Non-leptonic B-decays at two loops in QCD

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Outline

- Introduction
- Theoretical framework
- Two-loop penguin amplitudes
  - Computation: Master integrals in a canonical basis
  - Preliminary results
- The decay $B \to D\,\pi$ at two loops
- Conclusion

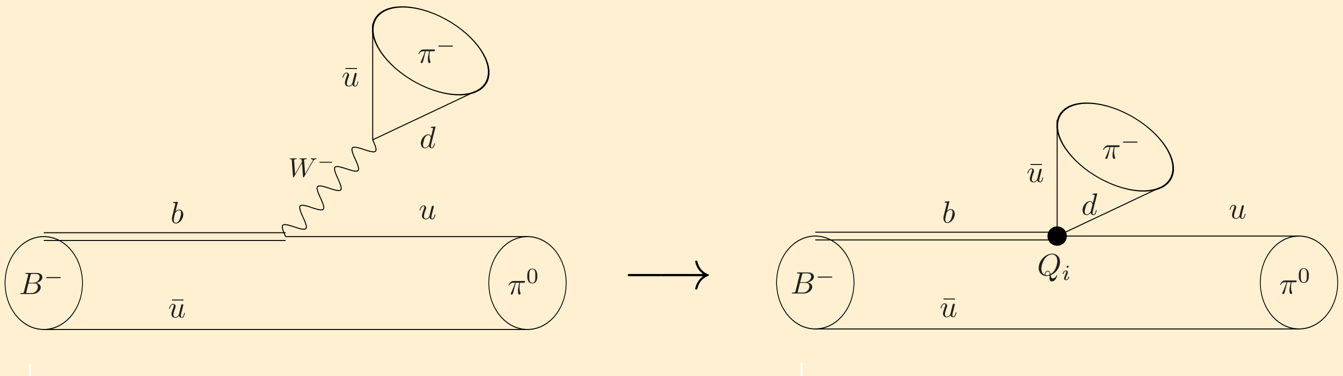
Introduction

- Non-leptonic B decays offer a rich and interesting phenomenology
  - Large data sets from B-factories, Tevatron, LHC, future Super-flavour factory
  - $\mathcal{O}(100)$ final states
  - Numerous observables:
    - branching ratios
    - CP asymmetries
    - polarisations
- Test of CKM mechanism (CP violation)
- Indirect search for New Physics
  - Not as sensitive as rare or radiative B decays, but large data sets

Introduction

- Theoretical description complicated by purely hadronic initial and final state
- QCD effects from many different scales
- Possible theory approaches
  - (QCD improved) Factorisation
    * Disentangles long and short distances systematically
    * Problems with factorisation of power suppressed and annihilation contributions. Endpoint divergences
  - Flavour symmetries: Isospin, U-Spin ($d \leftrightarrow s$), V-Spin ($u \leftrightarrow s$), Flavour SU(3)
    * Only few a priori assumptions about scales needed
    * Implementation of symmetry breaking difficult

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark

- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

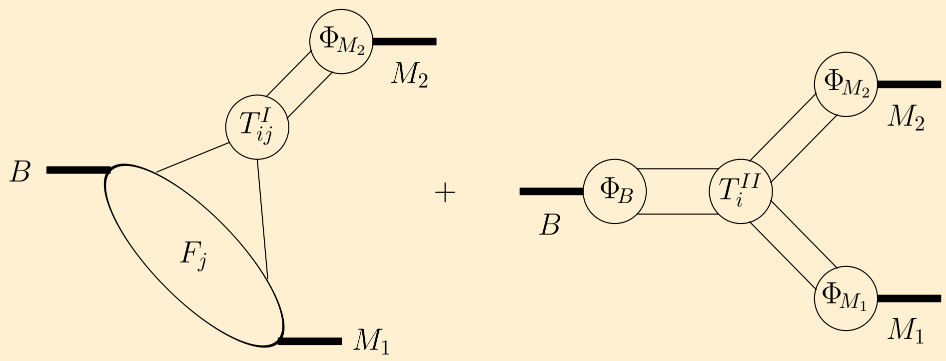
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2}\, m_b\, \bar{d}_L\, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

QCD factorisation



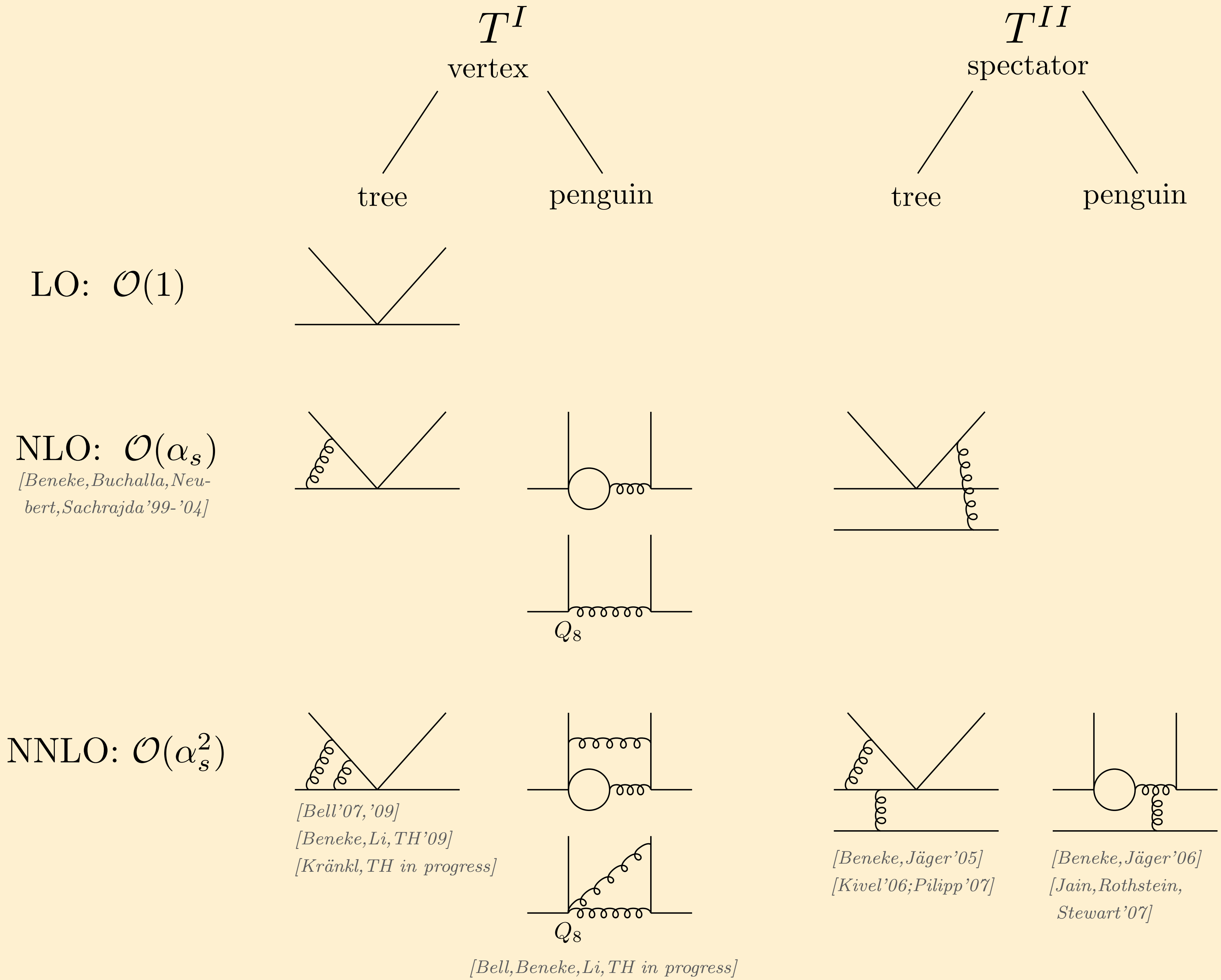
- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \;\; T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \;\; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable
- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes
  } Universal. From Sum Rules, Lattice
- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

Anatomy of QCD factorisation



Classification of amplitudes

$$\begin{aligned}
\sqrt{2}\,\langle \pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\pi}\,\lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big]\\
\langle \pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\}\\
-\,\langle \pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}\\
\langle \pi^-\bar{K}^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\bar{K}}\,\Big[\lambda_u^{(s)}\,\alpha_4^u+\lambda_c^{(s)}\,\alpha_4^c\Big]\\
\langle \pi^+K^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\bar{K}}\Big[\lambda_u^{(s)}\,(\alpha_1+\alpha_4^u)+\lambda_c^{(s)}\,\alpha_4^c\Big]
\end{aligned}$$

[Beneke,Neubert'03]

- Tree amplitudes α_1 and α_2 known analytically to NNLO *[Bell'07'09; Beneke,Li,TH'09]*
- Penguin amplitudes to NLO

$$\alpha_4^u(\pi\pi) = -0.029-[0.002+0.001i]_V+[0.003-0.013i]_P+[??\;+\;??\;i]_{\mathcal{O}(\alpha_s^2)}$$

$$+\left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.001]_{\rm LO}+[0.001+0.000i]_{HV+HP}+[0.001]_{\rm tw3}\big\} = -0.024^{+0.004}_{-0.002}+(-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029-[0.002+0.001i]_V-[0.001+0.007i]_P+[??\;+\;??\;i]_{\mathcal{O}(\alpha_s^2)}$$

$$+\left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.001]_{\rm LO}+[0.001+0.001i]_{HV+HP}+[0.001]_{\rm tw3}\big\} = -0.028^{+0.005}_{-0.003}+(-0.006^{+0.003}_{-0.002})i$$

Motivation for NNLO

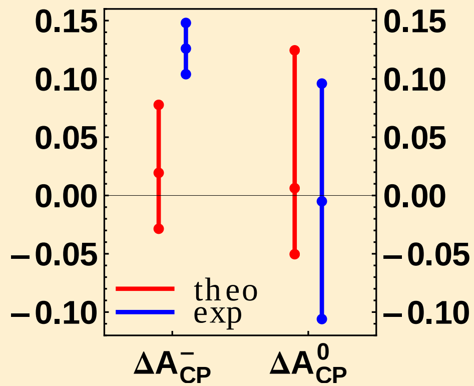
- Large cancellation in LO + NLO in color-suppressed tree amplitude α_2.
  - Particularly sensitive to NNLO
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$.
  - NNLO is only first perturbative correction
- Interesting quantities, e.g.

$$\Delta A_{\rm CP}(\pi K) = A_{\rm CP}(B^- \to \pi^0 K^-) - A_{\rm CP}(\bar{B}^0 \to \pi^+ K^-)$$

$\Delta A_{\rm CP}(\pi K) = (12.6 \pm 2.2)\,\%$ (exp.) *[HFAG'12]*

$\Delta A_{\rm CP}(\pi K) = (1.9^{+5.8}_{-4.8})\,\%$ (NLO QCDF) *[Hofer, Vernazza'12]*

Tension at the 2.2σ level



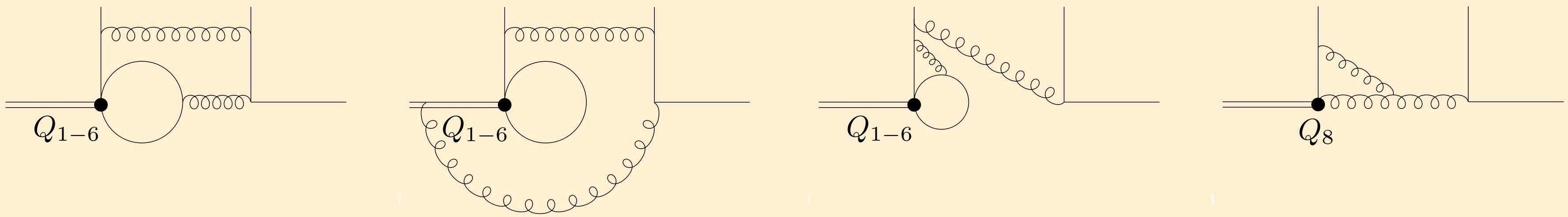
- Does factorisation hold at NNLO?

Penguin amplitudes at two loops

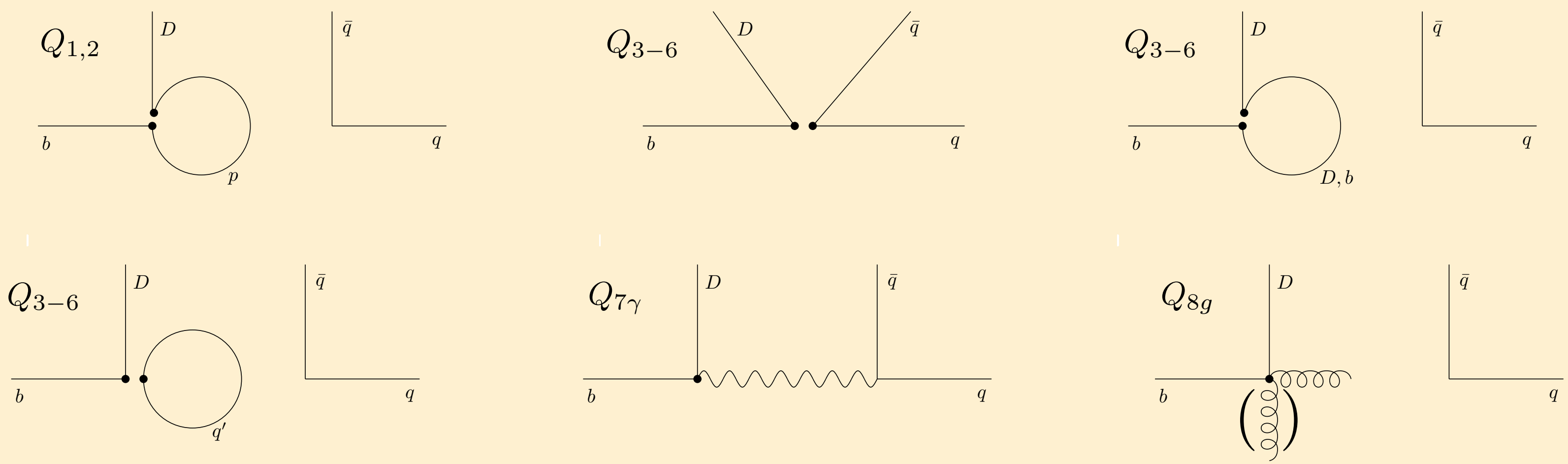
[Bell,Beneke,Li,TH, in preparation]

- NLO:

- $\mathcal{O}(70)$ diagrams at NNLO.

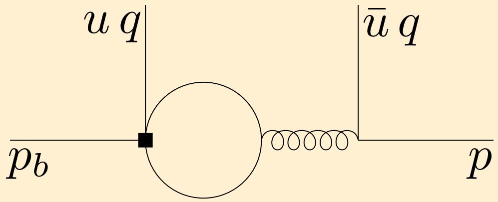


- Quite some book-keeping due to various insertions. Focus on $Q_1^{u,c}$ and $Q_2^{u,c}$.



Computational methods

- Kinematics:



$$p^2 = q^2 = 0$$
$$p_b^2 = m_b^2$$

  - Fermion loop with either $m = 0$ or $m = m_c$.
  - Suitable kinematic variables

$$s = \sqrt{1 - 4z_c/\bar{u}}\,,\ r = \sqrt{1 - 4z_c} \quad \longleftrightarrow \quad \bar{u}\,,\ z_c = \frac{m_c^2}{m_b^2} \quad \longleftrightarrow \quad s_1 = \sqrt{1 - 4/\bar{u}}\,,\ z_c$$

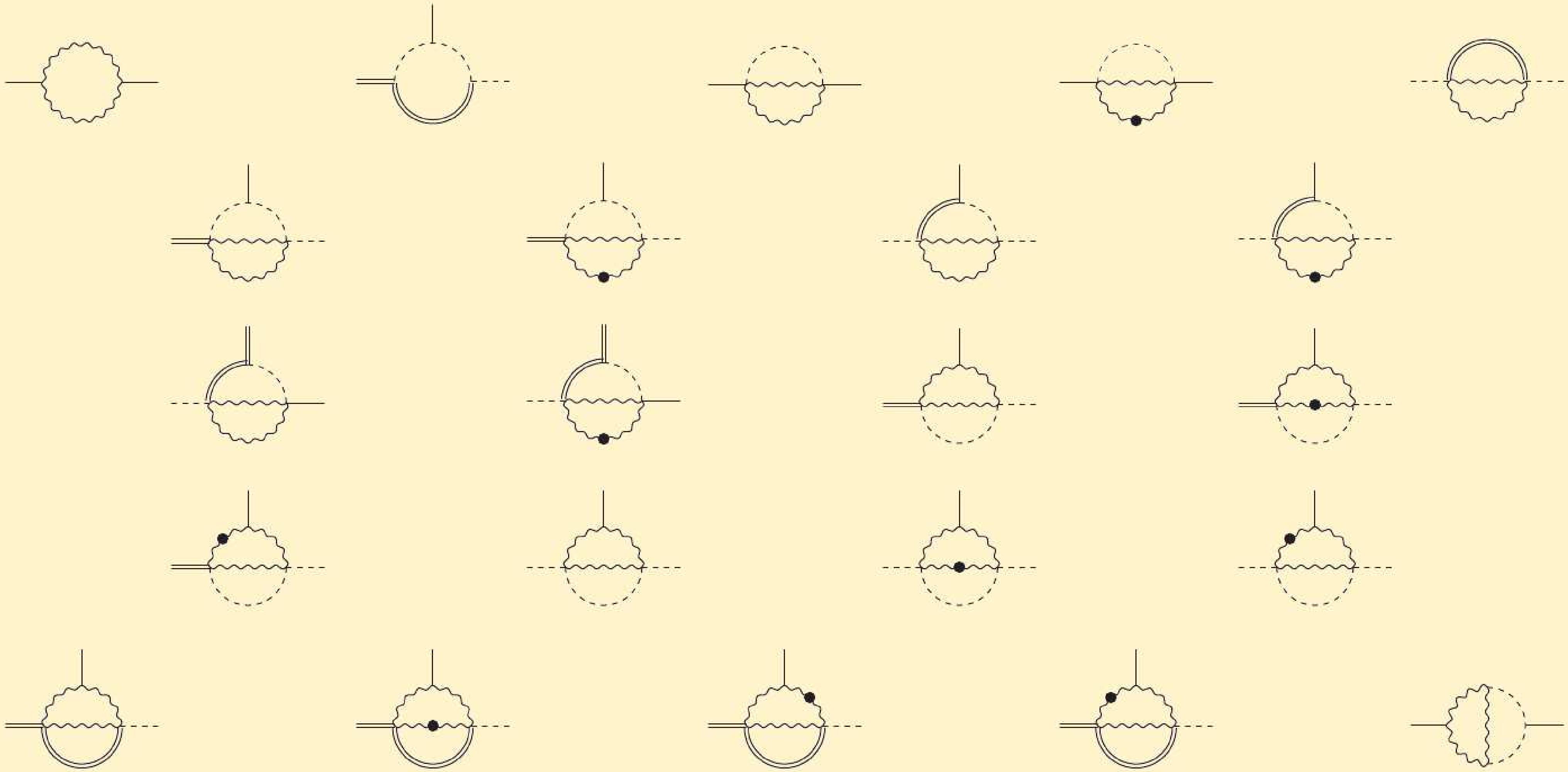
- Regularize UV and IR divergences dimensionally, $D = 4 - 2\epsilon$. Poles up to $1/\epsilon^3$.

- IBP relations, Laporta algorithm: Use AIR, FIRE, and in-house routine

  [Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08; Studerus,von Manteuffel'10,'12]

- Computation of 25 master integrals
  - Hypergeometric functions *[see e.g. Maitre,TH'05,'07]*
  - Mellin-Barnes representations *[Smirnov'99; Tausk'99; Czakon'05]*
  - Sector decomposition *[Borowka,Carter,Heinrich'12]*
  - Differential equations (see below) *[Kotikov'91; Remiddi'97]*

Master integrals for the penguin amplitudes



- Double: m_b^2 ,   wavy: m_c^2 ,   solid: $\bar{u}\, m_b^2$,   dashed: 0 .
- Genuine two-scale problem: u , $z_c \equiv m_c^2/m_b^2$
- Complication: Threshold at $\bar{u} = 4z_c$

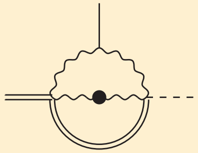
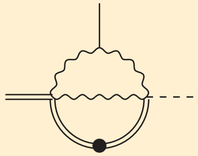
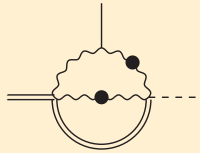
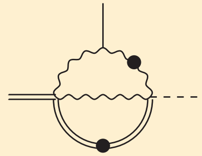
Computing the masters

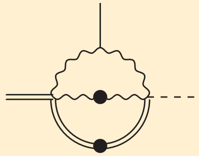
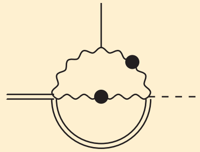
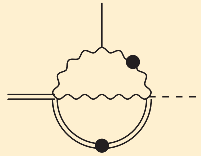
- Use differential equations in canonical form *[Henn'13]*
  - Found canonical basis for all masters ✓
  - First example of canonical basis in case of 2 different internal masses
  - Found solution in terms of iterated integrals wherever possible, including boundary conditions ✓
- Alphabet

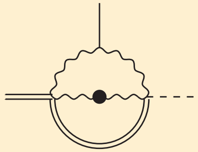
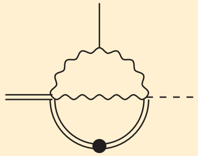
$$\left\{ s\ ,\ 1 \pm s\ ,\ r\ ,\ 1 \pm r\ ,\ r \pm s\ ,\ r^2 + 1 \pm 2s\ ,\ 1 + 2\sqrt{z_c} \pm s\ ,\ 1 - 2\sqrt{z_c} \pm s \right\}$$

- Benefits of canonical basis
  - No fake higher weights
  - QCD amplitude much simpler, especially denominators of pre-factors of masters
  - Suitable for convolution with LCDA

Canonical basis for master integrals I

$\frac{M_{18}}{u\epsilon^3} =$  $\quad \frac{M_{19}}{u\epsilon^3} =$  $\quad -\frac{2\,M_{20}}{u\bar{u}s\epsilon^2} =$  $+$ 

$\frac{M_{21}}{\epsilon^2} = \frac{2[(1+\bar{u})^2 z_c - \bar{u}^2]}{\bar{u}}$  $- \bar{u}s^2(1+\bar{u})\Big[$  $+$  $\Big]$

$+\frac{2\epsilon u}{m_b^2}\Big[$  $+$  $\Big]$

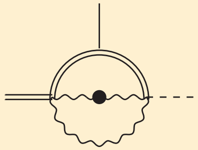
- Differential equation (sample)

$$\frac{dM_{19}}{ds} = \frac{4\epsilon M_{18}\, r\, \left(r^2+1\right)}{((r^2+1)^2-4s^2)} - \frac{2\epsilon M_{19}\, r\, \left(r^2+s^2-2\right)}{(1-r^2)(r^2-s^2)} + \frac{4\epsilon M_{20}\, r\, s}{\left((r^2+1)^2-4s^2\right)} - \frac{\epsilon M_{21}\, r\, \left(r^2+1\right)}{((r^2+1)^2-4s^2)}$$

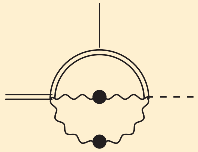
- Boundary conditions
  - M_{18} and M_{19} vanish in $s = r$ (i.e. in $u = 0$)
  - M_{20} and M_{21} vanish in $s = +i\infty$ (i.e. in $u = 1$)

Canonical basis for master integrals II

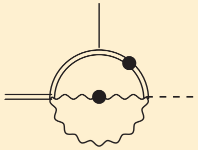
$$\frac{M_{23}}{u\epsilon^3} =$$



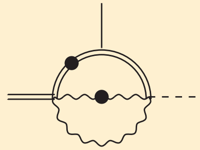
$$\frac{M_{24}}{\epsilon^2} = \frac{2\sqrt{8z_c(1-s_1)+(1+s_1)^2}}{1-s_1}\Bigg[$$



$$+\,2$$

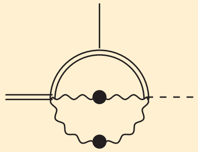


$$-\,\frac{2(1+s_1)}{1-s_1}$$

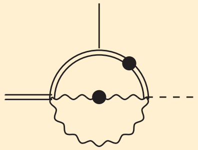


$$\Bigg]$$

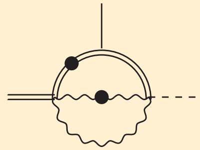
$$\frac{M_{25}}{\epsilon^2} = \frac{2\sqrt{8z_c(1+s_1)+(1-s_1)^2}}{1+s_1}\Bigg[$$



$$+\,2$$



$$-\,\frac{2(1-s_1)}{1+s_1}$$



$$\Bigg]$$

- Differential equation

$$\frac{dM_{23}}{ds_1} = \frac{2\,\epsilon\,M_{23}\,s_1\big(5-s_1^2\big)}{(1-s_1^2)\,(3+s_1^2)} - \frac{\epsilon\,M_{24}\,(3-s_1)}{4\,(1-s_1)\,\sqrt{8\,z_c(1-s_1)+(1+s_1)^2}} + \frac{\epsilon\,M_{25}\,(3+s_1)}{4\,(1+s_1)\,\sqrt{8\,z_c(1+s_1)+(1-s_1)^2}}$$

- Variable transformation to rationalize irrational factor?
- Does solution fall into the class of iterated integrals?
- Numerical convolution with LCDA no problem (no threshold)

Results

- Renormalisation of UV divergencies
- Subtraction of IR divergencies via matching onto SCET
  - Subtlety: Evanescent operators both on QCD and SCET side.

Results

- Renormalisation of UV divergencies
- Subtraction of IR divergencies via matching onto SCET
  - Subtlety: Evanescent operators both on QCD and SCET side.
- All poles cancel at first attempt ☺
- Analytical result for α_4^u ready. PRELIMINARY result
  - To NLO

$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? \; + \; ??\; i]_{\mathcal{O}(\alpha_s^2)}$$
$$+ \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002} + (-0.012^{+0.003}_{-0.002})i$$

  - NNLO shift

$$\alpha^u_{4,NNLO,Q1} = -0.0002 - 0.0006i$$
$$\alpha^u_{4,NNLO,Q2} = -0.0031 - 0.0067i$$

  - Interesting observation: Shift in Im-part is $\sim 50\%$, real part only $\sim 10\%$

The decays $B \to D^{(*)}\pi\,/\,\rho$

[Kränkl,TH, in progress]

- Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering power suppressed

- Applications
  - Ratios of decay widths

$$\frac{\Gamma(\bar{B}_d \to D^+\pi^-)}{\Gamma(\bar{B}_d \to D^{*+}\pi^-)} = \frac{(m_B^2 - m_D^2)^2 |\vec{q}|_{D\pi}}{4m_B^2 |\vec{q}|^3_{D^*\pi}} \left(\frac{F_0(m_\pi^2)}{A_0(m_\pi^2)}\right)^2 \left|\frac{a_1(D\pi)}{a_1(D^*\pi)}\right|^2$$

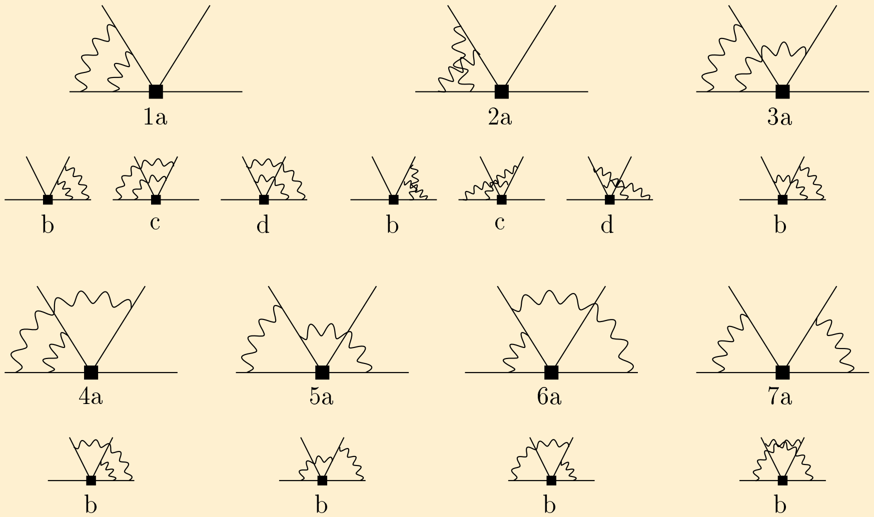
$$\frac{\Gamma(\bar{B}_d \to D^+\rho^-)}{\Gamma(\bar{B}_d \to D^+\pi^-)} = \frac{4m_B^2 |\vec{q}|^3_{D\rho}}{(m_B^2 - m_D^2)^2 |\vec{q}|_{D\pi}} \frac{f_\rho^2}{f_\pi^2} \left(\frac{F_+(m_\rho^2)}{F_0(m_\pi^2)}\right)^2 \left|\frac{a_1(D\rho)}{a_1(D\pi)}\right|^2$$

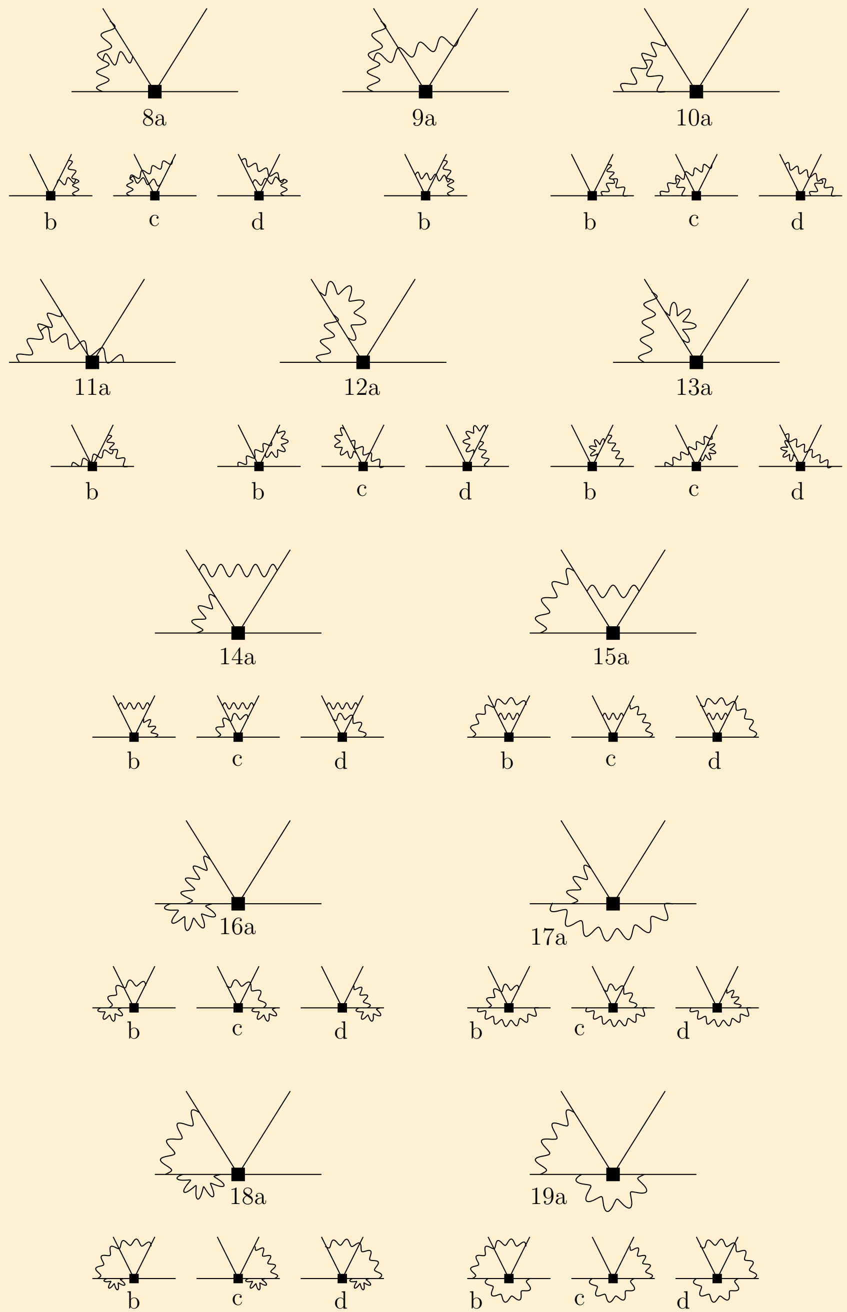
  - Test of factorisation

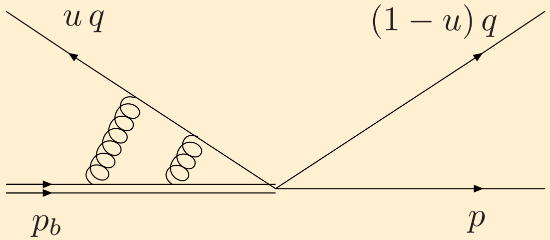
$$\frac{\Gamma(\bar{B}_d \to D^{(*)+}\pi^-)}{d\Gamma(\bar{B}_d \to D^{(*)+}l^-\bar{\nu})/dq^2\big|_{q^2=m_\pi^2}} = 6\pi^2 |V_{ud}|^2\, f_\pi^2\, |a_1(D^{(*)}\pi)|^2$$

  - Angular analysis in case of $D^*\rho$
  - Estimate size of power corrections

Two-loop diagrams for $B \to D\pi$



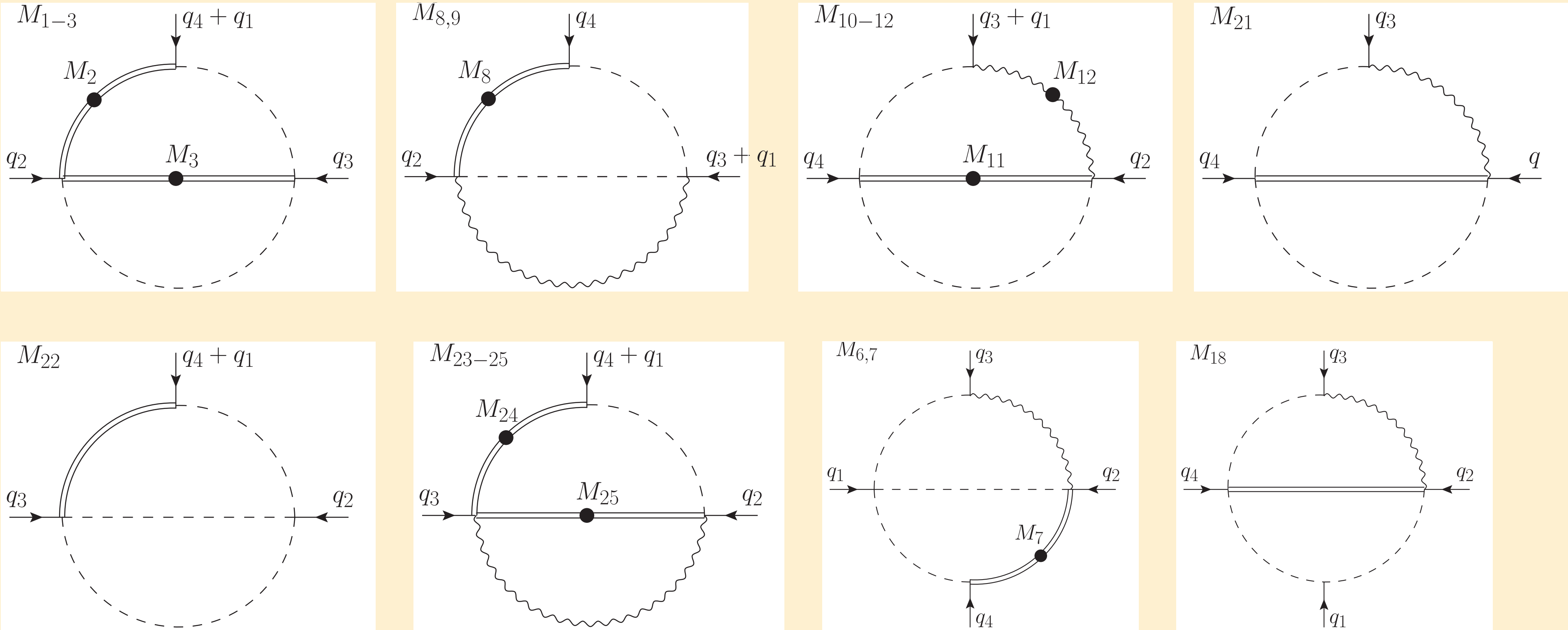




- Kinematics:

$$p_b^2 = m_b^2$$
$$q^2 = 0$$
$$p^2 = m_c^2$$

Master integrals for $B \to D\pi$ (selection)



- Double: m_b^2 , wavy: m_c^2 , dashed: 0 .
- Also here: Genuine two-scale problem: u , $z_c \equiv m_c^2/m_b^2$. Poles up to $1/\epsilon^4$.
- Masters completed: ${}_pF_q$, Mellin-Barnes, Differential equations, SecDec ✓
- Plan: Switch to canonical basis. Alphabet will be simpler than in penguin calculation since both m_b and m_c appear in external states.

Conclusion

- Field of nonleptonic B decays has reached the era of precision physics
  - Plethora of data from experiments
  - NNLO perturbative corrections almost available
  - Need also more precise non-perturbative input (Sum Rules, Lattice QCD)

- Computed penguin amplitudes α_4^u and α_4^c to two loops
  - Found solution to masters in a canonical basis ✓
  - Analytical result for α_4^u ready ✓
  - Result for α_4^c almost complete

- $B \to D\,\pi$ at two loops is work in progress
  - Hard scattering kernel almost complete

- Phenomenology based on NNLO results is largely overdue