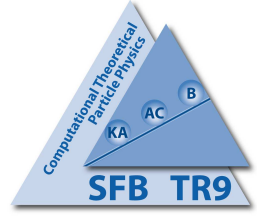
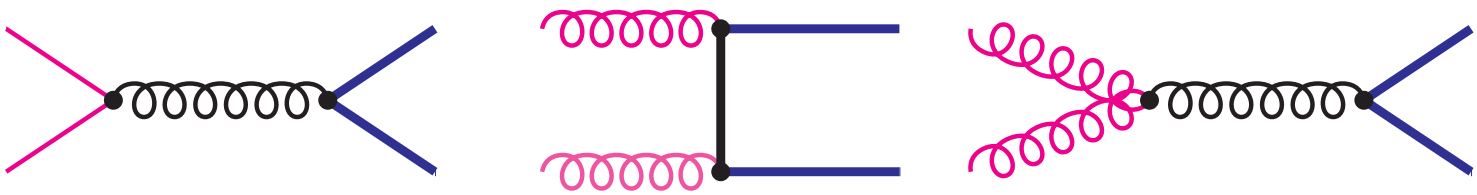
Top-pair production at NNLL

Jan Piclum



in collaboration with
M. Beneke, P. Falgari, S. Klein, C. Schwinn, M. Ubiali, F. Yan

Total Cross Section



$$\sigma_{t\bar{t}}(s) = \sum_{i,j} \int_{4m_t^2}^{s} \mathrm{d}\hat{s}\, \mathcal{L}_{ij}(s, \hat{s}, \mu_f)\, \hat{\sigma}_{ij}(\hat{s}, \mu_f, \mu_r)$$

two ways to compute cross section:

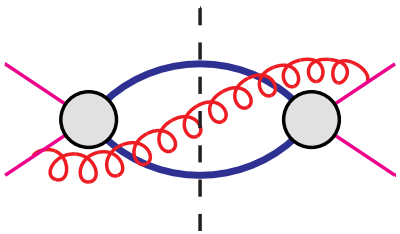
- fixed order
- sum dominant contributions to all orders

Dominant Terms

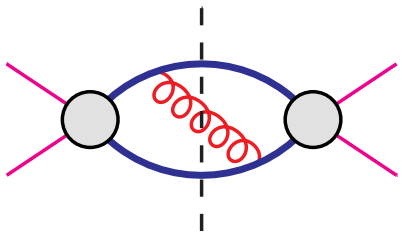
consider threshold limit: $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

Laenen et al. 1991; Catani et al.; Berger, Contopanagos; Kidonakis et al. 1996; Bonciani et al. 1998

Soft corrections:

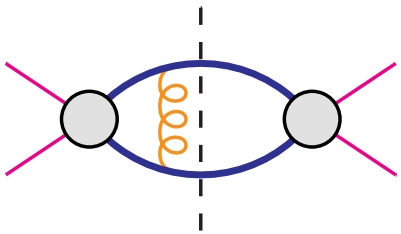


$\sim \alpha_s \ln^2 \beta$



$\sim \alpha_s \ln \beta$

Coulomb corrections:



$\sim \alpha_s/\beta$

Power counting: $\alpha_s/\beta \sim \alpha_s \ln \beta \sim 1$

Results at NNLO/NNLL

total cross section: $\sigma_{t\bar{t}}$

distributions: $\frac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}M_{t\bar{t}}}$, $\frac{\mathrm{d}\sigma_{t+X}}{\mathrm{d}p_T}$, ...

$\mathrm{NNLO}_{\mathrm{app}}$

Moch, Uwer 2008; Langenfeld, Moch, Uwer 2009;
Beneke, Czakon, Falgari, Mitov, Schwinn 2009;
Moch, Uwer, Vogt 2012

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011;
Kidonakis 2010

full NNLO

Bärnreuther, Czakon, Mitov; Czakon, Mitov 2012
Czakon, Fiedler, Mitov 2013

NNLL soft gluon resummation

Cacciari, Czakon, Mangano, Mitov, Nason 2011

Ahrens, Ferroglia, Neubert, Pecjak, Yang 2010, 2011

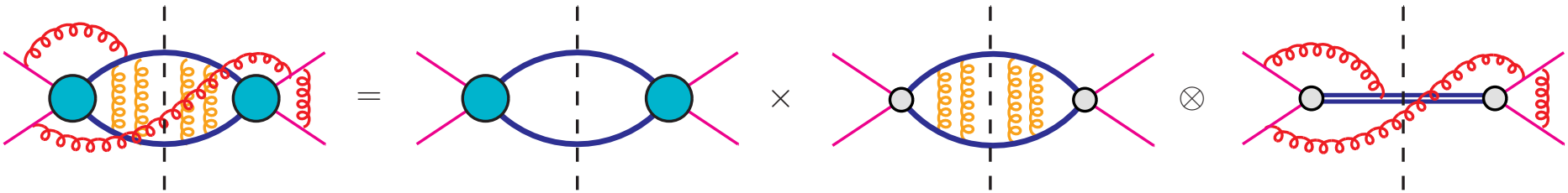
NNLL soft + Coulomb resummation

Beneke, Falgari, Klein, Schwinn 2011

Resummation in Momentum Space

Soft and Coulomb resummation: Beneke, Falgari, Schwinn 2009, 2010

$$\hat{\sigma}_{ij} = \sum_R H_{ij}^R \int \mathrm{d}\omega J^R(E - \omega/2)\, W^R(\omega)$$



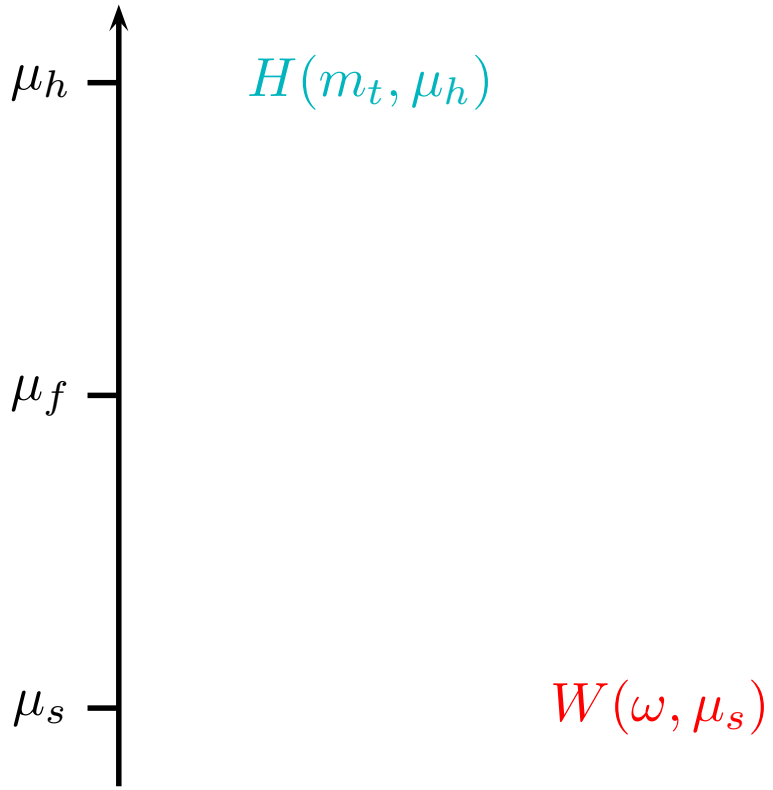
$$\hat{\sigma} \propto \hat{\sigma}^{(0)} \sum_k \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{LL}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{NLL}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{NNLL}} + \dots]$$

$$\times \{1\,(\text{LL, NLL})\,; \alpha_s, \beta\,(\text{NNLL})\,; \dots\}$$

Resummation of Soft Logarithms

Becher, Neubert 2006; Becher, Neubert, Xu 2007; Ferroglia Neubert, Pecjak, Yang 2009; Beneke, Falgari, Schwinn 2009; Czakon, Mitov, Sterman 2009

- typical scales:
  $\mu_h \sim 2m_t$, $\mu_s \sim 2m_t\beta^2$
- hard and soft function obey RGEs
- solve RGEs in momentum space



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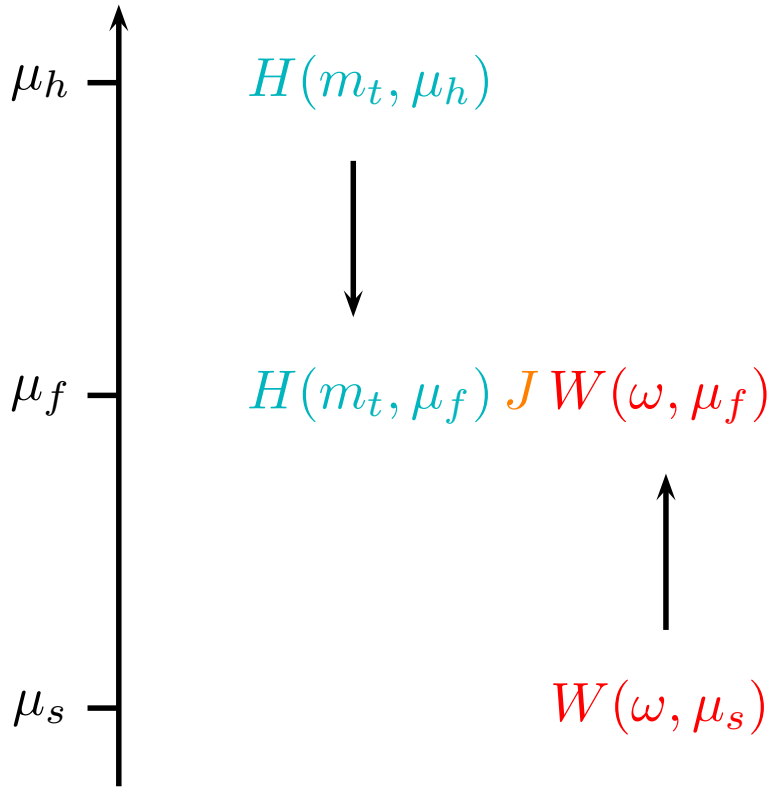
- evolve H from μ_h to μ_f
- evolve W from μ_s to μ_f



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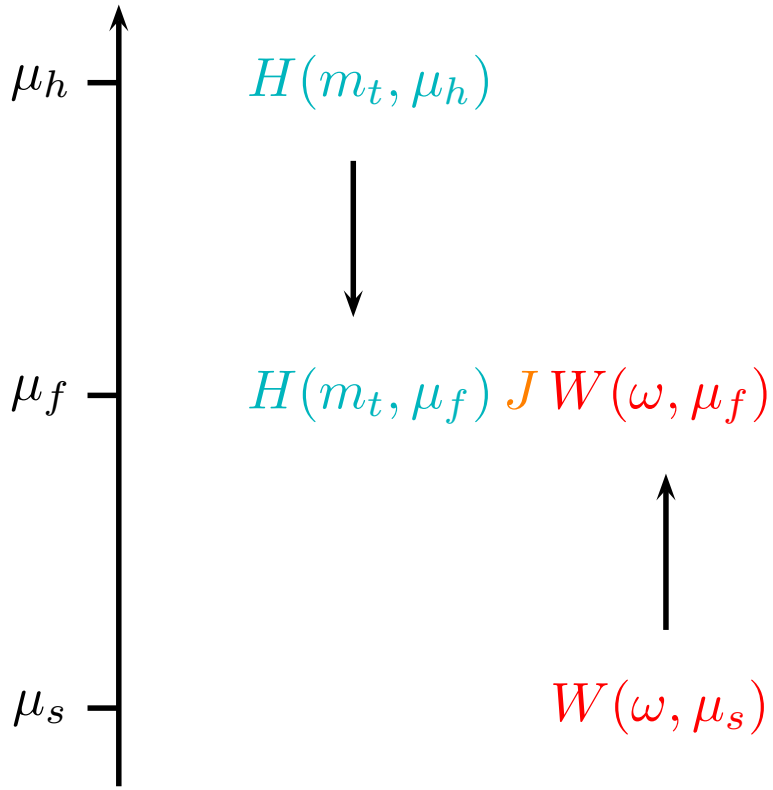
$\Rightarrow$ resums logarithms of β



NB: Resummation can also be done in Mellin space: $\ln\beta \to \ln N$

Sterman 1987; Catani, Trentadue 1989

Resummation of Coulomb Corrections

Resummation of Coulomb corrections from non-relativistic Green's function

Fadin, Khoze 1987; Peskin, Strassler 1990; ...

$$\left[-\frac{\vec{\nabla}^2}{m_t} - C_F\frac{\alpha_s}{r}\right] G_C^{(0)}(\vec{r},\vec{r}\,',E) = \delta(\vec{r}-\vec{r}\,')$$

$$J(E) = 2\,\mathrm{Im}\left[G_C^{(0)}(0,0;E) + \ldots\right]$$

- includes bound states below threshold ($E < 0$)
- depends on Coulomb scale: $\mu_C \sim m_t\beta$
- NNLL: need higher order Coulomb and non-Coulomb corrections

Matching to Fixed Order Cross Section

- $t\bar{t}$ production at Tevatron and LHC is not close to threshold: $\beta \approx 0.4$
- NLO: threshold expansion gives reasonable estimate of integral over β
- match to fixed order result to improve behaviour at large β

$$\hat{\sigma}^{\text{NNLO+NNLL}} = \hat{\sigma}^{\text{NNLL}} - \hat{\sigma}^{\text{NNLL}}\big|_{\text{NNLO}} + \hat{\sigma}^{\text{NNLO}}$$

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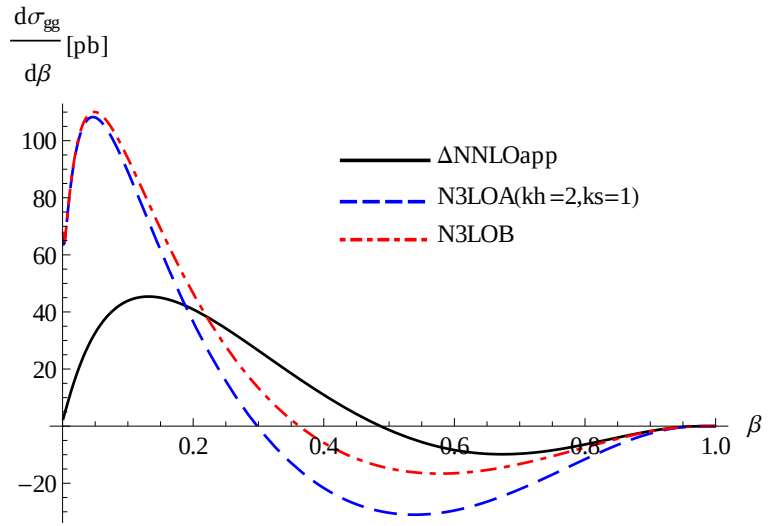
Is resummation necessary?

- potentially large corrections at N^3LO
- $1/\beta^4$ term at N^4LO

$\Rightarrow$ Yes



[Beneke, Falgari, Klein, Schwinn 2011]

TOPIXS

TOP-pair Inclusive X Section

http://users.ph.tum.de/t31software/topixs/

- user-friendly program for combined soft and gluon resummation
- flexible choice of input parameters ($\sqrt{s}$, m_t, ...)
- fully automatic evaluation of PDF+α_s error for predefined PDF sets
- current version: `TOPIXS 1.1`
- update with full NNLO result in progress

other programs are available:
`HATHOR` [Aliev et al. 2010], `TopNNLO` [Ahrens et al. 2011],
`Top++` [Czakon, Mitov 2011]

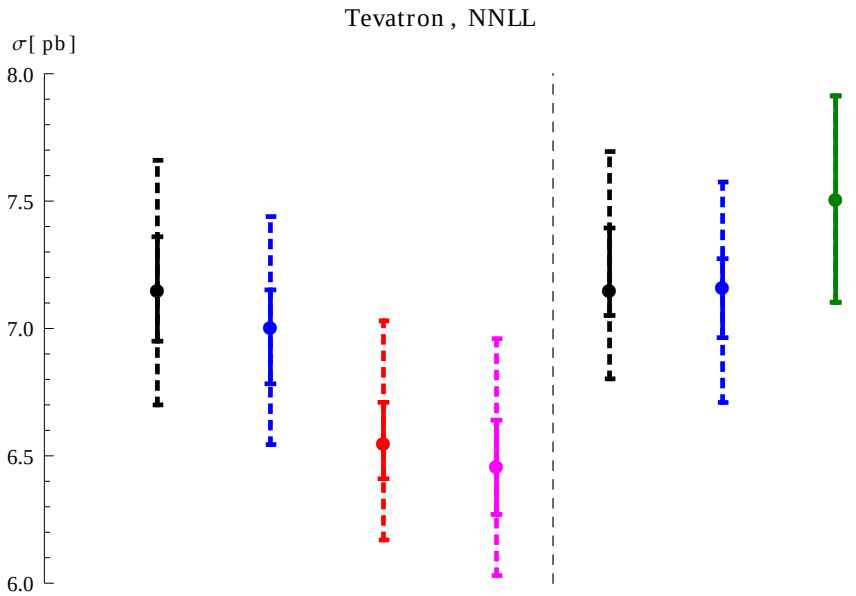
Results

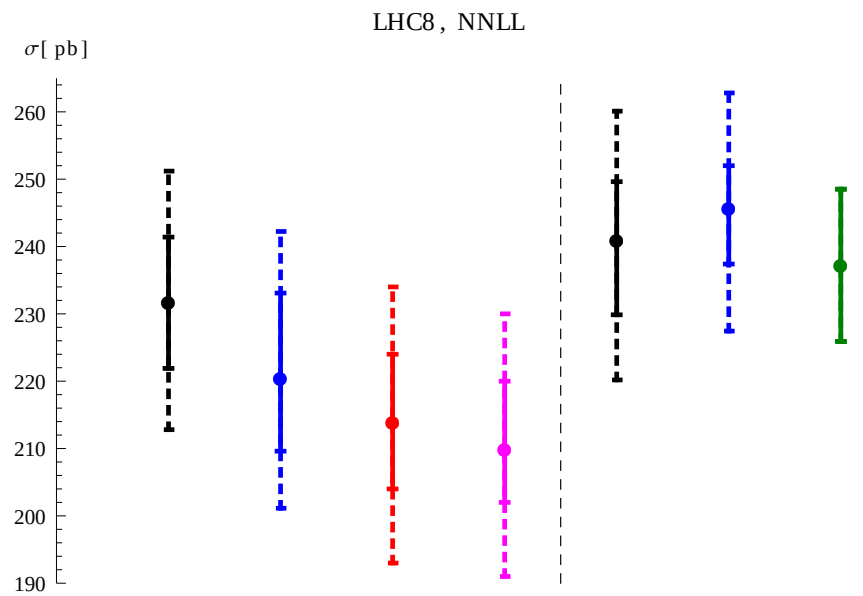
$\sigma_{t\bar{t}}$ [pb]	Tevatron	LHC @ 7 TeV	LHC @ 8 TeV
NLO	$6.68\,^{+0.36\ +0.23}_{-0.75\ -0.22}$	$158.1\,^{+19.5\ +6.8}_{-21.2\ -6.2}$	$226.2\,^{+27.8\ +9.2}_{-29.7\ -8.3}$
NNLO	$7.00\,^{+0.27\ +0.29}_{-0.37\ -0.24}$	$167.1\,^{+6.7\ +7.6}_{-10.7\ -7.1}$	$239.1\,^{+9.3\ +10.4}_{-14.8\ -9.6}$
NNLO+NNLL	$7.15\,^{+0.24\ +0.30}_{-0.10\ -0.25}$	$168.5\,^{+6.3\ +7.7}_{-7.5\ -7.2}$	$241.0\,^{+8.7\ +10.5}_{-11.1\ -9.7}$

m_t=173.3 GeV, MSTW 2008 NLO/NNLO, $\alpha_s(M_Z) = 0.1171$, TOPIXS 2.0 (preliminary)

- correction beyond NLO: Tevatron $\sim 7\%$, LHC $\sim 6\%$
- theory uncertainty: Tevatron $\sim \pm 3\%$, LHC $\sim \pm 5\%$
- PDF+α_s uncertainty $\sim \pm 5\%$

Comparison of NNLL Results





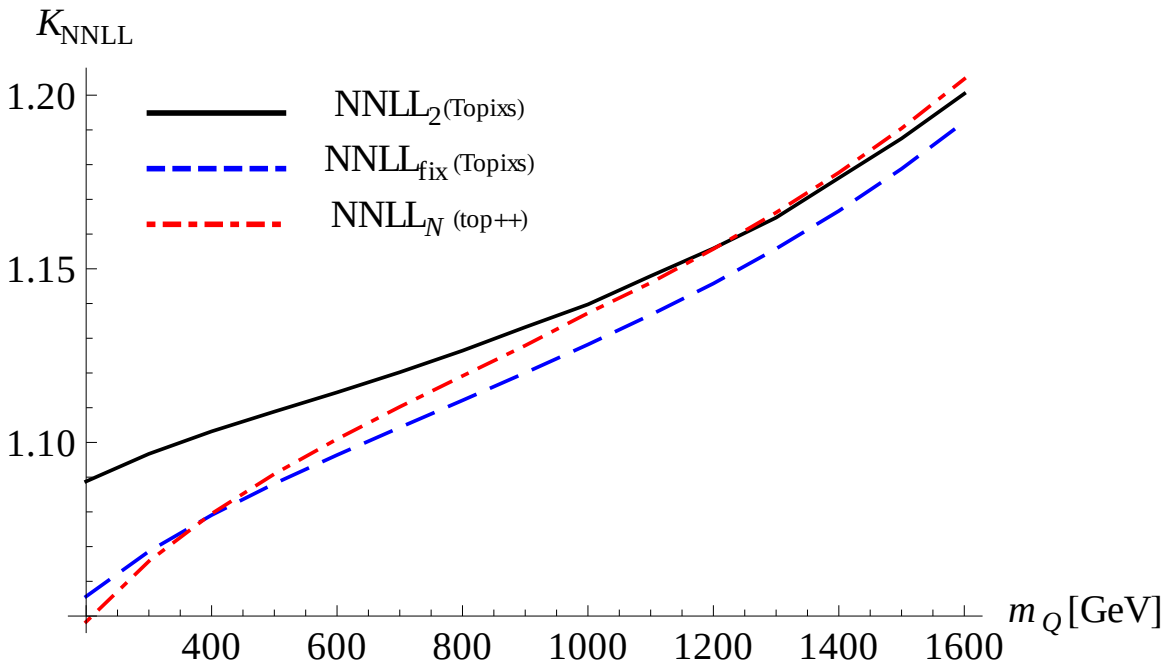
left of dashed line:
Beneke, Falgari, Klein, JP, Schwinn, Ubiali, Yan 2012 ($m_t = 173.3$ GeV)
Top++ 1.4 ($m_t = 173.3$ GeV; implements result by Cacciari, Czakon, Mangano, Mitov, Nason 2011)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (1PI, $m_t = 173.1$ GeV)
Ahrens, Ferroglia, Neubert, Pecjak, Yang 2011 (PIM, $m_t = 173.1$ GeV)

right of dashed line:
TOPIXS 2.0 (preliminary); Top++ 2.0

experimental results:
Tevatron combination from arXiv:1309.7570; ATLAS-CONF-2013-097 (rescaled to $m_t = 173.3$ GeV)

error bars: solid – theory, dashed – PDF(+α_s); PDF set: MSTW 2008 NNLO

Heavy "Top" Quarks



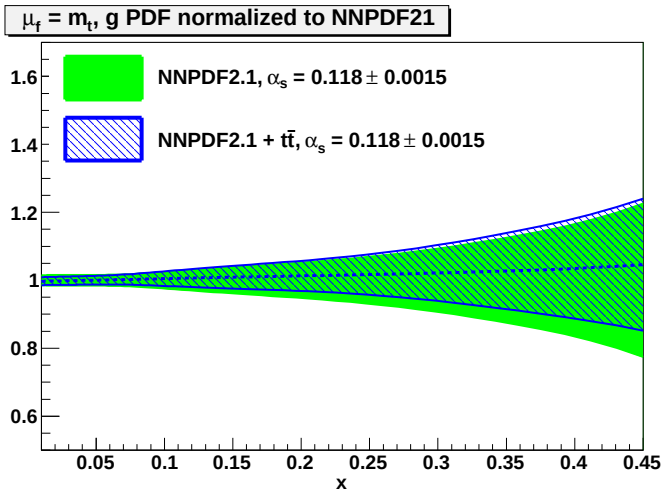
$$K_{\text{NNLL}} = \sigma_{t\bar{t}}^{\text{NNLL}} / \sigma_{t\bar{t}}^{\text{NLO}}$$

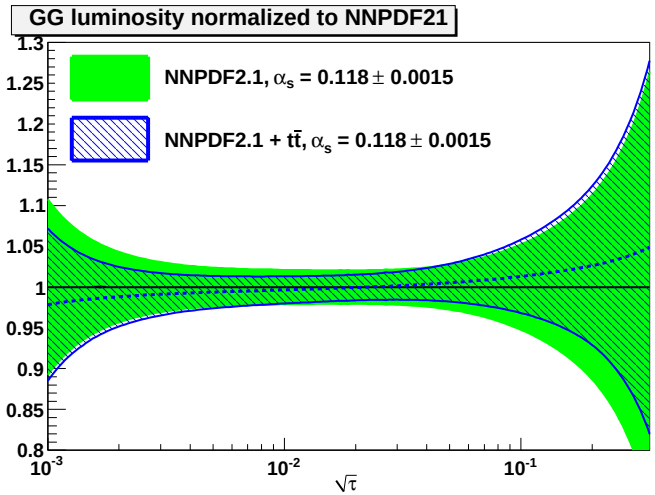
NNLL$_2$, NNLL$_{\text{fix}}$ computed with TOPXIS 1.0

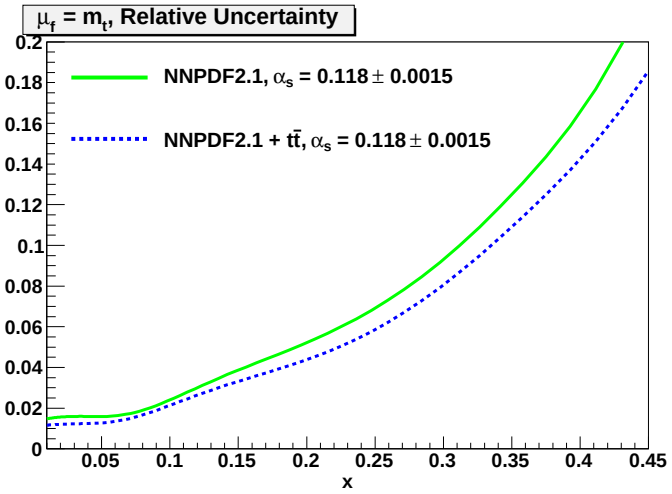
NNLL$_N$ computed with Top++ 1.2

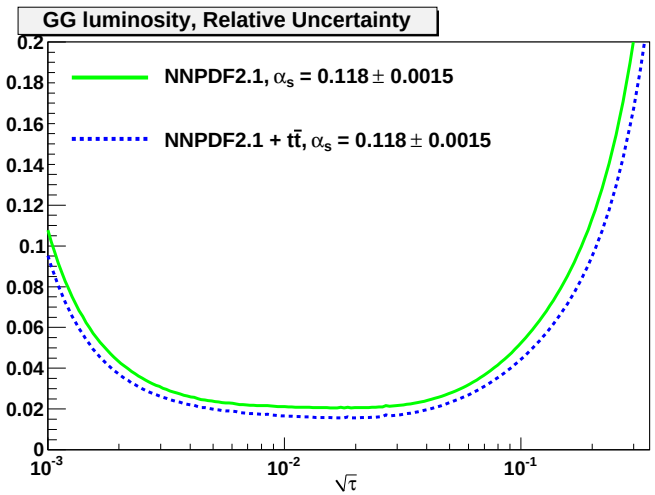
- different implementations agree better for larger masses
- Mellin- and momentum-space results agree within ambiguity of the latter

Gluon Distribution









Reweighting of NNPDF 2.1 gluon distribution using 350 replicas and

$$\sigma_{t\bar{t}}^{\exp} = 173.23 \pm 9.59 \text{ pb}, \; m_t = 173.3 \text{ GeV}, \; \alpha_s = 0.118 \pm 0.0015$$

see also [Czakon, Mangano, Mitov, Rojo 2013] and [Alekhin, Blümlein, Moch 2013]

Summary

- total cross section is available at NNLO+NNLL
- public programs are available for analyses

- theory uncertainty $\sim \pm 5\%$ at LHC and $\sim \pm 3\%$ at Tevatron
- PDF+α_s uncertainty $\sim \pm 5\%$
- LHC $t\bar{t}$ data has impact on PDF fits

Fixed vs. Running Soft Scale

fixed soft scale: Becher, Neubert, Xu 2007

- minimises relative fixed-order 1-loop soft correction to $\sigma_{t\bar{t}}$
- resums logarithms in hadronic cross section
- does not predict partonic cross section

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running soft scale: Beneke, Falgari, Klein, Schwinn 2011

- divide β integration into two regions
- $\beta < \beta_{\text{cut}}$: small ambiguities, $\mu_s = 2m_t\beta_{\text{cut}}^2$
- $\beta > \beta_{\text{cut}}$: no large logarithms, $\mu_s = 2m_t\beta^2$
- Tevatron: $\beta_{\text{cut}} = 0.35$; LHC: $\beta_{\text{cut}} = 0.54$

Kinematic Ambiguity in Fixed Soft Scale

cross section depends on energy $E = \sqrt{\hat{s}} - 2m_t$

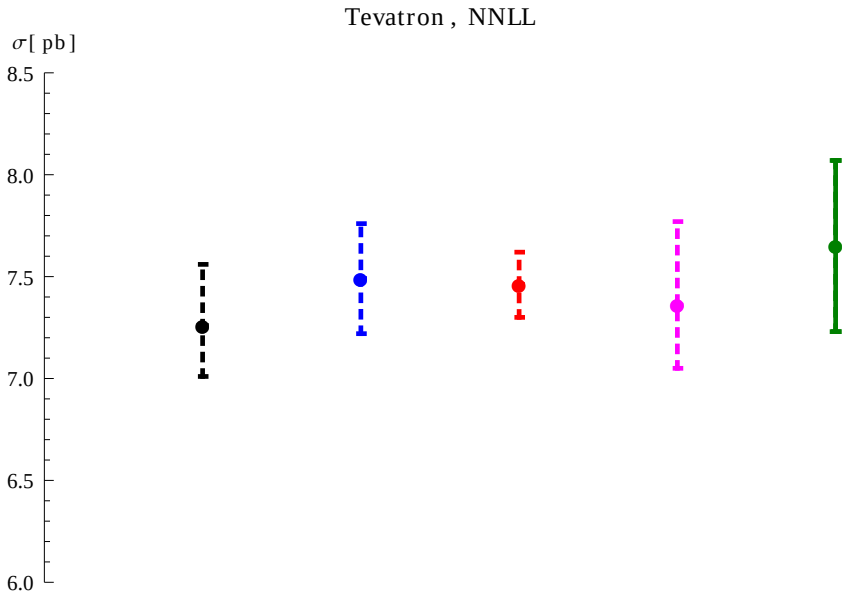
threshold limit: $E \approx m_t\beta^2 + \ldots$

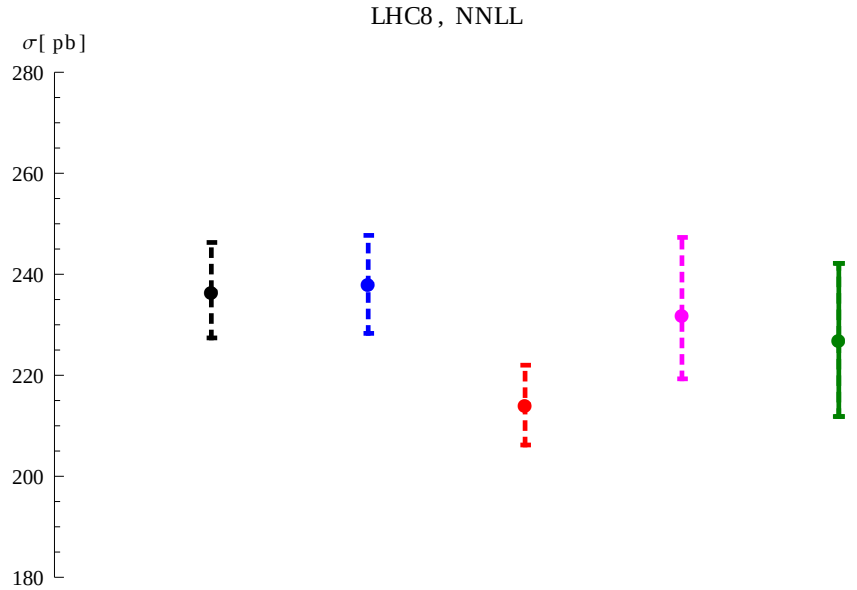
$\rightsquigarrow$ soft corrections can be expressed in terms of $\ln(E/\mu_s)$ or $\ln(m_t\beta^2/\mu_s)$

expressions agree in threshold limit, but lead to large differences at Tevatron and LHC energies:

	$\ln(E/\mu_s)$	$\ln(m_t\beta^2/\mu_s)$
Tevatron	$\mu_s = 52$ GeV	$\mu_s = 35$ GeV
LHC (8 TeV)	$\mu_s = 103$ GeV	$\mu_s = 60$ GeV

Comparison of PDF Sets



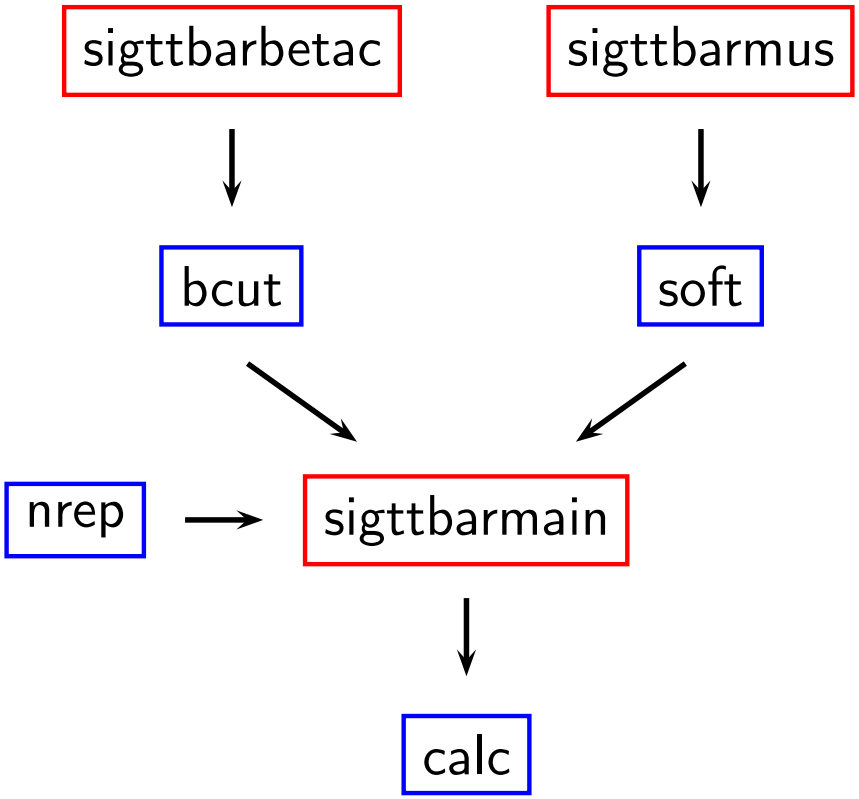


MSTW 2008
NNPDF 2.1
ABM 2011
CT 2010
Tevatron combination from TOP 2012; CMS result from ICHEP 2012

error bars: dashed – PDF+α_s

$\alpha_s(M_Z) = 0.118$, $m_t = 173.3$ GeV, NNLL$_2$ computed with TOPIXS 1.1 using NNLO PDFs

Program Structure



- 3 FORTRAN programs
- 4 C++ programs
- controlled by shell script
- configured with single text file

external libraries:

- LHAPDF — Whalley, Bourilkov, Group 2005
- hplog — Gehrmann, Remiddi 2001
- QUADPACK — Piessens et al. 1983
- GSL — Galassi et al. 2009

Example (version 2.0)

1. create config file:

example.cfg

```
OUTFILE=Tevatron
QUADPACK=../QUADPACK
CALCBETA=1
COLLIDER=0
SQRTS=1960
APPROX_NNLO=1
APPROX_NNLL2=1
```

2. run shell script:

```
./topixs example.cfg
```

3. ...

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1. create config file:

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2. run shell script:

```
./topixs example.cfg
```

3. ...

4. result:

Tevatron.res

```
proton-antiproton
sqrt(s) = 1960 GeV
mtop = 173.3 GeV
PDF set: MSTW08
running mu_s
beta_cut(ks = 1, NNLL) = 0.5700
beta_cut(ks = 2, NNLL) = 0.3800
beta_cut(ks = 4, NNLL) = 0.2400
full NNLO for qqbar channel
full NNLO for qq', qqbar' and qq channels
full NNLO for qg channel
full NNLO for gg channel

NNLO: 7.00564 pb
        +0.265619 -0.372279 (theory)
        +0.290501 -0.240203 (PDF+alpha_s)
  decomposition of theory error:
        +0.265619 -0.372279 (scale)
        +0 -0 (NNLO constant)

NNLL2BS: 7.15094 pb
        +0.243505 -0.0999633 (theory)
        +0.300005 -0.248672 (PDF+alpha_s)
  decomposition of theory error:
        +0.151686 -0.0122404 (scale)
        +0.190489 -0.0992111 (resummation)
        +0 -0 (NNLO constant)
```