Two-loop corrections to $t\bar{t}$ and VV production

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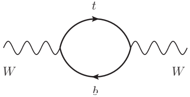
Top pair production at the LHC

LHC is top factory:

- **precise mass** determination: crucial input to many precision observables



- top decay: $t \to W^+ b$, leptonic W decay: $W^+ \to e^+ \nu_e$ (missing energy)

  background to **new physics** signatures (dark matter)

top quark pair production:

- LHC precision below NLO accuracy
- **threshold/resummations**:
  Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan; Aliev, Lacker, Langenfeld, Moch, Uwer, Vogt, Wiedermann; Cacciari, Czakon, Mangano, Mitov, Nason; Ahrens, Ferroglia, Neubert, Pecjak, Yang; Kidonakis
- **full NNLO** total σ (semi-numerical methods):
  Bärnreuther, Czakon, Fiedler, Mitov '08-'14

Analytic NNLO calculation

motivation for **analytic approach:**

- understand structure
- robust, fast, precise numerical evaluations
- cross-check

ingredients:

- **VV**: two-loop ME for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  leading N_c, (light) fermionic: Bonciani, Ferroglia, Gehrmann, Maitre, AvM, Studerus '08-'13
  poles: Ferroglia, Neubert, Pecjak, Yang '09
  small mass: Czakon, Mitov, Moch '06
  one-loop^2: Kniehl, Körner, Merebashvili, Rogal '05-'08, Anastasiou, Aybat '08
- **RV**: one-loop ME for $t\bar{t}$ + 1 parton
  Dittmaier, Uwer, Weinzierl '07, '09; Bevilacqua, Czakon, Papadopoulos, Worek '10, '11; Melnikov, Schulze '10
- **RR**: tree level ME for $t\bar{t}$ + 2 partons
- **subtraction terms**: up to 2 unresolved partons needed
  see talk by A. Gehrmann-de Ridder

Method of calculation

complexity:

- 2 independent ratios of scales, tensor rank 4
- 253 master integrals (w/products, wo/crossings)
- advanced mathematical functions

this talk:

- $gg \to t\bar{t}$: light fermionic (recently)
- $q\bar{q} \to t\bar{t}$: all contributions (in preparation)

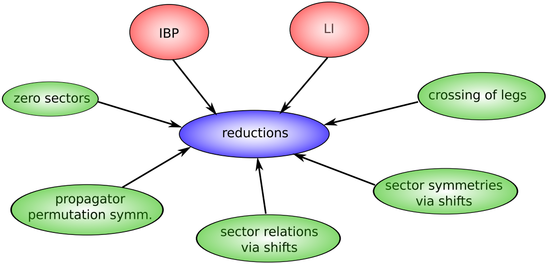
Recipe

1. generate **Feynman diagrams** with `QGRAF` by Nogueira
2. build **interference** terms with `Reduze 2`
3. **reduce** scalar integrals to masters via IBPs with `Reduze 2`
4. **renormalize**: $\overline{MS}$, pole mass
5. **solve masters** with differential equations
6. **optimisation of functional basis** for multiple polylogs

⇒ **analytic result** in terms of multiple polylogarithms,
allows fast **numerical evaluation**, **expansions**, . . .

Reduze 2 (AvM, C. Studerus)
arXiv:1201.4330, HepForge

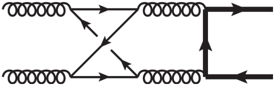
uses GiNaC (Bauer, Frink, Kreckel)
and Fermat (Lewis)



- distributed **Feynman integral reduction**
- advanced **shift finders**
- upcoming version features:
  - **bilinear propagators**
    (3-loop heavy flavour Wilson coefficients in DIS, see talk by A. de Freitas)
  - **phase space integrals**
    (RRV threshold contributions to N^3LO Higgs and DY (Li,AvM,Schabinger,Zhu '14))
  - **family finder**, . . .

Non-planar master integrals for light fermionic $gg \to t\bar{t}$

new masters integrals for



analytic calculation [AvM, C. Studerus '13]:

- solved with method of **differential equations**
- constants: regularity, symmetry, Mellin-Barnes
- used `MB.m` by Czakon '05, planar: `AMBRE` by Gluza, Kajda, Riemann '07 + Yundin '10
- checks: `SecDec 2.1` by Borowka, Heinrich '13

kinematics:

- diff. eq. contain $\sqrt{-s(4m^2 - s)}$, rationalize with Landau var. x:

$$s = -m^2(1-x)^2/x, \qquad t = -m^2 y, \qquad u = -m^2 z$$

- non-linear relations for crossed kinematics

$$(1-x)^2/x + y + z = -2$$

- involved functional identities, explicite imaginary parts

Multiple polylogarithms

[Remiddi, Gehrmann; Goncharov]

$$G(a_1, a_2, \ldots, a_n; x) = \int_0^x dt \; \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t),$$

with $G(;x) = 1$, complex weights a_i and complex argument x.

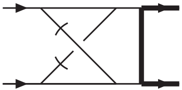
we employ also **generalised weights** $[f(o)]$:

$$G([f(o)], w_2, \ldots, w_n; x) = \int_0^x \mathrm{d}t \frac{f'(t)}{f(t)} G(w_2, \ldots, w_n; t)$$

example:

$$G([o^2+1]; x) = \int_0^x \mathrm{d}t \frac{2t}{t^2+1} = \int_0^x \mathrm{d}t \frac{1}{t-i} + \int_0^x \mathrm{d}t \frac{1}{t+i} = G(i; x) + G(-i; x)$$

see AvM, Schabinger, Zhu '13, related: Ablinger, Blümlein, Schneider '11 (cyclotomic polylogs)



$$= \frac{x^2}{m^4(1-x)^2(1-x+x^2+xy)} \sum_{i=-4}^{0} I_i \epsilon^i + \mathcal{O}(\epsilon)\,,$$

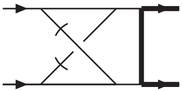
$$I_{-4} = \frac{7}{384}$$

$$I_{-3} = -\frac{5}{192} G(-(1-x+x^2)/x;y) + \frac{1}{64} G(-1;y) - \frac{5}{192} G([1-o+o^2];x) - \frac{1}{16} G(1;x) + \frac{11}{192} G(0;x) - \frac{5}{192} i\pi$$

$$I_{-2} = +\frac{1}{192} G(-(1-x+x^2)/x;y)^2 - \frac{1}{32} G(-(1-x+x^2)/x;y) G(-1;y) + \frac{1}{96} G(-(1-x+x^2)/x;y) G([1-o+o^2];x) + \frac{1}{8} G(-(1-x+x^2)/x;y) G(1;x) - \frac{7}{96} G(-(1-x+x^2)/x;y) G(0;x) - \frac{1}{64} G(-1;y)^2 - \frac{1}{32} G(-1;y) G([1-o+o^2];x) + \frac{1}{32} G(-1;y) G(0;x) + \frac{1}{192} G([1-o+o^2];x)^2 + \frac{1}{8} G([1-o+o^2];x) G(1;x) - \frac{7}{96} G([1-o+o^2];x) G(0;x) + \frac{1}{16} G(1;x)^2 - \frac{3}{16} G(0;x) G(1;x) + \frac{1}{12} G(0;x)^2 + \frac{1}{96} i\pi G(-(1-x+x^2)/x;y) - \frac{1}{32} i\pi G(-1;y) + \frac{1}{96} i\pi G([1-o+o^2];x) + \frac{1}{8} i\pi G(1;x) - \frac{7}{96} i\pi G(0;x) - \frac{35}{1152} \pi^2$$

$I_{-1} = \ldots$ (very lengthy)

where $x \equiv \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1}$, $y = -t/m^2$



$$= \frac{x^2}{m^4(1-x)^2(1-x+x^2+xy)} \sum_{i=-4}^{0} I_i \epsilon^i + \mathcal{O}(\epsilon)\,,$$

simplified representation:

$$I_{-4} = \frac{7}{384}$$

$$I_{-3} = -\frac{1}{32}\ln(y_1+z_1) + \frac{1}{64}\ln y_1 - \frac{5}{192}\ln z_1 + \frac{1}{32} i\pi$$

$$I_{-2} = \frac{1}{64}\ln^2(y_1+z_1) - \frac{1}{64}\ln^2 y_1 + \frac{1}{192}\ln^2 z_1 - \frac{1}{32}\ln y_1 \ln z_1 + \frac{1}{16}\ln z_1 \ln(y_1+z_1)$$
$$- \frac{1}{32} i\pi \ln(y_1+z_1) - \frac{1}{16} i\pi \ln z_1 - \frac{47}{1152}\pi^2$$

new functional basis for all poles: $\mathrm{Li}_3\left(\frac{y_1 z_1}{y_1+z_1}\right)$, $\mathrm{Li}_2\left(\frac{y_1 z_1}{y_1+z_1}\right)$, $\ln(y_1+z_1)$, $\log y_1$, $\ln z_1$

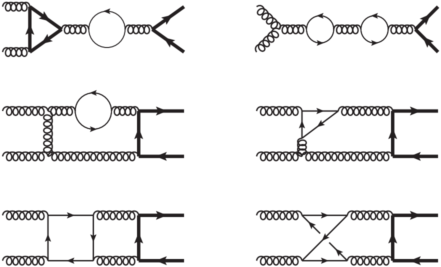
where $y_1 \equiv -t/m^2 + 1$, $z_1 \equiv -u/m^2 + 1$

instead of 28 (65 expanded) GPLs

we employ coproduct based and other algorithms for the multiple polylogarithms

- partly from Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
- numerical routines from Vollinga, Weinzierl '04

Light fermionic two-loop corrections to $gg \to t\bar{t}$



+ more

Bonciani, Ferroglia, Gehrmann, AvM, Studerus [arXiv:1309.4450]

gg **channel**: 789 two-loop diagrams (+ ghost init.)

$$\mathbf{2\,Re}\left\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \right\rangle = 2C_F N_c \big(N_c^3 \mathbf{A} + N_c \mathbf{B} + \frac{1}{N_c}\mathbf{C} + \frac{1}{N_c^3}\mathbf{D}$$

$$+ N_c^2 n_l \mathbf{E_l} + n_l \mathbf{F_l} + \frac{n_l}{N_c^2}\mathbf{G_l} + N_c n_l^2 \mathbf{H_l} + \frac{n_l^2}{N_c}\mathbf{I_l}$$

$$+ N_c^2 n_h \mathbf{E_h} + n_h \mathbf{F_h} + \frac{n_h}{N_c^2}\mathbf{G_h} + N_c n_h^2 \mathbf{H_h} + \frac{n_h^2}{N_c}\mathbf{I_h}$$

$$+ N_c n_l n_h \mathbf{H_{lh}} + \frac{n_l n_h}{N_c}\mathbf{I_{lh}} \big)$$

PRIMARY RESULT

$$G(\ldots;y), \quad \text{weights} \in \left\{-1, 0, -\frac{1}{x}, -x, -(1+x^2)/x, -(1-x+x^2)/x\right\}$$

$$G(\ldots;x), \quad \text{weights} \in \left\{-1, 0, 1, [1+o^2], [1-o+o^2]\right\}$$

note: jump in transcendentality weight from 2 for ϵ^{-1} to 4 for ϵ^0 for E_l, F_l, G_l

OPTIMISED FUNCTIONAL BASIS

choose real valued ln, $\mathrm{Li}_n(R_1)$, $\mathrm{Li}_{2,2}(R_1, R_2)$ with

$$|R_1| < 1, \qquad |R_1 R_2| < 1$$

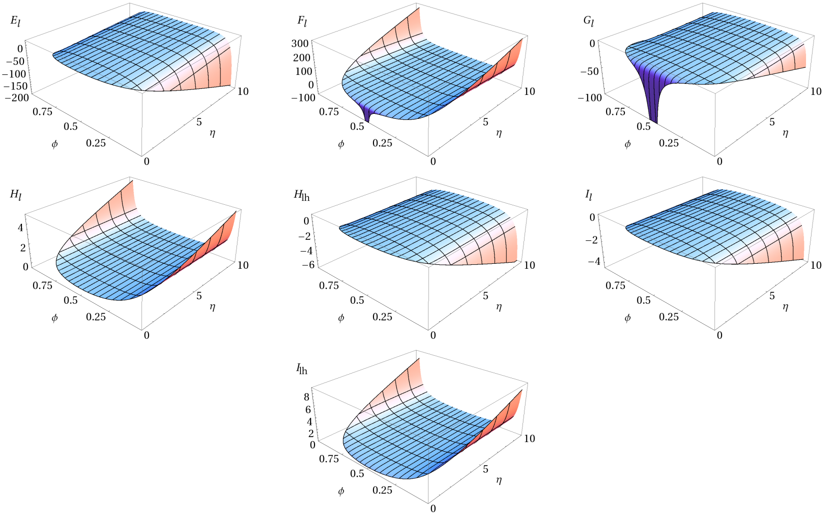
such that Li functions have convergent power series

$$\mathrm{Li}_n(R_1) = -\sum_{j_1=1}^{\infty} \frac{R_1^{j_1}}{j_1^n}, \qquad \mathrm{Li}_{2,2}(R_1, R_2) = \sum_{j_1=1}^{\infty}\sum_{j_2=1}^{\infty} \frac{R_1^{j_1}}{(j_1+j_2)^2} \frac{(R_1 R_2)^{j_2}}{j_2^2}$$

features:

- $R_i \in \left\{\pm x,\, x^2,\, \frac{1}{y+1},\, -xy,\, \frac{(1-x)y}{y+1},\, \frac{(1-x)z(x,y)}{z(x,y)+1}, \ldots\right\}$
- no spurious letters, no generalised weights
- **very fast and stable** numerical evaluation

Results for $gg \to t\bar{t}$ light N_f

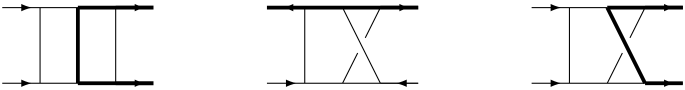


$$\eta = s/(4m^2) - 1, \quad \phi = -(t - m^2)/s$$

Next: complete $q\bar{q} \to t\bar{t}$

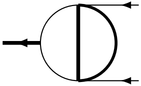
105 master integrals (w/ products, w/o crossings) in quark channel
J.M. Henn, AvM, V.A. Smirnov (in preparation)

new top level topologies:



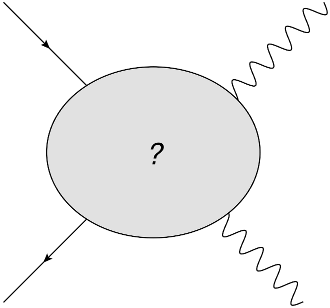
alphabet contains again the root $\sqrt{-s(4m^2 - s)}$ (rationalized by x)

note: heavy fermionic piece



contains $\sqrt{-s(4m^2+s)}$, known [Bonciani, Mastrolia, Remiddi '04]

Vector boson pair production



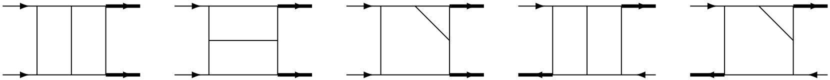
pair production of electroweak gauge bosons at LHC:

- precision tests of electroweak interactions and field content of SM
- here: $q\bar{q} \to ZZ,\ W^+W^-$
- missing ingredient for exact NNLO: two-loop contributions

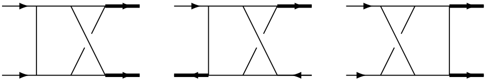
Master integrals for $q\bar{q} \to VV$

61 master integrals (w/ products, w/o crossings)

planar two-loop master integrals [Gehrmann,Tancredi,Weihs '13]



here: non-planar master integrals [Gehrmann,AvM,Tancredi,Weihs '14]



for VV', see [Caola, Henn, Melnikov, Smirnov '14]

Differential equations

- general form, $\epsilon = (4-d)/2$:

$$\frac{\partial}{\partial x_i} f_j(x_i, \varepsilon) = A^{(i)}_{jk}(x_i, \varepsilon) f_k(x_i, \varepsilon)$$

- in certain cases proper choice of basis achieves:

$$A^{(i)}_{jk}(x_i, \varepsilon) = \varepsilon\, \bar{A}^{(i)}_{jk}(x_i)$$

Henn '13; Henn, A. Smirnov, V. Smirnov '13

Canonical form

$$\mathrm{d}f(x_i, \varepsilon) = \varepsilon\, \mathrm{d}A(x_i)\, f(x_i, \varepsilon)$$

with

$$A(x_i) = A^{(l)}\, \ln R_l(x_i)$$

- decoupling, clear structure, general solution:

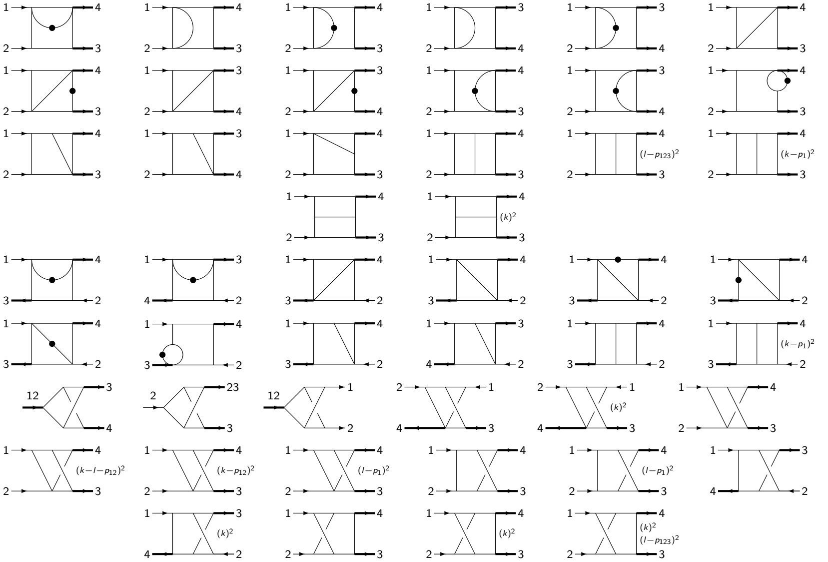
$$f = P\, e^{\varepsilon \int_{\mathcal{C}} \mathrm{d}A}\, f(\varepsilon = 0)$$

(pure functions for each order in ε)

- works for f expandable in terms of multiple polylogarithms, beyond: see talk by L. Tancredi

Master integrals for $q\bar{q} \to VV$

(planar bubbles, triangles and one-loop products not shown)



Gehrmann, AvM, Tancredi, Weihs '14

how to find **canonical basis** ?

- hints given in Henn '13; Smirnov, Smirnov, Henn '13: cuts, explicite bubble insertions
- basis change in Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi '14; Caron-Huot, Henn '14 for special cases

here:

PROCEDURE

construct canonical basis starting from rough first guess

1. **bottom up** strategy, assume subtopos in canonical form
2. for given sector, guess basis: **triangular for** $\epsilon = 0$, diff. eq. **linear in** ϵ (top level only)
3. integrate out **homogeneous part for** $\epsilon = 0$ (top level only)
4. remove unwanted terms $1/(u - v\epsilon)^n$, 1, ϵ^n iteratively
   simplifying assumption: restrict to minimal shifts

RESULT

vector of 75 master integrals in canonical basis with alphabet:

$$x,\ 1-x,\ 1+x,\ z,\ 1+z,\ x-z,\ 1-xz,\ 1+x^2-xz,\ 1+x+x^2-xz,\ z(1+x+x^2)-x$$

where $s = m^2(1+x)^2/x,\ \ u = -m^2 z$

integration and boundary terms:

- integrate partial diff. eq. in z if possible, otherwise in x
- uniform setup with same set of "physical" variables + functions
- **independent input** for a couple of very simple bubbles and triangles
- remaining boundary functions fixed by **regularity** (unphys. region):

$$z \to x, \qquad z \to 1/x, \qquad z \to -1, \qquad z \to (1+x+x^2)/x, \qquad x \to 1$$

- result in terms of G-functions with arguments z, x

checks:

- consistency of bound., diff. eq., known planar masters, real in Eulicidean region
- SecDec 2.1

optimisation for numerical evaluation:

- use real valued $\mathrm{Li}_{2,2}$, Li_n, ln
- choose Li with direct power series expansions
- evaluation time: $O(30ms)$ for generic point

RESULT

MC friendly analytic expressions, ready to be used for VV at NNLO

Conclusions & outlook

- technical progress
  - Reduze 2: reductions & shift finding for broad class of integrals
  - algorithms for multiple polylogarithms
  - procedure to put DEQ into normal form, works for typical cases
- $t\bar{t}$: analytic 2-loop
  - leading N_c + (light) fermionic available
  - full $q\bar{q} \to t\bar{t}$ in preparation
- $q\bar{q} \to ZZ,\ W^+W^-$: analytic 2-loop
  - complete set of two-loop master integrals
  - removes last major obstacle for first NNLO prediction
  - outlook: see talk by M. Grazzini on Tuesday

3 Algorithms for multiple polylogarithms

4 Expansions for light N_f corrections to $gg \to t\bar{t}$

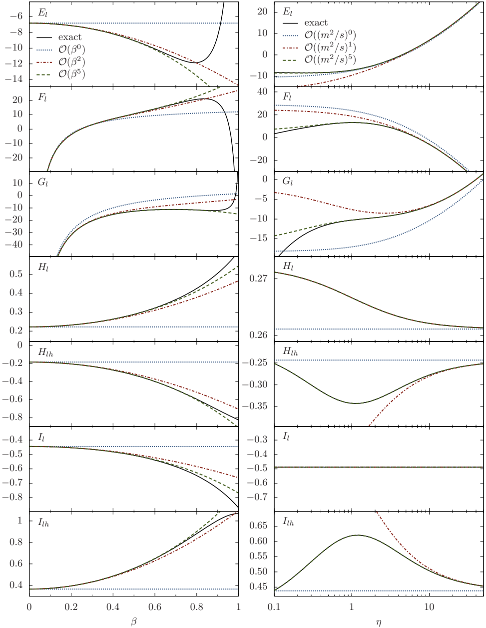
Algorithms for Multiple Polylogarithms

main algorithms:

1. **normal form for specific arguments**
   - independent of symbol calculus
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants

2. **coproduct based normal form** for general choice of basis
   - based on Goncharov '02, Brown '11, Duhr '12, Duhr, Gangl, Rhodes '11
   - handles generalised weights
   - identifies products (e.g. $G(0,1;x) + G(1,0;x) \to G(0;x)G(1;x)$)
   - matches irreducible factors *at symbol level*
   - uses Vollinga, Weinzierl '04 for numerical evaluation, fits constants

3. **construct new basis** with desired properties
   - based on Duhr, Gangl, Rhodes '11, apply to generalised weights



left: threshold expansion, $\beta = \sqrt{1 - 4m^2/s}$, $\cos\theta = 0.7$

right: "high energy" expansion, $\eta = s/(4m^2) - 1$, $\phi = -(t - m^2)/s = 0.35$