HATHOR for Single Top Production
- Updated Predictions and Uncertainty Estimates for Single Top Quark Production in Hadronic Collisions -

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Time consuming/costly analyses (some examples):

- PDF (uncertainty) studies
- b-PDF fits
- scale uncertainties
- analysis with respect to variation of e.g. top mass or other input parameters

⇒need for fast evaluation of cross sections!

One possible tool: HATHOR (for inclusive hadronic cross sections)

- fast by construction (only convolution with PDFs left to do)
- easy to use (specialized for the task, graphical interface, ...)
- aim for new release: extension by inclusive single top cross section up to approximate NNLO accuracy

What is HATHOR?

- HAdronic Top and Heavy quarks crOss section calculatoR
- so far able to calculate inclusive hadronic $t\bar{t}$-production cross section very fast up to NNLO [M. Aliev et al, Comput.Phys.Commun.182: 1034-1046,2011]
- separate variations of the factorization and renormalization scales and other parameters
- evaluation with different PDF-sets easy and fast
- possibility to obtain the cross section as a function of the running top-quark mass

Meyers Konversationslexikon (1885–90) [copyrights expired]

How does HATHOR work?

structure of inclusive hadronic cross sections

$$\sigma_{had} = \sum_{i,j} \int dx_1 dx_2 F_{i/H_1}(x_1, \mu_f) F_{j/H_2}(x_2, \mu_f) \hat{\sigma}^{ij}(\hat{s} \equiv x_1 x_2 s_{had}; \mu_f)$$

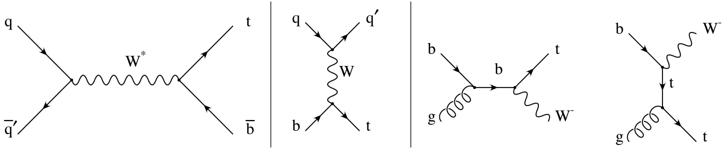
structure of inclusive partonic cross section (example: leading order)

$$\hat{\sigma}^{ij}_{LO}(\hat{s}) = \int d\Phi_n d\sigma^{ij}_{LO} = \frac{1}{2\hat{s}} \int d\Phi_n(\hat{s} \to p_1, ..., p_n) \left|\mathcal{M}_{LO}(i,j \to 1, ..., n)\right|^2$$

- phase space integration can be done once and for all
  $\Rightarrow$ inclusive partonic cross section as a function of $\hat{s}$
- only convolution with PDFs left to do
  $\Rightarrow$ inclusive hadronic cross section
- also obtain dependence on other parameters (top mass, couplings,...)
  $\Rightarrow$ fast analysis with respect to variations of these parameters
- however: restricted to total cross section

Theoretical home position

Single top production (leading order processes)



expansion up to NNLO accuracy (s,t: $k = 0$, Wt: $k = 1$)

$$\hat{\sigma}^{ij}(\hat{s};\alpha_s(\mu_R),\mu_f) = \alpha_s(\mu_R)^k\ \hat{\sigma}^{ij}_{LO}(\hat{s}) + \alpha_s(\mu_R)^{k+1}\ \hat{\sigma}^{ij}_{NLO}(\hat{s};\mu_R,\mu_f) + \alpha_s(\mu_R)^{k+2}\ \hat{\sigma}^{ij}_{NNLO}(\hat{s};\mu_R,\mu_f)$$

- NLO calculation known and documented in literature [e.g. Phys.Rev. D66 (2002) 054024] as well as implemented in public code [e.g. MCFM] but no analytic result for $\hat{\sigma}^{ij}_{NLO}$ available
- NNLO approximate predictions at NNLL (next-to-next-to-leading logarithm) accuracy given in literature [Phys. Rev. D 83, 091503]

How to calculate $\hat{\sigma}^{ij}_{NLO}$: a closer look

$$\text{"NLO} = \underbrace{\text{(virtual)} + \text{(real)} + \text{(factorization of initial state singularities)"}}_{\text{KLN-theorem} \Rightarrow \text{finite}}$$

$$\alpha_s(\mu_R)^{k+1}\,\hat{\sigma}^{ij}_{NLO} = \int d\Phi_n \underbrace{\frac{d\sigma^{ij}_V}{d\Phi_n}}_{\text{IR div.}} + \underbrace{\int d\Phi_{n+1} \frac{d\sigma^{ij}_R}{d\Phi_{n+1}}}_{\text{IR div.}} + \underbrace{\int dx\, d\Phi_n \frac{d\sigma^{ij}_{fac,x}}{d\Phi_n}}_{\text{IR div.}}$$

$$\overset{\text{Catani-}}{\underset{\text{Seymour}}{=}} \int d\Phi_n \underbrace{\left[\frac{d\sigma^{ij}_V}{d\Phi_n} + \int d\Phi_1 \frac{d\sigma^{ij}_A}{d\Phi_{n+1}}\right]}_{\text{finite}} + \underbrace{\int d\Phi_{n+1}\left[\frac{d\sigma^{ij}_R}{d\Phi_{n+1}} - \frac{d\sigma^{ij}_A}{d\Phi_{n+1}}\right]}_{\text{finite but time consuming}} + \underbrace{\int d\Phi_n \frac{d\sigma^{ij}_C}{d\Phi_n}}_{\text{finite}}$$

- partonic phase space integrations time consuming
  $\Rightarrow$ do once for fixed values of $\hat{s}$ and tabulate
- 3 different approaches: calculation from scratch (s- and t-channel), pseudo-PDFs with MCFM, "adjustment" of MCFM

1) Pseudo-PDFs (universally applicable)

[see also FastNLO for a somewhat similar approach]

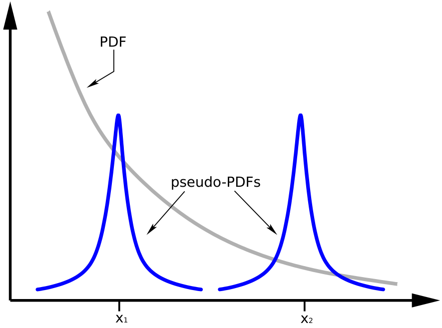
$$\sigma_{had} = \sum_{i,j} \int dx_1 dx_2 F_i(x_1, \mu_f) F_j(x_2, \mu_f) \hat{\sigma}^{ij}(\hat{s} \equiv x_1 x_2 s_{had}; \mu_f)$$

run MCFM with normalized, narrow Gaussian distributions

$F_i(x, \mu_0) = \frac{1}{\sqrt{2\pi}\delta} \exp\left(-\frac{(x-x_0)^2}{2\delta^2}\right)$ as PDFs to extract

$$\sigma_{had}(s; \mu_0, \mu_0)\Big|_{\text{pseudo-PDFs}} = \hat{\sigma}^{ij}(x_0^2 s; \mu_0, \mu_0) + \mathcal{O}(\delta^2)$$

Self-consistency check: use different values for δ



2) Hack MCFM (method used in the HATHOR single top extension)

$$\sigma_{had} = \sum_{i,j} \int dx_1 dx_2 F_i(x_1, \mu_f) F_j(x_2, \mu_f) \hat{\sigma}^{ij}(\hat{s} \equiv x_1 x_2 s_{had}; \mu_f)$$

- study internal structure of MCFM and identify partonic cross sections
- remove integration over PDFs and restrict numerical integration to the phase space and convolution integrals

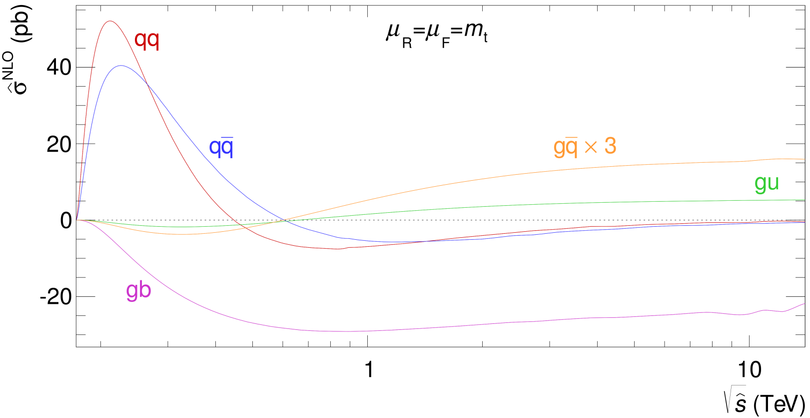
3) Do the math (at least for s- and t-channel)

- implement analytic form of $\hat{\sigma}^{ij}_{NLO}$ from building blocks given in literature

$\Rightarrow$ Results from the three methods agree within less than 1‰ ✓

- determination of $\hat{\sigma}^{ij}_{NLO}$ for a grid of $\hat{s}$ and m_t ($\mu_R = \mu_f = m_t$ fixed)
- interpolation $\Rightarrow \hat{\sigma}^{ij}_{NLO}(\hat{s}, m_t)$

$\hat{\sigma}^{ij}_{NLO}$ vs. $\sqrt{\hat{s}}$ for $m_t = 172.5$ GeV (t-channel, u : up-type, b : down-type)



Now we are able to evaluate:

$$\sigma_{had}^{NLO} = \alpha_s(\mu_R)^{k+1} \sum_{i,j} \int dx_1 dx_2 F_i(x_1, \mu_f) F_j(x_2, \mu_f) \hat{\sigma}^{ij}_{NLO}(\hat{s}; \mu_R, \mu_f)$$

for $\mu_R = \mu_f = m_t$ fixed!

What about evaluation at arbitrary scales?

need scale dependence of $\hat{\sigma}^{ij}_{NLO}(\hat{s}; \mu_R, \mu_f)$!

Use scale independence of σ_{had}:

$$\frac{d\sigma_{had}^{NLO}}{d\ln(\mu_R)} = \frac{d\sigma_{had}^{NLO}}{d\ln(\mu_f)} = \mathcal{O}(\alpha_s^{k+2})$$

$$\frac{d\sigma_{had}^{NNLO}}{d\ln(\mu_R)} = \frac{d\sigma_{had}^{NNLO}}{d\ln(\mu_f)} = \mathcal{O}(\alpha_s^{k+3})$$

$$\sigma_{had} = \sum_{i,j} \int dx_1 dx_2 F_i(x_1, \mu_f) F_j(x_2, \mu_f) \hat{\sigma}^{ij}(\hat{s}; \mu_f) = \sum_{ij} F_i \otimes F_j \otimes \hat{\sigma}^{ij}$$

$$\left[(f \otimes g)(x) := \int dy \int dz\; f(y) g(z) \delta(x - yz)\right]$$

$\Rightarrow$ scale dependence for a given order of $\hat{\sigma}^{ij}$ can be restored from:

DGLAP evolution equation of the PDFs F_i: P_{ij}: Altarelli-Parisi splitting kernels

$$\frac{dF_i(x,\mu)}{d\ln(\mu^2)} = \frac{\alpha_s(\mu)}{2\pi} \sum_j P_{ij} \otimes F_i(x, \mu) \qquad (\mu = \mu_f = \mu_R)$$

and the renormalization group equation (QCD beta function):

$$\frac{1}{2\pi} \frac{d\alpha_s(\mu_R)}{d\ln(\mu_R^2)} = -\beta\left(\frac{\alpha_s}{2\pi}\right) = -\frac{\alpha_s^2}{4\pi^2}\left(\beta_0 + \frac{\alpha_s}{2\pi}\beta_1 + \ldots\right)$$

which allows to deduce:

$$\alpha_s(\mu_f) = \alpha_s(\mu_R)\left(1 + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_f^2}\right)\frac{\alpha_s(\mu_R)}{2\pi} + \left[\beta_1 \ln\left(\frac{\mu_R^2}{\mu_f^2}\right) + \beta_0^2 \ln^2\left(\frac{\mu_R^2}{\mu_f^2}\right)\right]\frac{\alpha_s^2(\mu_R)}{4\pi^2} + \ldots\right)$$

so that μ_f and μ_R can be varied independently scale dependence up to NNLO (α_s^{k+2}):

$$\hat{\sigma}^{ij}_{LO}(\hat{s}) = f^{(0)}_{ij},$$

$$\hat{\sigma}^{ij}_{NLO}(\hat{s};\mu_R,\mu_f) = f^{(10)}_{ij} + \ln\left(\frac{\mu_f^2}{m_t^2}\right) f^{(11)}_{ij} + k\beta_0 \ln\left(\frac{\mu_R^2}{\mu_f^2}\right) f^{(0)}_{ij},$$

$$\hat{\sigma}^{ij}_{NNLO}(\hat{s};\mu_R,\mu_f) = f^{(20)}_{ij} + \ln\left(\frac{\mu_f^2}{m_t^2}\right) f^{(21)}_{ij} + \ln^2\left(\frac{\mu_f^2}{m_t^2}\right) f^{(22)}_{ij} + k\beta_1 \ln\left(\frac{\mu_R^2}{\mu_f^2}\right) f^{(0)}_{ij}$$
$$+(k+1)\beta_0 \ln\left(\frac{\mu_R^2}{\mu_f^2}\right)\left(f^{(10)}_{ij} + \ln\left(\frac{\mu_f^2}{m_t^2}\right) f^{(11)}_{ij}\right) + \frac{k(k+1)}{2}\beta_0^2 \ln^2\left(\frac{\mu_R^2}{\mu_f^2}\right) f^{(0)}_{ij}$$

with the scale independent functions

$$f^{(11)}_{ij} = -\sum_m \left(P^{(0)}_{mi} \otimes f^{(0)}_{mj} + P^{(0)}_{mj} \otimes f^{(0)}_{im}\right) + k\beta_0 f^{(0)}_{ij}, \qquad m \in \{q, \bar{q}, g\}$$

$$f^{(21)}_{ij} = -\sum_m \left(P^{(1)}_{mi} \otimes f^{(0)}_{mj} + P^{(1)}_{mj} \otimes f^{(0)}_{im}\right) + k\beta_1 f^{(0)}_{ij}$$
$$-\sum_m \left(P^{(0)}_{mi} \otimes f^{(10)}_{mj} + P^{(0)}_{mj} \otimes f^{(10)}_{im}\right) + (k+1)\beta_0 f^{(10)}_{ij},$$

$$2f^{(22)}_{ij} = -\sum_m \left(P^{(0)}_{mi} \otimes f^{(11)}_{mj} + P^{(0)}_{mj} \otimes f^{(11)}_{im}\right) + (k+1)\beta_0 f^{(11)}_{ij}$$

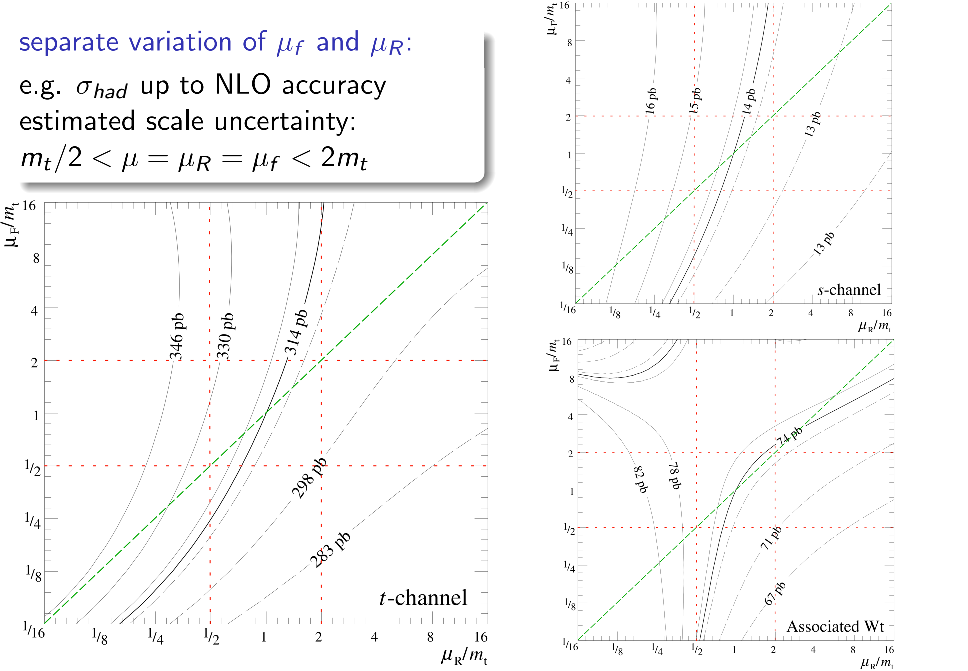
$\Rightarrow$ Scale dependence constructed from lower order building blocks!

separate variation of μ_f and μ_R:

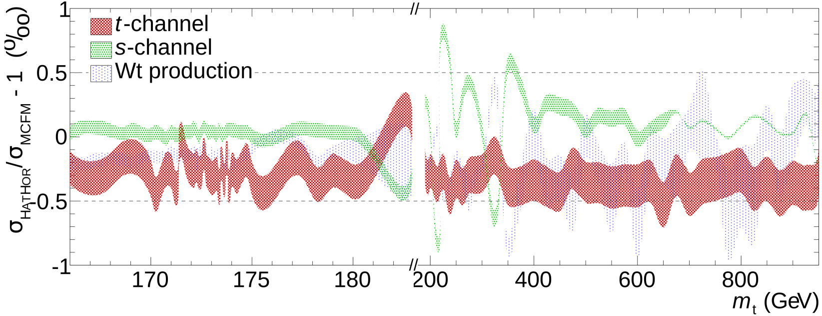
e.g. σ_{had} up to NLO accuracy

estimated scale uncertainty:

$m_t/2 < \mu = \mu_R = \mu_f < 2m_t$



Validation HATHOR/MCFM @ $\sqrt{s} = 14$ TeV



uncertainty bands indicate statistical integration error of MCFM

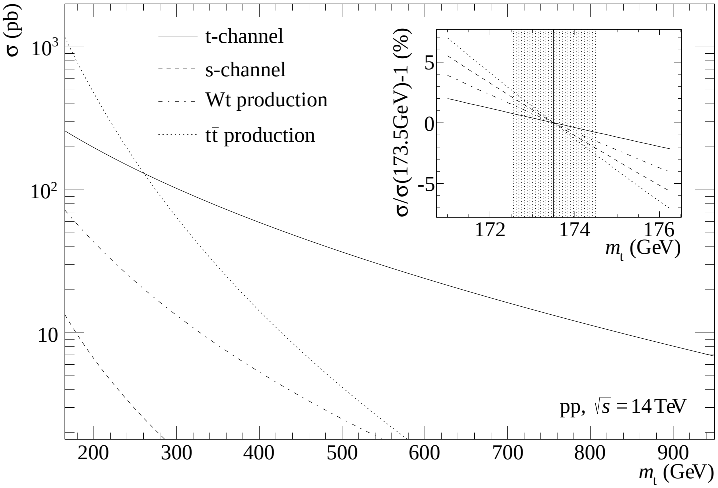
accuracy mostly better than 0.5‰

HATHOR computation time: ≈ 1 hour

Changeable and fixed parameters for single top production in HATHOR

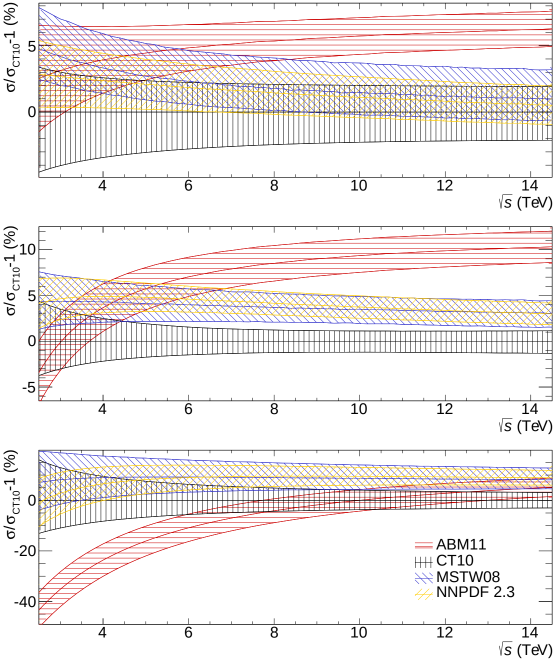
Variable	Default value	Fixed	Valid range
$\sqrt{s}$	1.96 TeV ($p\bar{p}$), 8 TeV (pp)	no	any
Colliding hadrons	pp	no	pp, $p\bar{p}$
Top charge	t	no	t, $\bar{t}$, $t+\bar{t}$
$(\hbar c)^2$	$3.89379323 \times 10^8\,\mathrm{GeV}^2\mathrm{pb}$	no	—
$\sin^2\theta_W$	0.2228972	no	—
$\alpha(m_Z^2)$	$1/132.2332298$	no	—
α_s	depends on chosen PDF	no	any appropriate PDF
m_W	80.385 GeV	yes	—
m_t (pole mass)	(172.5 GeV)	no	165 – 950 GeV
μ_R, μ_F	(m_t)	no	any
V_{ud}, V_{cs}	0.975	no	any
V_{us}, V_{cd}	0.222	no	any
V_{tb}	1	no	any
V_{td}, V_{ts}, V_{ub}, V_{cb}	0	no	any
p_t^b (assoc. Wt prod.)	≤ 25 GeV	yes	—
PDF set	(CT10 nlo)	no	any from LHAPDF

Mass dependence of different top quark production cross sections



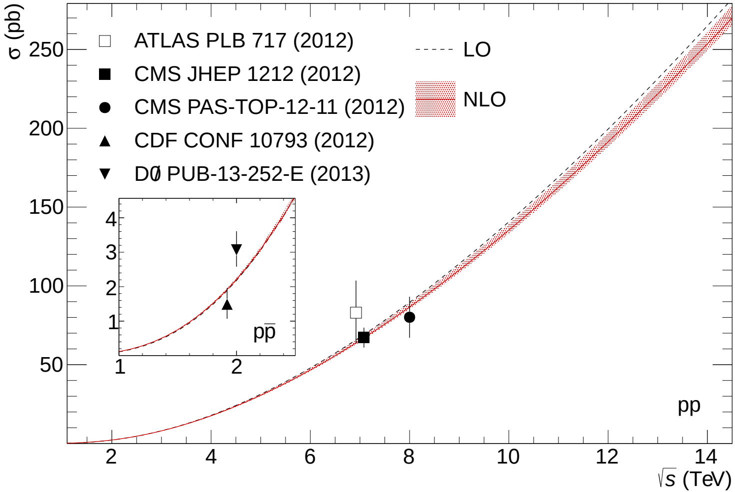
computed with CT10 PDF set, inner plot shows relative change around actual top mass

rule of thumb: $\frac{\Delta\sigma_t}{\sigma_t} = -1.4\frac{\Delta m_t}{m_t}$, $\frac{\Delta\sigma_s}{\sigma_s} = -3.4\frac{\Delta m_t}{m_t}$, $\frac{\Delta\sigma_{Wt}}{\sigma_{Wt}} = -2.7\frac{\Delta m_t}{m_t}$



- PDF uncertainties on the cross section (top down: s-, t-, wt-channel)
- uncertainty bands include uncertainties from PDF fits as well as α_s dependence
- time consuming/expensive analysis: $\approx$ 100 error-PDFs

Hadronic cross section for t-channel production vs. hadronic cms-energy



uncertainty bands indicate full scale uncertainty for NLO

HATHOR computation time on standard 2.4 GHz Xeon CPU: ≈ 35 min

Conclusion

HATHOR for single top/anti-top production (s-, t- and Wt-channel):

- NLO corrections to inclusive cross section (consistent with MCFM within 1‰)
- possible variation of
  - cms-energy
  - factorization and renormalization scale
  - couplings (α_s, α, $\sin\theta_W$)
  - PDFs
  - top mass
  - CKM matrix
- scale dependence up to NNLO
  - Outlook: inclusion of approximate NNLO corrections
  - Future goal: inclusion of NNLO corrections when available

Thanks to our colleagues from Wuppertal, in particular Wolfgang Wagner and Dominik Hirschbühl for constructive feedback on preliminary versions of the HATHOR program

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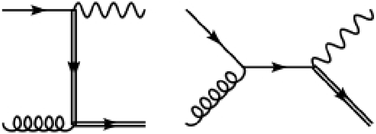
cut on transverse momentum of b-quark from top-decay in Wt-production at NLO necessary to suppresses the contribution from $t\bar{t}$-production (e.g. [JHEP 0911 (2009) 074])

Wt LO:



Wt NLO: