AQGC and Unitarisation in presence of a SM Higgs for Vector Boson Scattering

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Theoretische Physik I

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Theor. Physik 1

WHIZARD

Motivation

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2012/13: Discovery of new resonance

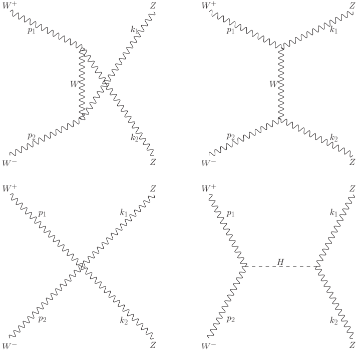
- Hot candidate for SM Higgs

⇒ Complete Higgs sector of SM ($w^{\pm}, z, h$)

⇒ unitarizes VBS-Amplitude

  - Naive definition:
    Amplitude does not rise with energy

- VBS is sensitive to New Physics contributions within the Higgs sector



Example $\mathcal{A}(WW \to ZZ)$

$$\frac{s}{v^2} - \frac{1}{v^2}\frac{s^2}{s-M_h{}^2} + \mathcal{O}(1)$$

Modelling New Physics

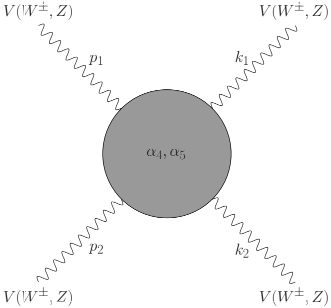
- Many models Beyond the Standard Model describe NP in VBS

$\Rightarrow$ Lots of work to test them all for NP

- Better: Looking for a model independent way

$\Rightarrow$ Ansatz of an Effective Field Theory!

- Modelling NP contributions via AQGC or additional resonances



Anomalous Quartic Gauge Couplings

- Unknown energy scale Λ of NP up to which the EFT is valid
- AQGC yield new contributions to VBS amplitude

$\Rightarrow$ Unitarity is not guaranteed

- EFT is limited through Unitarity conditions

Appelquist/Guo Hong: hep-ph/9304240

$$\mathcal{L}_4 = \alpha_4 \operatorname{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \operatorname{Tr}\left[\mathbf{V}_\nu \mathbf{V}^\nu\right]$$
$$\mathcal{L}_5 = \alpha_5 \operatorname{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\nu\right] \operatorname{Tr}\left[\mathbf{V}_\nu \mathbf{V}^\mu\right]$$

$\updownarrow$ Two equivalent representations

Eboli/Gonzalez-Garcia/Mizukoshi: hep-ph/0606118

$$\mathcal{L}_{S,0} = \frac{f_{S,0}}{\Lambda^4} \left[(\mathbf{D}_\mu \Phi)^\dagger \mathbf{D}_\nu \Phi\right] \left[(\mathbf{D}^\mu \Phi)^\dagger \mathbf{D}^\nu \Phi\right]$$
$$\mathcal{L}_{S,1} = \frac{f_{S,1}}{\Lambda^4} \left[(\mathbf{D}_\mu \Phi)^\dagger \mathbf{D}^\mu \Phi\right] \left[(\mathbf{D}_\nu \Phi)^\dagger \mathbf{D}^\nu \Phi\right]$$

Kinematical Aspects

- Rough estimate of energy-dependence for VBS amplitude:

Highest contribution

$$\epsilon_L^{\mu}(p_1)\epsilon_{L\,\mu}(p_2)\epsilon_L^{\nu}(k_1)\epsilon_{L\,\nu}(k_2) \sim \frac{s^2}{M_W^4}$$

- To respect Unitarity: terms of $\mathcal{O}(s)$ must cancel

Longitudinal polarized VB

$$\epsilon_L^{\mu}(k) = \left(\frac{k}{m}, 0, 0, \frac{E_k}{m}\right) \overset{k\to\infty}{\longrightarrow} \frac{k^{\mu}}{m} + \mathcal{O}\left(\frac{m}{E_k}\right)$$

Mandelstam variables

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - k_1)^2$$
$$u = (p_1 - k_2)^2$$

AQGC contributions

AQGC contribution to VBS amplitude (GBET):

- without Higgs: Alboteanu/Kilian/Reuter: arXiv:0806.4145

$$\boxed{\mathcal{A}(s,t,u) =:} \qquad \mathcal{A}(w^+w^- \to zz) = 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2+u^2}{v^4}$$

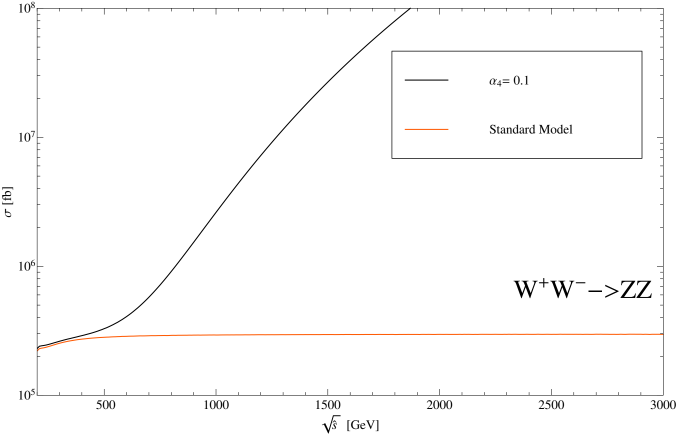
$$\mathcal{A}(w^+z \to w^+z) = 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2+u^2}{v^4}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = (4\alpha_4 + 2\alpha_5) \frac{s^2+t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4}$$

$$\mathcal{A}(w^+w^+ \to w^+w^+) = 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5) \frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8(\alpha_4 + \alpha_5) \frac{s^2+t^2+u^2}{v^4}$$

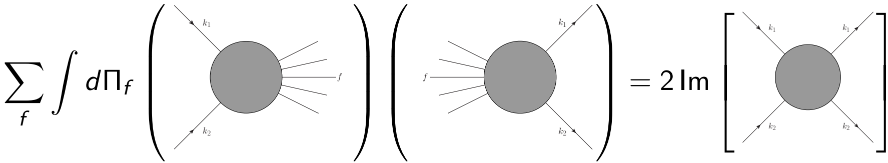
$\equiv$ with Higgs: Kilian/Reuter/Sekulla: in prep



AQGC breaks Unitarity obviously

Cannot use whole LHC energy region for data analysis

Optical Theorem



Unitarity of S-Matrix ⇒ Optical Theorem

$$\sigma_{\text{tot}} = \frac{\mathrm{Im}\left[\mathcal{M}_{\text{ii}}(\text{t}=0)\right]}{s}$$

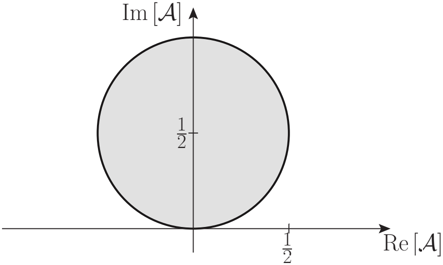
The imaginary part of a forward scattering amplitude arises from a sum of contributions from all possible intermediate state particles

- right(left) side: matrix elements are (not) dependent on t
- Get relation for matrix elements → partial waves $\mathcal{A}_\ell$

Argand-Circle

Argand-Circle condition

$$\left|\mathcal{A}_\ell(s) - \frac{\mathrm{i}}{2}\right| \leq \frac{1}{2}$$



- Out of Argand-Circle: Break Unitarity
- In/On Argand-Circle : Fullfill Unitarity
  - in: Inelastic scattering ($<$)
  - on: Elastic scattering ($=$)

! VBS for physical states is inelastic

Unitarity Bound on AQGC

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- VBS interaction matrix has non diagonal elements:

$$\begin{pmatrix} \mathcal{A}_{w^+w^+\to w^+w^+} & 0 & 0 & 0 \\ 0 & \mathcal{A}_{w^+z\to w^+z} & 0 & 0 \\ 0 & 0 & \mathcal{A}_{w^+w^-\to w^+w^-} & \mathcal{A}_{w^+w^-\to zz} \\ 0 & 0 & \mathcal{A}_{zz\to w^+w^-} & \mathcal{A}_{zz\to zz} \end{pmatrix}$$

$\Rightarrow$ Using Isospin-Symmetry to diagonalize interaction matrix

- Partial wave decomposition into Isospin-Spin eigenamplitudes $\mathcal{A}_{I\ell}$
- Example for $\mathcal{A}(WW \to ZZ)$

$$\mathcal{A}(w^+w^- \to zz) = \frac{1}{3}(\mathcal{A}_{00}(s) - \mathcal{A}_{20}(s)) - \frac{10}{3}(\mathcal{A}_{02}(s) - \mathcal{A}_{22}(s)) + 5(\mathcal{A}_{02}(s) - \mathcal{A}_{22}(s))\frac{t^2+u^2}{s^2}$$

- All $\mathcal{A}_{I\ell}$ have to lie on the Argand-Circle

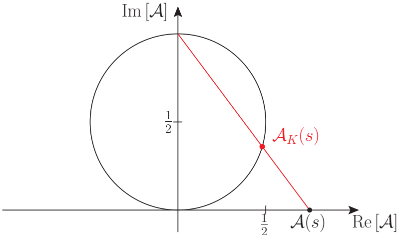
K-Matrix

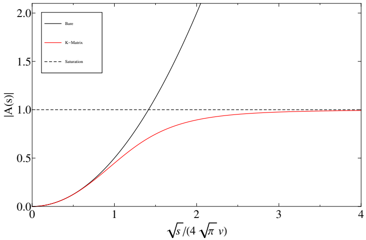
K-Matrix Unitarisation

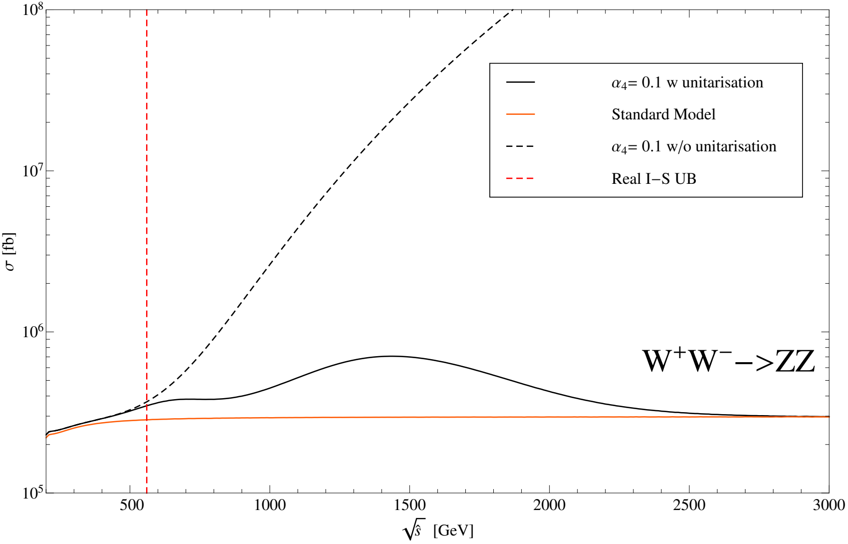
$$\mathcal{A}^K_{I\ell}(s) = \frac{1}{\mathrm{Re}\left(\frac{1}{\mathcal{A}_{I\ell}(s)}\right) - \mathrm{i}}$$

$$= \frac{\mathcal{A}_{I\ell}(s)}{1 - \mathrm{i}\mathcal{A}_{I\ell}(s)} \quad \text{if } \mathcal{A}_{I\ell}(s) \in \mathbb{R}$$

- Projection of unphysical elastic amplitudes onto Argand-Circle
- At high energies the amplitude saturises
- Choose of Unitarisation scheme introduces model dependence
- Implemented in Monte-Carlo Generator WHIZARD







10^8
10^7
10^6
10^5
σ [fb]
500
1000
1500
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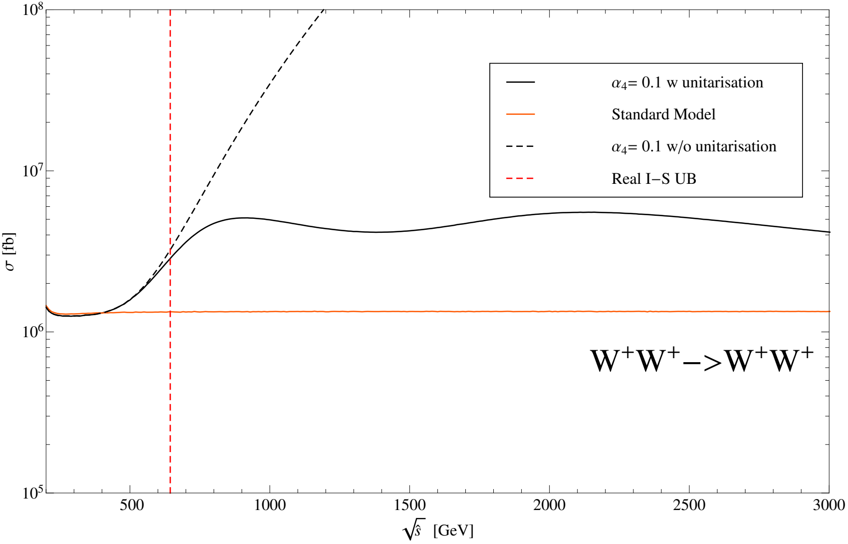
3000
$\sqrt{\hat{s}}$ [GeV]
α_4= 0.1 w unitarisation
Standard Model
α_4= 0.1 w/o unitarisation
Real I−S UB
$W^+W^- \to ZZ$



α_4= 0.1 w unitarisation
Standard Model
α_4= 0.1 w/o unitarisation
Real I−S UB
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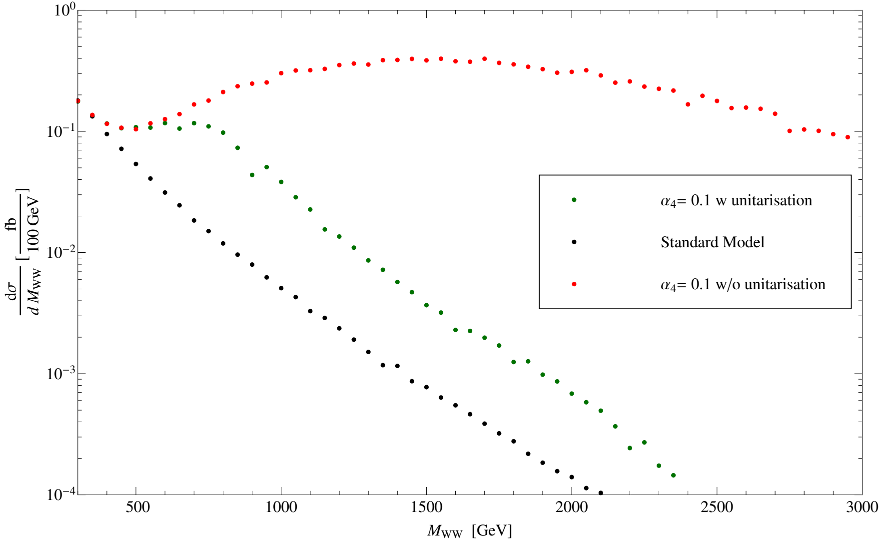
$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 8$ TeV

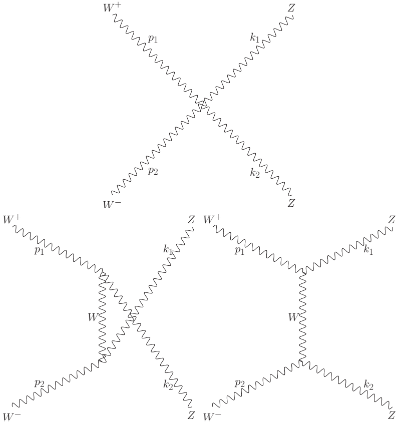


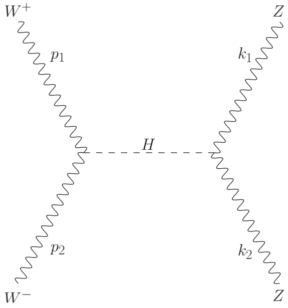
Conclusion

- Want to set model independent limits to NP contributions
- Unknown scale Λ_{NP} at which EFT breaks down
- But: Energy range for testing AQGC is bound by Unitarity
- Unitarisation scheme (K-Matrix):
  - Number of events generated by MC fulfill unitarity condition
  - Unitarisation scheme introduces model dependence
  - Nevertheless: Use complete LHC energy range for analysis

Backup Slides

Example: $WW \to ZZ$





VB self-interaction

$$\frac{s}{v^2} + \mathcal{O}(1)$$

Higgs resonance

$$-\frac{1}{v^2}\frac{s^2}{s-M_h{}^2} + \mathcal{O}(1)$$

SM: VBS Amplitude unitarized by Higgs

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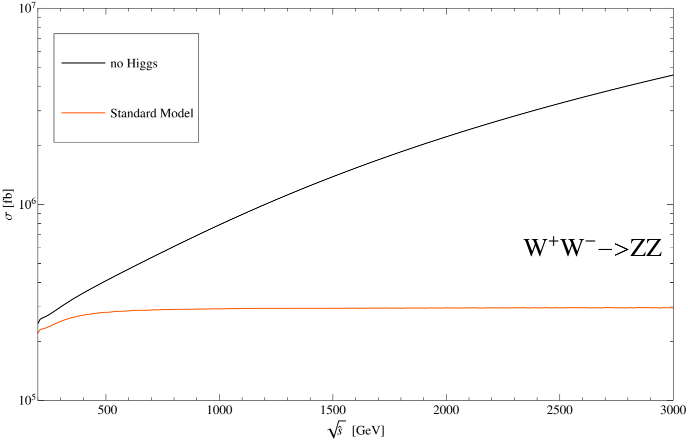
Higgsless Model

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} + \mathcal{O}(1)$$

SM

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} - \frac{1}{v^2}\frac{s^2}{s-{M_h}^2} + \mathcal{O}(1)$$



Definition of Unitarity

Unitarity of S(cattering)-Matrix

$$S^\dagger S = 1$$

$\Rightarrow$ Condition for interaction matrix T:

$$T^\dagger T = -\mathrm{i}\left(T - T^\dagger\right)$$

- With special case of forward scattering $\left(t = -s\frac{1-\cos\Theta}{2}\right)$:

$$\sum_f \int d\Pi_f \, |\mathcal{M}_{if}|^2 = 2\,\mathrm{Im}\left[\mathcal{M}_{ii}(t=0)\right]$$

From S-Matrix to matrix element $\mathcal{M}$

$$\mathrm{S} = 1 + \mathrm{i}\,T$$

$$\mathcal{M}_{if} = \langle f|\, T\, |i\rangle$$

Unitarity Condition

Partial waves

$$\mathcal{M}_{if}(s,t,u) = 32\pi \sum_{\ell} (2l+1)\mathcal{A}_{if}^{\ell}(s) P^{\ell}(\cos\Theta)$$

$$\sigma_{\mathrm{tot}} = \frac{1}{64\pi^2 s}\int d\Omega \left|\mathcal{M}_{if}\right|^2 = \sum_{f,\ell} \frac{32\pi(2l+1)}{s}\left|\mathcal{A}_{if}^{\ell}\right|^2$$

$$\frac{\mathrm{Im}\left[\mathcal{M}_{\mathrm{ii}}(\mathrm{t}=0)\right]}{s} = \sum_{\ell} \frac{32\pi(2l+1)}{s}\,\mathrm{Im}\left[\mathcal{A}_{ii}^{\ell}\right]$$

Unitarity condition for partial wave amplitudes

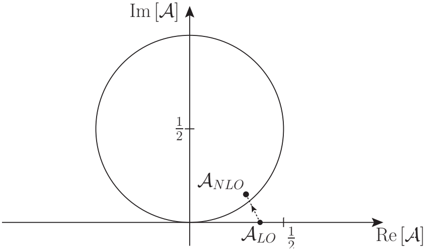
$$\left|\mathcal{A}_{ii}^{\ell}\right|^2 \overset{!}{\leq} \mathrm{Im}\left[\mathcal{A}_{ii}^{\ell}\right]$$

Tree Level Amplitudes

- Simple tree level amplitudes are real
- Higher order corrections add imaginary contributions

→ Corrected amplitude will get inside of Argand-Circle, if it is physical

- You can set bounds on real part of amplitude!



Unitarity Bound

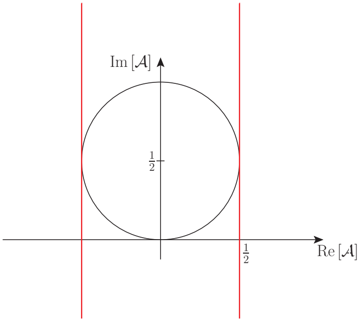
Real Unitarity Bound

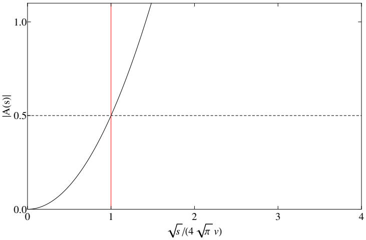
$$|\mathrm{Re}\,(\mathcal{A}(s))| \leq \frac{1}{2}$$

Example: Higgsless model

- $\mathcal{A}_0(s) = \frac{s}{32\pi v^2}$
- Viable up to

$$\sqrt{s} \leq 4\sqrt{\pi}v \approx 1.7 \text{ TeV}$$





Unitarity Bound for α_4 AQGC

Bounds for α_4

$$\ell = 0 : \sqrt{s} \leq \left(\frac{6\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.5\ \text{TeV}}{\sqrt[4]{\alpha_4}}$$

$$\ell = 2 : \sqrt{s} \leq \left(\frac{60\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.9\ \text{TeV}}{\sqrt[4]{\alpha_4}}$$

- Bound is dependent on coupling α_4
- Use strongest bound

α_4 AQGC contribution to $WW \to ZZ$

$$\mathcal{A}(s,t,u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

Bounds on Eigenamplitudes

AQGC amplitudes (GBET):

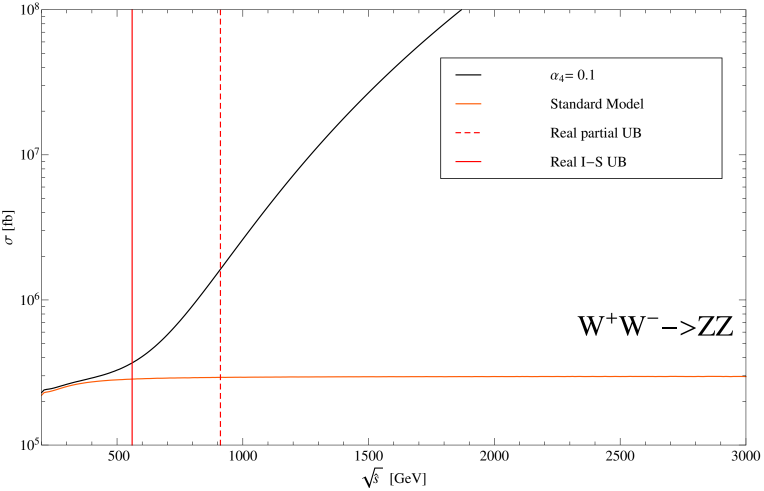
$$\mathcal{A}_{00}(s) = \frac{1}{12\pi}\left(7\alpha_4 + 11\alpha_5\right)\frac{s^2}{v^4}$$
$$\mathcal{A}_{02}(s) = \frac{1}{60\pi}\left(2\alpha_4 + \alpha_5\right)\frac{s^2}{v^4}$$
$$\mathcal{A}_{11}(s) = \frac{1}{24\pi}\left(\alpha_4 - 2\alpha_5\right)\frac{s^2}{v^4}$$
$$\mathcal{A}_{20}(s) = \frac{1}{6\pi}\left(2\alpha_4 + \alpha_5\right)\frac{s^2}{v^4}$$
$$\mathcal{A}_{22}(s) = \frac{1}{120\pi}\left(2\alpha_4 + \alpha_5\right)\frac{s^2}{v^4}$$

Bounds on α_4

$$00 : \sqrt{s} \lesssim 320\ \text{GeV} \cdot \alpha_4^{-\frac{1}{4}}$$
$$20 : \sqrt{s} \lesssim 360\ \text{GeV} \cdot \alpha_4^{-\frac{1}{4}}$$

- Bounds depend on linear combination of AQGC

$\Rightarrow$ Stronger constraint than partial wave amplitude ($\ell = 0 : \sqrt{s} \lesssim 500\text{GeV} \cdot \alpha_4^{-\frac{1}{4}}$) (Assumption: Isospin is preserved)



10^8
10^7
10^6
10^5
σ [fb]
500
1000
1500
2000
2500
3000
$\sqrt{\hat{s}}$ [GeV]
α_4= 0.1
Standard Model
Real partial UB
Real I−S UB
W^+W^-−>ZZ

Cut-Off Method

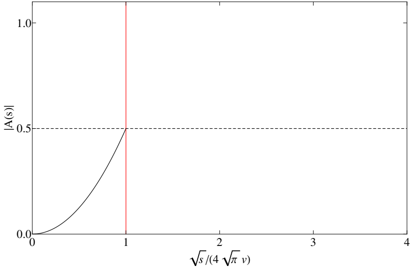
Cut-Off function

$$\Theta\left(\Lambda_C^2 - s\right)$$

Cut-Off energy Λ_C

Λ_C equates unitarity bounds (often 0th partial wave)

- Naive prevention of Unitarity violation
- No continuous transition at Λ_C
- Ignore any interesting physics above Unitary bound
- Better: Use observables, which do not conflict unitarity condition



Form Factor

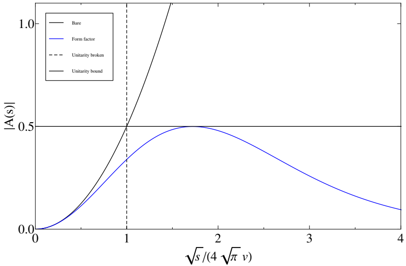
Form Factor

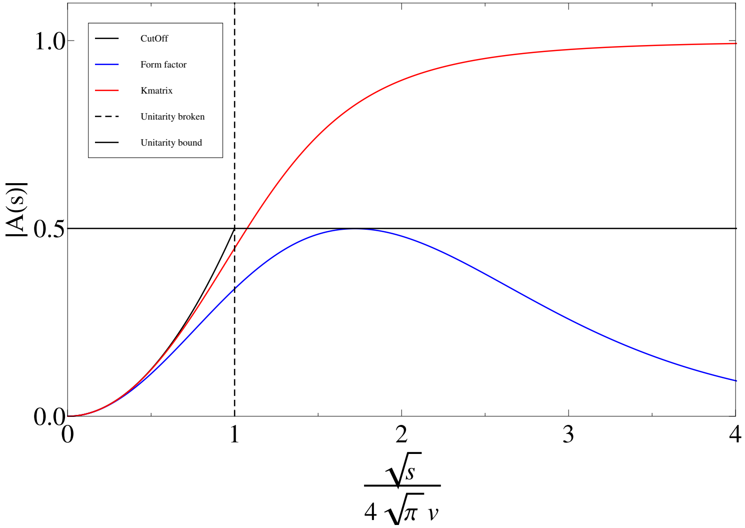
$$\frac{1}{\left(1+\frac{s}{\Lambda_{FF}^2}\right)^n}$$

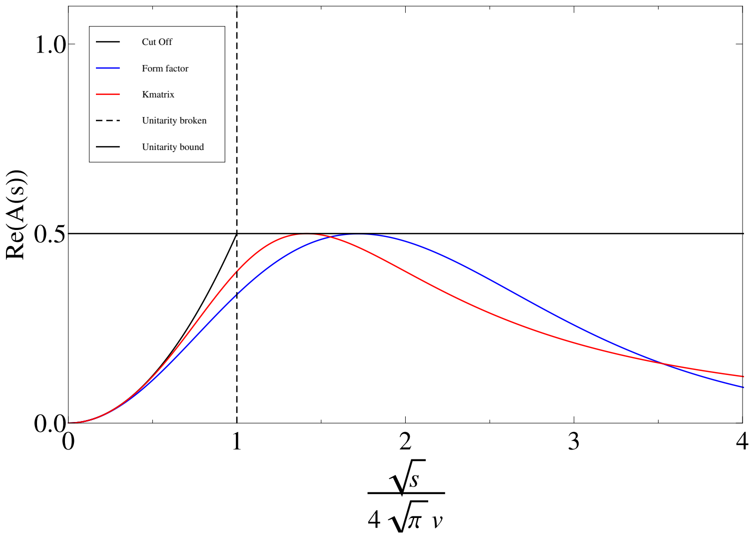
Parameters

- n Chosen to prevent breaking of Unitarity
- Λ_{FF} Calculate highest possible value that satisfy real Unitarity bound (0*th* partial wave)

- Use Form Factor to suppress breaking of unitarity
- Can be generally used for arbitrary anomalous operator
- Need "Fine Tuning"




CutOff
Form factor
Kmatrix
Unitarity broken
Unitarity bound
|A(s)|
1.0
0.5
0.0
0
1
2
3
4
$\frac{\sqrt{s}}{4\sqrt{\pi}\,v}$


Cut Off
Form factor
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Re(A(s))
1.0
0.5
0.0
0
1
2
3
4
$\frac{\sqrt{s}}{4\sqrt{\pi}\,v}$

"Comparison"

- Which Unitarisation scheme provides the best description?

→ All of them:
Unitarisation schemes are an arbitrary way to guarantee Unitarity

Form Factor

- Suppression of amplitude to get below Unitarity bound

MC Generate less events than possible

K-Matrix

- Saturation of amplitude to achieve Unitarity

MC Generate maximal possible number of events

Isospin-Spin Eigenamplitudes

$$\mathcal{A}(w^+w^+ \to w^+w^+) = \mathcal{A}_{02}(s) - 10\mathcal{A}_{22}(s) + 15\mathcal{A}_{22}(s)\frac{t^2+u^2}{s^2}$$

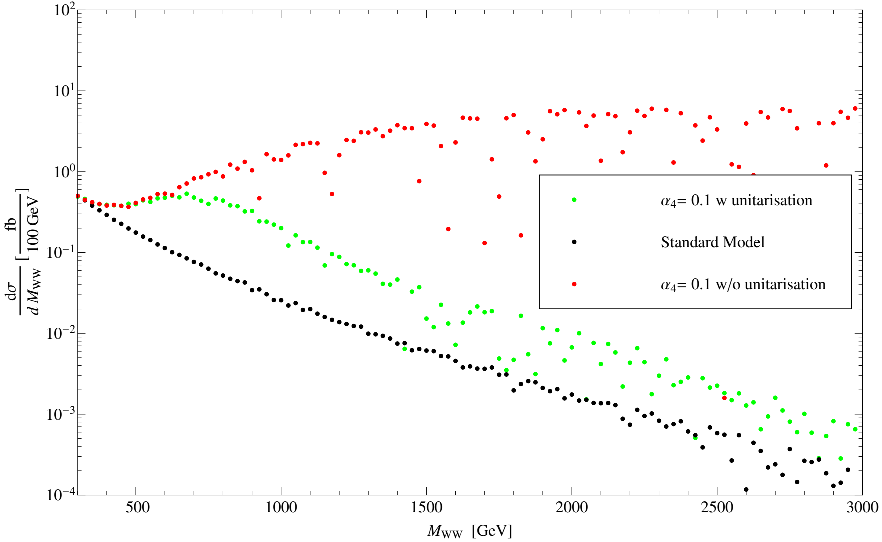
$$\mathcal{A}(w^+w^- \to zz) = \frac{1}{3}\left(\mathcal{A}_{00}(s) - \mathcal{A}_{20}(s)\right) - \frac{10}{3}\left(\mathcal{A}_{02}(s) - \mathcal{A}_{22}(s)\right) + 5\left(\mathcal{A}_{02}(s) - \mathcal{A}_{22}(s)\right)\frac{t^2+u^2}{s^2}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{1}{2}\mathcal{A}_{20}(s) - 5\mathcal{A}_{22}(s) + \left(-\frac{3}{2}\mathcal{A}_{11}(s) + \frac{15}{2}\mathcal{A}_{22}(s)\right)\frac{t^2}{s^2} + \left(\frac{3}{2}\mathcal{A}_{11}(s) + \frac{15}{2}\mathcal{A}_{22}(s)\right)\frac{u^2}{s^2}$$

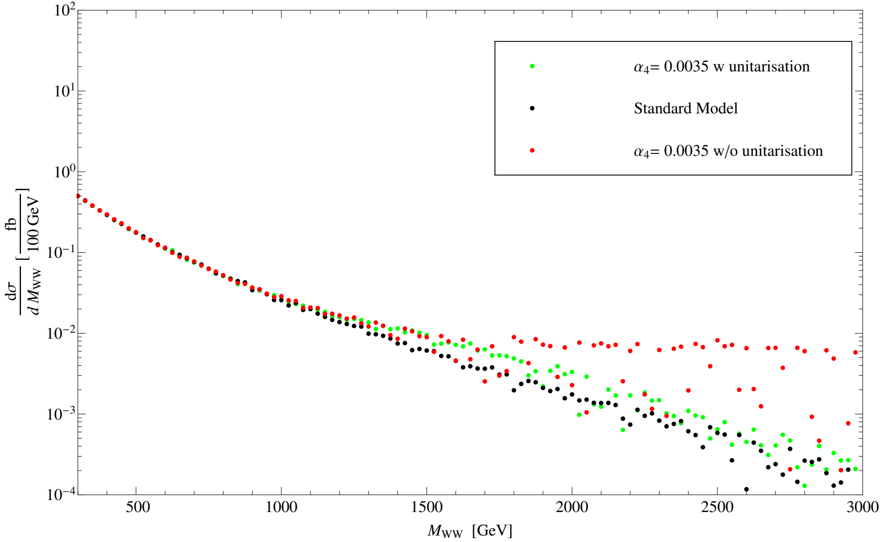
Isospin-Spin Eigenamplitudes

$$\begin{aligned}
\mathcal{A}(w^+w^- \to w^+w^-) &= \frac{1}{6}\left(2\mathcal{A}_{00}(s) + \mathcal{A}_{20}(s)\right) - \frac{5}{3}\left(2\mathcal{A}_{02}(s) + \mathcal{A}_{22}(s)\right) \\
&\quad + \left(5\mathcal{A}_{02}(s) - \frac{3}{2}\mathcal{A}_{11}(s) + \frac{5}{2}\mathcal{A}_{22}(s)\right)\frac{t^2}{s^2} \\
&\quad + \left(5\mathcal{A}_{02}(s) + \frac{3}{2}\mathcal{A}_{11}(s) + \frac{5}{2}\mathcal{A}_{22}(s)\right)\frac{u^2}{s^2} \\
\mathcal{A}(zz \to zz) &= \frac{1}{3}\left(\mathcal{A}_{00}(s) + 2\mathcal{A}_{20}(s)\right) - \frac{10}{3}\left(\mathcal{A}_{02}(s) + 2\mathcal{A}_{22}(s)\right) \\
&\quad + 5\left(\mathcal{A}_{02}(s) + 2\mathcal{A}_{22}(s)\right)\frac{t^2+u^2}{s^2}
\end{aligned}$$

$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV



$PP \to e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV



EW Chiral Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} - \sum_\psi \overline{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}_0' + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots$$

$$\mathcal{L}_0' = \frac{v^2}{4} \operatorname{tr}[\mathbf{T}\mathbf{V}_\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \operatorname{tr}[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_2 = \mathrm{i} \operatorname{tr}[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_3 = \mathrm{i} \operatorname{tr}[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_4 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr}[\mathbf{V}^\mu \mathbf{V}^\nu]$$

$$\mathcal{L}_5 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr}[\mathbf{V}_\nu \mathbf{V}^\nu]$$

$$\mathcal{L}_6 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_7 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}_\nu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_8 = \tfrac{1}{4} \operatorname{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr}[\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_9 = \tfrac{\mathrm{i}}{2} \operatorname{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr}[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_{10} = \tfrac{1}{2} \left(\operatorname{tr}[\mathbf{T}\mathbf{V}_\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu]\right)^2$$

with $\mathbf{T} = \Sigma \tau^3 \Sigma^\dagger$

indirect information for new physics in $\beta_1, \alpha_i, \dots$

(flavor Physic only in M)

Integrate out Resonances

Leading order effects for resonances in EW sector:

$$\mathcal{L}_\Phi = z\left[\Phi\left(M_\Phi^2 + DD\right)\Phi + 2\Phi J\right] \qquad \Rightarrow \qquad \mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2} JJ + \frac{z}{M^4} J(DD)J + \mathcal{O}(M^{-6})$$

- Simple Example: scalar Singlett σ:

$$\mathcal{L}_\sigma = -\frac{1}{2}\left[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] - h_\sigma \operatorname{tr}[\mathbf{TV}_\mu] \operatorname{tr}[\mathbf{TV}^\mu]\right]$$

- effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2}\left[g_\sigma \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] + h_\sigma \operatorname{tr}[\mathbf{TV}_\mu] \operatorname{tr}[\mathbf{TV}^\mu]\right]^2$$

- leads to following AQGC

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_7 = 2 g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_{10} = 2 h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right)$$

- special case: SM higgs with $g_\sigma = 1$ und $h_\sigma = 0$

$$\begin{aligned}\mathcal{L}_{QGC} = & e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+\right] \\ & + e^2 \frac{c_\mathrm{w}}{s_\mathrm{w}} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^-\right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ & + e^2 \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ & + \frac{e^2}{2 s_\mathrm{w}^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+\right)^2\right] + \frac{e^2}{4 s_\mathrm{w}^2 c_\mathrm{w}^4} h^{ZZ} (Z^\mu Z_\mu)^2\end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

SM Lagrangian

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\,\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\,\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \qquad W^{\pm},\ Z$$

$$+\left(\partial_{\mu}\phi\right)^{\dagger}\partial^{\mu}\phi - V(\phi) \qquad h$$

$$+\frac{v^2}{4}\,\text{tr}\left[(\mathbf{D}_{\mu}\Sigma)^{\dagger}(\mathbf{D}^{\mu}\Sigma)\right] \qquad w^{\pm},\ z$$

$$-\frac{g_h v}{2}\,\text{tr}\left[\mathbf{V}^{\mu}\mathbf{V}_{\mu}\right] h$$

Vector Bosons

$$\mathbf{W}_{\mu\nu} = \partial_{\mu}\mathbf{W}_{\nu} - \partial_{\nu}\mathbf{W}_{\mu} + \mathrm{i}g\left[\mathbf{W}_{\mu}, \mathbf{W}_{\nu}\right]$$

$$\mathbf{B}_{\mu\nu} = \partial_{\mu}\mathbf{B}_{\nu} - \partial_{\nu}\mathbf{B}_{\mu}$$

$$\mathbf{W}_{\mu} = W^{a}_{\mu}\frac{\tau^{a}}{2} \qquad \mathbf{B}_{\mu} = B_{\mu}\frac{\tau^{3}}{2}$$

$$\mathbf{D}_{\mu} = \partial_{\mu} + \mathrm{i}g\mathbf{W}_{\mu} - \mathrm{i}g'\mathbf{B}_{\mu}$$

Higgs Sector

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\Sigma = \exp\left[-\frac{\mathrm{i}}{v} w^{a}\tau^{a}\right]$$

$$\mathbf{V}_{\mu} = \Sigma\left(\mathbf{D}_{\mu}\Sigma\right)^{\dagger}$$

Unitary Gauge

- Goldstone bosons are absorbed by vector bosons as longitudinal degrees of freedom
- $w^a \equiv 0 \rightarrow \Sigma \equiv 1$
- $\mathbf{D}_\mu = \partial_\mu - \mathbf{V}_\mu = \partial_\mu + \frac{ig}{2}\left(\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_w} Z\tau^3\right)$

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\,\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\,\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \underbrace{(\partial_\mu\phi)^\dagger\,\partial^\mu\phi - \frac{v^2}{4}\,\text{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] - \frac{g_h v}{2}\,\text{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h}_{\underset{g_h=1}{\widehat{=}}\,(\mathbf{D}_\mu\phi)^\dagger\mathbf{D}^\mu\phi} - V(\phi)$$

- Coincides with known SM parametrisation

Gaugelesse Limit

- $g \to 0,\ g' \to 0$
- $\mathbf{D}_\mu = \partial_\mu$
- $\mathbf{V}_\mu = \Sigma\,(\partial_\mu \Sigma)$

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\,\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\,\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + (\partial_\mu \phi)^\dagger\,\partial^\mu \phi - V(\phi) + \frac{v^2}{4}\,\text{tr}\left[(\partial_\mu \Sigma)^\dagger(\partial^\mu \Sigma)\right] - \frac{g_h v}{2}\,\text{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] h$$

- Decoupling of Higgs Sector