Mixed QCD×EW corrections to Drell–Yan processes in the resonance region

Alexander Huss

in collaboration with
S. Dittmaier and C. Schwinn

7th Annual Helmholtz Alliance Workshop on
“Physics at the Terascale”

2.–4. December 2013, Karlsruhe

December 3rd, 2013

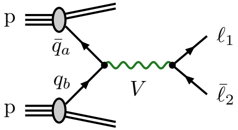
Outline

1. Motivation
2. Pole Approximation: NLO EW $\mathcal{O}(\alpha)$
3. Pole Approximation: NNLO QCD×EW $\mathcal{O}(\alpha_s \alpha)$
4. Summary and Outlook

$\mathrm{W}^{\pm}$ and Z production at the LHC

Large cross section & clean experimental signature

$\hookrightarrow$ One of the most precise probes to test the Standard Model (SM)



$$q\,\bar{q} \to \mathrm{Z} \to \ell^-\,\ell^+$$
$$u\,\bar{d} \to \mathrm{W}^+ \to \nu_\ell\,\ell^+$$
$$d\,\bar{u} \to \mathrm{W}^- \to \ell^-\,\bar{\nu}_\ell$$

Standard candle

- detector calibration
- luminosity monitor
- constraining quark PDFs

Precise measurement of M_V & Γ_V

- Tevatron: $M_\mathrm{W} = 80.387 \pm 0.016\mathrm{GeV}$
- LHC: aimed precision of $\Delta M_\mathrm{W} \lesssim 10\mathrm{MeV}$
- fits to kinematic distributions

$\Rightarrow$ **Precise theoretical predictions mandatory!**

Theoretical Status

- NNLO QCD $\mathcal{O}(\alpha_s^2)$

  [Hamberg, v. Neerven, Matsuura '91] [Harlander, Kilgore '02] [Anastasiou, Dixon, Melnikov, Petriello '04] [Melnikov, Petriello '06] [Catani, Cieri, Ferrera, de Florian, Grazzini '09] [Gavin, Li, Petriello, Quackenbush '11] [Gavin, Li, Petriello, Quackenbush '13]
- NLO EW $\mathcal{O}(\alpha)$ $\mathrm{W}^{\pm}$

  [Wackeroth, Hollik '97] [Baur, Keller, Wackeroth '99] [Dittmaier, Krämer '02] [Baur, Wackeroth '04] [Arbuzov, et al. '06] [Calame, Montagna, Nicrosini, Vicini '07] [Brensing, Dittmaier, Kramer, Muck '08]
- NLO EW $\mathcal{O}(\alpha)$ Z

  [Baur, Keller, Sakumoto '98] [Baur, Brein, Hollik, Schappacher, Wackeroth '02] [Dittmaier, Huber '10]
- Approaches to combination

  [Cao, Yuan '04] [Richardson, et al. '12] [Bernaciak, Wackeroth '12] [Barze, et al. '12, '13] [Li, Petriello '12]
- Steps towards NNLO QCD×EW $\mathcal{O}(\alpha_s\alpha)$ (far from complete)

  [Kotikov, Kühn, Veretin '08] [Bonciani '11] [Kilgore, Sturm '12] [Kara '13]
- NLO EW $\mathcal{O}(\alpha)$ to $V+j$

  [Kühn, Kulesza, Pozzorini, Schulze '07] [Kühn, Kulesza, Pozzorini, Schulze '07] [Hollik, Kasprzik, Kniel '08] [Denner, Dittmaier, Kasprzik, Mück '09, '11]
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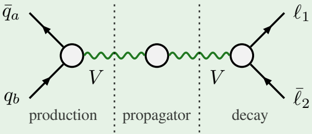
Largest theoretical unknowns: $\mathcal{O}(\alpha_s\alpha)$
particularly important in resonance region
$\hookrightarrow$ Pole expansion

Aim: Improve the theoretical prediction in resonance region
↪ Expansion about complex pole $\mu_V^2 = M_V^2 - \mathrm{i}M_V\Gamma_V$

[Stuart '91] [H.Veltman '94]
[Aeppli, v.Oldenborgh, Wyler '94]

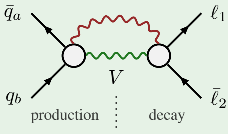
Leading pole approximation (PA): $\sim (p_V^2 - \mu_V^2)^{-1}$

Factorizable corrections:



- on-shell production & decay
  ↪ fact. ini $(2 \to 1)$
  ↪ fact. fin $(1 \to 2)$
  ↪ propagator $(1 \to 1)$
  (taken care by on-shell scheme)

Non-factorizable corrections:



- connect production & decay
  resonant contribution
  ↔ only soft-photon exchange
  ↪ non-fact. $(2 \to 2)$

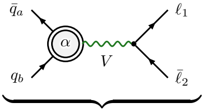
⇒ Simplifications compared to the full off-shell calculation $(2 \to 2)$ [Dittmaier, Krämer '01], etc.

Virtual Factorizable Corrections

Cut a V propagator $\longrightarrow$ two disconnected diagrams

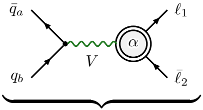
$$\mathcal{M}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}},\text{fact}} = \sum_\lambda \frac{\mathcal{M}^{\bar{q}_a q_b \to V}_{\mathrm{V_{ew}}}(\lambda)\,\mathcal{M}^{V \to \ell_1 \bar{\ell}_2}_{\mathrm{B}}(\lambda) + \mathcal{M}^{\bar{q}_a q_b \to V}_{\mathrm{B}}(\lambda)\,\mathcal{M}^{V \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}}}(\lambda)}{p_V^2 - \mu_V^2}$$

$=$



$\rightsquigarrow$ fact. ini

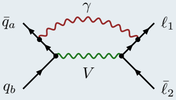
$+$



$\rightsquigarrow$ fact. fin

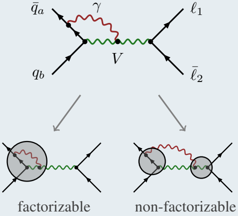
- explicit propagator $(p_V^2 - \mu_V^2)^{-1}$
- $\sum_\lambda$: spin correlation
- gauge invariance $\leftrightarrow$ on-shell projection $(p_V^2 = M_V^2)$

Manifestly non-factorizable



- no "explicit" propagator $(p_V^2 - \mu_V^2)^{-1}$

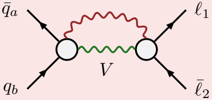
Not manifestly non-factorizable



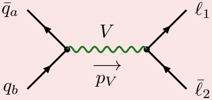
- explicit propagator $(p_V^2 - \mu_V^2)^{-1}$
- gauge-invariant definition: $\Big[\text{full} - \text{fact.}\Big]_{p_V^2 \to M_V^2}$
- origin: $\left\{ \begin{array}{c} \text{infrared regularization } (1/\epsilon) \\ \text{vs.} \\ \text{on-shell limit } (p_V^2 \to M_V^2) \end{array} \right\}$ do not commute!

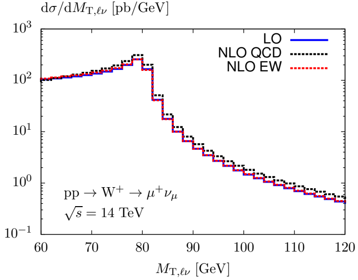
Only soft region ($|q^\mu| \lesssim \Gamma_V$) leads to resonant contributions!

$\Rightarrow$



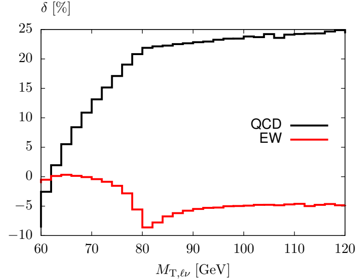
$\sim \delta_{\mathrm{V_{ew},nf}}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}$



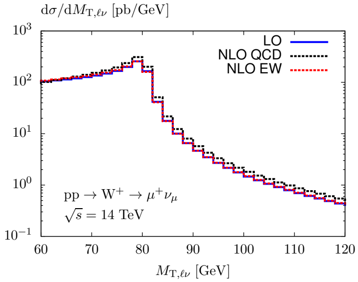


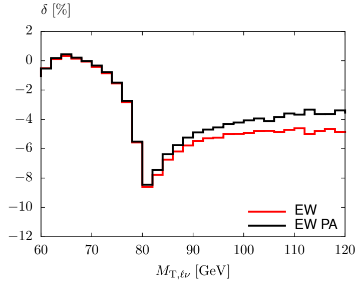
$$M_{T,\ell\nu} = \sqrt{2(E_{T,\ell}\not{E}_T - \mathbf{p}_{T,\ell} \cdot \not{\mathbf{p}}_T)}$$

- most important distribution for the determination of M_W



- **EW**: significant shape distortion





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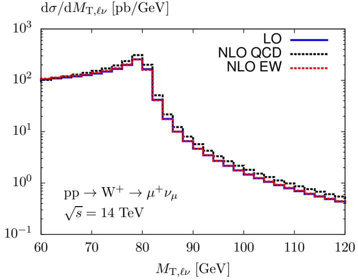
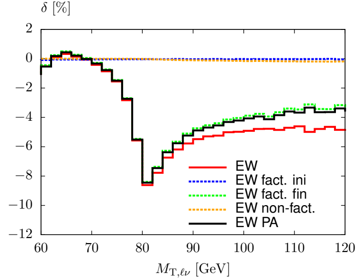
- most important distribution for the determination of M_W

- good agreement between **full** & **PA**





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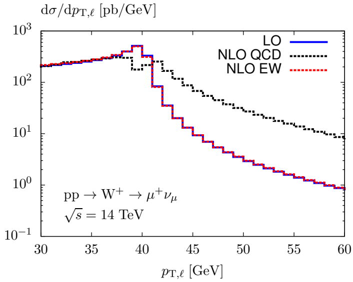
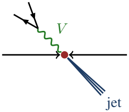
- most important distribution for the determination of M_W

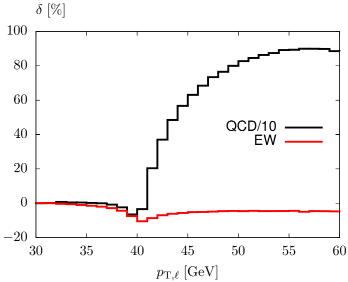
- good agreement between **full** & **PA**
- **fact. ini** & **non-fact.** small and flat
- corrections mainly from **fact. fin**

W^+: $p_{T,\ell}$ distribution

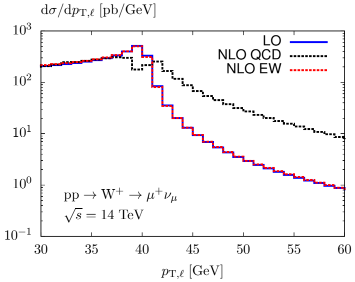


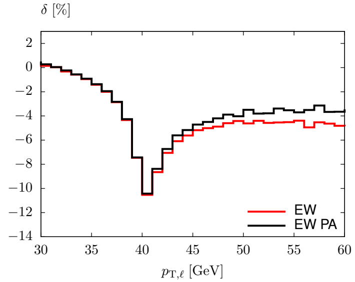
- also important for M_W measurement
- sensitive to initial-state radiation
- jet veto





- **QCD**: huge corrections above threshold $\leftrightarrow$ recoil of the jet
- **EW**: also shape distortion





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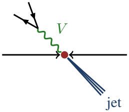
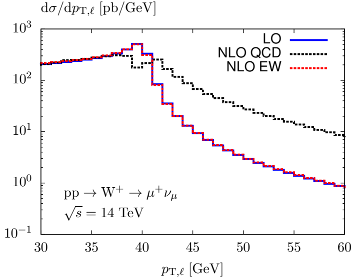
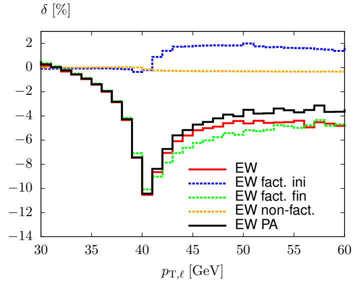


- good agreement between **full** & **PA**

W^+: $p_{\mathrm{T},\ell}$ distribution





- also important for M_{W} measurement
- sensitive to initial-state radiation
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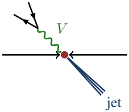
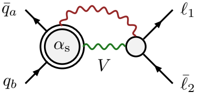
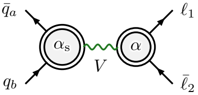


- good agreement between **full** & **PA**
- **non-fact.** again flat and small
- **fact. ini** contributes significantly
  $\leftrightarrow$ sensitivity to initial-state radiation
- again largest contribution from **fact. fin**



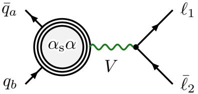
Non-factorizable (nf) corrections*

- **discussed in the following**



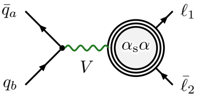
Factorizable Initial–Final corrections*

- large corrections
  & shape distortion expected
- work in progress



Factorizable Initial–Initial corrections*

- no significant shape distortion expected
  c.f. M_T distribution for fact. ini corrections $\mathcal{O}(\alpha)$
- no $\mathcal{O}(\alpha_s\alpha)$ PDFs



Factorizable Final–Final corrections*

- only a constant factor: $\mathcal{O}(\alpha_s\alpha)$ counterterm
  $\hookrightarrow$ no impact on shape

*only virtual contributions indicated $\rightsquigarrow$ also real-, double-real emission, interferences,. . .

Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}$$

$$+ \int_{3} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_{2} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_{2} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

NLO QCD

$$\hat{\sigma}^{\mathrm{QCD}} = \int_{3} \mathrm{d}\sigma^{\mathrm{R_s}} + \int_{2} \mathrm{d}\sigma^{\mathrm{V_s}} + \int_{2} \mathrm{d}\sigma^{\mathrm{C_s}}$$

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathrm{EW}} = \iint_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_{ew}}} + \int_{2} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_{ew}}}$$

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$$+ \int_{3} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_{2} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_{2} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

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$$\hat{\sigma}^{\mathrm{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}$$

NLO EW: non-factorizable corrections

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based on eikonal currents modified by off-shell effects

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\text{C}_\text{s}\otimes\text{R}_\text{ew}}$$
$$+ \int_3 \mathrm{d}\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\text{C}_\text{s}\otimes\text{V}_\text{ew}}$$

NLO QCD

$$\hat{\sigma}^{\text{QCD}} = \int_3 \mathrm{d}\sigma^{\text{R}_\text{s}} + \int_2 \mathrm{d}\sigma^{\text{V}_\text{s}} + \int_2 \mathrm{d}\sigma^{\text{C}_\text{s}}$$

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\text{nf}}^{\text{EW}} = \iint_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\text{R}_\text{ew}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\text{V}_\text{ew}} = \iint_{2+\gamma} \mathrm{d}\sigma^{\text{B}}\ \delta_{\text{R}_\text{ew},\text{nf}}^{2\to2+\gamma} + \int_2 \mathrm{d}\sigma^{\text{B}}\ 2\,\text{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}\right\}$$

Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

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$$+ \int_3 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

NLO QCD

$$\hat{\sigma}^{\mathrm{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}},$$

$$\mathrm{d}\sigma_a^{\mathrm{C_s}} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \frac{1}{\epsilon} \left(\frac{4\pi\mu^2}{\mu_{\mathrm{F}}^2}\right)^{\epsilon} \sum_b \int_0^1 \mathrm{d}z \; \mathrm{d}\sigma_b^{\mathrm{B}} \; P^{ab}(z)$$

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{EW}} = \iint_{2+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_{ew}}} = \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{B}} \; \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to 2+\gamma} + \int_2 \mathrm{d}\sigma^{\mathrm{B}} \; 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to 2}\right\}$$

Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} d\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} d\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} d\sigma^{\text{C}_\text{s}}\, \delta_{\text{R}_\text{ew},\text{nf}}^{2\to 2+\gamma}$$

$$+ \int_3 d\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 d\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 d\sigma^{\text{C}_\text{s}}\, 2\,\text{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to 2}\right\}$$

NLO QCD

$$\hat{\sigma}^{\text{QCD}} = \int_3 d\sigma^{\text{R}_\text{s}} + \int_2 d\sigma^{\text{V}_\text{s}} + \int_2 d\sigma^{\text{C}_\text{s}},$$

$$d\sigma_a^{\text{C}_\text{s}} = \frac{\alpha_\text{s}}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_\text{F}^2}\right)^{\epsilon}\sum_b \int_0^1 dz\, d\sigma_b^{\text{B}}\, P^{ab}(z)$$

NLO EW: non-factorizable corrections

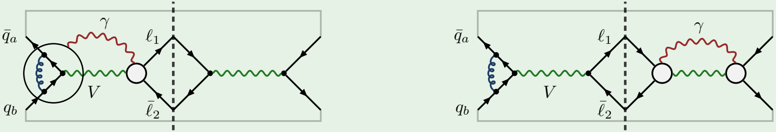
$$\hat{\sigma}_{\text{nf}}^{\text{EW}} = \iint_{2+\gamma} d\sigma_{\text{nf}}^{\text{R}_\text{ew}} + \int_2 d\sigma_{\text{nf}}^{\text{V}_\text{ew}} = \iint_{2+\gamma} d\sigma^{\text{B}}\, \delta_{\text{R}_\text{ew},\text{nf}}^{2\to 2+\gamma} + \int_2 d\sigma^{\text{B}}\, 2\,\text{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to 2}\right\}$$

Non-Factorizable $\mathcal{O}(\alpha_s\alpha)$ Corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} d\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} d\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} d\sigma^{\text{C}_\text{s}}\, \delta_{\text{R}_\text{ew},\text{nf}}^{2\to2+\gamma}$$

$$+ \int_3 d\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 d\sigma_{\text{nf}}^{\text{V}_\text{s}\otimes\text{V}_\text{ew}} + \int_2 d\sigma^{\text{C}_\text{s}}\, 2\,\text{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}\right\}$$

(Virtual QCD) × (Virtual EW)



- $\propto \delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}$ due to non-trivial cancellations
- cf. $\mathcal{O}(\alpha)$ corrections $\propto \delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}$

Non-Factorizable $\mathcal{O}(\alpha_s\alpha)$ Corrections

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\}$$

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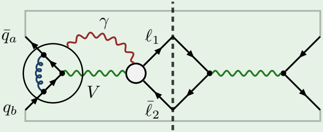
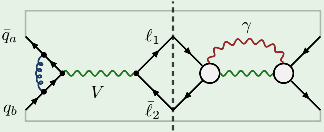




- $\propto \delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}$
  due to non-trivial cancellations
- cf. $\mathcal{O}(\alpha)$ corrections $\propto \delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}$

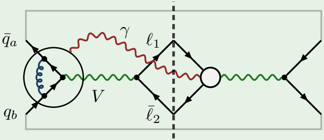
$$\mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\} \mathrm{d}\sigma^{\mathrm{V_s}}$$

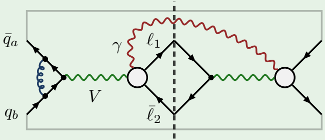
Non-Factorizable $\mathcal{O}(\alpha_s\alpha)$ Corrections

$$\hat\sigma_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} d\sigma_{\text{nf}}^{\text{R}_s\otimes\text{R}_{ew}} + \iint_{2+\gamma} d\sigma_{\text{nf}}^{\text{V}_s\otimes\text{R}_{ew}} + \iint_{2+\gamma} d\sigma^{\text{C}_s}\,\delta^{2\to2+\gamma}_{\text{R}_{ew},\text{nf}}$$

$$+ \int_3 d\sigma_{\text{nf}}^{\text{R}_s\otimes\text{V}_{ew}} + \int_2 d\sigma^{\text{V}_s}\,2\,\text{Re}\left\{\delta^{2\to2}_{\text{V}_{ew},\text{nf}}\right\} + \int_2 d\sigma^{\text{C}_s}\,2\,\text{Re}\left\{\delta^{2\to2}_{\text{V}_{ew},\text{nf}}\right\}$$

(Virtual QCD)×(Real EW)



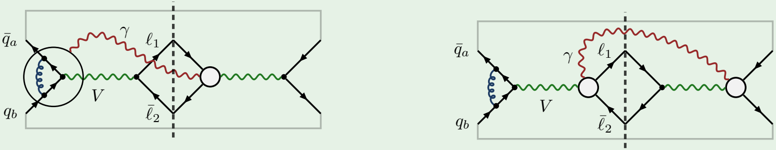


- same eikonal currents as in $\mathcal{O}(\alpha)$ corrections (only external legs are relevant!)

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\}$$

(Virtual QCD)×(Real EW)



- same eikonal currents as in $\mathcal{O}(\alpha)$ corrections (only external legs are relevant!)

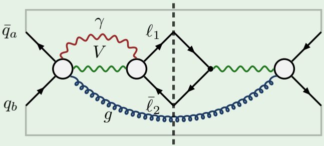
$$\mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}\right\} \mathrm{d}\sigma^{\mathrm{V_s}}$$

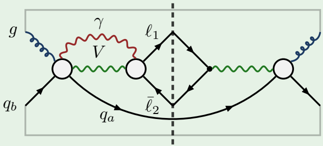
Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\}$$

(Real QCD) × (Virtual EW)



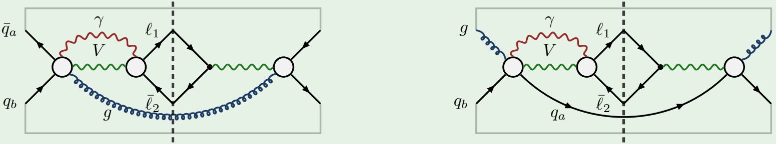


- additional kinematic dependence from QCD emission

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to3}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to2}\right\}$$

(Real QCD) × (Virtual EW)



- additional kinematic dependence from QCD emission

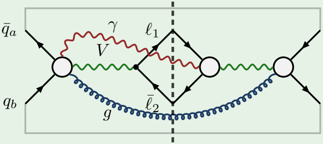
$$\mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}^{2\to3}\right\}\mathrm{d}\sigma^{\mathrm{R_s}}$$

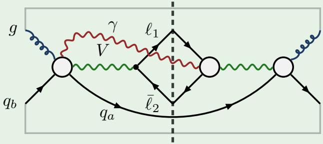
Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\text{R}_\text{s}\otimes\text{R}_\text{ew}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\text{V}_\text{s}}\, \delta_{\text{R}_\text{ew},\text{nf}}^{2\to2+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\text{C}_\text{s}}\, \delta_{\text{R}_\text{ew},\text{nf}}^{2\to2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma^{\text{R}_\text{s}}\, 2\,\mathrm{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to3}\right\} + \int_2 \mathrm{d}\sigma^{\text{V}_\text{s}}\, 2\,\mathrm{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}\right\} + \int_2 \mathrm{d}\sigma^{\text{C}_\text{s}}\, 2\,\mathrm{Re}\left\{\delta_{\text{V}_\text{ew},\text{nf}}^{2\to2}\right\}$$

(Real QCD)×(Real EW)



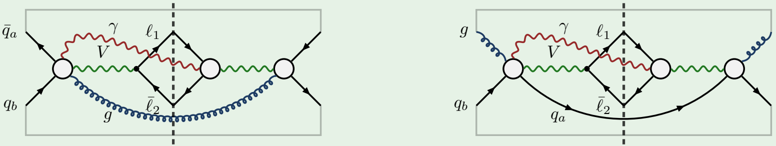


- new kinematic dependence on gluon emission (tree level)

$$\hat\sigma_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\, \delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}$$

$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to3}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\}$$

(Real QCD) × (Real EW)



- new kinematic dependence on gluon emission (tree level)

$$\mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} = 2\,\mathrm{Re}\left\{\delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}}\right\} \mathrm{d}\sigma^{\mathrm{R_s}}$$

Non-Factorizable $\mathcal{O}(\alpha_s \alpha)$ Corrections

$$\hat{\sigma}^{\mathrm{QCD}\otimes\mathrm{EW}}_{\mathrm{nf}} = \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\, \delta^{2\to3+\gamma}_{\mathrm{Rew,nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta^{2\to2+\gamma}_{\mathrm{Rew,nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta^{2\to2+\gamma}_{\mathrm{Rew,nf}}$$

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Infrared singularities—QCD corrections: dipole subtraction formalism

$$\hat{\sigma}^{\mathrm{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}$$

$$= \int_3 \left[\left(\mathrm{d}\sigma^{\mathrm{R_s}}\right)_{\epsilon=0} - \left(\mathrm{d}\sigma^{\mathrm{A_s}}\right)_{\epsilon=0}\right] + \int_2 \left[\mathrm{d}\sigma^{\mathrm{V_s}} + \mathrm{d}\sigma^{\mathrm{C_s}} + \int_1 \mathrm{d}\sigma^{\mathrm{A_s}}\right]_{\epsilon=0}$$

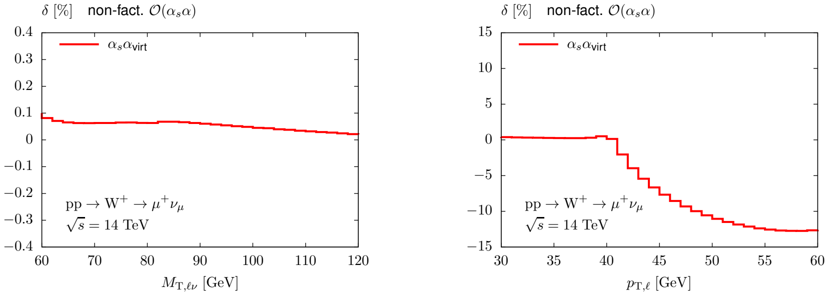
Infrared singularities—EW corrections: phase-space slicing method $\Delta E \ll \Gamma_V$

$$\int_\gamma \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathrm{QCD}}\, \delta^{\gamma}_{\mathrm{Rew,nf}} = \int_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathrm{QCD}}\, \delta^{\gamma}_{\mathrm{Rew,nf}} + \int_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathrm{QCD}}\, \delta^{\gamma}_{\mathrm{Rew,nf}}$$

$$= \underbrace{\int_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\, \delta^{\gamma}_{\mathrm{eik}}}_{=\,\delta_{\mathrm{soft}}(\Delta E)}\, \mathrm{d}\sigma^{\mathrm{QCD}} + \int_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathrm{QCD}}\, \delta^{\gamma}_{\mathrm{Rew,nf}}$$

$$\iint_{n+\gamma} \mathrm{d}\sigma^{\mathrm{QCD}}\,\delta^{\gamma}_{\mathrm{R_{ew}},\mathrm{nf}} \;+\; \int_n \mathrm{d}\sigma^{\mathrm{QCD}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}\right\}$$

$$= \iint_{\substack{n+\gamma \\ E_\gamma > \Delta E}} \mathrm{d}\sigma^{\mathrm{QCD}}\,\delta^{\gamma}_{\mathrm{R_{ew}},\mathrm{nf}} \;+\; \int_n \mathrm{d}\sigma^{\mathrm{QCD}}\left[2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}\right\} + \delta_{\mathrm{soft}}(\Delta E)\right] \quad , \quad \Delta E \ll \Gamma_V$$



$$\iint_{n+\gamma} \mathrm{d}\sigma^{\mathrm{QCD}}\, \delta^{\gamma}_{\mathrm{R_{ew}},\mathrm{nf}} + \int_n \mathrm{d}\sigma^{\mathrm{QCD}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathrm{nf}}\right\}$$

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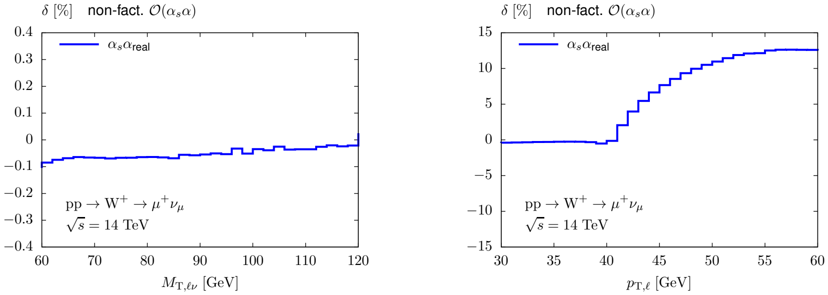
$$\hookrightarrow \mathcal{O}\left(\alpha_{\mathrm{s}}\alpha_{\mathrm{virt}}\right)$$



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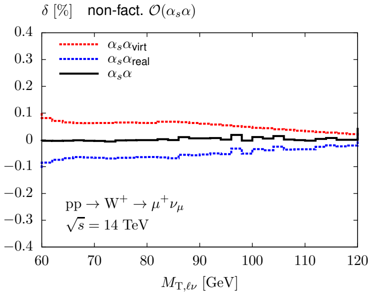
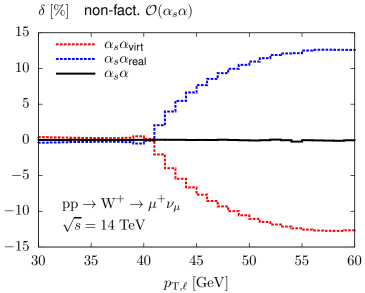
$\hookrightarrow \mathcal{O}\left(\alpha_{\mathrm{s}}\alpha_{\mathrm{real}}\right)$





- almost perfect cancellation between different contributions
- tiny & flat corrections!

1. Motivation
2. Pole Approximation: NLO EW $\mathcal{O}(\alpha)$
3. Pole Approximation: NNLO QCD×EW $\mathcal{O}(\alpha_s \alpha)$
4. Summary and Outlook

Summary and Outlook

Largest theoretical unknown in Drell–Yan processes: $\mathcal{O}(\alpha_s \alpha)$
important in distributions around resonance (M_W measurement) $\rightsquigarrow$ Pole expansion

Pole approximation @ $\mathcal{O}(\alpha)$

PA reproduces full result near resonance

- fact. ini: small and flat in M_T distributions, larger for p_T
- fact. fin: dominant contribution
- non-fact.: small and flat

Pole approximation @ $\mathcal{O}(\alpha_s \alpha)$

- establish concept of PA at this order
- calculation of non-factorizable corrections $\rightarrow$ negligible
  $\hookrightarrow$ factorizable corrections are dominant
- largest contribution expected from
  (QCD initial state) $\times$ (EW final state) factorizable corrections
  $\hookrightarrow$ work in progress

Summary and Outlook

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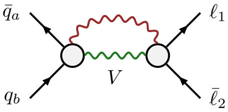
Thank you

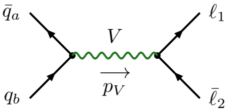
Backup Slides

Only soft region ($|q^\mu| \lesssim \Gamma_V$) leads to resonant contributions!

- neglect q^μ everywhere except in divergent propagators
- only scalar integrals
- corrections factorize off from the lower-order diagram



$\sim \delta^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew},nf}}$



$u\bar{d} \to \mathrm{W}^+ \to \nu_\ell \ell^+$:

$$\delta^{\bar{d}u \to \nu_\ell \ell^+}_{\mathrm{V_{ew},nf}} = -\frac{\alpha}{2\pi}\Bigg\{ -2 + Q_d \,\mathrm{Li}_2\left(1 + \frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u \,\mathrm{Li}_2\left(1 + \frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right) + \left[2\ln\left(\frac{\mu_\mathrm{W}^2 - \hat{s}_\mathrm{res}}{M_\mathrm{W}^2}\right) - \frac{c_\epsilon}{\epsilon} - \ln\left(\frac{\mu^2}{M_\mathrm{W}^2}\right)\right] \left[1 + Q_d \ln\left(-\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u \ln\left(-\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)\right]\Bigg\}$$

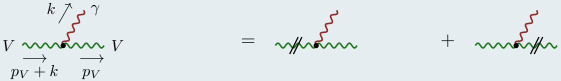
[Dittmaier, Krämer '02]

Real Corrections in the Pole Approximation

Real corrections in the PA

Decompose into initial-state and final-state radiation

$$\frac{1}{(p_V+k)^2-M_V^2}\cdot\frac{1}{p_V^2-M_V^2} = \frac{1}{2p_V\cdot k}\left[\frac{1}{p_V^2-M_V^2} - \frac{1}{(p_V+k)^2-M_V^2}\right]$$



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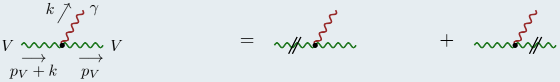

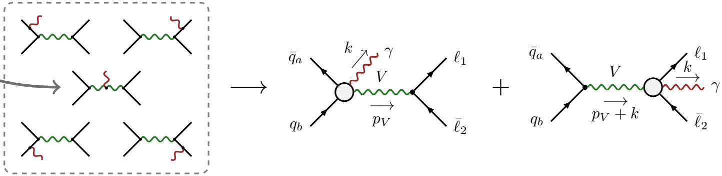

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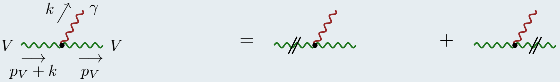
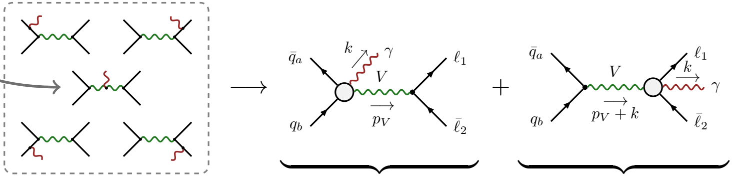
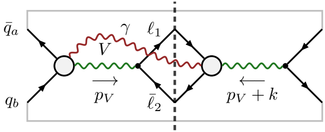
Decompose into initial-state and final-state radiation

$$\frac{1}{(p_V+k)^2-M_V^2}\cdot\frac{1}{p_V^2-M_V^2} = \frac{1}{2p_V\cdot k}\left[\frac{1}{p_V^2-M_V^2} - \frac{1}{(p_V+k)^2-M_V^2}\right]$$



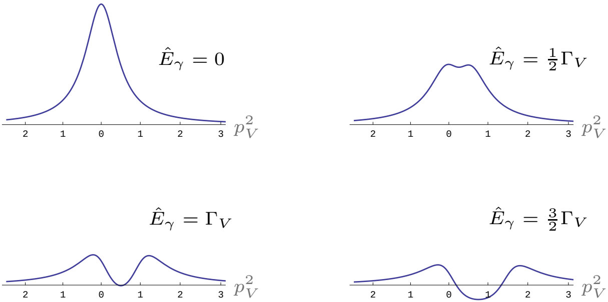


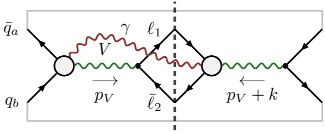
$|\ldots|^2 \rightsquigarrow$ fact. ini. $\qquad |\ldots|^2 \rightsquigarrow$ fact. fin.



- only soft photons! $(E_\gamma \sim \Gamma_V \ll M_V)$

$$2\,\mathrm{Re}\left\{ \frac{1}{p_V^2 - \mu_V^2} \left(\frac{1}{(p_V + k)^2 - \mu_V^2} \right)^* \right\}$$





- only soft photons! $(E_\gamma \sim \Gamma_V \ll M_V)$

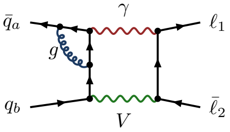
modified eikonal currents $\rightsquigarrow$ factorizes off from diagram without γ emission

$$\left|\mathcal{M}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma}_{\mathrm{Rew}}\right|^2 \Big|_{\text{non-fact}} = \delta^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma}_{\mathrm{Rew},\text{nf}} \left|\mathcal{M}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{B}}\right|^2$$

$$\delta^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma}_{\mathrm{Rew},\text{nf}} = -e^2 \; 2\,\mathrm{Re}\left\{ (J^\mu_{\text{prod}})^* J_{\text{dec},\mu} \right\}$$

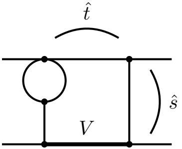
$$J^\mu_{\text{prod}} = -Q_a \frac{p_a^\mu}{k \cdot p_a} + Q_b \frac{p_b^\mu}{k \cdot p_b} + (Q_a - Q_b) \frac{(p_a + p_b)^\mu}{k \cdot (p_a + p_b)}$$

$$J^\mu_{\text{dec}} = \left[-Q_1 \frac{k_1^\mu}{k \cdot k_1} + Q_2 \frac{k_2^\mu}{k \cdot k_2} + (Q_1 - Q_2) \frac{(k_1 + k_2)^\mu}{k \cdot (k_1 + k_2)} \right] \frac{p_V^2 - \mu_V^2}{(p_V + k)^2 - \mu_V^2}$$



$$\sim -\frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{2\pi}\,\frac{Q_q Q_l \alpha}{2\pi}\,\mathcal{M}^0\,(1-\epsilon)\,(-\hat{t})\,(\mu_V^2-\hat{s}^2)\;I(\hat{s},\hat{t})$$

$I(\hat{s},\hat{t}) =$

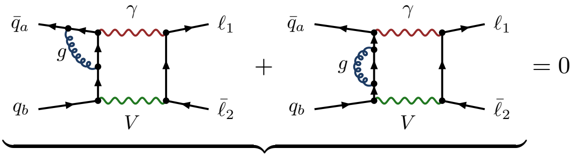


- Mellin–Barnes representation
- method of regions
- generalization of [Yennie, Frautschi, Suura '61]

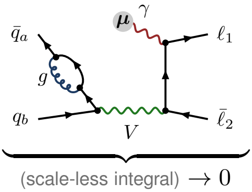
$$= \frac{(4\pi)^{2\epsilon}\Gamma^2(1+\epsilon)}{(\mu_V^2-\hat{s})(-\hat{t})}\left(\frac{\mu_V^2-\hat{s}}{M_V^2}\right)^{-3\epsilon}\left(\frac{-\hat{t}}{\mu^2}\right)^{-2\epsilon}\left\{\frac{1}{2\epsilon^3}+\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left[2+\frac{5\pi^2}{12}+\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]\right.$$
$$+2\,\mathrm{Li}_3\left(\frac{-\hat{t}}{M_V^2}\right)+\mathrm{Li}_3\left(1+\frac{\hat{t}}{M_V^2}\right)-6\zeta(3)-2\ln\left(\frac{-\hat{t}}{M_V^2}\right)\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]$$
$$\left.+\ln^2\left(\frac{-\hat{t}}{M_V^2}\right)\ln\left(1+\frac{\hat{t}}{M_V^2}\right)+\frac{5\pi^2}{6}+2\,\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)+4+\mathcal{O}(\epsilon)+\mathcal{O}\left(\hat{s}-\mu_V^2\right)\right\}$$

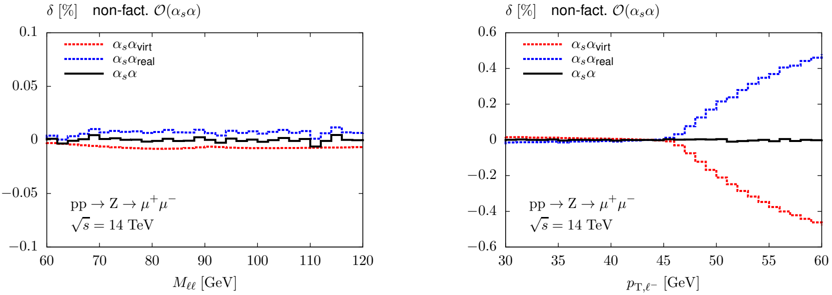
YFS argument

[Yennie, Frautschi, Suura '61]



$$\frac{(2\pi\mu)^{2\epsilon}}{\mathrm{i}\pi^2}\int \mathrm{d}q^D\,(-eQ_a)\frac{2q_a^{\mu}}{(q+p_a)^2}$$





δ [%] non-fact. $\mathcal{O}(\alpha_s\alpha)$
$\alpha_s\alpha_{\text{virt}}$
$\alpha_s\alpha_{\text{real}}$
$\alpha_s\alpha$
$\text{pp} \to \text{Z} \to \mu^+\mu^-$
$\sqrt{s} = 14$ TeV
$M_{\ell\ell}$ [GeV]
0.1
0.05
0
−0.05
−0.1
60
70
80
90
100
110
120
δ [%] non-fact. $\mathcal{O}(\alpha_s\alpha)$
$\alpha_s\alpha_{\text{virt}}$
$\alpha_s\alpha_{\text{real}}$
$\alpha_s\alpha$
$\text{pp} \to \text{Z} \to \mu^+\mu^-$
$\sqrt{s} = 14$ TeV
p_{T,ℓ^-} [GeV]
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
30
35
40
45
50
55
60