The new PV prescription for IR singularities in NLO splitting functions

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Loops and Legs in Quantum Field Theory,
Weimar 27.04-2.05.2014

Plan

- Motivation
  - The KrkMC project
- Re-calculation of the NLO kernels
- New Principal Value prescription
- Results for NLO kernels
- The Axiloop package
- Summary

(Very) Long-term perspective

The NNLO + NLO Parton Shower for LHC

- LO Hard process + LO Shower
  Pythia, Herwig (1980-s)
- NLO Hard process + LO Shower
  MC@NLO, PowHEG (2000-s)

- NLO Hard process + NLO Shower
  KrkMC (Jadach et.al., ongoing)

- NNLO Hard process + NLO Shower
  ????????????????

Other developments: MINLO, Z. Nagy et.al., H. Tanaka et.al. ...

The KrkMC project

Based on collinear factorization. Requires:

- Reformulation of factorization in fully exclusive way
- Recalculation of the evolution kernels
  - exclusive
  - in four dimensions
  - well defined relation to MS-bar
- Kinematical mappings
- Reweighting procedure (positive, convergent)

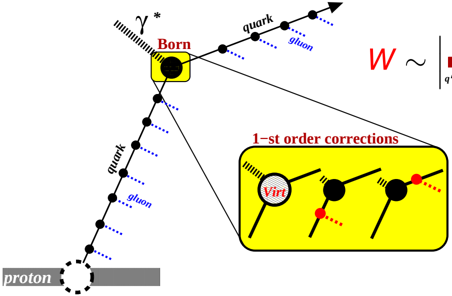
Axial gauge instrumental – allows for physical interpretation

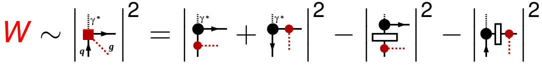
Here we discuss recalculation of the NLO evolution kernels

real-real ones – [JHEP 1108 (2011) 012]

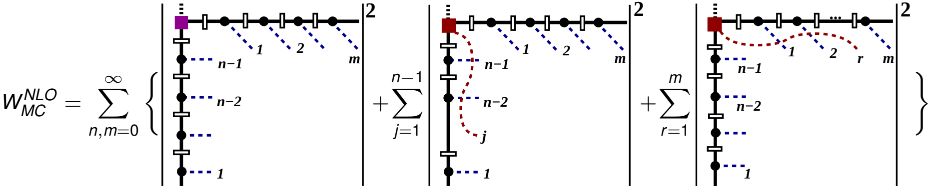
virtual-real ones – HERE, [PL B732 (2014) 218-222, arXiv:1403.6897]

NLO-corrected Hard process





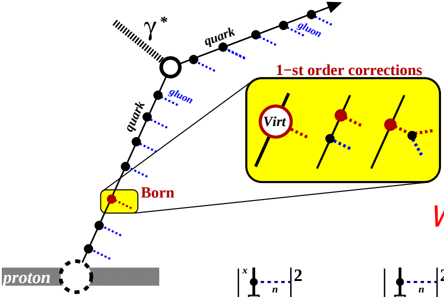
The KrkMC project

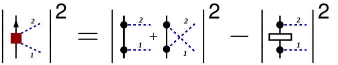


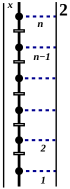
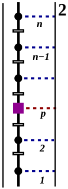
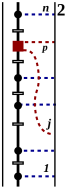
$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Fj})}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Bj})}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega},$$

NLO-corrected middle-of-the-ladder kernel, C_F^2

The KrkMC project



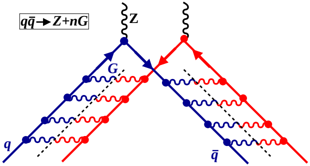
$W \sim$ 

$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$  $+ \sum_{p=1}^{n}$  $+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$  $\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$

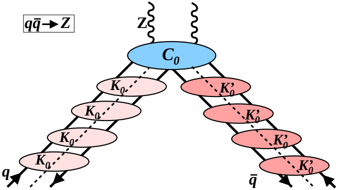
$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}.$$

Construction of the evolution kernel
in collinear factorization

LO cascade and
construction of the ladder

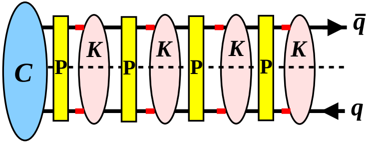


Include NLO and
group graphs in the ladder into
"two-particle-ireducible" sets K

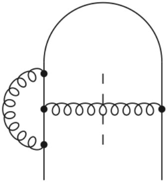


Use "projection operators" P to
split the ladder and extract kernels

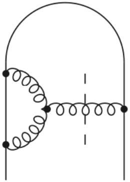
$$\Gamma_{qq} = \mathrm{Tr}\left[\frac{\hat{n}}{4nq} K \hat{p}\right]$$



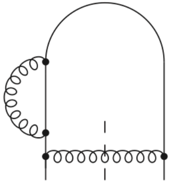
Contributions to Non Singlet P_{qq} kernel



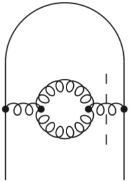
(c): $C_F^2 - \frac{1}{2} C_F C_A$



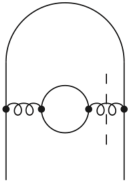
(d_{qq}): $\frac{1}{2} C_F C_A$



(e): C_F^2



(f): $C_F C_A$



(g): $C_F T_F$

PV prescription

☺ Axial gauge = physical interpretation as parton shower
☹ Axial gauge = spurious (unphysical) singularities

$$\text{gluon propagator:} \quad \frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{nl}\right)$$

Spurious singularities cancel in full set of diags, but need regularization
Curci, Furmanski, Petronzio [80] Ellis, Vogelsang [96], Heinrich, Kunszt [97]:

$$\text{Principal Value:} \quad \left[\frac{1}{nl}\right]_{PV} = \frac{nl}{(nl)^2 + \delta^2 (pl)^2}$$

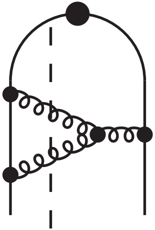
PV is more like "phenomenological rule"

Rigorous prescription: Mandelstam [83], Leibbrandt [84].
Difficult in calculations: Bassetto, Heinrich, Kunszt, Vogelsang [97].

Side remark: Linear denominators used in NNLO calculations of rapidity distributions of EW bosons [Anastasiou, Dixon, Melnikov, Petriello, 2004]

$$d^m k\, \delta\!\left(\frac{kp_1}{kp_2} - u\right) \to d^m k\, \frac{kp_2}{k(p_1 - up_2) - i0} - c.c.$$

Problem with real emission graph



Standard Heinrich, Kunszt [1998]:

$$N(\epsilon, Q^2)\left[\frac{P_{qq}(x)}{\epsilon^3} - 2I_0\frac{P_{qq}(x)}{\epsilon^2} + \frac{p_{qq}(x)}{\epsilon}\Big(-2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big)\right] + \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Not good for Parton Shower, needs Real-Virt. cancel.

Parton Shower oriented Jadach, et.al. [2011]:

$$\frac{p_{qq}(x)}{\epsilon}\Big(+2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\Big) + \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Good for PS, δ is a cut-off in 4-dimensions, easy to generate in MC:

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim \int_\delta^1 \frac{dx}{x} = -\ln\delta, \qquad I_1 = \int_0^1 \frac{dx\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta,$$

$$P_{qq}(x) = p_{qq} + \epsilon(1-x), \qquad p_{qq} = \frac{1+x^2}{1-x}$$

New use of PV prescription

Standard: regularize with PV only the gluon propagator
leave other singularities in (+)-component of integration momenta

$$\frac{d^m l}{l_+^{1-\epsilon}}, \qquad l_+ = \frac{nl}{np}$$

New proposal: regularize with PV all singularities of the integrand
in (+)-component of integration momenta, real & virtual

$$\frac{d^m l}{l_+^{1-\epsilon}} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \left(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \ldots\right)$$

All (+)-singularities cancel in the final expression (kernel), so extension of "phenomenological PV rule" of Curci-Furmanski-Petronzio possible

Example: virtual three point integral

Must perform (+)-integral as the last one, Ellis, Vogelsang [1996]:

$$\int \frac{d^m l}{(2\pi)^m} \; \frac{f(l_+)}{l^2(l-q)^2(l-p)^2} =$$

$$= \frac{-i}{16\pi^2 q^2}\left(\frac{4\pi}{-q^2}\right)^{-\epsilon}\frac{\Gamma(1-\epsilon)}{-\epsilon}\left[\int_0^x dy f(l_+)\frac{z^\epsilon(1-z)^\epsilon}{1-y}\left(1+2\epsilon\ln\frac{1-y}{1-z}\right)\right.$$

$$\left. + 2\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}(1-x)^{-\epsilon}\int_x^1 dy f(l_+)(1-y)^{-1+2\epsilon}\right],$$

$$x = q_+/p_+, \quad y = l_+/p_+, \quad z = y/x, \quad p^2 = (p-q)^2 = 0, \quad m = 4+2\epsilon,$$

Singularities at $y = 0$ and $y = x$: only from gluon propagator.
Singularity at $y = 1$: not from gluon propagator!

Proposal: treat all (+)-singularities on equal footing

Note: (+)-singularities lead to $1/\epsilon^3$ poles in kernel

Example: scalar non-axial integral

kinematics: $p^2 = (p-q)^2 = 0$

$$J_3^{\mathrm{F}} = \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (q-l)^2 (p-l)^2}$$

The PV prescription:

$$J_3^{\mathrm{F}} = C\left(-\frac{1}{\epsilon^2} + \frac{\pi^2}{6}\right), \qquad C = i\frac{\Gamma(1-\epsilon)}{(4\pi)^2 |q^2|}\left(\frac{4\pi}{|q^2|}\right)^{-\epsilon}$$

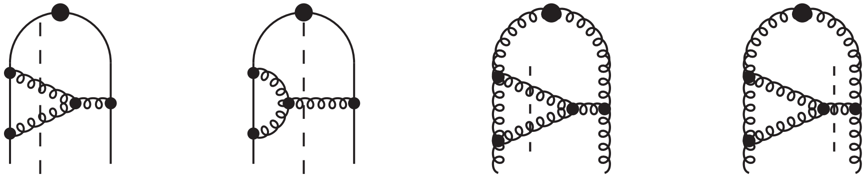
New PV prescription:

$$J_3^{\mathrm{F}} = C\left(-\frac{2I_0 + \ln(1-x)}{\epsilon} - 4I_1 + 2I_0 \ln(1-x) + \frac{\ln^2(1-x)}{2}\right),$$

$\frac{1}{\epsilon^2}$ replaced by I_1 and $\frac{1}{\epsilon} I_0$

NLO kernels P_{qq} and P_{gg} in New PV scheme

There are only four graphs with $1/\epsilon^3$ singularity:

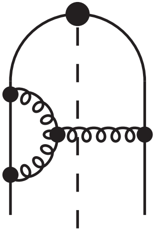


$\tilde{\Gamma}^{(d_R)}_{qq}(x,\epsilon)$ $\tilde{\Gamma}^{(d_V)}_{qq}(x,\epsilon)$ $\tilde{\Gamma}^{(d_R)}_{gg}(x,\epsilon)$ $\tilde{\Gamma}^{(d_V)}_{gg}(x,\epsilon)$

related to P_{qq} and P_{gg} splitting functions in a standard way:

$$\tilde{\Gamma}_{qq(gg)}(x,\epsilon) = \delta_{1-x} + \frac{1}{\epsilon}\left(\frac{\alpha_S}{2\pi}P^{LO}_{qq(gg)}(x) + \frac{1}{2}\left(\frac{\alpha_S}{2\pi}\right)^2 P^{NLO}_{qq(gg)}(x) + \dots\right) + \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$

Virtual contribution to NLO P_{qq} kernel in NPV



The virtual graph contributes:

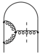
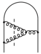
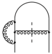
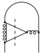
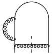
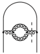
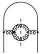
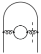
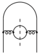
$$\tilde{\Gamma}_{qq}^{(d_V)}(x,\epsilon) = -\frac{1}{\epsilon^2} P_{qq}\big(1+\epsilon \ln(1-x)\big)\tilde{Z}_{d_V}$$

$$-\frac{1}{\epsilon} p_{qq}\Big[I_0\big(2\ln x + 2\ln(1-x)\big) - 6 I_1 - \mathrm{Li}_2(1-x) + \ln^2 x - 3 + \frac{8}{12}\pi^2 \Big] + \frac{1}{\epsilon}\frac{1}{2}\frac{1+x}{1-x},$$

$$\tilde{Z}_{d_V} = 4 I_0 + 2\ln(1-x) + \ln x - \frac{3}{2},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}_{qq}^{(d)}$ identical as in standard PV scheme

Contributions to inclusive kernel P_{qq} in NPV prescr.

			SUM			SUM				SUM			SUM
	$(d):\ {}^1\!/_2\, C_F C_A$			$(c):\ C_F^2 - {}^1\!/_2\, C_F C_A$			$(e):\ C_F^2$	$(f):\ {}^1\!/_2\, C_F C_A$			$(g):\ C_F T_F$		
Double poles													
p_{qq}	-6	0	-6	-6	0	-6	6	44/3	$-22/3$	22/3	$-8/3$	4/3	$-4/3$
$p_{qq}\ \ln x$	4	0	4	4	0	4	-8	0	0	0	0	0	0
$p_{qq}\ \ln(1-x)$	8	0	8	0	0	0	0	-16	8	-8	0	0	0
$p_{qq}\ I_0$	16	0	16	8	0	8	-8	-16	8	-8	0	0	0
Single poles													
p_{qq}	-7	-4	-11	-7	0	-7	7	0	103/9	103/9	0	$-10/9$	$-10/9$
$p_{qq}\ \ln x$	0	$-3/2$	$-3/2$	0	$-3/2$	$-3/2$	0	0	11/3	11/3	0	$-2/3$	$-2/3$
$p_{qq}\ \ln(1-x)$	-3	8	5	-3	0	-3	3	22/3	$-34/3$	-4	$-4/3$	4/3	0
$p_{qq}\ \ln^2 x$	2	-1	1	2	-1	1	-2	0	0	0	0	0	0
$p_{qq}\ \ln x \ln(1-x)$	2	4	6	2	0	2	-4	0	-4	-4	0	0	0
$p_{qq}\ \ln^2(1-x)$	4	-2	2	0	0	0	0	-8	6	-2	0	0	0
$p_{qq}\ \mathrm{Li}_2(1)$	8	-2	6	4	0	4	-4	0	-4	-4	0	0	0
$p_{qq}\ \mathrm{Li}_2(1-x)$	-2	2	0	2	-2	0	0	0	0	0	0	0	0
$1-x$	$-5/2$	3/2	-1	$-7/2$	$-15/2$	-11	3	22/3	-4	10/3	$-4/3$	0	$-4/3$
$(1-x)\ \ln x$	2	0	2	2	0	2	-4	0	0	0	0	0	0
$(1-x)\ \ln(1-x)$	4	0	4	0	0	0	0	-8	4	-4	0	0	0
$1+x$	$-1/2$	1/2	0	1/2	$-1/2$	0	0	0	0	0	0	0	0
$(1+x)\ \ln x$	0	1/2	1/2	0	$-7/2$	$-7/2$	0	0	0	0	0	0	0
Spurious poles													
$p_{qq}\ I_0$	0	8	8	0	0	0	0	0	-4	-4	0	0	0
$p_{qq}\ I_0 \ln x$	4	4	8	4	0	4	-4	0	-4	-4	0	0	0
$p_{qq}\ I_0 \ln(1-x)$	12	-4	8	4	0	4	-4	-8	4	-4	0	0	0
$p_{qq}\ I_1$	-12	4	-8	-4	0	-4	4	0	4	4	0	0	0
$(1-x)\ I_0$	8	0	8	4	0	4	-4	-8	4	-4	0	0	0

Columns "SUM" agree with standard PV scheme

Exclusive contributions to P_{qq}

Part C_F^2:

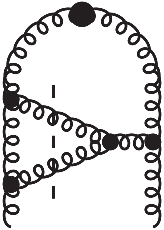
$$\alpha_S^2\, C_F^2\, \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\, \frac{1}{|k^2|} \left\{ \frac{1}{\epsilon}\, 4 \ln x \left(\left(\frac{|k^2|}{\mu_R^2} \right)^\epsilon - 1 \right) P_{qq} \right.$$
$$\left. + \left(p_{qq}\, 4\, \mathrm{Li}(1-x) - (1-x) + (1+x) \right) \left(\frac{|k^2|}{\mu_R^2} \right)^\epsilon \right\}$$

Parts $C_F C_A$, $C_F T_F$:

$$\alpha_S^2\, C_F\, \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\, \frac{1}{|k^2|} \left\{ \frac{1}{\epsilon} \left[C_A \frac{11}{3} - T_F \frac{4}{3} - 4\, C_A \left(\ln(1-x) + I_0 \right) \left(\frac{|k^2|}{\mu_R^2} \right)^\epsilon \right] P_{qq} \right.$$
$$\left. + 4\, C_A \left[p_{qq} \left(\mathrm{Li}(1) - \mathrm{Li}(1-x) + I_0 \ln(1-x) - 2 I_1 \right) - \frac{x}{2} \right] \left(\frac{|k^2|}{\mu_R^2} \right)^\epsilon \right\}$$

Monte Carlo friendly

Real contribution to NLO P_{gg} kernel in NPV



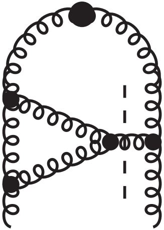
Only ϵ^{-1} poles
→ the calculation can be done in 4-dimensions,
→ much simpler than in standard PV

The real graph in New PV prescription:

$$\tilde{\Gamma}_{gg}^{(d_R)}(x,\epsilon) = C_S^{(d_R)} \frac{1}{\epsilon} \Bigg[P_{gg} \Bigg(-4 I_1 + 4 I_0 \big(\ln(1-x) - \ln(x) - 2\big) + 2\ln^2(1-x) + 2\ln^2(x) - 4\ln(x)\ln(1-x) - 8\ln(1-x) + \frac{11}{3}\ln(x) + 2\frac{\pi^2}{6} + 4 \Bigg) + \ln(x)\Big(\frac{11}{3}x^2 + \frac{23}{6}x + \frac{23}{6} + \frac{11}{3x}\Big) - \frac{22}{3}x^2 + \frac{24}{3}x - \frac{25}{3} + \frac{22}{3x} \Bigg].$$

Virtual contribution to NLO P_{gg} kernel in NPV

The virtual graph in New PV prescription:



$$\Gamma^{(d_V)}_{gg,NPV}(x,\epsilon) = C_S^{(d_V)} P_{gg}\Bigg[\frac{1}{\epsilon^2}\big(1+\epsilon\ln(1-x)\big)\tilde{Z}^V_{GS} + \frac{1}{\epsilon}\Big(4I_0\ln(1-x) + 8I_0\ln(x) - 16I_1 + 4\ln^2(x) + 12\frac{\pi^2}{6} - \frac{134}{9}\Big)\Bigg] - C_S^{(d_V)}\frac{1}{3\epsilon}x$$

$$\tilde{Z}^V_{GS} = 12I_0 + 4\ln(1-x) + 4\ln(x) - \frac{22}{3},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}^{(d)}_{gg}$
identical as in standard PV scheme

Both schemes, PV and New PV,
give the same P_{qq} and P_{gg} kernels

Axiloop

Fully automated package for symbolic calculation
of NLO kernels in axial (light-cone) gauge

- Written in `Mathematica`

```
$ math
Mathematica 9.0 for Linux x86 (64-bit)
Copyright 1988-2013 Wolfram Research, Inc.

In[1]:= << Axiloop`;

In[2]:= Axiloop`$Version

Out[2]= Axiloop 2.3 (Mar 2014)
```

- Can be installed from here

```
curl http://raw.github.com/gituliar/Axiloop/master/install.sh|sh
```

The Axiloop package

- Library of integrals, in PV and NPV prescriptions
- One-loop integration and renormalisation
  (keeping track of the UV and IR poles)

```
In[2]:= $Get[
  IntegrateLoop[ l.k/(l.l (l+k).(l+k) (l+p).(l+p)), l,
    SimplifyNumeratorAndDenominator -> True]
  ,
  {"integrated", "short"}
]

             Qv[k] R0[eir]   Qv[p] T0[euv]   Qv[q] T0[euv]
Out[2]= -   ------------- + ------------- - -------------
                  2               2               2
```

The `Axiloop` package

- One-particle FS integration

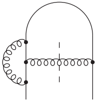
```
In[1]:= WrL0qq = 2 g^2 (1+x^2 + (1-x)^2 eps)/((1-x) k.k);

In[2]:= IntegrateLeg[ WrL0qq, k ]

                2    2 eps          2             2
        - 2 g   (Q )     Qr (1+x  + eps (1-x) ) (1 + eps Log[1-x])
Out[2]= ---------------------------------------------------------
                               eps (1-x)
```

Axiloop - example

Input



```
x G[n]/(4k.n)**FP[k]**FV[i1]**FP[l+k]**FV[mu]**FP[l+p]**
  FV[i2]**GP[i1,i2,l]**FPx[p]**GPx[mu,nu,p-k]**FV[nu]**FP[k]
```

Exclusive kernel (bare)

```
I g^4 / ((1-x)(-k.k)) (
  Qv[p](
    B0[euv] (-2(1+x^2 + (1-x)^2eps)) +
    B1[euv] 2x(x - (1-x)eps) +
    T0[euv] (3-2x^2 + (1-2x^2)eps - 2(1-x)eps^2)) +
  Qv[k](
    P0[euv] (-6(1+x^2 + (1-x)^2eps)) +
    T0[euv] x(4+5x - (2+x)eps + 2(1-x)eps^2) +
    R0[eir] (-2(1+x^2 + (1-x)^2eps)) +
    S0[eir] 2(1+x^2 + (1-x)^2eps) +
    T0[eir] (6-4x+x^2 + (4-8x+3x^2)eps - 2(2-3x+x^2)eps^2)) +
  Qv[q](
    K0[euv] (-2(1-x+x^2 + (1-3x+2x^2)eps)/(1-x)) +
    T0[euv] (1-x)(3-x + (1-3x)eps - 2(1-x)eps^2) +
    V1[euv] 2x^2(x - (1-x)eps) +
    V2[euv] 2x^2(x - (1-x)eps)))
```

Summary

- We calculated NLO virtual one-loop components of NS P_{qq} kernel in new, MC-friendly scheme, in inclusive and exclusive forms.
- We modified the way of using the PV prescription in the light-cone gauge by applying it to all the singularities in the plus component of the integration momentum.
- In the New PV prescription the inclusive NLO kernels P_{qq} and P_{gg} agree with the standard PV ones.
- Partial results of four graphs contributing to the kernels, differ in PV and New PV: the $1/\epsilon^3$ poles, present in PV, are replaced by $(1/\epsilon)\ln^2\delta$ etc.
- Most of real graphs, free of $1/\epsilon^3$ and $1/\epsilon^2$ poles, can be calculated in four dimensions and are usable for the stochastic simmulations.
- The higher order poles cancel separately for real and for virtual components.