Electroweak precision tests in the LHC era and beyond

A. Freitas
University of Pittsburgh

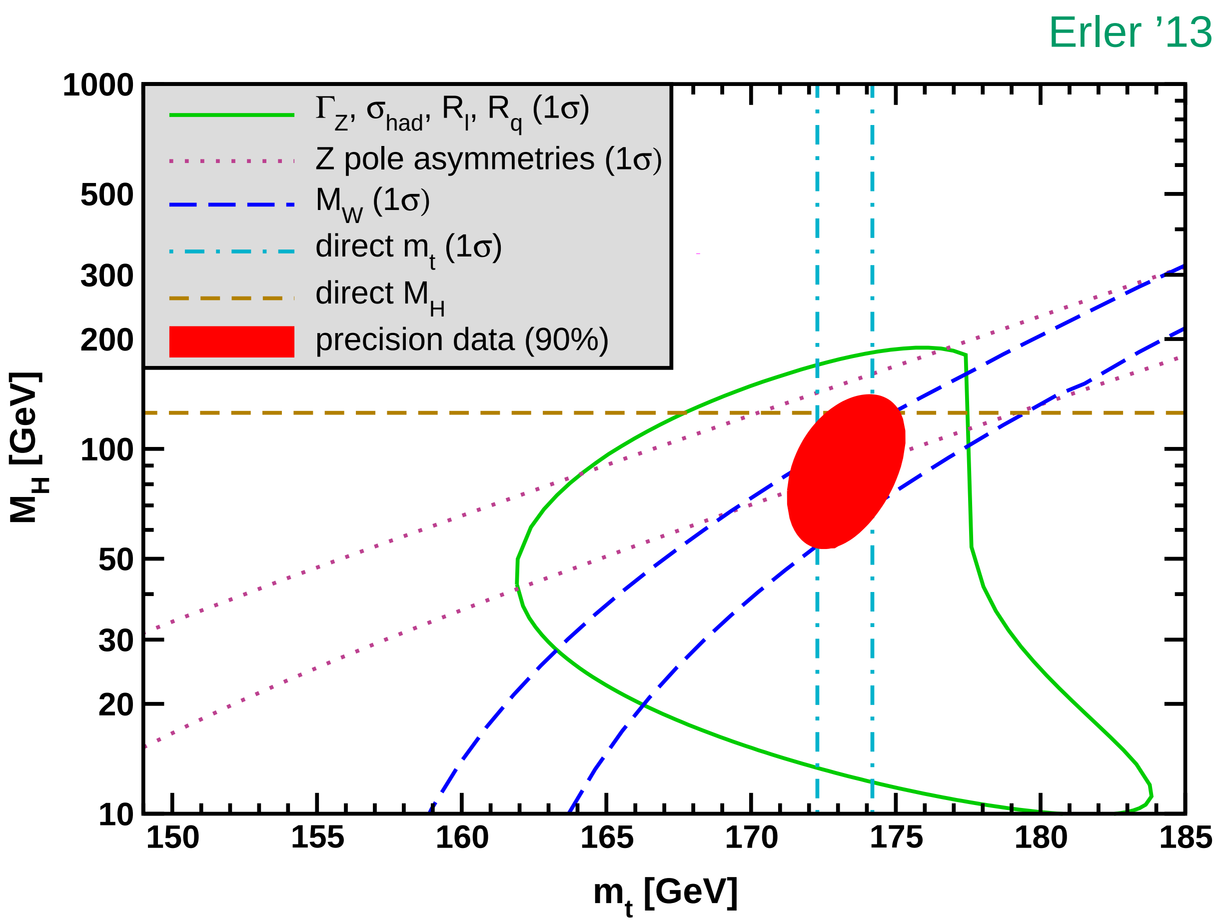
based on arXiv:1310.2256, 1401.2447;
1307.3962 (Snowmass white paper)

1. **Introduction**
2. **Electroweak precision observables**
3. **Z-boson width and cross-section at 2-loop**
4. **Outlook: beyond LHC**

Introduction

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables



Direct measurements:

$M_{\mathrm{H}} = 125.6 \pm 0.4$ GeV

$m_{\mathrm{t}} = 173.24 \pm 0.95$ GeV

Indirect prediction:

$M_{\mathrm{H}} = 123.7 \pm 2.3$ GeV (with LHC BRs)

$M_{\mathrm{H}} = 89^{+22}_{-18}$ GeV (w/o LHC data)

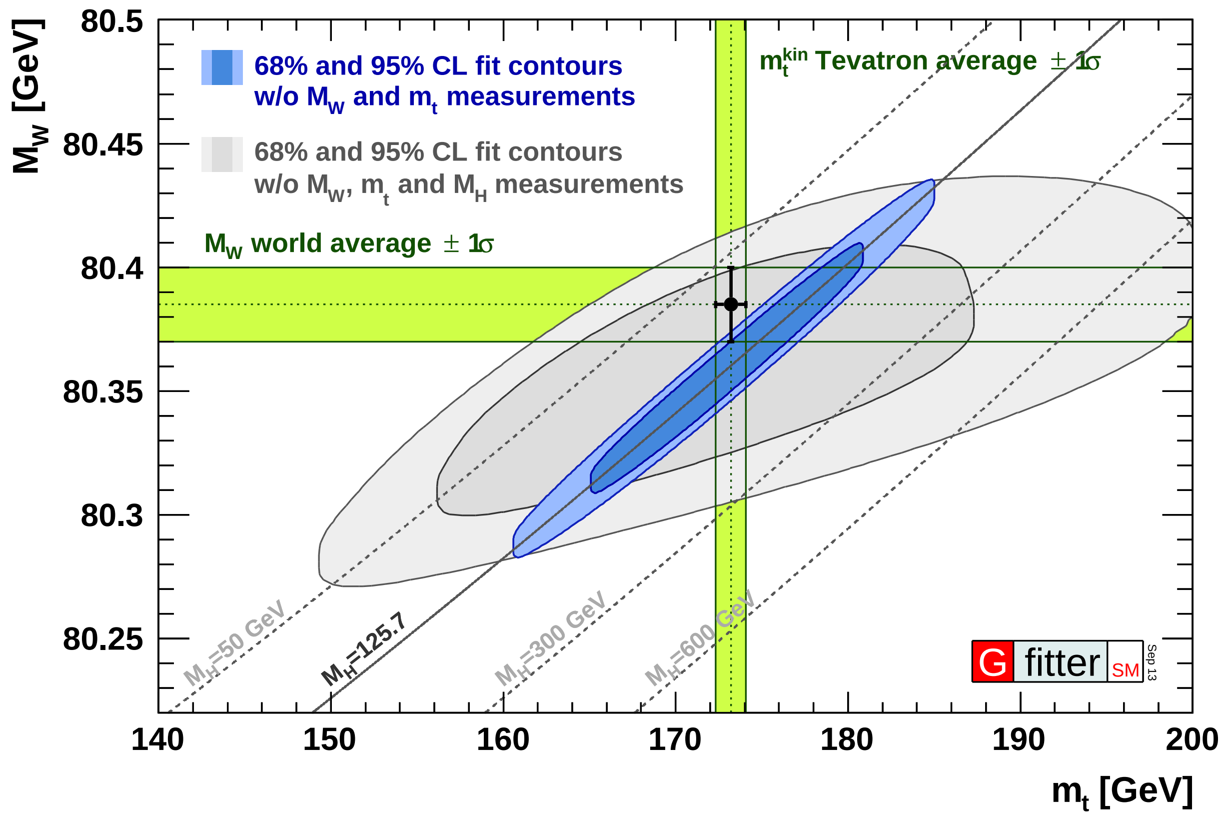
$m_{\mathrm{t}} = 177.0 \pm 2.1$ GeV

Introduction

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

GFitter '13



Direct measurements:

$M_W = 80385 \pm 15$ MeV

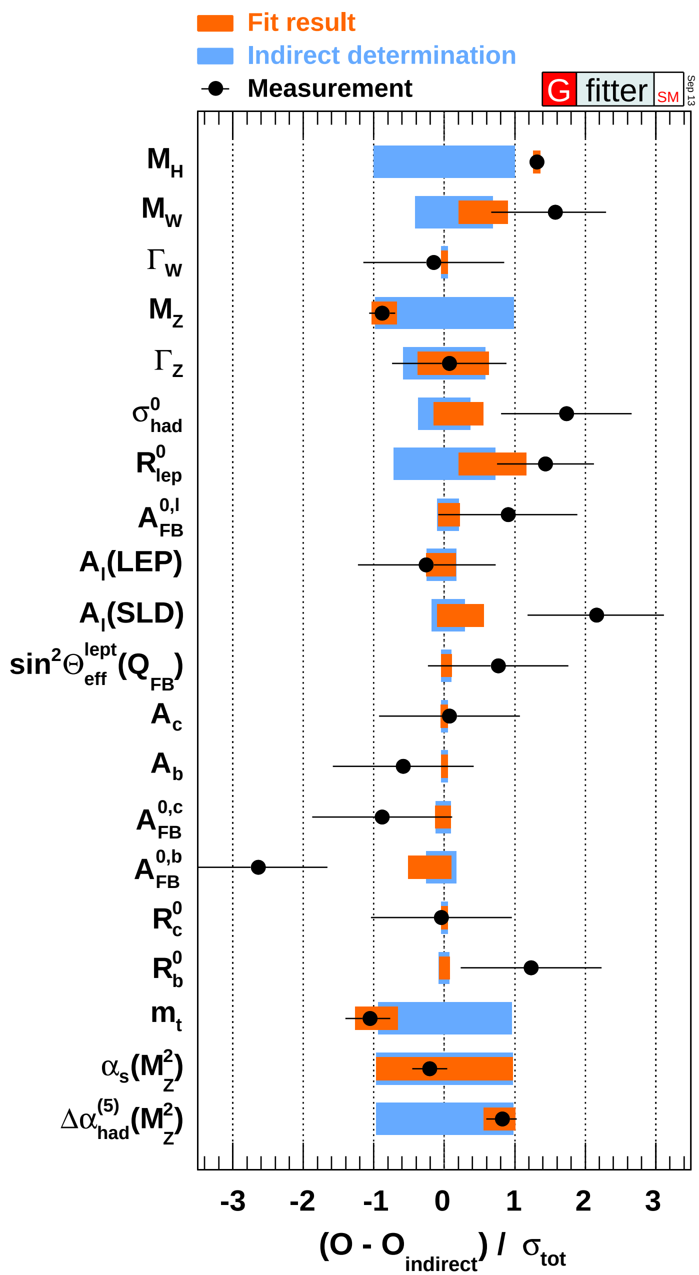
$m_t = 173.24 \pm 0.95$ GeV

Indirect prediction:

$M_W = 80358 \pm 7$ MeV

$m_t = 177.0 \pm 2.1$ GeV

Introduction

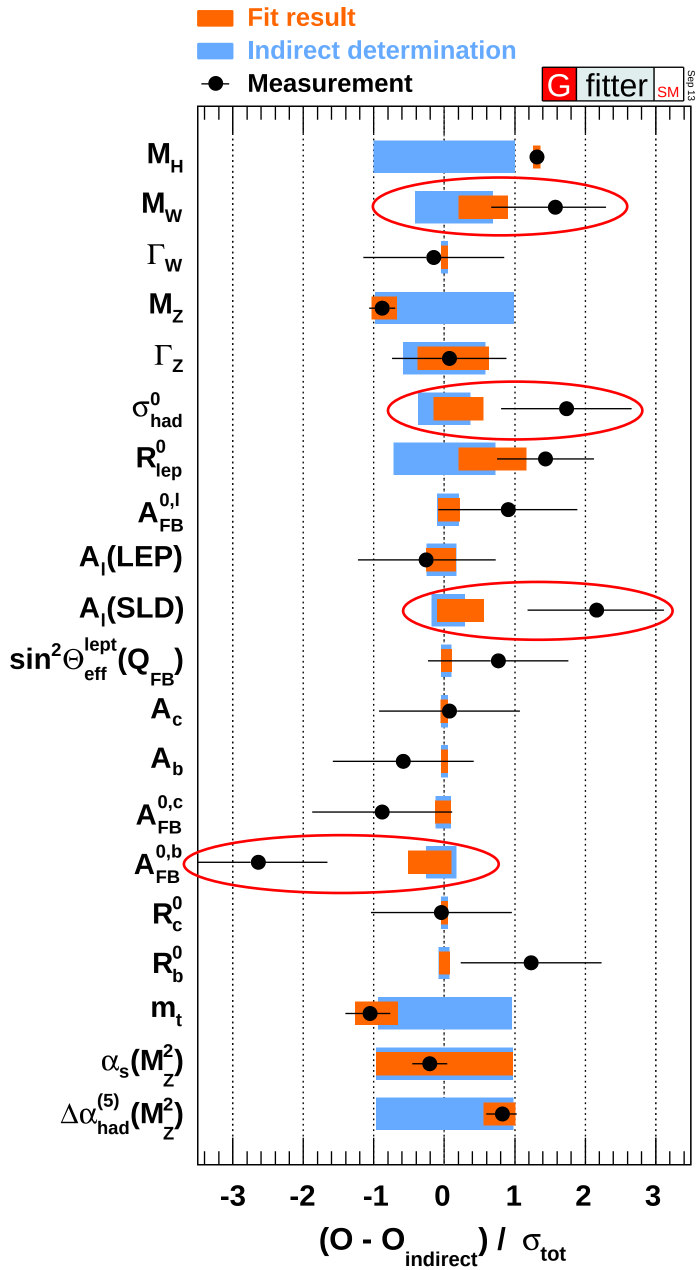


Surprisingly good agreement:
$\chi^2/\text{d.o.f.} = 18.1/14$ ($p = 20\%$)

Most quantities measured with
1%–0.1% precision

GFitter ’13

Introduction



Surprisingly good agreement:
χ^2/d.o.f. $= 18.1/14$ ($p = 20\%$)

Most quantities measured with 1%–0.1% precision

A few interesting deviations:

M_W	$(\sim 1.5\sigma)$
σ^0_{had}	$(\sim 1.7\sigma)$
A_ℓ(SLD)	$(\sim 2\sigma)$
A^b_{FB}	$(\sim 2.5\sigma)$
$(g_\mu - 2)$	$(\gtrsim 3\sigma)$

GFitter '13

Electroweak precision observables

Experimental precision requires inclusion of **radiative corrections** in theory

1-loop:	few %	needed
2-loop:	few $\times 10^{-3}$	needed
3-loop:	few $\times 10^{-4}$	partially needed

Electroweak precision tests

Most important quantities:

	Exp. error	Th. error
M_W	15 MeV	4 MeV
Γ_Z	2.3 MeV	0.5 MeV
$\sigma^0_\text{had} = \sigma[e^+e^- \to Z \to \text{had.}]$	37 pb	6 pb
$R_b = \Gamma[Z \to b\bar{b}]/\Gamma[Z \to \text{had.}]$	6.6×10^{-4}	1.5×10^{-4}
$\sin^2\theta^\ell_\text{eff}$ (from A_LR and A_FB)	1.6×10^{-4}	0.5×10^{-4}

- Complete NNLO or *fermionic* NNLO corrections known

  Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Freitas, Huang '12; Freitas '13

- Partial 3/4-loop corrections

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05; Schröder, Steinhauser '05
  Chetyrkin et al. '06; Boughezal, Czakon '06

Z-pole observables

After deconvolution of initial-state QED radiation and subtraction of γ-exchange:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)S'$$

$$s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

$s_0,\ R,\ S,\ S'$ are gauge-invariant

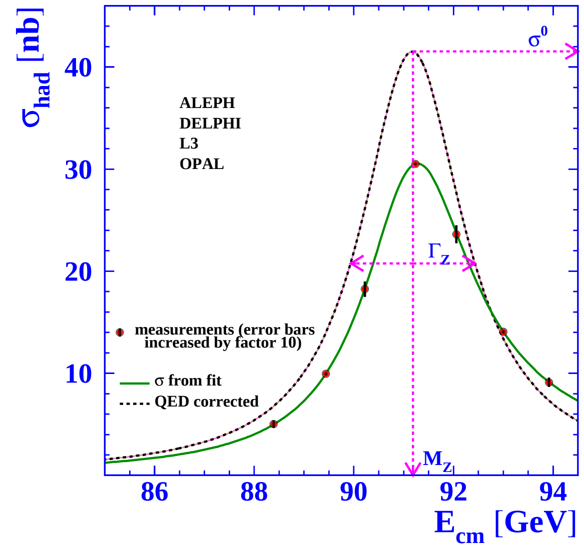
Willenbrock, Valencia '91; Sirlin '91; Stuart '91

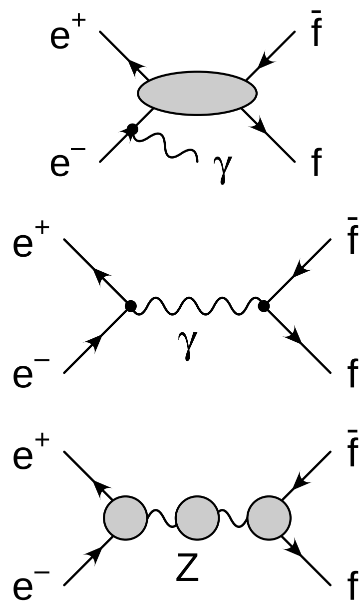
Gambino, Grassi '00

$$\sigma \sim \frac{1}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \text{non.res.}$$

$$M_Z = M_Z^{\text{exp}} - 34\text{ MeV}$$
$$\Gamma_Z = \Gamma_Z^{\text{exp}} - 0.9\text{ MeV}$$

LEP EWWG '05





Z-boson width and cross-section at 2-loop

Z width:

$$\Gamma_Z = \frac{1}{M_Z}\text{Im}\,\Sigma_Z(s_0)$$
$$= \frac{1}{M_Z}\left[\frac{\text{Im}\,\Sigma_Z}{1+\text{Re}\,\Sigma'_Z} - \tfrac{1}{2}M_Z\Gamma_Z(\text{Im}\,\Sigma_Z)(\text{Im}\,\Sigma''_Z)\right]_{s=M_Z^2} + \mathcal{O}(\Gamma_Z^3).$$

Optical theorem:

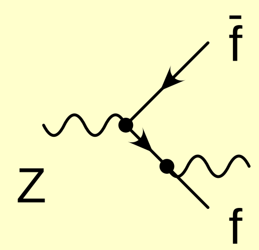
$$\Gamma_Z = \sum_f \Gamma_f, \qquad \Gamma_f \approx \frac{N_c M_Z}{12\pi}\left[\left(\mathcal{R}_V^f|v_f|^2 + \mathcal{R}_A^f|a_f|^2\right)\frac{1}{1+\text{Re}\,\Sigma'_Z}\right]_{s=M_Z^2}$$

$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

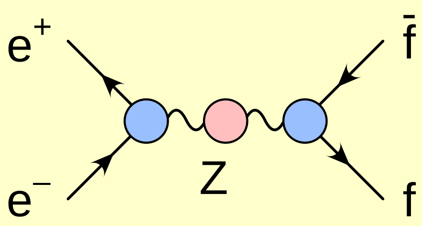
Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

Baikov, Chetyrkin, Kühn, Rittinger '12



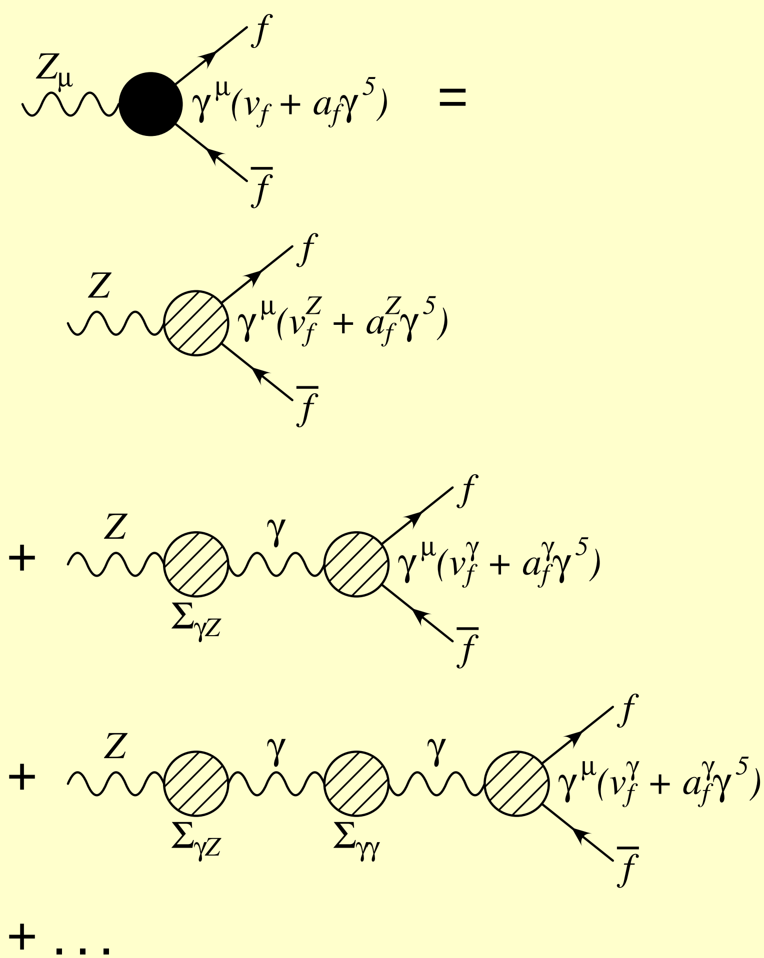
v_f, a_f, Σ'_Z: Electroweak corrections

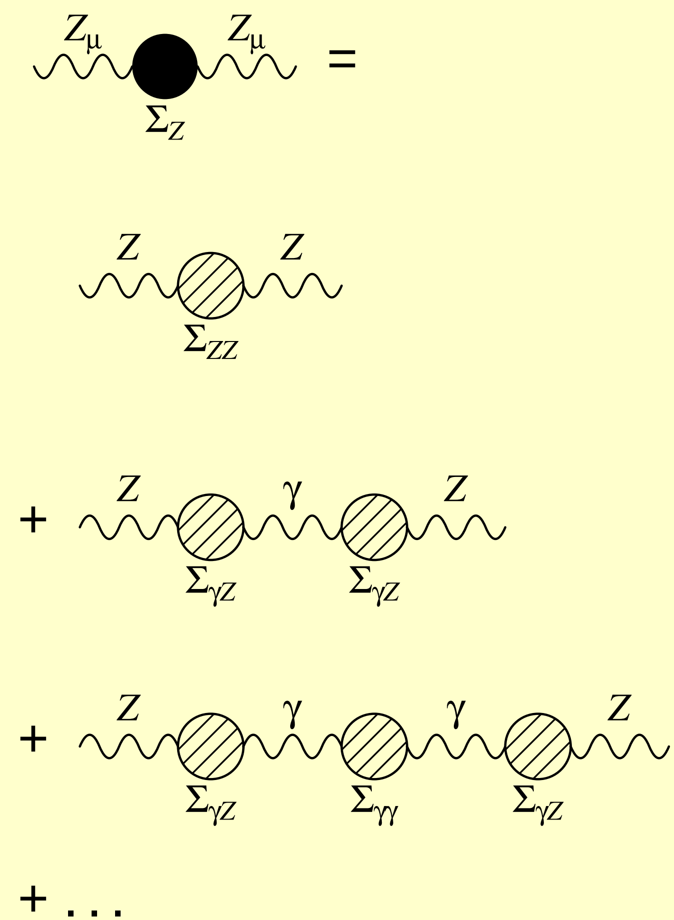


Z-boson width

$$v_f(s) = v_f^Z(s) - v_f^\gamma(s)\,\frac{\Sigma_{\gamma Z}(s)}{s + \Sigma_{\gamma\gamma}(s)}, \qquad \Sigma_Z(s) = \Sigma_{ZZ}(s) - \frac{[\Sigma_{\gamma Z}(s)]^2}{s + \Sigma_{\gamma\gamma}(s)},$$

(similar for a_f)





Z-pole cross-section

Peak cross section:

$$\sigma^0_{\rm had} = \frac{1}{64\pi^2 s} \sum_q \int d\Omega \, \left| \mathcal{A}(e^+e^- \to q\bar{q}) \right|^2_{s=M_Z^2}$$

Explicit calculation: Freitas '13

$$\sigma^0_{\rm had} = \frac{12\pi}{M_Z^2} \sum_q \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} (1 + \delta X)$$

Correction term first at NNLO: Grassi, Kniehl, Sirlin '01

$$\delta X_{(2)} = -(\mathrm{Im}\,\Sigma'_{Z(1)})^2 - 2\Gamma_Z M_Z \,\mathrm{Im}\,\Sigma''_{Z(1)}$$

Bulk of electroweak corrections for Γ_Z and $\sigma^0_{\rm had}$ contained in g_V^f, g_A^f, Σ'_Z;

known to $\mathcal{O}(\alpha\alpha_s)$, $\mathcal{O}(N_f\alpha^2)$ Czarnecki, Kühn '96

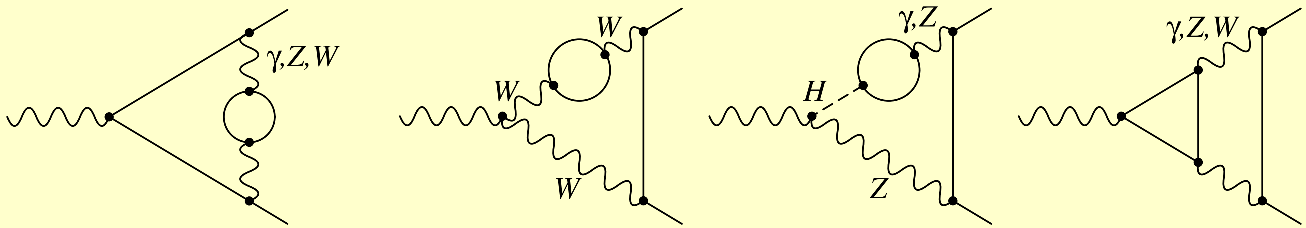
Harlander, Seidensticker, Steinhauser '98; Freitas '13

and partial 3-/4-loop

Two-loop Z-vertex corrections

Dominant contributions:

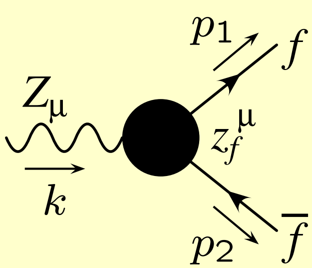
Diagrams with closed fermion loops (enhanced by N_f and/or m_t^2)



- Generation of diagrams with *FeynArts* — Küblbeck, Eck, Mertig '92, Hahn '01
- Projection technique:

$$v_f(k^2) = \frac{1}{2(2-d)k^2}\,\mathrm{Tr}[\gamma_\mu \not{p}_1\, z_f^\mu(k^2)\, \not{p}_2]$$

$$a_f(k^2) = \frac{1}{2(2-d)k^2}\,\mathrm{Tr}[\gamma_5\,\gamma_\mu \not{p}_1\, z_f^\mu(k^2)\, \not{p}_2]$$



- On-shell renormalization:
  M_W, M_Z, M_H, m_t defined through (complex) propagator poles
- Dirac/Lorentz algebra, reduction to basis integrals
- Evaluation of 2-loop integrals in general not possible analytically

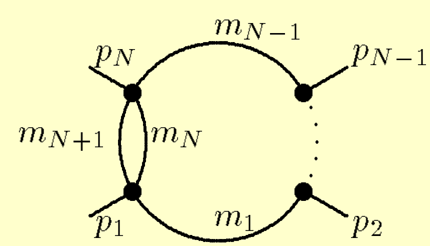
Dispersion relations

Topologies with **self-energy sub-loop** can easily be integrated by using dispsersion relation for B_0 function: S. Bauberger et al. '95

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

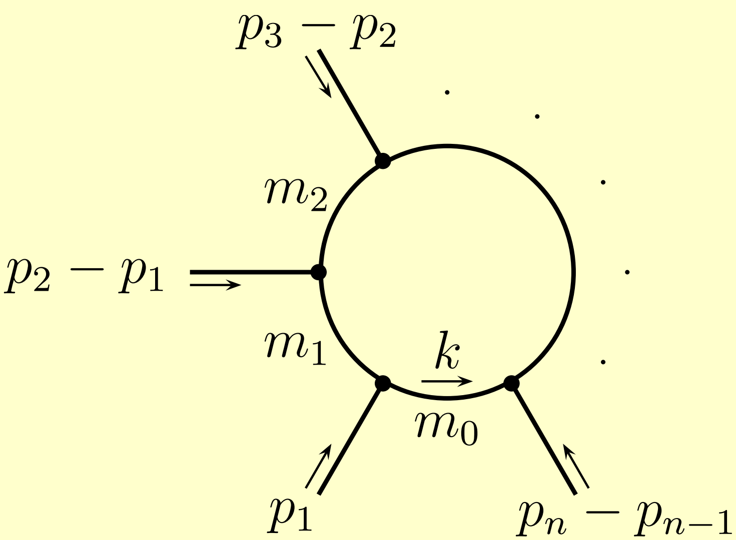
with $$\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2-1)}{\Gamma(D-2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}},$$

$\lambda(a, b, c) = (a-b-c)^2 - 4bc$



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2)$$
$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \, \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

Feynman parameter integration



$$I^{(1)} = \int \widetilde{dk}\, \frac{N(k)}{D^{(1)}(k)}$$

$$\widetilde{dk} = e^{\gamma_E (4-d)/2} \frac{d^d k}{i\pi^{d/2}}$$

$$D^{(1)}(k) = [k^2 - m_0^2][(k-p_1)^2 - m_1^2] \cdots [(k-p_n)^2 - m_n^2]$$

Introduce Feynman parameters and expand in ε:

$$I^{(1)} = \int_0^1 dx_1 \ldots dx_{n-1} \Big[D_0 \Big(\frac{1}{\varepsilon} + \log(A - i\epsilon) \Big) + D_1 (A - i\epsilon)^{-1} + D_2 (A - i\epsilon)^{-2} + \ldots \Big]$$

UV poles

Extension to two loops

UV divergencies: in either or both *subloops*, and also *global*

$\longrightarrow$ Direct evaluation not possible, need **UV subtraction terms** Freitas '12

Global UV divergency:

$$G^{(2)}_{\text{glob}} = \left. \frac{N(k_1, k_2)}{D^{(2)}(k_1, k_2)} \right|_{p_i = 0}$$

Works for all two-loop N-point functions except selfenergies

$\int \widetilde{dk}_1 \widetilde{dk}_2 \, G^{(2)}_{\text{glob}}$ consists of two-loop vacuum integrals (known analytically)

Davydychev, Tausk '92

$I^{(2)}_{\text{gs}} \equiv I^{(2)} - \int \widetilde{dk}_1 \widetilde{dk}_2 \, G^{(2)}_{\text{glob}}$ can have singularities in one subloop (both subloops only for tadpoles and selfenergies)

Extension to two loops

Subloop UV divergency in k_1 loop:

- Introduce Feynman parameters for k_1 subloop and perform k_1 tensor reduction (as before)

contain all k_2 dependence

$$I^{(2)}_{\text{gs}} = \int_0^1 dx_1 ... dx_{m-1} \int \widetilde{dk_1} \widetilde{dk_2} \left[\frac{C_1}{[k_1^2 - A]^m} + \frac{C_2}{[k_1^2 - A]^{m-1}} + \ldots \frac{C_j}{[k_1^2 - A]^2} \right],$$

of k_1 propagators

UV diverg.

- UV subloop subtraction term:

$$G^{(2)}_{\text{sub}} = \int_0^1 dx_1 \ldots dx_{m-1} \frac{C_j}{[k_1^2 - \mu^2]^2},$$

($\mu =$ suitably chosen mass parameter)

Extension to two loops

Numerical integration over Feynman parameters:

Physical thresholds: A changes sign in integration region
$\longrightarrow$ Problematic for numerical integrators

$\longrightarrow$ Deform Feynman parameter integration into comple x plane:

Nagy, Soper '06

$$x_i = z_i - i\lambda z_i(1-z_i)\frac{\partial A}{\partial z_i}, \qquad 0 \le z_i \le 1.$$

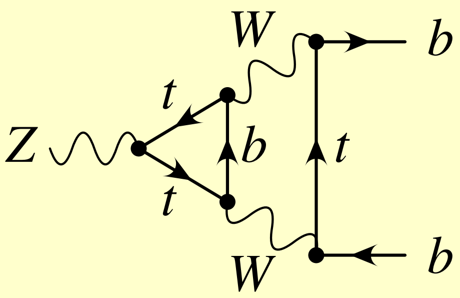
$$A(\vec{x}) = A(\vec{z}) - i\lambda \sum_i z_i(1-z_i)\left(\frac{\partial A}{\partial z_i}\right)^2 + \mathcal{O}(\lambda^2).$$

Typical choice: $\lambda \sim 0.5 - 1$

Two-loop example

Diagram contributing to $Z \to b\bar{b}$:

(global and subloop UV singularities, no IR singularity)



	this work*	BT method **
$\mathcal{O}(\varepsilon^{-2})$	-2.30183413	-2.30183413
$\mathcal{O}(\varepsilon^{-1})$	5.07108758	5.07108758
$\mathcal{O}(\varepsilon^{0})$	8.326(1)	8.3259

$M_Z = 1$, $M_W = 80/91$,
$m_t = 180/91$

$N = 10^6$, $\lambda = 0$ (no cuts)

* Freitas '12, public code *Nicodemos*

**Bernstein-Tkachov method from Awramik, Czakon, Freitas, Kniehl '08

Results for Γ_Z and σ^0_{had}

- Checks against existing calculations
  for $\sin^2\theta^\ell_{\text{eff}}$, $\sin^2\theta^b_{\text{eff}}$, R_b — Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas, Kniehl '09
  Freitas, Huang '12

- Known corrections to vertex form factors:
  $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$ — Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  $\mathcal{O}(m_t^2\alpha\alpha_s^2)$, $\mathcal{O}(m_t^4\alpha^2\alpha_s)$, $\mathcal{O}(m_t^6\alpha^3)$, $\mathcal{O}(m_t^2\alpha\alpha_s^3)$
  Avdeev, Fleischer, Mikhailov, Tarasov '94; Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03; Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal, Czakon '06
  $\mathcal{O}(N_f\alpha^2)$ — Freitas '13

- Input parameters:
  M_Z, M_H, m_t, $\Delta\alpha$, α_s, G_μ
  For final-state effects: m_b, m_c, m_τ
  M_W computed from G_μ — Awramik, Czakon, Freitas, Weiglein '04

Results for Γ_Z and σ^0_{had}

	Γ_Z [GeV]	σ^0_{had} [nb]	R_ℓ	R_c	R_b
Born + $\mathcal{O}(\alpha)$	2.49769	41.4687	20.8031	0.17230	0.21558
$\mathcal{O}(\alpha\alpha_s)$	-0.00121	0.0071	-0.0068	-0.00008	0.00035
leading 3/4-loop	-0.00089	0.0012	-0.0020	<0.00001	<0.00001
$\mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	-0.00136	0.0114	-0.0434	0.00001	-0.00013
Total*	2.49430	41.4882	20.7512	0.17223	0.21580
Large-m_t exp.**	2.49485	41.4840	20.7484	0.17220	0.21579

* excluding $\mathcal{O}(m_t^2\alpha\alpha_s^3)$

** best known previous result $\mathcal{O}(m_t^4\alpha^2)$, $\mathcal{O}(m_t^2\alpha^2)$ (except for $b\bar{b}$)

Degrassi, Gambino, Vicini '96; Degrassi, Gambino, Sirlin '97; Degrassi, Gambino '99

Barbieri et al. '92; Fleischer, Tarasov, Jegerlehner '93,95

Outlook

- SM is experimentally confirmed, but no evidence for new physics at collider
  $\longrightarrow$ "Low"-energy precision measurements may lead the way

- Experimental precision from LEP/SLC demands SM prediction with
  2-loop corrections and **partial 3-loop corrections**

- Much progress during last 10–20 years:
  Complete or *fermionic* 2-loop corrections available for M_W and **all Z-pole observables**; $\Delta_\mathrm{th} < \Delta_\mathrm{exp}$

- **LHC** will provide independent results for $\sin^2\theta_\mathrm{eff}$ and M_W,
  but overall precision not improved

- **ILC/TLEP** with $\sqrt{s} \sim M_\mathrm{Z}$ will reduce experimental error by at least $\mathcal{O}(2-10)$
  $\longrightarrow$ Challenge for theorists!

Outlook

Precision at proposed future colliders:

	Current exp.	ILC	TLEP	Current th. error
$M_{\rm W}$ [MeV]	15	3–5	~ 1	4
Γ_Z [MeV]	2.3	~ 1	$\lesssim 1$	0.5
R_b $[10^{-5}]$	66	15	$\lesssim 5$	~ 15
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	16	1.3	0.3	4.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/TLEP!

Outlook

Precision at proposed future colliders:

	ILC	TLEP	Th. error with 3-loop[†]	Param. error ILC*	Param. error TLEP**
M_W [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	$\lesssim 1$	$\lesssim 0.2$	0.5	?
R_b [10^{-5}]	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2\theta^\ell_\mathrm{eff}$ [10^{-5}]	1.3	0.3	1.5	2	2

[†] Theory improvements: $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^3\alpha^3)$, $\mathcal{O}(N_f^2\alpha^3)$

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_\mathrm{s} = 0.001$, $\delta M_\mathrm{Z} = 2.1$ MeV

****TLEP:** $\delta m_t \lesssim 50$ MeV, $\delta\alpha_\mathrm{s} =?$, $\delta M_\mathrm{Z} = 0.1$ MeV

also: $\delta(\Delta\alpha) = 5 \times 10^{-5}$

Backup slides

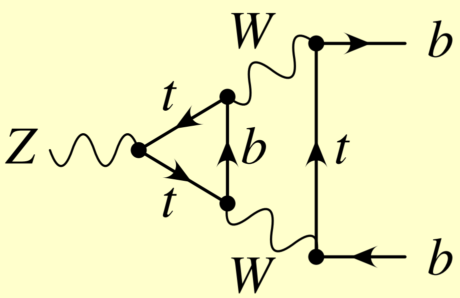
Z-pole observables

	Experiment	Theory error	Main source
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	$\alpha^2\alpha_s$, $\alpha\alpha_s^2$, α^3, α_{bos}^2
σ_{had}^0	41540 ± 37 pb	6 pb	$\alpha^2\alpha_s$, α^3, α_{bos}^2
R_b	0.21629 ± 0.00066	0.00015	$\alpha^2\alpha_s$, α^3, $\alpha\alpha_s^2$, α_{bos}^2
$\sin^2\theta_{eff}^\ell$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^2\alpha_s$, α^3

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scheme dependence (use with care!)
- Experience from similar calculations

γ_5



$$\{\gamma^\mu, \gamma_5\} = 0$$

In 4 dim.: $\mathrm{tr}\{\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma_5\} = 4i\epsilon^{\alpha\beta\gamma\delta}$

In d dim.: $\mathrm{tr}\{\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma_5\} = 0$

Contributions with ϵ-tensor are UV-finite and gauge-invariant — Adler, Bardeen '69; Jones, Leveille '82; Jegerlehner '00

$\longrightarrow$ Compute this part in 4 dim.

Use photon mass to avoid (spurious) IR divergencies — Awramik, Czakon, Freitas '06

Reduction to master integrals

- Passarino-Veltman reduction (for sub-loop bubbles) — Weiglein, Scharf, Böhm ’94
- Integration by parts & Lorentz-invariance identities — Chertyrkin, Tkachov ’81; Gehrmann, Remiddi ’00
- Derivative of 2-loop self-energy:

$$T(p^2; m_1^2, m_2^2, m_3^2, m_4^2, m_5^2; \nu_1, \nu_2, \nu_3, \nu_4, \nu_5) = -\frac{(4\pi^2\mu^2)^{4-d}}{\pi^4}$$

$$\times \int \frac{d^d q_1 d^d q_2}{[q_1^2 - m_1^2]^{\nu_1}[(q_1+p)^2 - m_2^2]^{\nu_2}[(q_1-q_2)^2 - m_3^2]^{\nu_3}[q_2^2 - m_4^2]^{\nu_4}[(q_2+p)^2 - m_5^2]^{\nu_5}}$$

$$\frac{\partial T}{\partial(p^2)} = \frac{1}{2p^2} p^\mu \frac{\partial T}{\partial p^\mu} = -\frac{1}{2p^2}\Big[(\nu_2+\nu_5)T - \nu_2 T(\nu_1 - 1, \nu_2 + 1) - \nu_5 T(\nu_4 - 1, \nu_5 + 1)$$
$$+ (m_2^2 - m_1^2 + p^2)T(\nu_2 + 1) + (m_5^2 - m_4^2 + p^2)T(\nu_5 + 1)\Big].$$

$\longrightarrow$ Same class of functions, can then be reduced with i.b.p. identities

Two-loop UV divergencies

Subloop UV divergency in k_1 loop:

- Integrated subtraction term:

$$\int \widetilde{dk}_1 \widetilde{dk}_2\, G^{(2)}_{\rm sub} = -\Gamma(\varepsilon - 2)\mu^{2-\varepsilon} \int_0^1 dy_1 \ldots dy_{m-1} \underbrace{\int \widetilde{dk}_2\, C_j}.$$

one-loop integral
(same procedure as above)

- Remainder $I^{(2)}_{\rm rem} \equiv I^{(2)} - \int \widetilde{dk}_1 \widetilde{dk}_2\, G^{(2)}_{\rm glob} - \int \widetilde{dk}_1 \widetilde{dk}_2\, G^{(2)}_{\rm sub}$ is finite

- Feynman parameters for k_2 subloop
- Tensor reduction for k_2 terms
- Perform k_1 and k_2 integrals
  $\longrightarrow$ Numerical integral over Feynman parameters
- Deform integration contour as necessary

Electroweak precision tests: new physics

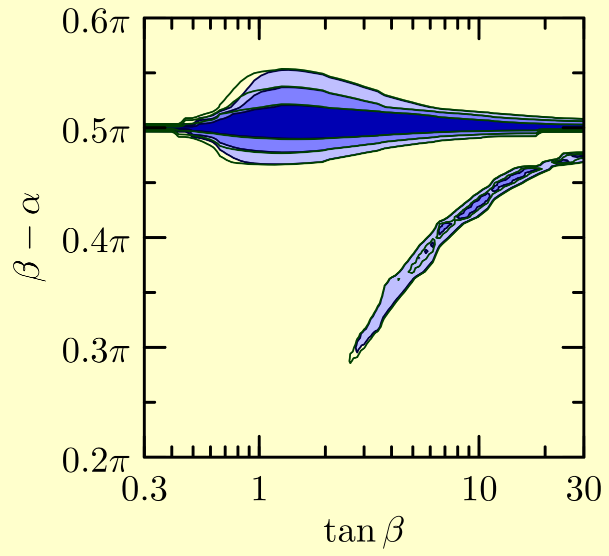
Constraints on Higgs physics:

- Indirect determination of M_{H}

- Couplings: need to go beyond SM, *e.g.* THDM:

$$\left|\frac{g_{hVV}^{\mathrm{THDM}}}{g_{hVV}^{\mathrm{SM}}}\right| = \sin(\beta-\alpha),$$

$$\left|\frac{g_{hff}^{\mathrm{THDM}}}{g_{hff}^{\mathrm{SM}}}\right| = \frac{\cos\alpha}{\sin\alpha} \text{ or } \frac{\sin\alpha}{\cos\alpha}$$



Eberhardt, Nierste, Wiebusch '13

- Couplings: effective operators

$$\mathcal{O}_W = (D^\mu\Phi)^\dagger W_{\mu\nu}(D^\nu\Phi)$$
$$\mathcal{O}_{WW} = \Phi^\dagger W^{\mu\nu}W_{\mu\nu}\Phi$$

etc.

Mebane, Greiner, Zhang, Willenbrock '13

Chen, Dawson, Zhang '13