum-rule

$$e.g. \qquad |U_{e1}|^2 = \frac{1-\eta}{2} \quad \Longrightarrow \quad \cos^2\theta_{12} = \frac{1-\eta}{2\cos^2\theta_{13}}.$$

Atmospheric sum-rules

- To simplify our expressions we introduce the following parameters (King 2007)

$$\sin\theta_{12} \equiv \frac{1+s}{\sqrt{3}}, \qquad \sin\theta_{23} \equiv \frac{1+a}{\sqrt{2}}, \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}},$$

  which have the following 1σ ranges (Fogli 2012)

$$-0.07 \le s \le -0.01, \qquad 0.21 \le r \le 0.23, \qquad -0.15 \le a \le -0.07.$$

- We then expand the atmospheric sum-rule to first order in r, this allows us to express *all phenomenologically interesting* models by the constraint

$$a = a_0 + \lambda r \cos\delta + \mathcal{O}(r^2, a^2).$$

Viable atmospheric sum-rules

Type	Group	Sum-rule
$\lambda \approx 1$	S_4	$a = r\cos\delta$
	A_5	$a = \sqrt{\frac{1+\varphi}{2}}\, r\cos\delta$
$\lambda \approx -\frac{1}{2}$	A_4	$a = -\frac{1}{2} r\cos\delta$
	S_4	$a = -\frac{1}{\sqrt{6}} r\cos\delta \pm \frac{2}{3}(\sqrt{3} - 2)$
	A_5	$a = -\frac{1}{\sqrt{2(1+\varphi)}} r\cos\delta$
	A_5	$a = -\sqrt{\frac{3+2\varphi}{22}}\, r\cos\delta \pm \frac{2}{11}(7+\varphi)s$

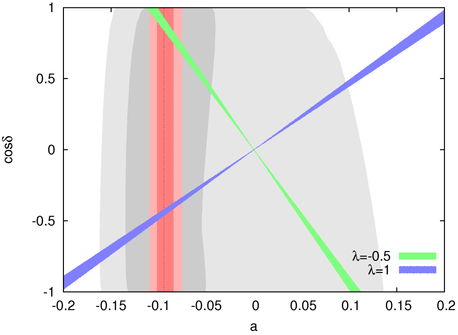
We find 8 viable sum-rules from the construction discussed above. These divide neatly into two classes based on their approximate values of λ.

Sum-rules and current data

- The linearized sum-rule can be seen as a prediction of the model for the parameter $\cos\delta$.

$$\cos\delta = \frac{a}{\lambda r}$$

- The grey bands show the current global-fit data (NuFit 1.0 2012), whilst the pink bands show the projected sensitivity to *a* in 2025 with the current generation of experiments (Huber et al. 2009).

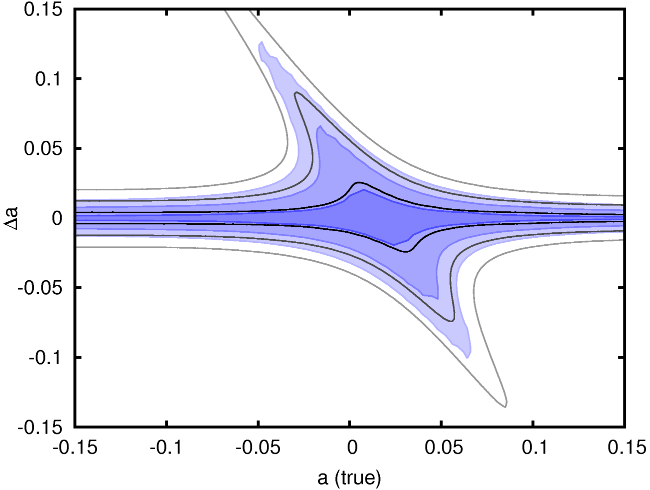


Outline of talk

Simulation details

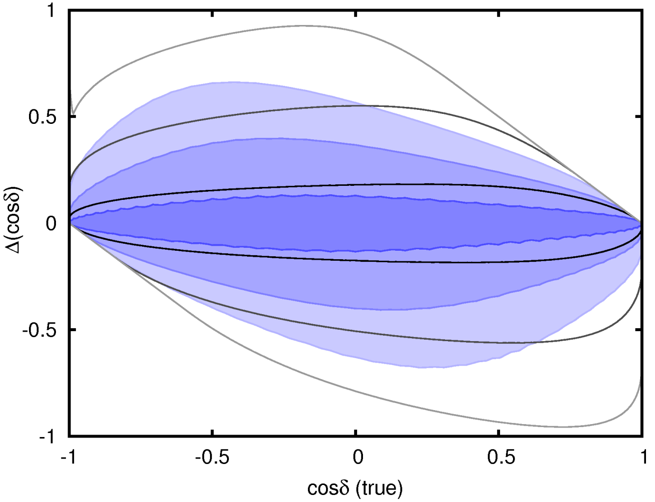
- We have simulated the measurement of sum-rules for some representative next-generation facilities using the GLoBES package (see 0407333 and 0701187).
- A superbeam based on the LAGUNA-LBNO proposal of a beam from CERN to Pyhäsalmi (Finland). This has a baseline distance of 2300 km and a 100 kton liquid Argon detector (for more info. see CERN-SPSC-2012-021).
- We also consider a Low-Energy Neutrino Factory (LENF) with a baseline of 2000 km and a stored-muon energy of 10 GeV. We have run simulations for both a 100 kton MIND and a more optimistic 70 kton liquid Argon detector (LAr) (for more info. see IDS-NF-020).

Precision in relevant parameters: a



The 1, 3 and 5σ allowed regions for $a = \sqrt{2}\sin\theta_{23} - 1$ as a function of the true value of a. Solid regions are for the LENF, empty regions for the superbeam.

Precision in relevant parameters: $\cos\delta$

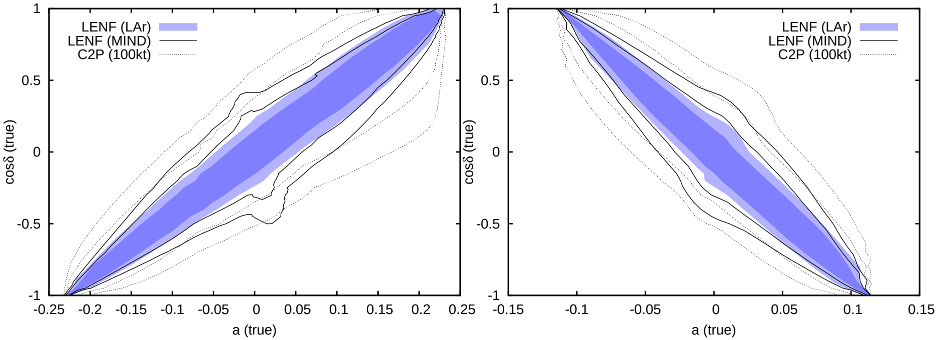


The 1, 3 and 5σ allowed regions for $\cos\delta$ as a function of the true value of $\cos\delta$. Solid regions are for the LENF, empty regions for the superbeam.

Excluding sum-rules

- Combining single parameter determinations (as in the previous slide) can only tell us so much about the ability to exclude parameter combinations.
- In general, parameter correlations can lead these sensitivities to change.
- We have scanned over true values of a and $\cos\delta$. For each pair, we have plotted the $\Delta\chi^2$ value of the best-fitting solution obeying a given sum-rule. When this becomes higher than a certain significance threshold, we can say that the sum-rule hypothesis is excluded.

Excluding $a = r\cos\delta$ and $a = -\frac{1}{2}r\cos\delta$



- These plots show 2 and 3σ allowed regions for the given sum-rules as a function of the true parameters.
- There is a central bump which is due to trivial solutions to the sum-rule close to the origin with $a \approx 0$ and $\cos\delta \approx 0$.

PB et al. (2013) *in preparation.*

Outline of talk

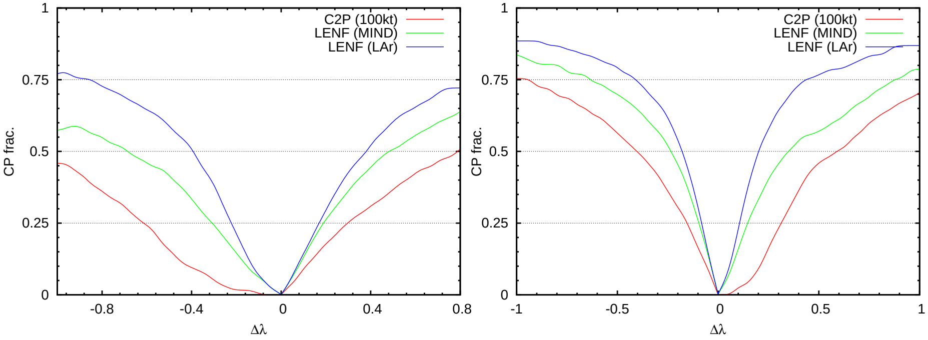
Discriminating between sum-rules

- We have seen that the 8 sum-rules of interest can be classified into one of two types $\lambda \approx 1$ and $\lambda \approx -\frac{1}{2}$.

- What degree of precision would be necessary to discriminate between close lying sum-rules?

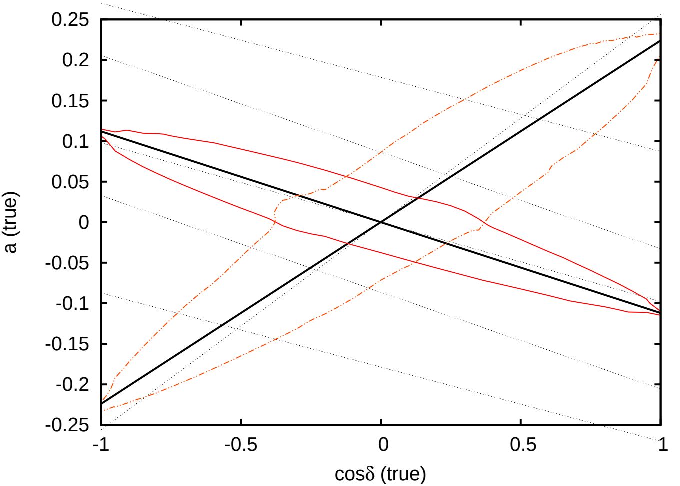
$\lambda \approx 1$	$a = r\cos\delta$ $a = \sqrt{\frac{1+\varphi}{2}}\, r\cos\delta$	$\Longrightarrow$	$\Delta\lambda \approx 0.144$
$\lambda \approx -\frac{1}{2}$	$a = -\frac{1}{2} r\cos\delta$ $a = -\frac{1}{\sqrt{2(1+\varphi)}}\, r\cos\delta$	$\Longrightarrow$	$\Delta\lambda \approx 0.063$

Determining λ (for $a_0 = 0$)



- CP fraction is defined here as the fraction of values which obey $a = \lambda_T r \cos\delta$ for which the sum-rule $a = \lambda_F r \cos\delta$ can be excluded.
- We see for $\lambda_F = 1$, a CP fraction of 50% is possible with the most optimistic facility only if $|\Delta\lambda| \approx 0.4$. For $\lambda_F = -0.5$, the required deviation roughly halves.

Excluding competing sum-rules



To exclude all sum-rules of this type will be very challenging. However, for large parts of parameter space the problem may be reduced to a low-multiplicity degeneracy.

Outline of talk

Conclusions

- With increased precision in the neutrino flavour sector, the next generation of experiments will enable us to start to test a number of proposed new physics models which address leptonic flavour.

- There is a large literature of models which use discrete symmetries to predict correlations amongst the parameters of the PMNS matrix. A quite general class of models can have these constraints expressed as atmospheric sum-rules.

- Individual sum-rules can be excluded for a significant fraction of the parameter space. Differentiating between the sum-rules that we have identified will be challenging but possible at a aggressive facility.

Thank you.