Determination of the top quark mass

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Top quark mass

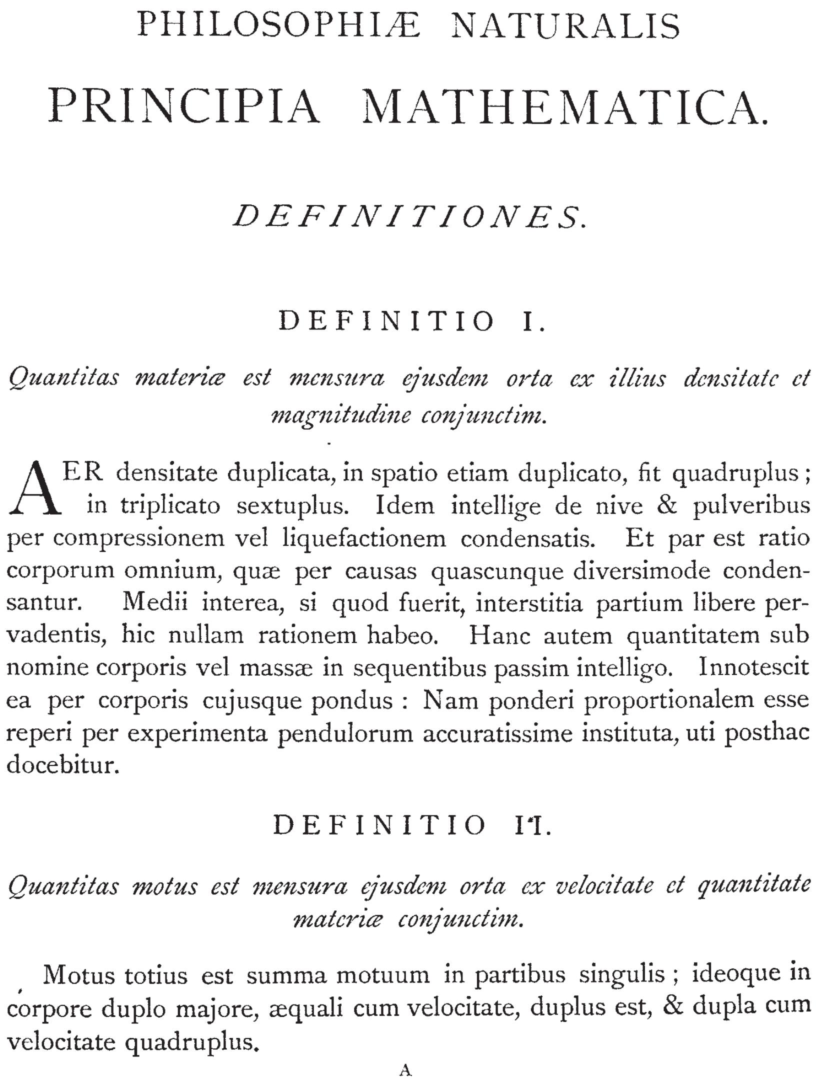
What is the value of the top quark mass ?

$$\mathsf{m_t} = ?$$

Mass definitions – a classical concept

Classical mechanics

- Mass is defined as product of density and volume of matter
- *The quantity of matter is that which arises jointly from its density and magnitude. A body twice as dense in double the space is quadruple in quantity. This quantity I designate by the name of body or of mass.* Newton



PHILOSOPHIÆ NATURALIS

PRINCIPIA MATHEMATICA.

DEFINITIONES.

DEFINITIO I.

Quantitas materiæ est mensura ejusdem orta ex illius densitate et magnitudine conjunctim.

AER densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine corporis vel massæ in sequentibus passim intelligo. Innotescit ea per corporis cujusque pondus: Nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.

DEFINITIO II.

Quantitas motus est mensura ejusdem orta ex velocitate et quantitate materiæ conjunctim.

Motus totius est summa motuum in partibus singulis; ideoque in corpore duplo majore, æquali cum velocitate, duplus est, & dupla cum velocitate quadruplus.

A

Atomic theory

- Mass is conserved Lavoisier
- Mass of body is sum of mass of its constituents
  $M(X) = N_A m_a(X)$ Avogadro

Special relativity

- Equivalence principle
  $E = mc^2$ Einstein

Quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

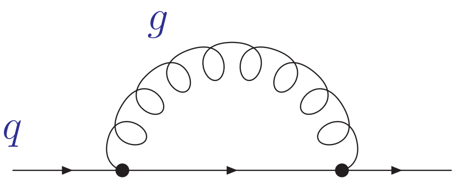
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Ambiguity Δm_q in definition of pole mass up to corrections $\mathcal{O}(\Lambda_{QCD})$
  Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97
  - lattice QCD bound: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda '11

Short distance mass

- Short distance masses ($\overline{MS}$, 1S Hoang, Teubner '99, PS Beneke '98, . . .) probe at scale of hard interaction: $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
- Conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ in perturbation theory
  Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99

Top quark mass

What is the value of the top quark mass ?

$$\mathsf{m_t} = \mathsf{?}$$

Some Answers

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

$$m_t = 173.76 \pm 0.76\,\mathrm{GeV}$$

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All in all I believe that it is justified to assume that MC mass parameter is interpreted as mpole, within the ambiguity intrinsic in the definition of mpole, thus at the level of $\sim 250 - 500$ MeV. M. Mangano @ Top2103

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That is, we can state as the final result for the likely relation between the top quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m_{\mathrm{pole}} = m_{\mathrm{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \ldots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

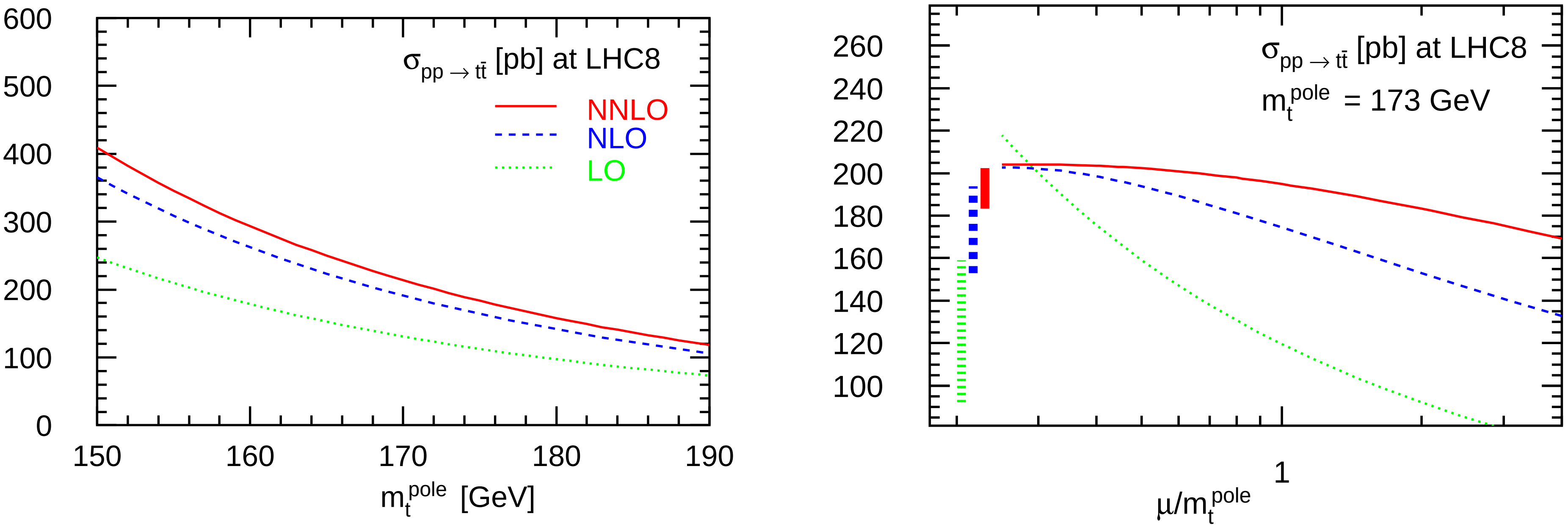
Rates and shapes

- Rates and shapes of distributions offer possibility for top mass determination with well-defined renormalization scheme
- Requirements:
  - theory predictions at least to NLO in QCD
  - sufficient sensitivity to m_t (kinematics)
- Observables (examples):
  - inclusive cross section
  - leptonic decay: m_{lb} distribution
  - jet rates

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13

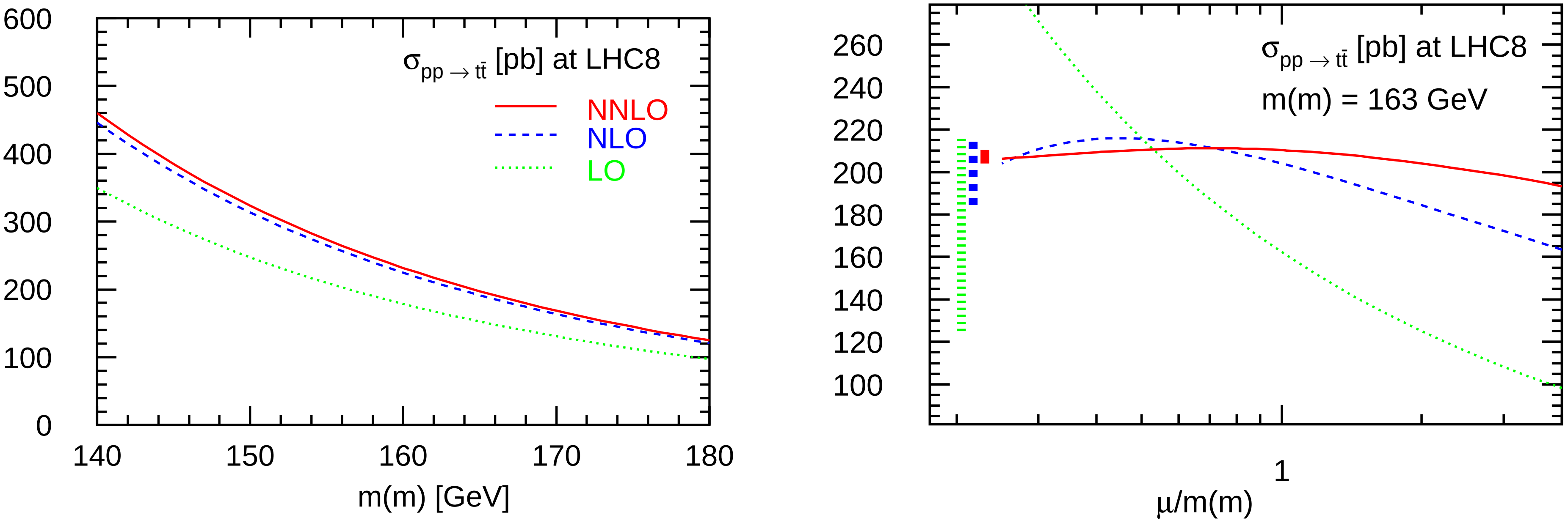


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO → NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with running mass

Comparison pole mass vs. $\overline{MS}$ mass

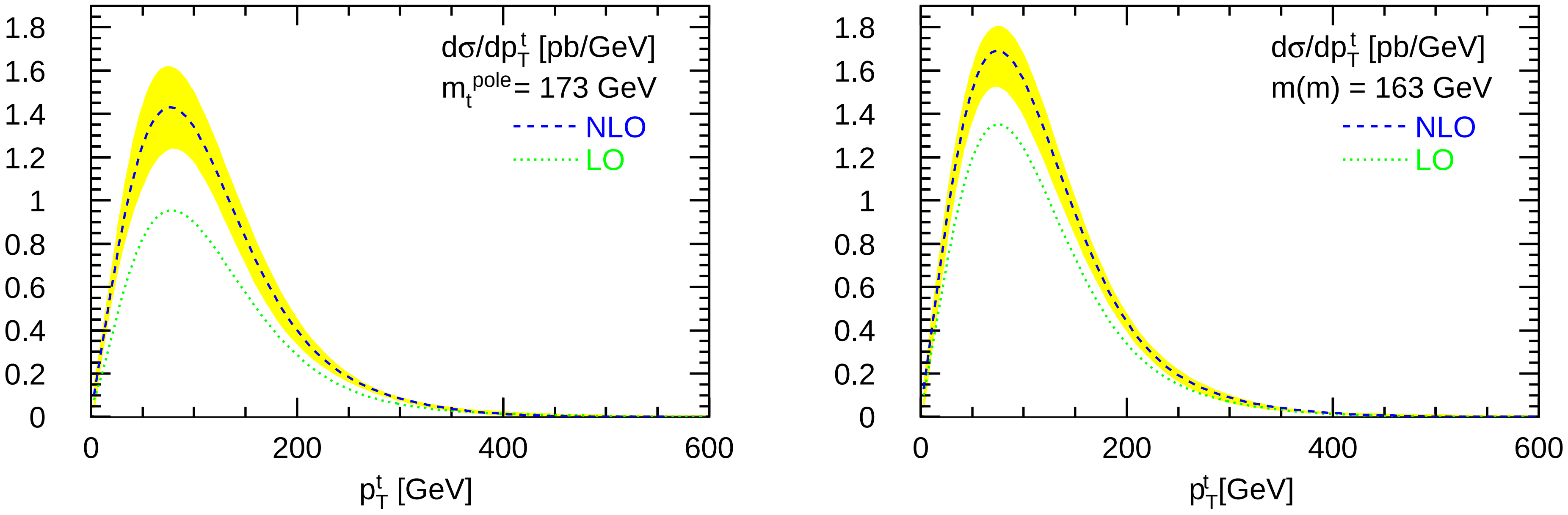
Dowling, S.M. '13



- good apparent convergence of perturbative expansion
- small theoretical uncertainity form scale variation

Differential cross sections

NLO in QCD



- Running mass for differential distributions shows same features, e.g. p_T^t-distribution Dowling, S.M. '13

Differential cross sections

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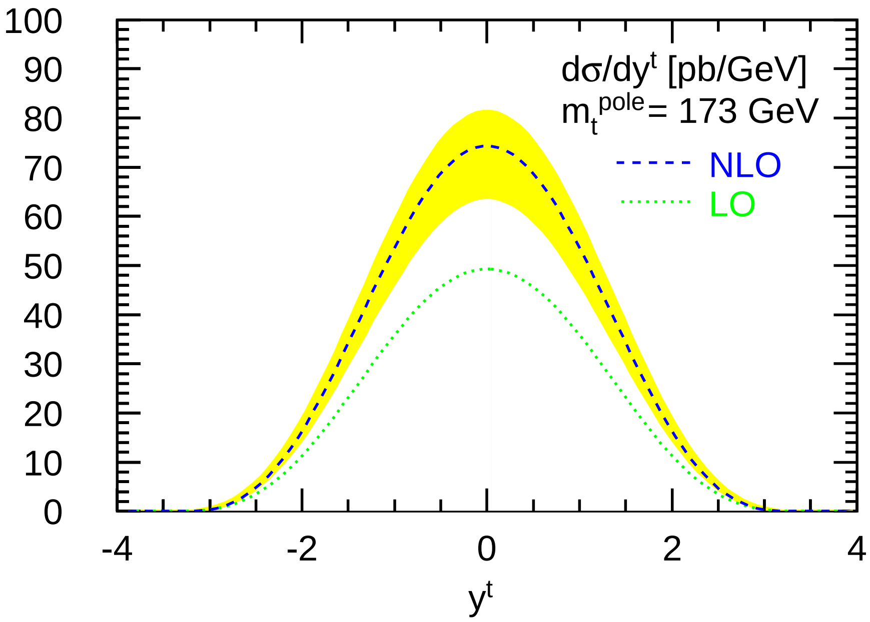
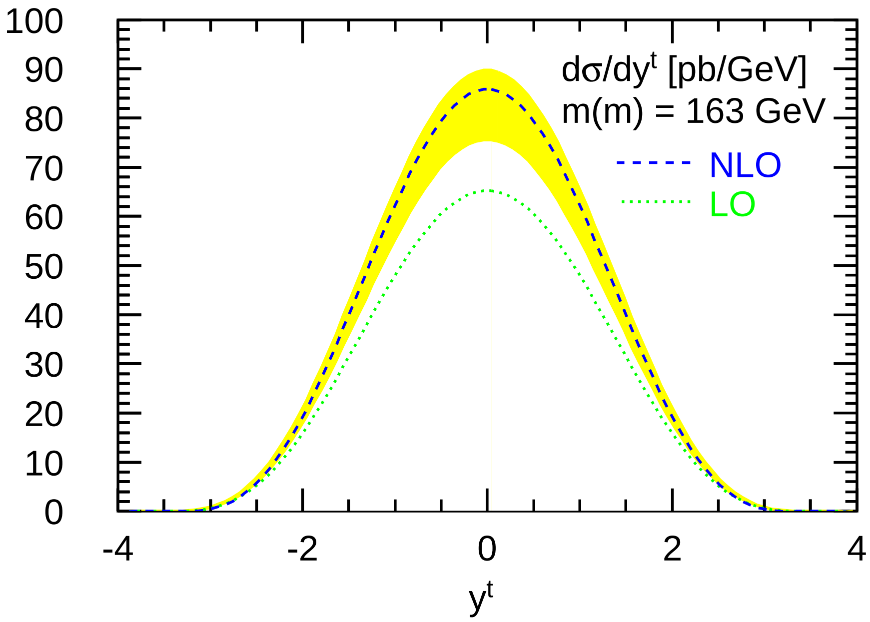
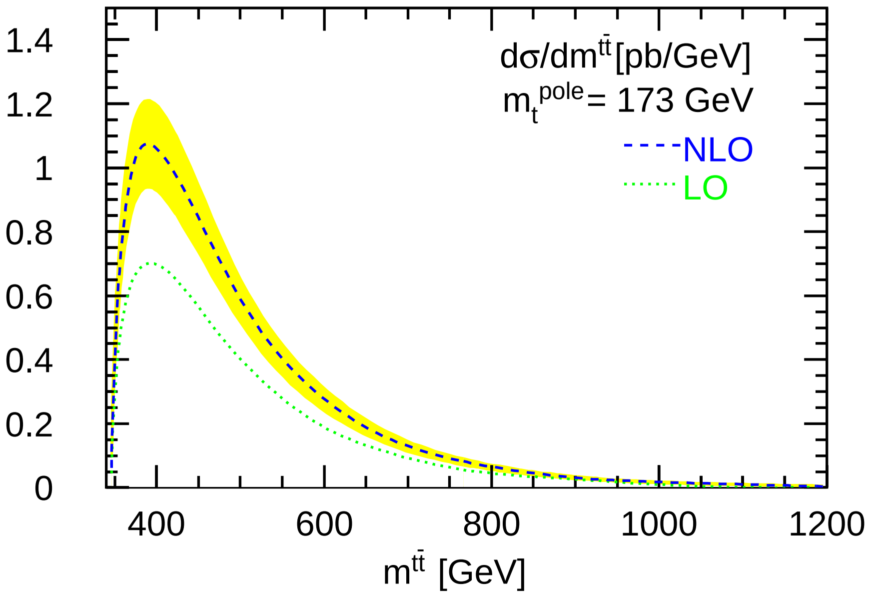


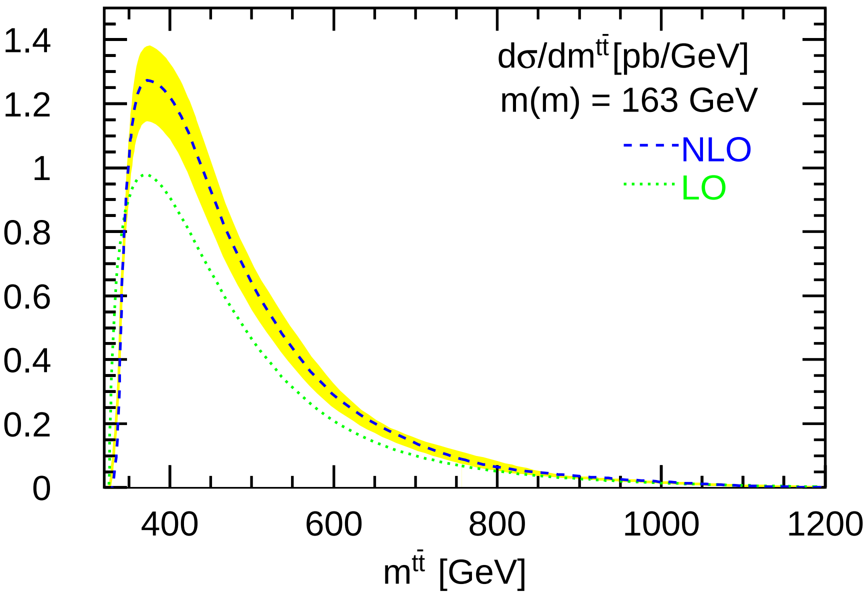


- Running mass for differential distributions shows same features, e.g. y^t-distribution Dowling, S.M. ‘13

Differential cross sections

NLO in QCD





- Running mass for differential distributions shows same features, e.g. $m_{t\bar{t}}$-distribution Dowling, S.M. ‘13

Top mass from total cross section

- Intrinsic limitation of sensitivity in total cross section

$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

- QCD factorization for cross section

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

  - joint dependence on non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, masses m_X

Correlations are essential

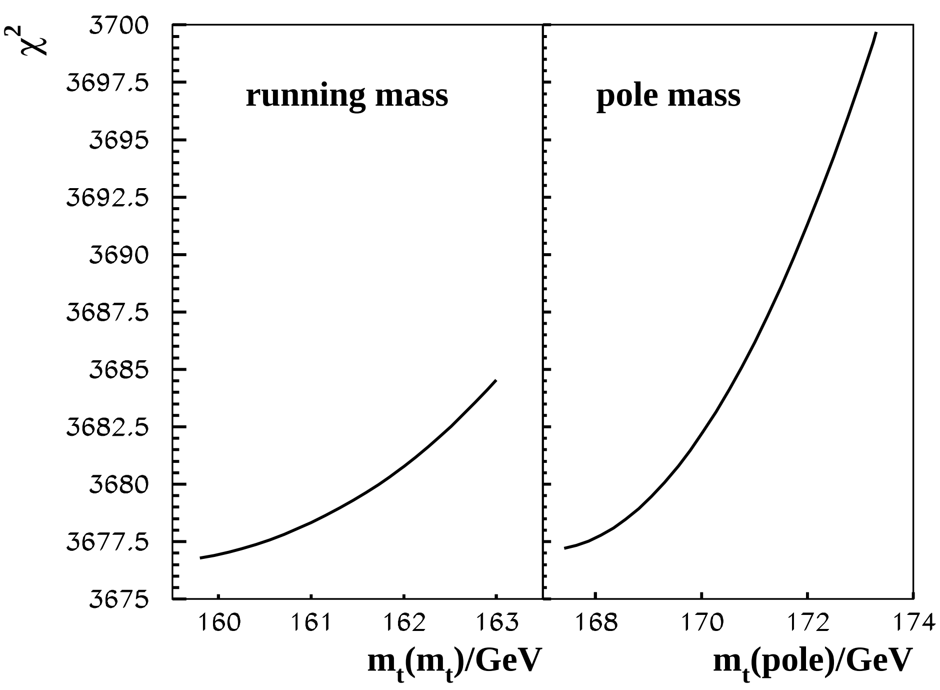
- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$ and gluon PDF

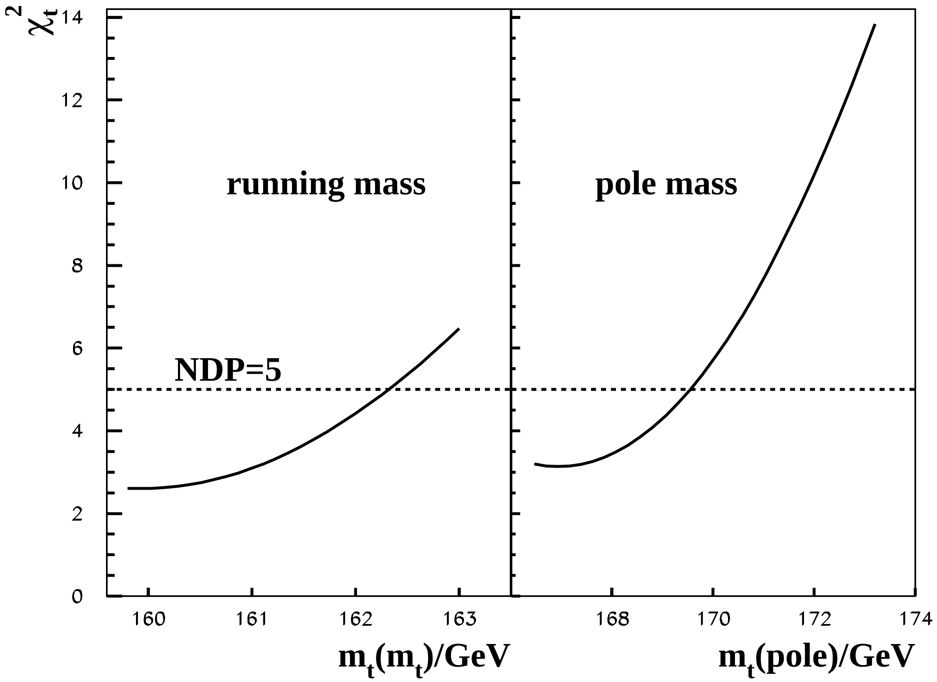
$$\sigma_{t\bar{t}} \sim \alpha_s^2\, m_t^2\, g(x) \otimes g(x)$$

  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo '13
  - fit with PDF re-weighting and fixed values of m_t insufficient
    Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

Top cross section data in ABM12 fit

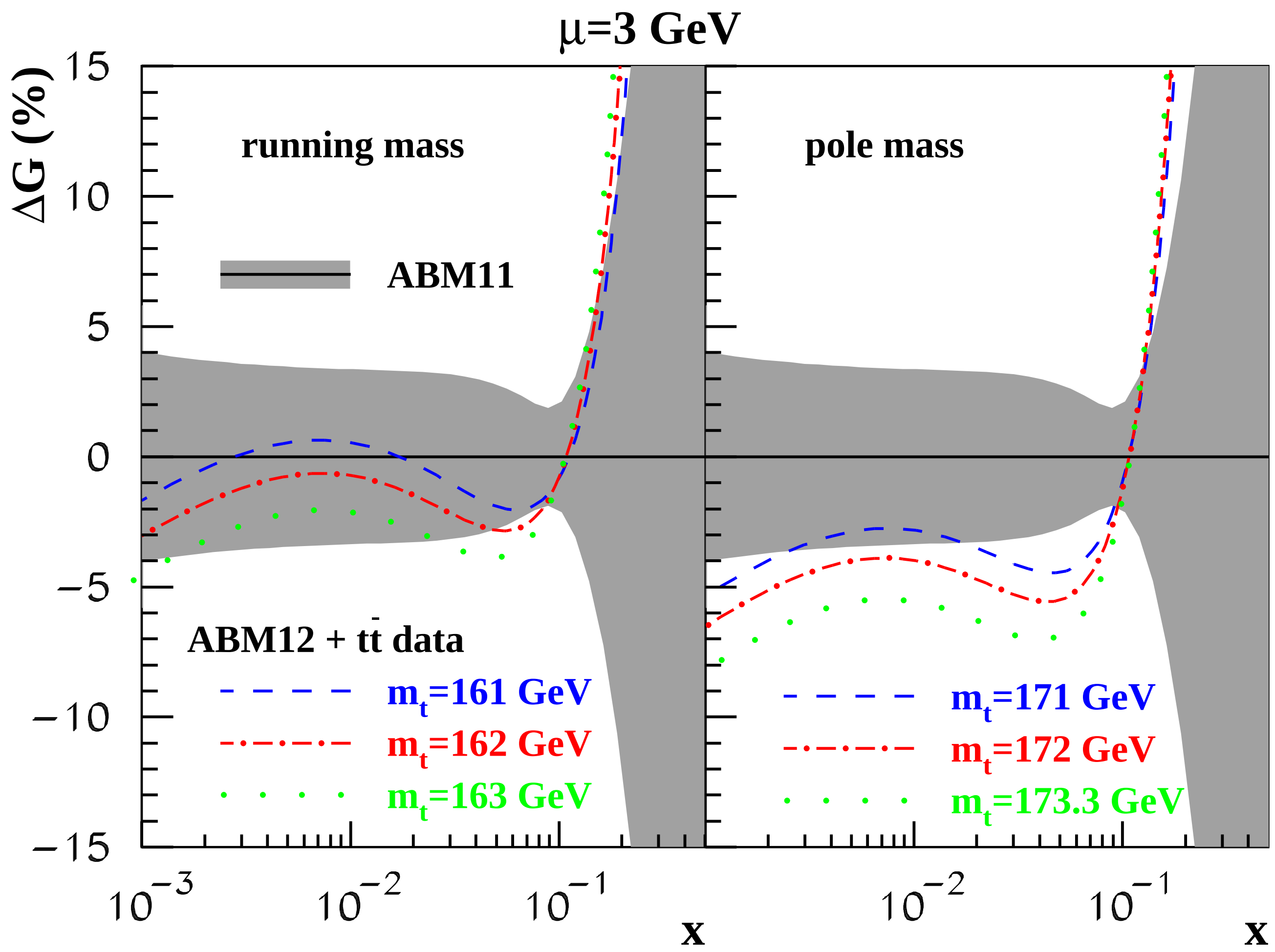
- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. '13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)





Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - correlation of gluon PDF with value of m_t (illustration of bias in analysis Czakon, Mangano, Mitov, Rojo ‘13)



Top mass from leptonic decay

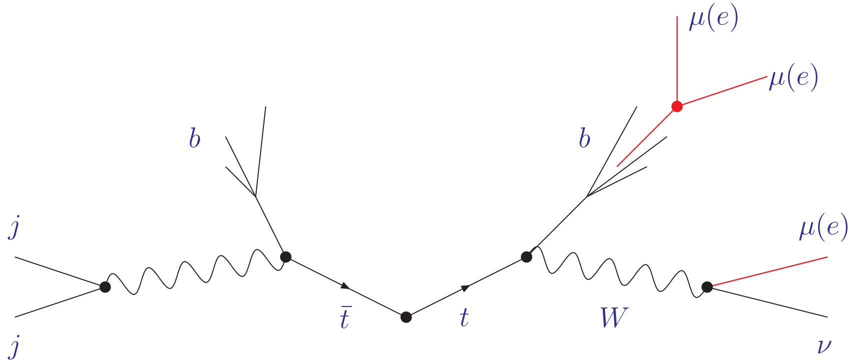
- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

  - identification of μ-pair in J/ψ decay; leptonic or hadronic decay of W

    Kharchilava '00

    Chierici, Dierlamm '06



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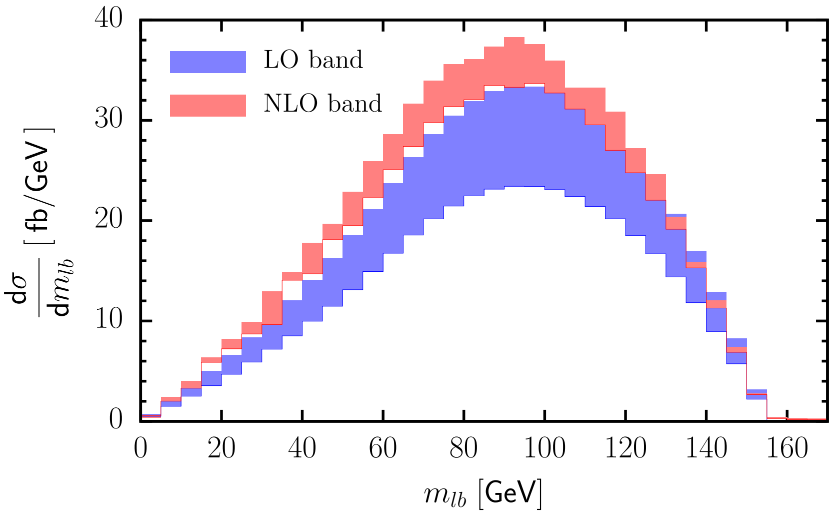
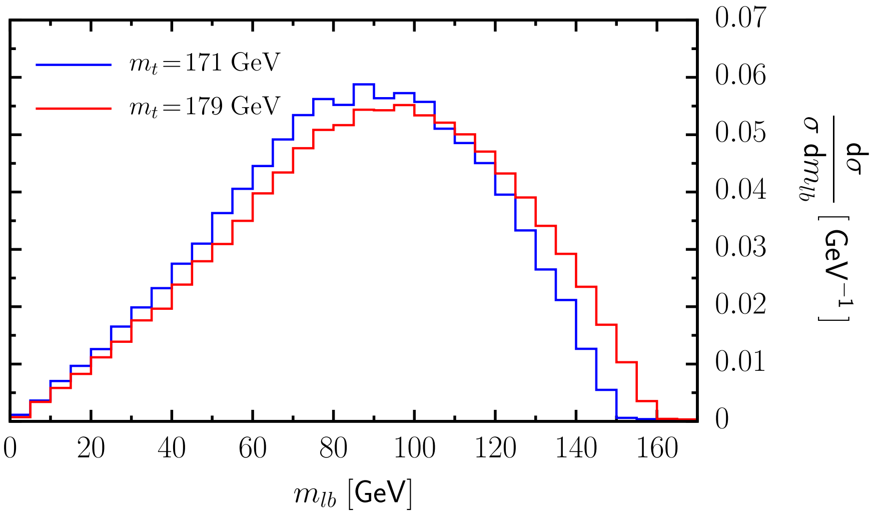
$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

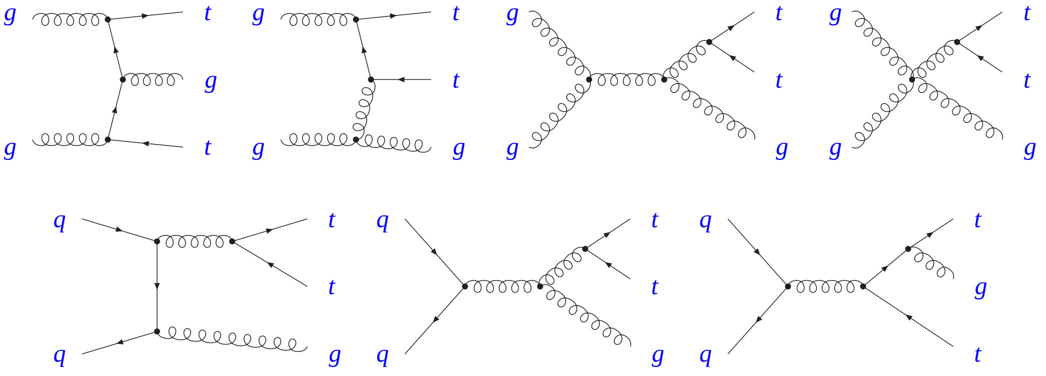
- Study of m_{lb} distribution at NLO in QCD Biswas, Melnikov, Schulze ‘10
  - NLO QCD corrections to production **and** decay very important for value of m_t (effects of order $\Delta m_t = \mathcal{O}$(few) GeV
- Invariant mass distribution of lepton and b-jet (LHC14)
  - scale dependence at LO and NLO (left)
  - normalized m_{lb} distributions, $m_t = 171\ \mathrm{GeV}$ and $179\ \mathrm{GeV}$ (right)

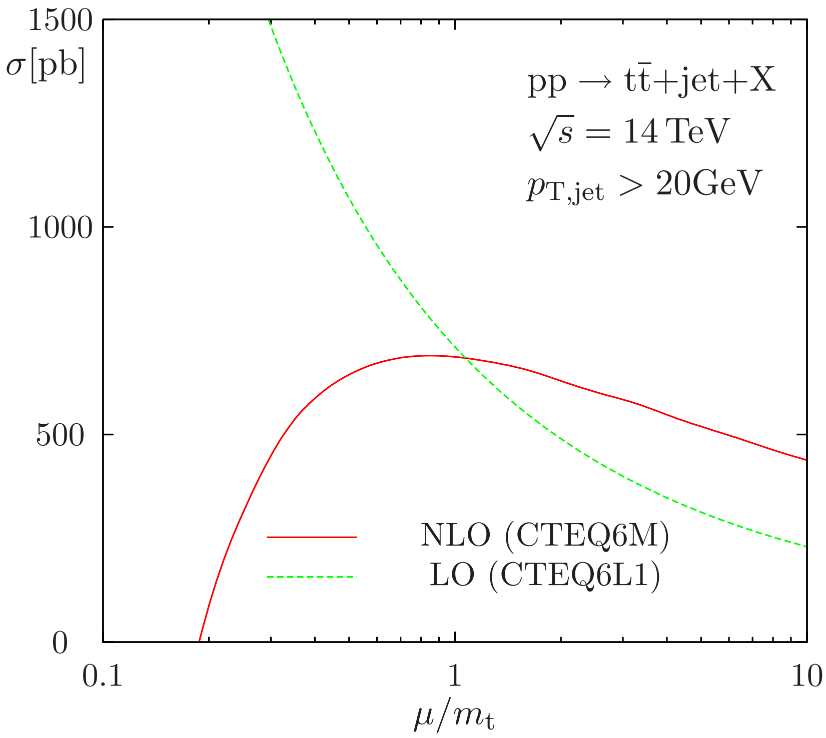




Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\mathrm{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\mathrm{jet}}}$

Mass measurement with $t\bar{t} + \mathrm{jet}$*-samples*

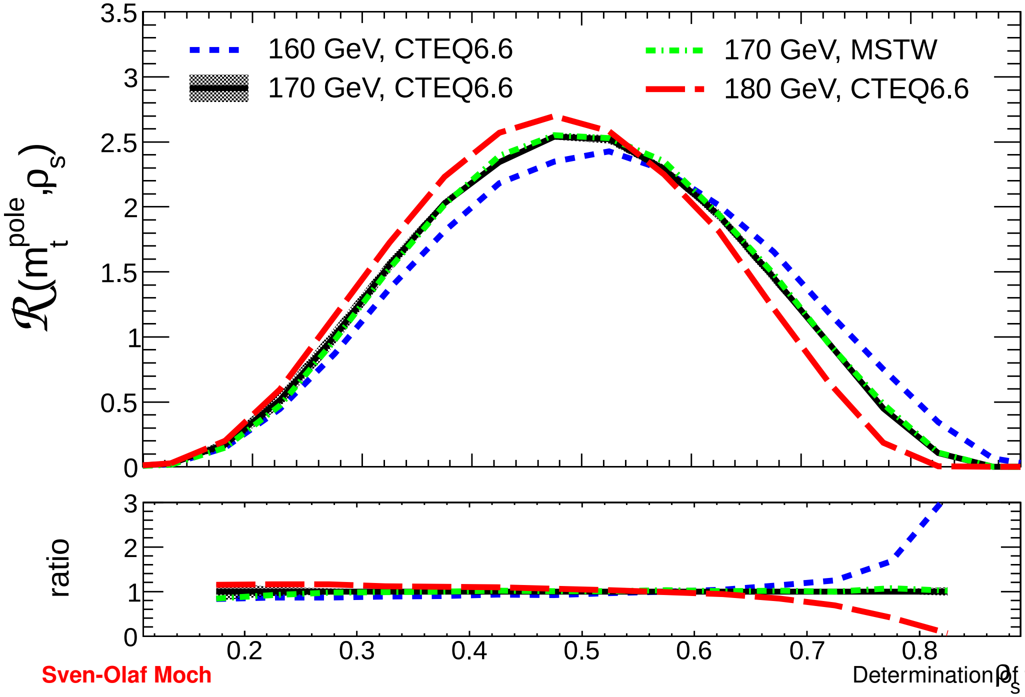
- Mass measurement with new observable

  Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos ‘13

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t} + 1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$
- Normalized-differential $t\bar{t} + \mathrm{jet}$ cross section

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

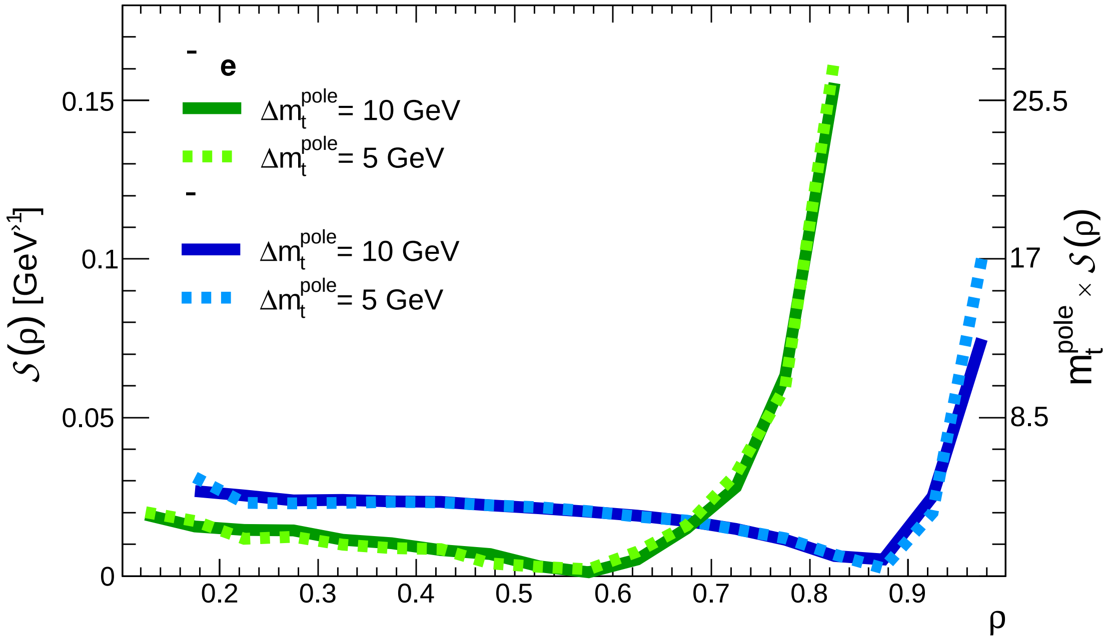
  - significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Sensitivity to top-quark mass very good

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$

  - increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared to $t\bar{t}$



Upshot

- Precision determination of well-defined top-quark mass m_t possible
  - alternative to inclusive cross sections

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
    - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

- scale of new physics smaller than Λ to ensure vacuum stability
- lower bound on m_H for fixed Λ

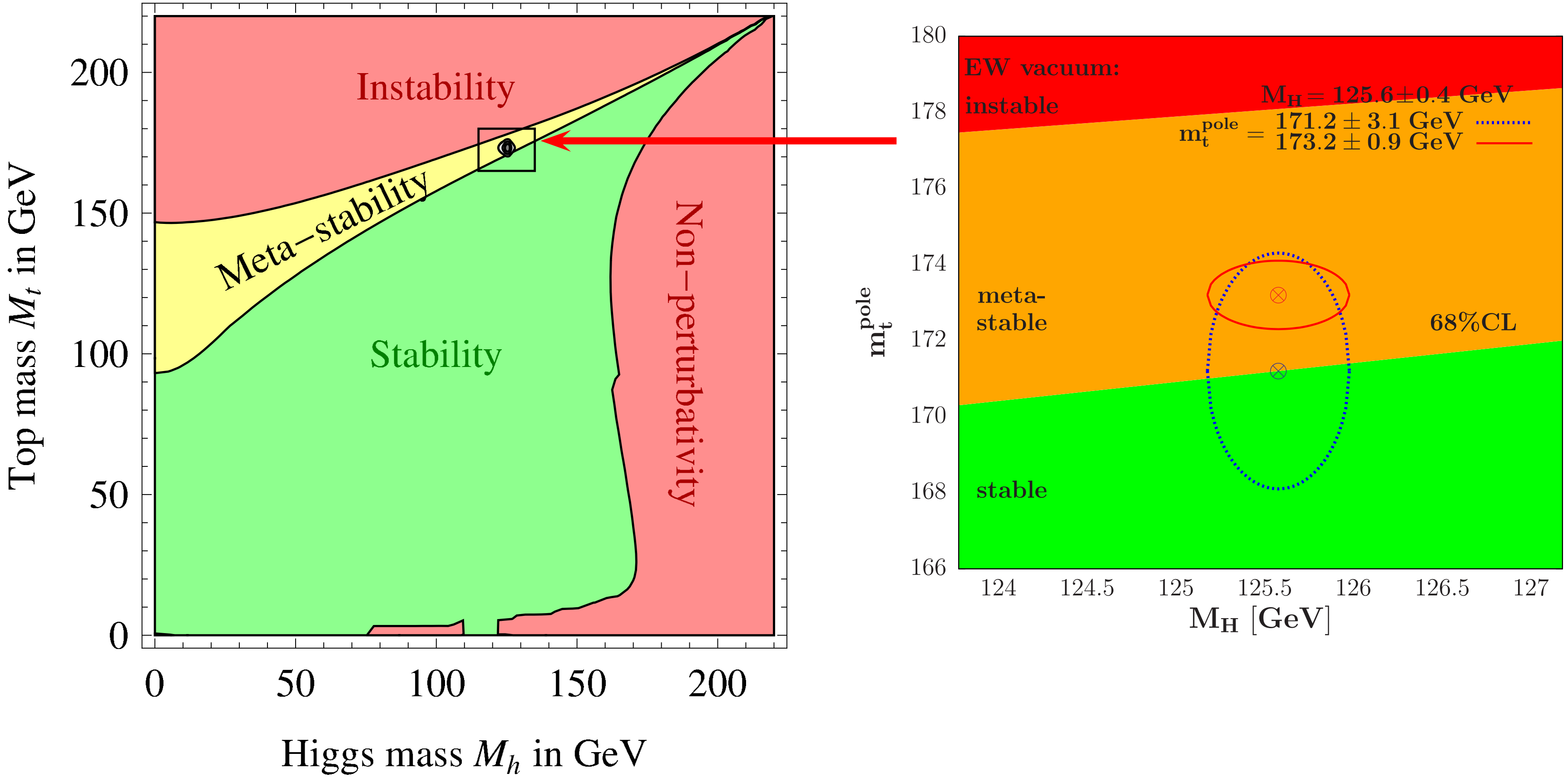
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\mathrm{pole}} - 173.2\ \mathrm{GeV}}{0.9\ \mathrm{GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0\ \ \mathrm{GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\mathrm{MS}}}(m_t) \; = \; 162.3 \pm 2.3 \; \pm 0.7\ \mathrm{GeV}$ implies in pole mass scheme $m_t^{\mathrm{pole}} \; = \; 171.2 \pm 2.4 \pm 0.7\ \mathrm{GeV}$
  - good consistency of mass value between different PDF sets

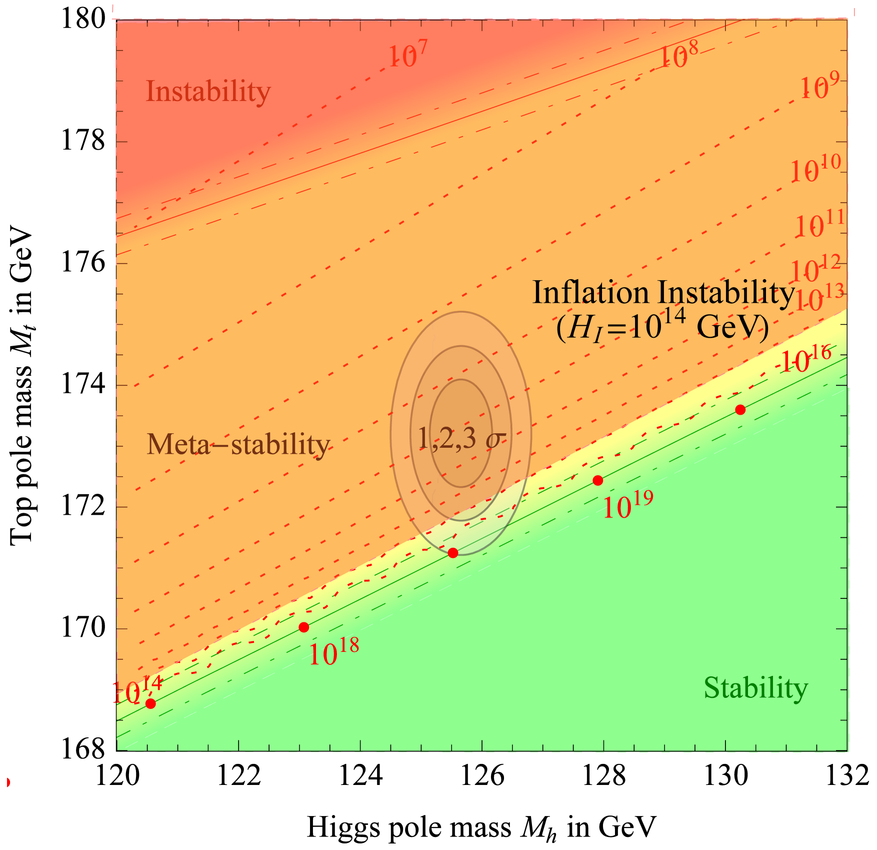
Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 125.2 \pm 6.2$ GeV
  - “fate of universe” still undecided

Fate of the universe (after BICEP2)



Espinosa '14

- BICEP2 suggests large value of the Hubble rate during inflation $H_I \sim 10^{14}$ GeV
- Large H_I cause large amplitude fluctuations $\sim H_I$ of Higgs field (if light during inflation)
- Large amplitude fluctuations $\sim H_I$ trigger vacuum decay if H_I is larger than the instability scale

Summary

Top quark mass

- Running mass ($\overline{\mathrm{MS}}$ scheme) at NNLO in QCD

$$m_t(m_t) = 163.3 \pm 2.3\mathrm{GeV}$$

- On-shell scheme (pole mass) at NNLO in QCD

$$m_t = 171.2 \pm 2.4\mathrm{GeV}$$

Summary

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition
- Correlations in data analysis are important, e.g. with α_s and PDFs

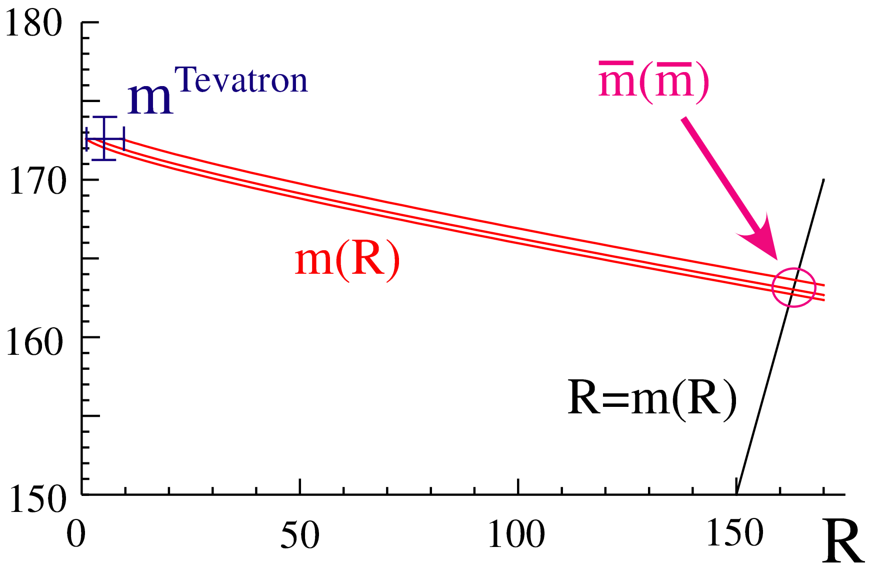
LHC measurements

- Inclusive and differential observables
- Very precise data and well-defined renormalization scheme definition
- $\overline{\mathrm{MS}}$ scheme for mass exhibits better convergence

Future challenge

- Joint effort theory and experiment

Top-quark Monte Carlo mass

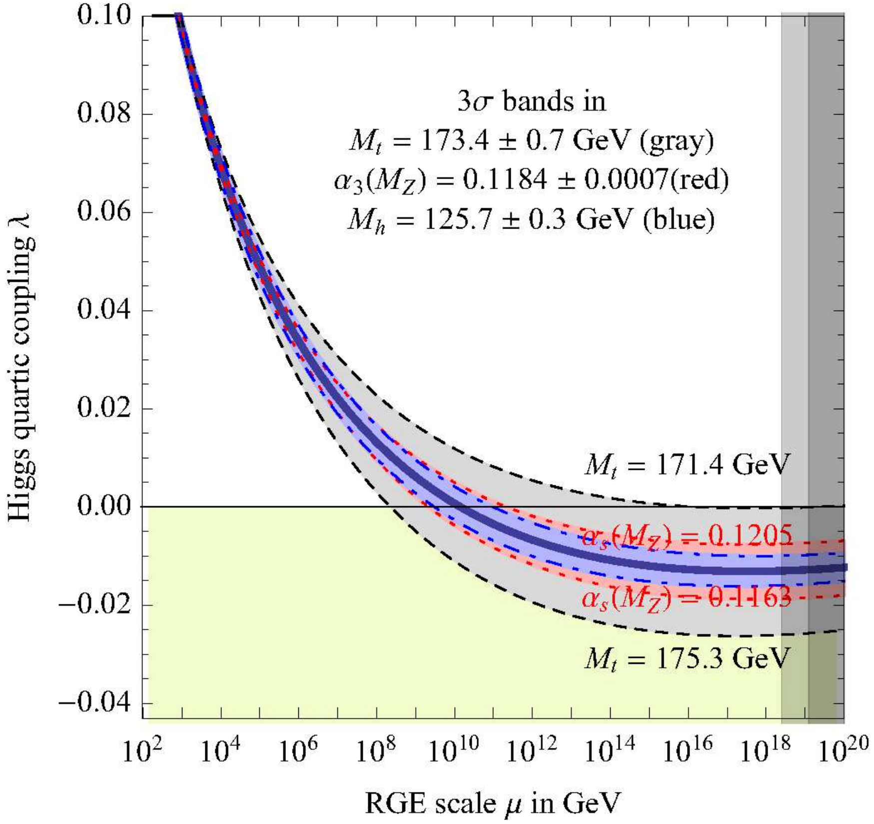


Hoang, Stewart '08

- Running of Monte Carlo mass from low scale $\mathcal{O}(1)\ \text{GeV}$ to $\mu = m_t$
- Argument based on analogy to peak position of invariant jet-mass distribution M^{peak} with decaying top quark for short distance mass m_t

$$M^{\text{peak}} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\Lambda_{\text{QCD}}}{m_t}$$

Higgs self-coupling



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ‘13

- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s
  - top-quark mass least precise parameter
- Vacuum stability bound at M_P in terms of m_t

$$m_t \leq (171.36 \pm 0.15 \pm 0.25_{\alpha_3} \pm 0.17_{m_h})\,\text{GeV} = (171.36 \pm 0.46)\,\text{GeV}$$