Renormalization of Vacuum Expectation Values and IR structure of FDH/DRED at 2-loop order

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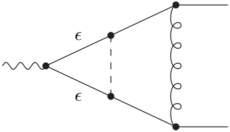
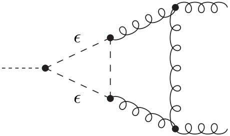
- with Marcus Sperling and Alexander Voigt [JHEP 1307, 1401]
- with Christoph Gnendiger and Adrian Signer [arxiv 1404]

Outline

1. Renormalization of VEVs
2. IR structure in FDH and DRED
3. Conclusions





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1. Renormalization of VEVs
   - Aim: better understanding of $\delta\hat{Z}$, general 2-loop computation
2. IR structure in FDH and DRED
3. Conclusions

Renormalization of VEVs

Renormalization such that $\langle\phi\rangle = 0$, i.e. tadpoles vanish

$$\phi \to \sqrt{Z}\phi, \quad v \to v + \delta v = \sqrt{Z}(v + \delta\bar{v}) = \sqrt{Z}\sqrt{\hat{Z}}v$$

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- Aim: better understanding of $\delta\hat{Z}$, general 2-loop computation

Possibility and question

Could get $\delta\hat{Z}$ from e.g.

$$\frac{\delta m_t}{m_t} = \frac{\delta y_t}{y_t} + \frac{\delta Z}{2} + \frac{\delta\hat{Z}}{2}$$

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Question: Can we first renormalize the theory and then compute v, so v should be finite???

Depends on the gauge: No for R_ξ: $\partial^\mu A_\mu - \xi e v (2\,\mathrm{Im}\phi)$

Investigation of δv in a nutshell

Trick: [Kraus,Sibold '95][Kraus '97] [Hollik,Kraus,Roth,Rupp,Sibold,DS '02]

Introduce background field $\hat{\phi}(x)$, only at the end: $\hat{\phi}(x) = \hat{v} =$const

$$\phi + \hat{\phi} \to \sqrt{Z}(\phi + \sqrt{\hat{Z}}\hat{\phi})$$

Modified R_ξ gauge fixing:

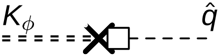
$$F = \partial^\mu A_\mu + ie\xi(\hat{\phi}^\dagger\phi - \phi^\dagger\hat{\phi})$$

BRS transformation

$$s\hat{\phi} = \hat{q}$$

Computation

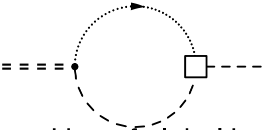
Global gauge invariance+Slavnov-Taylor identity restrict divergences



$$= -\frac{i}{2}\delta\hat{Z}$$

Meaning: renormalization of composite operator corresponding to breaking of global symmetry by gauge fixing

Very few Feynman rules for with well-defined origin



[BRS transform$\propto e$] [breaking of global invariance$\propto \xi e(\hat{\phi}^\dagger\phi - \phi^\dagger\hat{\phi})$]

General model

Scalar, spinor, vector fields ϕ_a, ψ_p, V^A with Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{inv}}|_{\phi \to \phi_{\text{eff}}} + \mathcal{L}_{\text{fix,gh}} + \mathcal{L}_{\text{ext}}$$

with

$$\begin{aligned}
\mathcal{L}_{\text{inv}} = & -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \frac{1}{2} (D_\mu \phi)_a (D^\mu \phi)_a + i \psi^\alpha_p \sigma^\mu_{\alpha\dot{\alpha}} \left(D^\dagger_\mu \bar{\psi}^{\dot{\alpha}} \right)_p \\
& - \frac{1}{2!} m^2_{ab} \phi_a \phi_b - \frac{1}{3!} h_{abc} \phi_a \phi_b \phi_c - \frac{1}{4!} \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \\
& - \frac{1}{2} \left[(m_f)_{pq} \, \psi^\alpha_p \psi_{q\alpha} + \text{h.c.} \right] - \frac{1}{2} \left[Y^a_{pq} \psi^\alpha_p \psi_{q\alpha} \phi_a + \text{h.c.} \right] \\
\mathcal{L}_{\text{fix,gh}} = & \; s \left[\bar{c}^A \left(F^A + \xi B^A / 2 \right) \right] \\
\mathcal{L}_{\text{ext}} = & \; K_{\phi_a} s \phi_a + K_{V^A_\mu} s V^A_\mu + K_{c^A} s c^A + [K_{\psi_p} s \psi_p + \text{h.c.}]
\end{aligned}$$

General, modified R_ξ gauge fixing:

$$F^A = \partial^\mu V^A_\mu + i g \xi (\hat{\phi})_a T^A_{ab} \phi_b$$

Results: $\delta Z \to \gamma \quad \delta\hat{Z} \to \hat{\gamma} \quad \delta v = \frac{v}{2}(\delta Z + \delta\hat{Z}) \to \beta_v$

$$\hat{\gamma}^{(1)}(S) = \frac{\xi}{(4\pi)^2} 2g^2 C^2(S)$$

$$\hat{\gamma}^{(2)}(S) = \frac{\xi}{(4\pi)^4} \Big\{ g^4 \Big[2\,(1+\xi)\, C^2(S) C^2(S) + \frac{7-\xi}{2} C_2(G) C^2(S) \Big] - 2g^2 C^2(S) Y^2(S) \Big\}$$

$$\beta_v = [\gamma(S) + \hat{\gamma}(S)]\, v$$

can now be implemented into spectrum generator (generators) Sarah [Staub], FlexibleSUSY

Prior status for RGE coefficients

Model	$\beta_v^{(1)}$	$\beta_v^{(2)}$
MSSM	✓[Chankowski Nucl.Phys. B423]	✓[Yamada 94] $O(g^2 Y^2)$
E_6SSM	✓[Athron, DS, Voigt '12]	✗
∀ gauge theory	?	✗
∀ SUSY model	?	✗

Model	β(phys. parameter)	γ(fields)
∀ gauge theory	✓[Machacek,Vaughn '83, Luo et al '03]	✓
∀ SUSY model	✓[Martin, Vaughn; Jack, Jones; Yamada '93]	✓ partially

Note in SUSY: $\gamma\left(\text{scalar in WZ gauge+Landau or } R_\xi \text{ gauge}\right) \neq \gamma\left(\text{superfield}\right) \stackrel{?}{=} \gamma\left(\text{light cone gauge}\right)$

Here: filled the gaps

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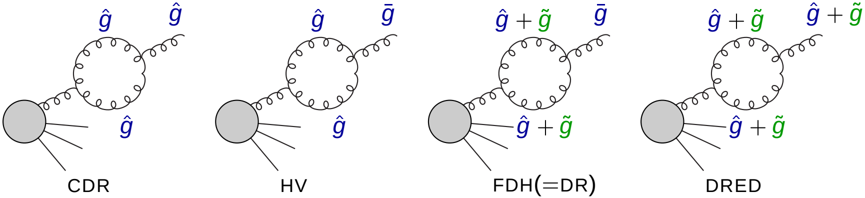
Scheme definitions and differences

[Signer, DS ’08]



CDR: $P_{\hat g \to \hat g \hat g}$

HV: $\dots$

FDH: $\dots$

$+P_{\hat g \to \tilde g \tilde g}$

[Kunszt, Signer, Trocsanyi ’94]

[Catani, Seymour, Trocsanyi ’97]

DRED: $\dots$

$\dots$

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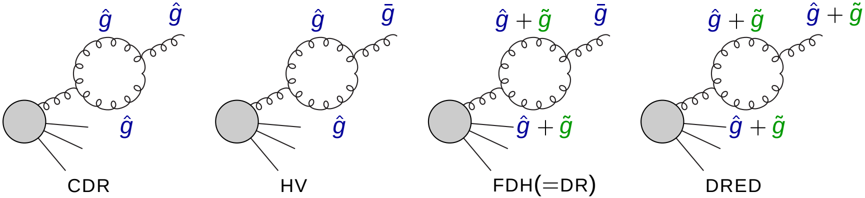

$\gamma(\hat{g}) = \int P_{\hat{g}\to\hat{g}\hat{g}}$ ... ...

$+\int P_{\hat{g}\to\tilde{g}\tilde{g}}$ ...

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$$\gamma(\tilde{g}) = \int P_{\tilde{g}\to\hat{g}\tilde{g}} + \int P_{\tilde{g}\to\tilde{g}\hat{g}}$$

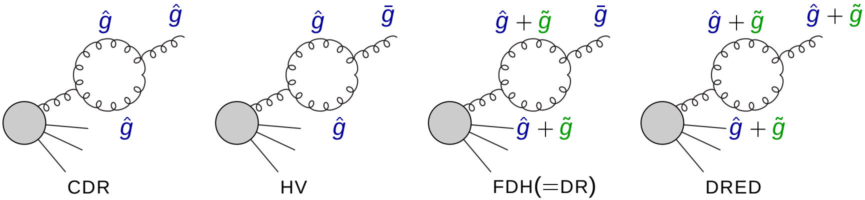
[Signer, DS '08]

- RS dependence:

FDH: additional state $\tilde{g}$: *value* of $\gamma(\bar{g})$ changes
DRED: additional splitting of $\tilde{g}$: additional $\gamma(\tilde{g})$
DRED: split $g = \hat{g} + \tilde{g}$ required to understand factorization [Signer, DS '08]
("problem" of [Beenakker, Kuijf, v Neerven, Smith '88][v Neerven, Smith '04])

IR structure at one-loop

$$F_i^{1L}|_{IR} = \ln Z = 2\left(\frac{\alpha_s}{4\pi}\right)\left(-\frac{\gamma_1^{cusp} C_i}{4\epsilon^2} + \frac{\gamma_1^i}{2\epsilon}\right)$$



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$$\gamma(\tilde{g}) = \int P_{\tilde{g}\to\hat{g}\tilde{g}} + \int P_{\tilde{g}\to\tilde{g}\hat{g}}$$

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Renormalization in FDH and DRED

- Different couplings α_s, α_e, $\alpha_{4\epsilon}$ (can be set equal in the end)

$$\delta\alpha_s \neq \delta\alpha_e, \beta^s \neq \beta^e, \ldots$$

- Required, otherwise divergent/non-unitary results

[Jack, Jones, Roberts '94][Harlander, Kant, Mihaila, Steinhauser '06][Kilgore '11]

FDH or DRED

$\Updownarrow$

HV or CDR of theory with new, ϵ-scalar of multiplicity $N_\epsilon = 2\epsilon$ with independent couplings and γ's

Aim: IR structure at 2-loop, quark/gluon form factors (*Hgg*) in FDH

IR structure (form factors) [Becher, Neubert][Gardi, Magnea]

Result:

$$\ln \mathbf{Z} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\Gamma_1'}{4\epsilon^2} + \frac{\Gamma_1}{2\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\beta_{20}\Gamma_1'}{16\epsilon^3} + \frac{\Gamma_2' - 4\beta_{20}\Gamma_1}{16\epsilon^2} + \frac{\Gamma_2}{4\epsilon}\right) + \ldots$$

Derivation:

$$\frac{d}{d\ln\mu}\mathbf{Z} = -\Gamma\mathbf{Z}$$

$$\Gamma = -2C_i\,\gamma^{\text{cusp}}\big(\alpha_s(\mu)\big)\ln\mu^2 + 2\gamma^i\big(\alpha_s(\mu)\big)$$

$$= \sum_n \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \left[\Gamma_n'\ln\mu^2 + \Gamma_n\right]$$

Integrate over $\mu \rightsquigarrow$ IR structure

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Integrate over $\mu \rightsquigarrow$ IR structure

Changes in FDH

- Everything depends on $\alpha_s(\mu)$ and $\alpha_e(\mu)$
- Additional contributions to Γ_n from ϵ-scalars of order $N_\epsilon = 2\epsilon$

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Integrate over $\mu \rightsquigarrow$ IR structure

IR structure (form factors)

Changes in result:

$$\ln \mathbf{Z}^{\text{FDH}}_{\text{2-loop}} = \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_{20}\Gamma'_{10}}{16\epsilon^3} + \frac{\Gamma'_{20} - 4\beta_{20}\Gamma_{10}}{16\epsilon^2} + \frac{\Gamma_{20}}{4\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\left(-\frac{3\beta^e_{11}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{11} - 4\beta^e_{11}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{11}}{4\epsilon}\right) + \left(\frac{\alpha_e}{4\pi}\right)^2 \left(-\frac{3\beta^e_{02}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{02} - 4\beta^e_{02}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{02}}{4\epsilon}\right) + \mathcal{O}(\alpha^3).$$

and $\Gamma_{10} = \Gamma_1 + \mathcal{O}(N_\epsilon)$ etc

- Leads to translation rules between CDR, HV, FDH
- Logic and result fully compatible with [Kilgore '12]

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$$+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\left(-\frac{3\beta^e_{11}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{11} - 4\beta^e_{11}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{11}}{4\epsilon}\right)$$
$$+ \left(\frac{\alpha_e}{4\pi}\right)^2 \left(-\frac{3\beta^e_{02}\Gamma'_{01}}{16\epsilon^3} + \frac{\Gamma'_{02} - 4\beta^e_{02}\Gamma_{01}}{16\epsilon^2} + \frac{\Gamma_{02}}{4\epsilon}\right) + \mathcal{O}(\alpha^3).$$

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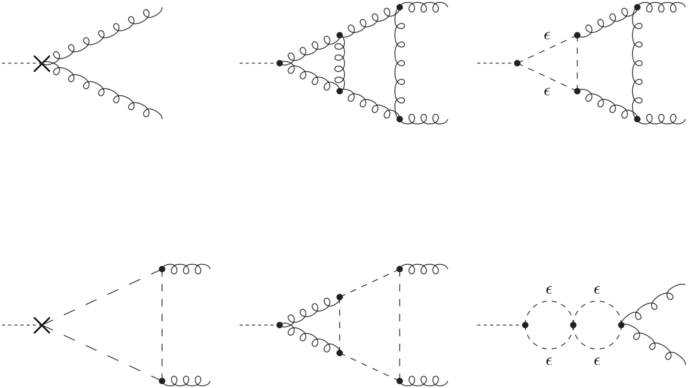
- Logic and result fully compatible with [Kilgore '12]
- Differences to [Kilgore '12]: slightly different γ^i, other sample processes

Hgg in FDH: Computation and renormalization



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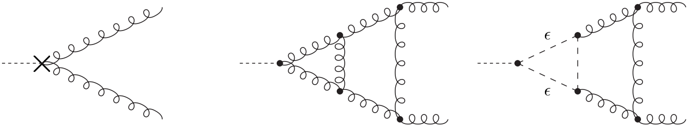
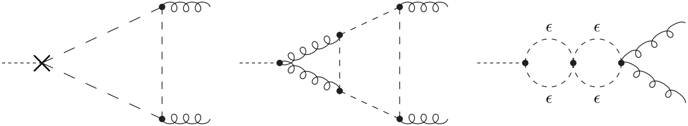


$$\delta\lambda^{(2)} = \lambda\left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{(\beta^s_{20})^2}{\epsilon^2} - \frac{\beta^s_{30}}{\epsilon}\right) + \left(\lambda + \lambda_\epsilon\right)\left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\left(- \frac{\beta^s_{21}}{2\epsilon}\right)$$

[Spiridonov '84] applicable, but mixing occurs



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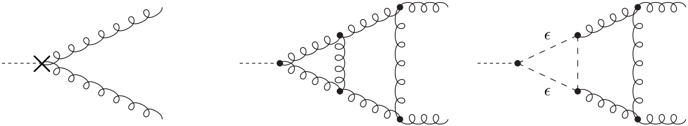

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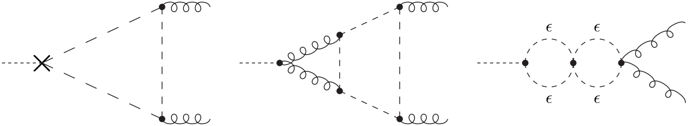

$$\delta\lambda^{(1)}_\epsilon = \lambda_\epsilon\left[\left(\frac{\alpha_s}{4\pi}\right)\left(- \frac{3C_A}{\epsilon}\right) + \left(\frac{\alpha_e}{4\pi}\right)\frac{N_F}{\epsilon} + \left(\frac{\alpha_{4\epsilon}}{4\pi}\right)C_A\left(\frac{-1 + N_\epsilon}{\epsilon}\right)\right]$$

explicit calculation

Results

$$\bar{G}^{(2)}(\alpha_s,\alpha_e) = G^{(2)}(\alpha_s) \quad + \left(\frac{\alpha_s}{4\pi}\right)^2 N_\epsilon \left\{ C_A^2 \left[-\frac{1}{4\epsilon^3} + \frac{-\frac{7}{18}+\frac{N_\epsilon}{72}}{\epsilon^2} + \frac{\frac{49}{27}-\frac{\pi^2}{72}}{\epsilon} \right] + C_A N_F \frac{1}{9\epsilon^2} \right\}$$

$$+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon} \right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

$$G^{(2)}(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)^2 \quad \left\{ C_A^2 \left[\frac{11}{2\epsilon^3} + \frac{3+\frac{\pi^2}{6}}{\epsilon^2} + \frac{-\frac{346}{27}+\frac{11\pi^2}{36}+\zeta(3)}{\epsilon} + \frac{5105}{162} + \frac{67\pi^2}{36} - \frac{143\zeta(3)}{9} \right] \right.$$

$$+ C_A N_F \left[-\frac{1}{\epsilon^3} - \frac{17}{9\epsilon^2} + \frac{\frac{64}{27}-\frac{\pi^2}{18}}{\epsilon} - \frac{916}{81} - \frac{5\pi^2}{18} - \frac{46\zeta(3)}{9} \right]$$

$$\left. + C_F N_F \left[\frac{1}{\epsilon} - \frac{67}{6} + 8\zeta(3) \right] + N_F^2 \frac{2}{9\epsilon^2} \right\} + \mathcal{O}\left(\epsilon^1\right)$$

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$$+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) N_\epsilon \left\{ -\frac{C_F N_F}{2\epsilon} \right\} + \mathcal{O}\left(N_\epsilon \epsilon^0\right).$$

- Difference CDR-FDH of order $\frac{N_\epsilon}{\epsilon}$ in agreement with prediction of IR structure if, e.g. $\bar{\gamma}_{20}^{g} = \gamma_{20}^{g} + N_\epsilon\left(\frac{98}{27} - \frac{\pi^2}{36}\right)C_A^2$
- $\Rightarrow$ translation rules

Summary and extensions

- *Hgg* computed and renormalized in FDH (independent couplings!)
- IR structure in FDH understood, leads to IR translation rules
- rules apply if the same renormalization scheme is used ($\overline{MS}$)
- Also possible: IR structure prediction in FDH with $\overline{DR}$ renormalization

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Conclusions

Divergence structure of $\sqrt{\hat{Z}}v$ in gauge theories

- from gauge fixing, $\propto$ squared gauge couplings and $\propto \xi$
- $\rightarrow \delta\hat{Z}_{H_u} - \delta\hat{Z}_{H_d}$ =finite in MSSM at 1-loop accidentally
- 2-loop β functions, γ^{SUSY} complete

IR structure in FDH understood, transition rules at higher orders

$$\left[\mathsf{G}^{(2)} - \ln \mathbf{Z}^{(2)}\right]^{\mathsf{FDH}} = \left[\mathsf{G}^{(2)} - \ln \mathbf{Z}^{(2)}\right]^{\mathsf{CDR}} + \mathcal{O}(N_\epsilon \epsilon^0)$$

- all ingredients known: β, γ's, $H \rightarrow gg$ renormalization at 2-loop
- full agreement with [Kilgore '12]
- Outlook: further checks, γ's for ϵ-scalars soon