Top and bottom production near threshold at third order

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Loops and Legs in Quantum Field Theory
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Outline

- Introduction and theoretical framework
- Upsilon(1S) leptonic width

  MB, Kiyo, Marquard, Penin, Piclum, Seidel, Steinhauser, 1401.3005 [hep-ph]
- Top production near threshold in e^+e^- collisions

  MB, Kiyo, Schuller, 1312.4791 [hep-ph] and in preparation;
  MB, ..., Steinhauser, in preparation
- Effective field theory of unstable particle production, non-resonant and P-wave effects

  MB, Piclum, Rauh, 1312.4792 [hep-ph]; MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph];
  Jantzen, Ruiz-Femenia 1307.4337 [hep-ph]

- Bound states (below)
  - $\Upsilon(1S)$
  - Quarkonium production and annihilation (only partly weak coupling)
- Production (above)
  - $e^+e^- \to t\bar{t}X$
  - Top-pair and sparticle pair production in hadron collisions ("threshold resummation")
- Annihilation (above)
  - Dark matter annihilation

Third order only for e^+e^-.

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2) = i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(j_\mu(x)j_\nu(0))|0\rangle, \qquad j^\mu(x) = [\bar{Q}\gamma^\mu(\gamma_5)Q](x)$$

Spectral function

Non-perturbative but weak coupling. Expansion in α_s and $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{\sqrt{q^2} - 2m_t}{m_t}}$, while $\alpha_s/v = O(1)$

$$R \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \Big\{ 1\,(\text{LO});\, \alpha_s, v\,(\text{NLO});\, \alpha_s^2, \alpha_s v, v^2\,(\text{NNLO}); \ldots \Big\}$$

Summation through Schrödinger equation.

$$\text{Im}\,\Pi(E) = \frac{N_c}{2m^2} \underbrace{\sum_{n=1}^{\infty} Z_n \times \pi\delta(E_n - E)}_{\text{bound states}} + \Theta(E)\, \underbrace{\text{Im}\,\Pi(E)_{\text{cont}}}_{\text{continuum}}$$

$$R \equiv \frac{\sigma_{e^+e^- \to WWb\bar{b}X}}{\sigma_0} = 12\pi e_t^2\, \text{Im}\,\Pi(E + i\Gamma_t) + \text{non-resonant}$$

First calculate $\Pi(\mathcal{E})$ for complex $\mathcal{E} = E + i\Gamma$, then take $\Gamma \to 0$.

Tools

Non-relativistic effective field theory and threshold expansion (defines the matching procedure!)
See arXiv:1312.4791 [hep-ph]

Relevant scales: $m_t \approx 175\,\mathrm{GeV}$ (hard), $m_t\alpha_s \approx 30\,\mathrm{GeV}$ (soft, potential) and the ultrasoft scale (us) $m_t\alpha_s^2 \approx 2\,\mathrm{GeV}$.

$$\mathcal{L}_{\mathrm{QCD}}\,[Q(h,s,p),\; g(h,s,p,us)] \qquad \mu > m_t$$

$$\downarrow$$

$$\mathcal{L}_{\mathrm{PNRQCD}}\,[Q(p),\; g(us)] \qquad \mu < m_t v$$

Ultrasoft scale appears explicitly only at NNNLO. Calculation of the (non-logarithmic) NNNLO correction is needed.

$$\Pi^{(v)}(q^2) = \frac{N_c}{2m^2}\,c_v\left[c_v - \frac{E}{m}\left(c_v + \frac{d_v}{3}\right)\right]G(E) + \ldots$$

$$G(E) = \frac{i}{2N_c(d-1)}\int d^dx\, e^{iEx^0}\,\langle 0|\,T(\,[\chi^\dagger\sigma^i\psi](x)\,[\psi^\dagger\sigma^i\chi](0))|0\rangle_{|\mathrm{PNRQCD}}\,,$$

3rd order ingredients

- Bound state quantities (S-wave)
  - E_n – Kniehl, Penin, Smirnov, Steinhauser (2002); MB, Kiyo, Schuller (2005); Penin, Sminrov, Steinhauser (2005)
  - $|\psi_n(0)|^2$ – MB, Kiyo, Schuller (2007); MB, Kiyo, Penin (2007)
- Matching coefficients
  - a_3 – Anzai, Kiyo, Sumino (2009); Smirnov, Sminrov, Steinhauser (2009)
  - c_3 – Marquard, Piclum, Seidel, Steinhauser (2013) [2009]
- Continuum (PNRQCD correlation function)
  - ultrasoft – MB, Kiyo (2008)
  - potential – MB, Kiyo, Schuller, in preparation [2007]

Note: logarithmically enhanced 3rd order terms known before or resummed [Hoang et al. 2001-2013; Pineda et al. 2002-2007]
2nd order available since end of 1990s.

I. $\Upsilon(1S)$ leptonic width

MB, Kiyo, Marquard, Penin, Piclum, Seidel, Steinhauser,
1401.3005 [hep-ph], PRL **112** 151801 (2014)

Simplest quantity that involves bound-state and short-distance dynamics (annihilation) already at leading order.

$$\Gamma(\Upsilon(nS) \to \ell^+\ell^-) \;=\; \frac{4\pi\alpha^2}{9m_b^2}\,|\psi_n(0)|^2\, c_v \left[c_v - \frac{E_n^p}{m_b}\left(c_v + \frac{d_v}{3}\right) + \ldots\right]$$

$$+ \text{ non-perturbative condensate corrections}$$

$$\left|\psi_n^{\rm LO}(0)\right|^2 = \frac{8m_b^3\alpha_s^3}{27\pi n^3} \qquad E_n^{p,\rm LO} = -\frac{4m_b\alpha_s^2}{9n^2}$$

Only the $n = 1$ state is dominated by weak-coupling bound-state dynamics. Even here $m_b v^2 \gg \Lambda_{\rm QCD}$ is questionable.

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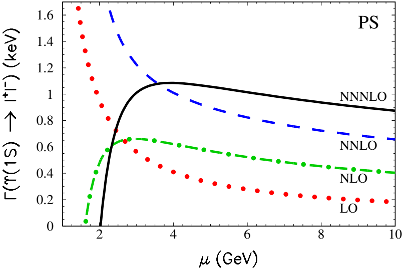
Condensate corrections [Leutwyler (1981), Voloshin (1982), Pineda (1997)]

$$\delta_{\rm np}|\psi_1(0)|^2 = |\psi_1^{\rm LO}(0)|^2 \times 17.54\pi^2 K \qquad \text{with} \quad K = \frac{\langle \frac{\alpha_s}{\pi} G^2\rangle}{m_b^4(\alpha_s C_F)^6}$$

$$M_{\Upsilon(1S)} = 2m_b + E_1^{\rm p} + \frac{624\pi^2}{425}\,m_b(\alpha_s C_F)^2\,K\,,$$

$\Upsilon(1S)$ leptonic width at 3rd order

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm pole}$$
$$= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1 + \alpha_s\,(-2.003 + 3.979\,L)$$
$$+ \alpha_s^2\left(9.05 - 7.44\ \ln\alpha_s - 13.95\,L + 10.55\,L^2\right)$$
$$+ \alpha_s^3\Big(-0.91 + 4.78_{a_3} + 22.07_{b_2\epsilon} + 30.22_{c_f}$$
$$- 134.8(1)_{c_g} - 14.33\ \ln\alpha_s - 17.36\ \ln^2\alpha_s$$
$$+ (62.08 - 49.32\ \ln\alpha_s)\,L - 55.08\,L^2$$
$$+ 23.33\,L^3\Big) + \mathcal{O}(\alpha_s^4)\Big]$$
$$= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1 + 1.166\alpha_s + 15.2\alpha_s^2 + \Big(66.5 + 4.8_{a_3}$$
$$+ 22.1_{b_2\epsilon} + 30.2_{c_f} - 134.8(1)_{c_g}\Big)\,\alpha_s^3 + \mathcal{O}(\alpha_s^4)\Big]$$
$$= \frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\,[1 + 0.28 + 0.88 - 0.16]$$



at $\mu = 3.5$ GeV. $L = \ln\left(\mu/(m_b C_F\alpha_s)\right)$.
Similar behaviour in PS mass scheme.

$\Upsilon(1S)$ leptonic width at 3rd order

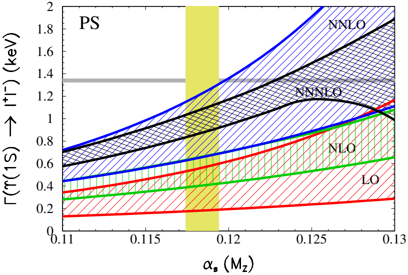
$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm PS}$$
$$= \frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}\,[1 + 0.37 + 0.95 - 0.04]$$
$$= [1.08 \pm 0.05(\alpha_s)^{+0.01}_{-0.20}(\mu)]\ \text{keV}$$

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm exp} = 1.340(18)\ \text{keV}$$

Can the missing contribution be reliably estimated from the condensate expansion?

- $K = \dfrac{\langle \frac{\alpha_s}{\pi} G^2\rangle}{m_b^4(\alpha_s C_F)^6}$ uncertain by factor 20.
- $\delta_{\rm np}\Gamma_{\ell\ell}(\Upsilon(1S)) = \dfrac{4\alpha^2\alpha_s}{9}\,\dfrac{17.54\times 425}{3744}\,\delta M^{\rm np}_{\Upsilon(1S)}$

$$\approx\ [1.28^{+0.17}_{-0.18}(\alpha_s) \pm 0.42(m_b)^{+0.20}_{-0.57}(\mu) \pm 0.12(m_c)]\ \text{keV}\,.$$



Unfortunately not. (But depends on precise value of m_b!)

m_b from Upsilon(1S) mass and Upsilon sum rules

- Determine the PS bottom mass from the $\Upsilon(1S)$ mass.

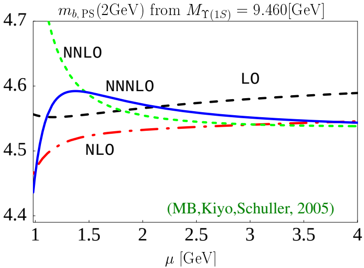
  $\overline{\text{MS}}$ mass: $\bar{m}_b(\bar{m}_b) = 4.22(8)$ GeV

  (including charm mass effect)

  Error dominated by unknown condensate correction. Attribute difference to

  $\bar{m}_b(\bar{m}_b) = 4.163(16)$ GeV [Chetyrkin et al., 2009]

  to $\delta M^{\text{np}}_{\Upsilon(1S)}$.



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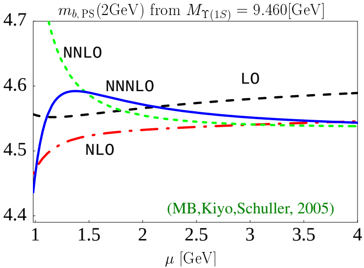


- Bottom mass from sum rules

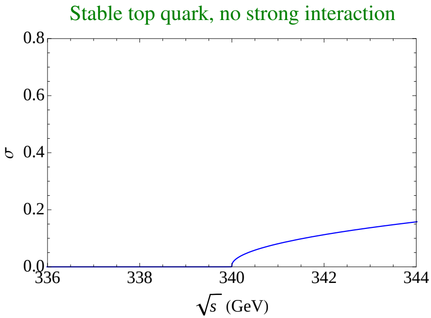
$$\int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)_{|\text{th}} = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)_{|\text{exp}}$$

  - Non-perturbative corrections negligible (bound states + continuum). Relies on global duality $\Longrightarrow$ cannot chose n too large ($n \sim 10$).
  - Third order continuum spectral function is needed. In preparation. [MB, Kiyo, Schuller; MB, Maier, Piclum, Rauh]. N3LO bound states + NNLO continuum: [Penin, Zerf, 2014]

II. Top production near threshold in e^+e^- collisions

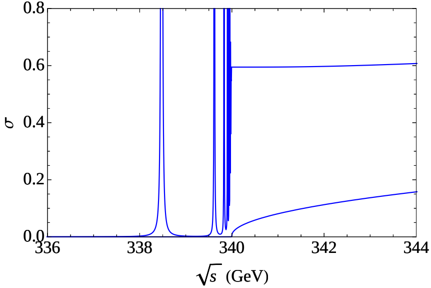
MB, Kiyo, Schuller, 1312.4791 [hep-ph] and in preparation;
MB, ..., Steinhauser, in preparation

Ultra-precise mass measurement
Unique QCD dyanmics



Ultra-precise mass measurement
Unique QCD dyanmics

Stable top quark, with strong Coulomb force



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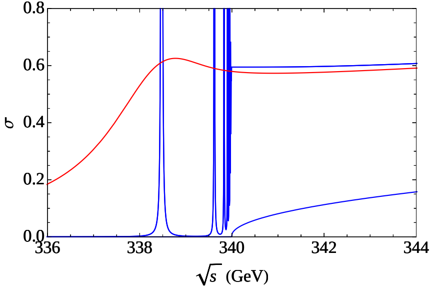
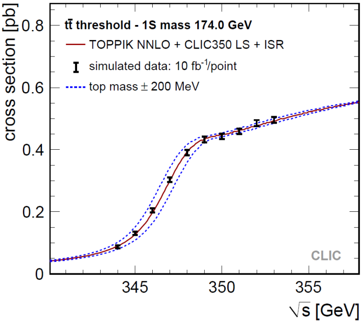


Smallest structure in particle physics known to exist (10^{-17}m).
Direct "spectroscopic" mass and width measurement.

Threshold scan at ILC

Additional smearing of the resonance due to beam luminosity spectrum (collider-specific) and ISR



Most recent study for ILC/CLIC [Seidel, Simon, Tesai, Poss, 2013] assumes $10\,\mathrm{fb}^{-1}$ at 10 points.

$$[\delta m_t]_{\rm thr} = 27\,\mathrm{MeV} \qquad \text{[simultaneous fit of } \alpha_s\text{]}$$

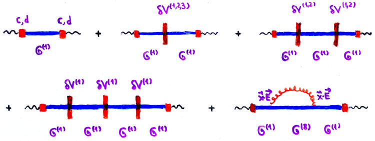
Key items for theoretical precision

- Mass definition: Potential-subtracted mass [MB, 1998]

$$m_{\rm PS}(\mu_f) \equiv m_{\rm pole} + \frac{1}{2}\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\, \tilde{V}_{\rm Coulomb}(\vec{q}\,)$$

Cancellation of large perturbative contributions from the IR.
$m_{t,\rm PS}(20\,{\rm GeV}) = 171.5\,{\rm GeV}$.

- 3rd order perturbation theory around Coulomb background. Consistent extraction of the divergence in dim. reg.

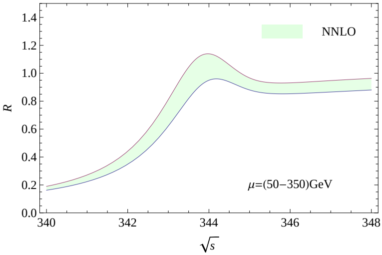


$$G_c^{(1,8)}(\boldsymbol{r},\boldsymbol{r}\,',E) = \frac{my}{2\pi} e^{-y(r+r')} \sum_{l=0}^{\infty}(2l+1)(2yr)^l(2yr')^l P_l\left(\frac{\boldsymbol{r}\cdot\boldsymbol{r}\,'}{rr'}\right) \sum_{s=0}^{\infty} \frac{s!L_s^{(2l+1)}(2yr)L_s^{(2l+1)}(2yr')}{(s+2l+1)!(s+l+1-\lambda)}$$

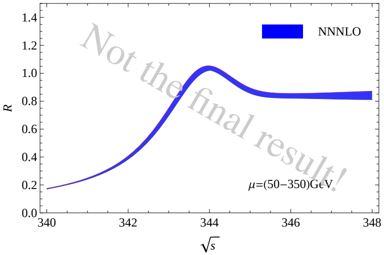
$$y = \sqrt{-m(E+i\epsilon)},\ \lambda = \frac{m\alpha_s}{2y} \times \{C_F\,(\text{singlet});\, C_F - C_A/2\,(\text{octet})\}$$

Cross section near threshold, from 2nd to 3rd order

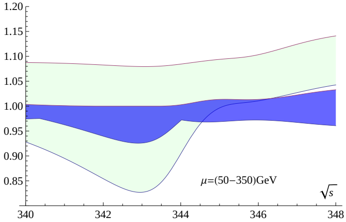
[MB, Signer, Smirnov, 1999]



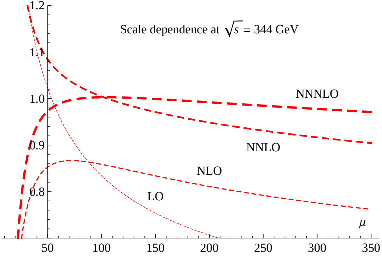
[MB, Kiyo, Schuller; MB ... Steinhauser, in progress]



$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV},\ \Gamma_t = 1.33\,\mathrm{GeV}$

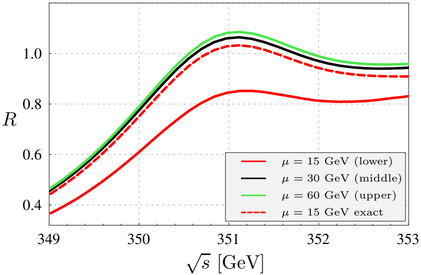


Scale issues



No convergence for $\mu \lesssim 50\,\text{GeV}$.

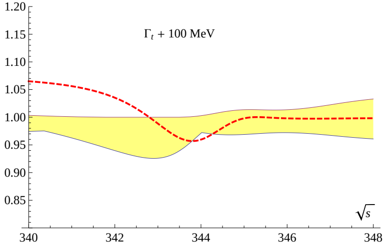
[MB, Kiyo, Schuller (2005)]

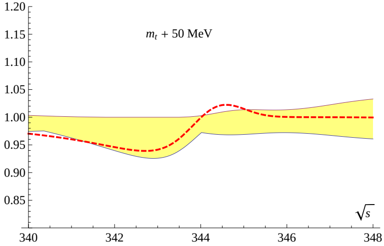


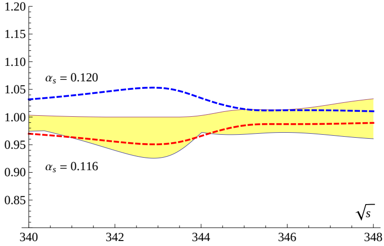
Compare recent NNLO+(N)NLL analysis [Hoang, Stahlhofen, 2013]

Coulomb corrections only (for $m_{t,\text{PS}} = 175\,\text{GeV}$, $\Gamma_t = 1.5\,\text{GeV}$). Scale depedence at third order and exact solution.

Parameter dependence







III. Non-resonant, finite-width, P-wave effects

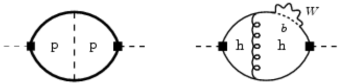
MB, Piclum, Rauh, 1312.4792 [hep-ph]; MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph]; Jantzen, Ruiz-Femenia 1307.4337 [hep-ph]

Finte-width divergences and "electroweak effects"

The QCD-only result usually discussed is far from reality

- Finite-width divergences (overall log divergence, already at NNLO):

$$[\delta G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$



Since $E = \sqrt{s} - 2m_t + i\Gamma$, the divergence survives in the imaginary part:

$$\text{Im}\,[\delta G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon}$$

- Electroweak effect. Must consider $e^+e^- \to W^+W^-b\bar{b}$.

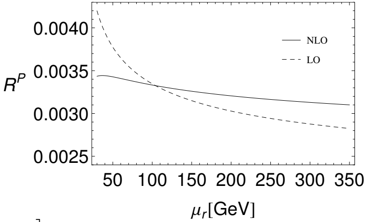
$$\sigma_{e^+e^- \to W^+W^-b\bar{b}} = \underbrace{\sigma_{e^+e^- \to [t\bar{t}]_{\text{res}}}(\mu_w)}_{\text{pure (PNR)QCD}} + \sigma_{e^+e^- \to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$$

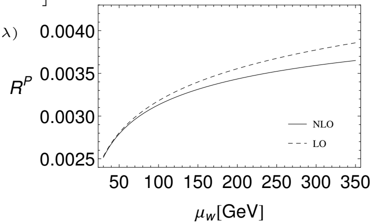
Non-resonant starts at NLO (overall linear divergence) [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]. Finite-width scale dep must cancel. Need consistent dim reg calculation.

P-wave contribution at NLO (N3LO for total cross section)

$$R = 12\pi\,\mathrm{Im}\left[e_t^2\Pi^{(v)}(q^2) - \frac{2q^2}{q^2-M_Z^2}v_e v_t e_t \Pi^{(v)}(q^2) + \left(\frac{q^2}{q^2-M_Z^2}\right)^2 (v_e^2+a_e^2)(v_t^2\Pi^{(v)}(q^2) + a_t^2\Pi^{(a)}(q^2))\right]$$

$$\begin{aligned}
\delta_1 G^P(E) = &-\frac{m_t^4\alpha_s^4 C_F^3}{64\pi^2\lambda^2}\\
\times\Bigg\{&\beta_0\Bigg[\left(-\frac{1}{12\epsilon^2}+\frac{59}{9}+\frac{5\pi^2}{72}+4L^r_\lambda+2L^r_\lambda L^w_\lambda-\left(L^w_\lambda\right)^2\right)+\left(9+6L^r_\lambda\right)\lambda\\
&+\left(\frac{3}{40\epsilon^2}+\frac{1}{20\epsilon}-\frac{344}{15}-\frac{\pi^2}{8}-\frac{21}{2}L^r_\lambda+\frac{1}{2}L^w_\lambda-6L^w_\lambda L^r_\lambda+3\left(L^w_\lambda\right)^2\right)\lambda^2\\
&+\left[-4-6\lambda+10\lambda^2+2(3\lambda^2-1)L^r_\lambda\right]\hat\psi(2-\lambda)\\
&+\left(\lambda-\lambda^3\right)\left[\psi_1(2-\lambda)\left(\frac{22}{3}+2L^r_\lambda-2\hat\psi(2-\lambda)\right)+\psi_2(2-\lambda)\right]\\
&+\left(3\lambda^2-1\right)\left[3\psi_1(2-\lambda)-\hat\psi(2-\lambda)^2\right]+\frac{3}{4-2\lambda}\,{}_4F_3(1,1,4,4;5,5,3-\lambda;1)\Bigg]\\
&+a_1(\epsilon)\left[\frac{1}{6\epsilon}+2+L^w_\lambda+3\lambda-\left(\frac{3}{10\epsilon}+\frac{26}{5}+3L^w_\lambda\right)\lambda^2+\left(3\lambda^2-1\right)\hat\psi(2-\lambda)\right.\\
&\left.+\left(\lambda-\lambda^3\right)\psi_1(2-\lambda)\right]\Bigg\}.
\end{aligned}$$





[MB, Piclum, Rauh 2013]

Earlier numerical, non-dim reg result [Penin, Pivovarov, 1999]

From QCD $t\bar{t}$ threshold to a realistic prediction

At 3rd order QCD effects under control. Then

- $\sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$ to (at least) NNLO in consistent scheme.
  Mostly inclusive.

  NLO [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]
  Partial results only at NNLO [Hoang, Reisser, Ruiz-Femenia, 2010; Jantzen, Ruiz-Femenia, 2013]

- QED effects (from NLO) [Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010]

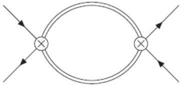
- Electroweak matching coefficients absorptive parts [Hoang, Reisser, 2004]

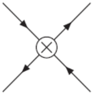
- Initial state radiation (formalism in MB, Falgari, Schwinn, Signer, Zanderighi, 2007).

- Higgs contributions [Eiras, Steinhauser, 2006]

Finally, need four-loop (formally even five-loop) relation between the pole and the MSbar mass to convert the precise value for the PS mass to an equally precise value of the MSbar mass.

Unstable particle EFT provides a systematic expansion of the amplitude in powers of Γ/m.





Resonant contributions

Production of an on-shell, non-relativistic $t\bar{t}$ pair and subsequent decay $t \to W^+ b$. Effective non-relativistic propagator contains on-shell width.

Non-resonant contributions

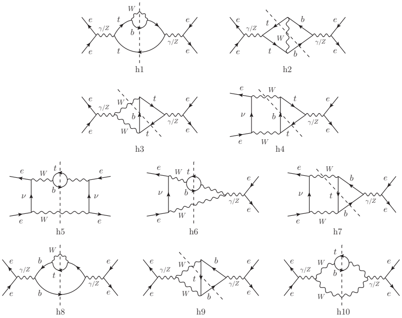
All-hard region. Off-shell lines. Full theory diagrams expanded around $s = 4m_t^2$. No width in propagators.

$$i\mathcal{A} = \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x\, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0)\, i\mathcal{O}_p^{(l)}(x)] | e^- e^+ \rangle + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

$$\mathcal{O}_p^{(v,a)} = \bar{e}_{c_2} \gamma_i (\gamma_5) e_{c_1}\, \psi_t^\dagger \sigma^i \chi_t \qquad \mathcal{O}_{4e}^{(k)} = \bar{e}_{c_1} \Gamma_1 e_{c_2}\, \bar{e}_{c_2} \Gamma_2 e_{c_1},$$

$$\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \mathrm{Im}\left[C_{4e}^{(k)}\right] \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

Separately divergent and factorization ("finite-width") scale-dependent.



Equivalent to the dimensionally regulated $e^+e^- \to bW^+\bar{t}$ process with $\Gamma_t = 0$, expanded in the hard region around $s = 4m_t^2$.

$$\int_{\Delta^2}^{m_t^2} dp_t^2\,(m_t^2 - p_t^2)^{\frac{d-3}{2}} H_i\Big(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\Big)$$

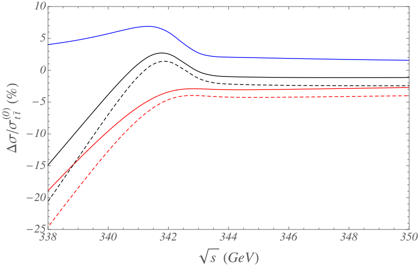
$$p_t^2 \equiv (p_b + p_{W+})^2$$

$$H_1\Big(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\Big) \overset{p_t^2 \to m_t^2}{\to} \text{const} \times \frac{1}{(m_t^2 - p_t^2)^2}$$

Linearly IR divergent. Finite in dim reg.

Can impose invariant mass cuts on top decay products. $\Delta^2 = M_W^2$ for inclusive cross section. EFT works differently for loose and wide cuts [Actis, MB, Falgari, Schwinn, 2008]

[MB, Jantzen, Ruiz-Femenia, 2010]

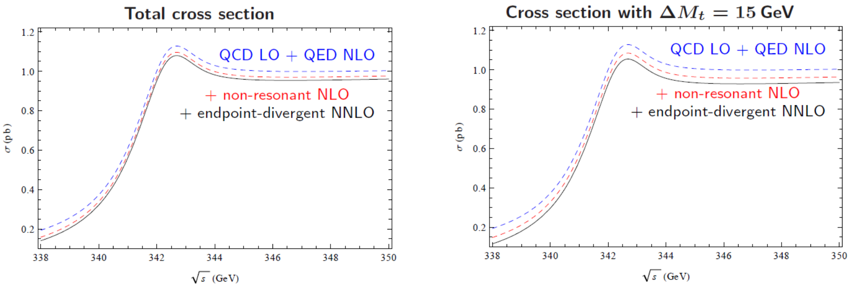


QED and non-resonant corrections relative to the $t\bar{t}$ LO cross section in percent: $\sigma^{(1)}_{\rm QED}/\sigma^{(0)}_{t\bar{t}}$ (upper solid blue), $\sigma^{(1)}_{\rm non\text{-}res}/\sigma^{(0)}_{t\bar{t}}$ for the total cross section (lower solid red) and $\Delta M_t = 15$ GeV (lower dashed red). The relative size of the sum of the QED and non-resonant corrections is represented by the middle (black) lines, for $\Delta M_{t,\rm max}$ (solid) and $\Delta M_t = 15$ GeV (dashed). $m_{t,\rm pole} = 172$ GeV.

Large correction below threshold.
Much larger than QCD scale-dependence at 3rd order ($\pm(2-3)\%$)

$e^+e^- \to W^+W^-b\bar{b}$ near $s = 4m_t^2$

NLO + NNLO singular terms [Jantzen, Ruiz-Femenia, 2013; see also Hoang, Reisser, Ruiz-Femenia, 2010]
(Singular refers to expansion in Λ/m_t where Λ is an invariant mass cut such that $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$.)



NNLO non-resonant still -2% at threshold and larger below.
Accurate description of region below peak is required for precise determination of m_t.

Summary

I $\Upsilon(1S)$ leptonic width to 3rd order.

- Bound-state and short-distance dynamics at the weak-to-strong coupling interface in QCD
- Perturbation theory seems to be under control
- 30% missing "non-perturbative" contribution not under control

II $e^+e^- \to t\bar{t}X$ cross section near threshold now computed at NNNLO in (PNR)QCD

Sizeable 3rd order corrections and reduction of theoretical uncertainty.
Ultra-precise mass measurement.

III Realistic predictions for $e^+e^- \to W^+W^-b\bar{b}$ near top-pair threshold

Soon available with sufficient accuracy (NNLO), including cuts.

Note: Need four-loop (formally even five-loop) relation between the pole and the MSbar mass to convert the precise value for the PS mass to an equally precise value of the MSbar mass.