$\mathcal{O}(\alpha\alpha_{\rm s})$ corrections to Drell–Yan processes in the resonance region

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(see also arXiv:1403.3216 [hep-ph])

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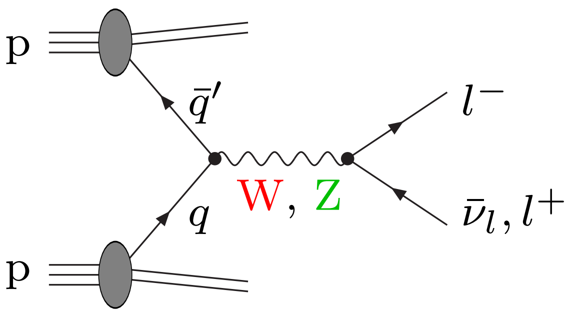
Introduction

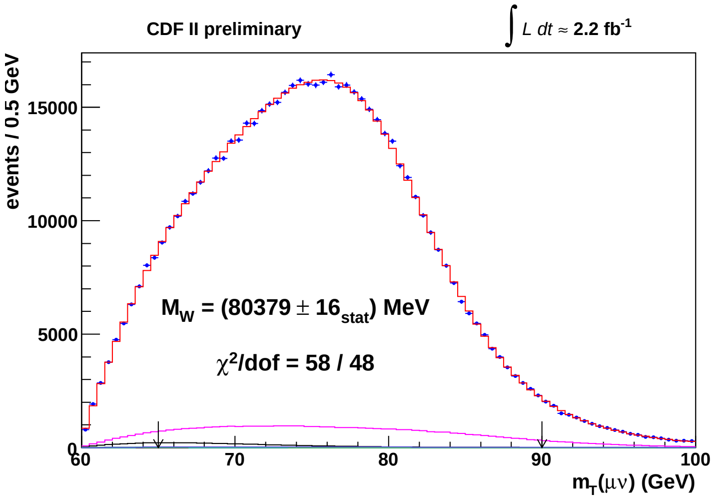
W- and Z-boson production at hadron colliders $\rightarrow$ important standard candles



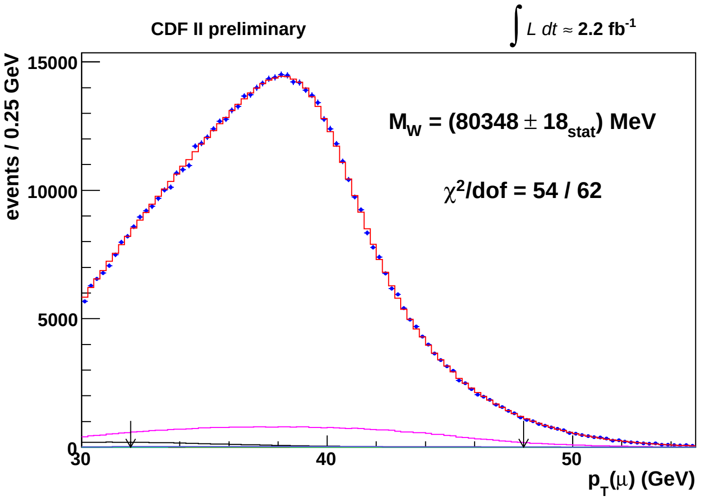
- M_Z, $\sigma_{W/Z}$ $\rightarrow$ calibration, PDFs, ...
- M_W, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ $\rightarrow$ precision measurements
- W′, Z′ searches at high M_{ll} or $M_{\mathrm{T},l\nu_l}$

Example: M_W @ CDF (2012) $\rightarrow$ $M_W^{\mathrm{CDF}} = 80.387\,\mathrm{GeV} \pm 19\,\mathrm{MeV}$

a) fit to transverse W-boson mass



b) transverse lepton momentum $p_{\mathrm{T},l}$



$$M_{\mathrm{T},l\nu} = \sqrt{2(E_{\mathrm{T},l}\,\not{E}_{\mathrm{T}} - \mathbf{p}_{\mathrm{T},l}\cdot\not{\mathbf{p}}_{\mathrm{T}})}$$

Note: LHC sensitivity $\Delta M_W \sim 7\,\mathrm{MeV}$

Besson et al. ’08

Combination of NLO QCD and EW corrections

Issue unambiguously fixed only by calculating the 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections, until then rely on approximations and estimate the uncertainties:

Comparison of two extreme alternatives:

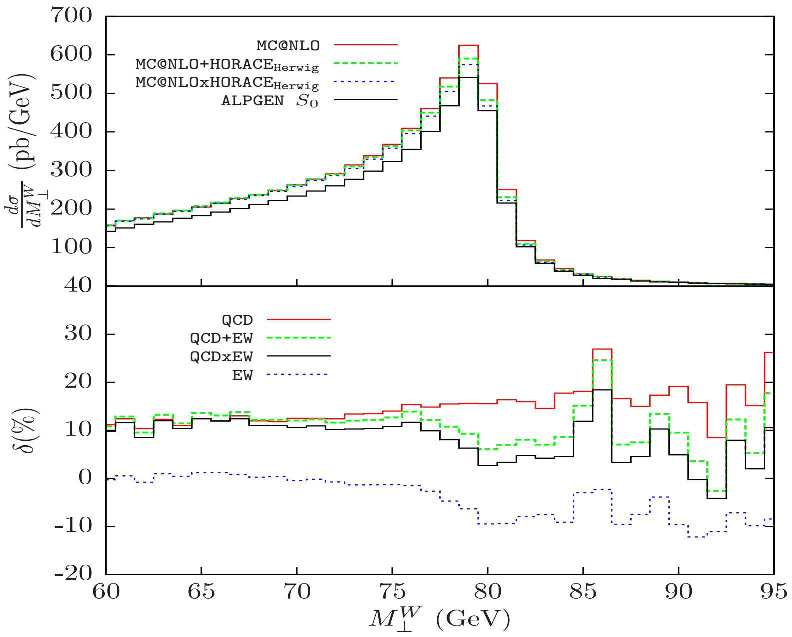
$$(1+\delta_{\rm QCD}^{\rm NLO}+\delta_{\rm EW}^{\rm NLO})$$

versus

$$(1+\delta_{\rm QCD}^{\rm NLO})\times(1+\delta_{\rm EW}^{\rm NLO})$$

$\hookrightarrow$ difference at %-level with shape distortion

Balossini et al. '09 (HORACE)



$\Rightarrow$ $\mathcal{O}(\alpha\alpha_s)$ corrections should be known at least in resonance region !

QCD and EW corrections to W/Z production:

NNLO QCD + NLO EW

+ QCD resummations / parton-shower matching

+ improvements known

Steps towards $\mathcal{O}(\alpha\alpha_s)$ corrections

- NLO EW for $\mathrm{W/Z}$ production with a hard jet
  - $\mathrm{W/Z/\gamma + 1\,jet}$, stable W/Z bosons — Maina, Moretti, Ross '04; Kühn, Kulesza, Pozzorini, Schulze '04–'07; Hollik, Kasprzik, Kniehl '07
  - off-shell W/Z bosons with decays — Denner, S.D., Kasprzik, Mück '09–'12

$$\mathrm{W + 1\,jet} \to l\nu_l + \mathrm{1\,jet}, \qquad \mathrm{Z}/\gamma^* + \mathrm{1\,jet} \to l^+l^-/\bar{\nu}_l\nu_l + \mathrm{1\,jet}$$

- further partial results
  - on-shell $\mathrm{Z}f\bar{f}$ vertex — Kotikov, Kühn, Veretin '07
  - virtual corrections to $q\bar{q}'\to\mathrm{W/Z}\to l\bar{l}'$ — Bonciani '11; Kilgore, Sturm '11
  - inclusive $\Gamma_{\mathrm{W}\to q\bar{q}'}$ — Kara '13
- resonance expansion for $q\bar{q}'\to\mathrm{W/Z}\to l\bar{l}'$ — S.D., Huss, Schwinn '13/'14 **This talk !**

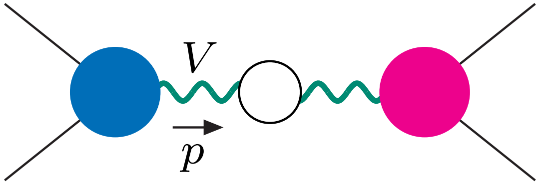
Pole expansion @ $\mathcal{O}(\alpha)$

Pole expansion of loop amplitudes – general idea

Stuart '91; H.Veltman '92
Aeppli, v.Oldenborgh, Wyler '94

Starting point: Dyson-summed matrix element

$$\mathcal{M} = \underbrace{\frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)}} + N(p^2)$$



resonant part with complex pole at $p^2 = \mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V$ gauge invariant

Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

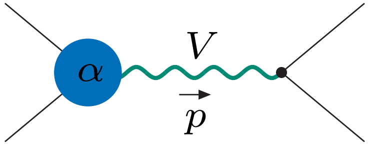
$$= \underbrace{\frac{W(\mu_V^2)}{p^2 - \mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)}} + \underbrace{\left[\frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)} - \frac{W(\mu_V^2)}{p^2 - \mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)}\right] + N(p^2)}$$

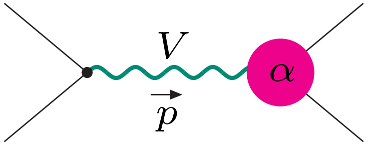
resonance pole = gauge invariant
$\hookrightarrow$ "factorizable contributions"

resonant "non-factorizable" corrections
$+$ non-resonant continuum

Virtual factorizable corrections

$$\mathcal{M}^{(1)}_{\text{fact}} = \sum_\lambda \frac{\mathcal{M}^{(1)}_{\text{production}}(\lambda)\mathcal{M}^{(0)}_{\text{decay}}(\lambda) + \mathcal{M}^{(0)}_{\text{production}}(\lambda)\mathcal{M}^{(1)}_{\text{decay}}(\lambda)}{p^2 - \mu_V^2}$$





Comments:

respect V-spin correlations; $W(\mu_V^2) \to W(M_V^2)$ possible in $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_s\alpha)$

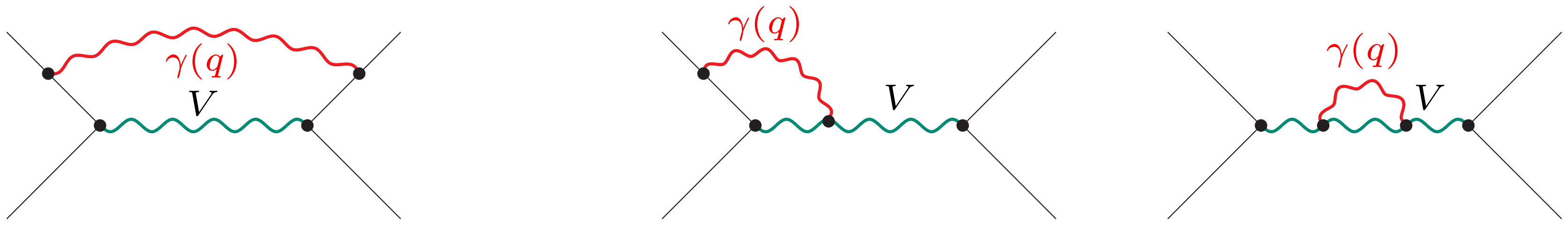
Virtual non-factorizable corrections

Fadin, Khoze, Martin '94; Melnikov, Yakovlev '96; Beenakker, Berends, Chapovsky '97; Denner, Dittmaier, Roth '97,'98

Origin:

on-shell limit ($p^2 \to M_V^2$) and IR regularization (e.g. $m_\gamma \to 0$) do not commute

for γ exchange between external and/or resonant lines:



"manifestly non-factorizable"

- resonant IR-divergent contribution

"not manifestly non-factorizable" diagrams

- fact. contribution: $W(M_V^2)$
- non-factorizable part:
  $W_{\mathrm{non-fact}}(p^2) \equiv \left[W(p^2) - W(M_V^2)\right]_{p^2 \to M_V^2}$

General features: Fadin, Khoze, Martin '94

- contributions only from soft momenta $|q^\mu| \sim \Gamma_V \ll M_V$

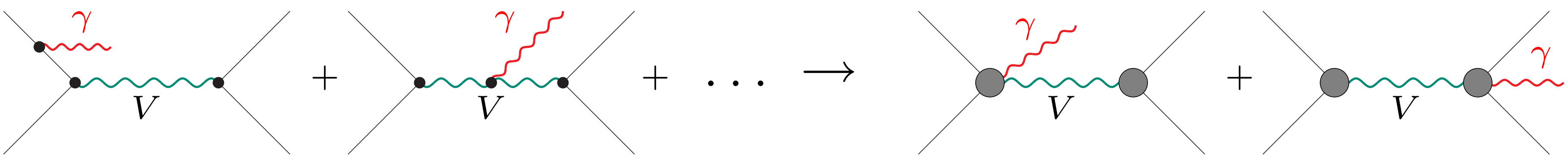
  $\hookrightarrow$ calculation within "extended soft-photon approximation" (keep off-shell V propagators)
- result factorizes from Born amplitude: $\mathcal{M}^{\mathrm{virt}}_{\mathrm{non-fact}} = \delta^{\mathrm{virt}}_{\mathrm{non-fact}} \, \mathcal{M}^{(0)}$
- virtual + real non-fact. corrections cancel in inclusive quantities such as σ_{tot}

Pole expansion of real photonic corrections

Split diagrams with radiating resonances (2 resonant propagators) as follows:

$$\frac{1}{[(p+k)^2-\mu_V^2](p^2-\mu_V^2)} = \frac{1}{2pk}\left[\frac{1}{p^2-\mu_V^2} - \frac{1}{(p+k)^2-\mu_V^2}\right]$$

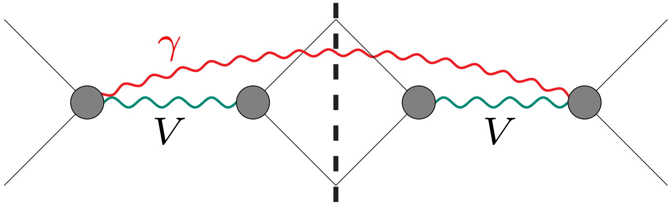
$\hookrightarrow$ decomposition of $\mathcal{M}_{i\to f+\gamma}$ into initial- and final-state radiation:



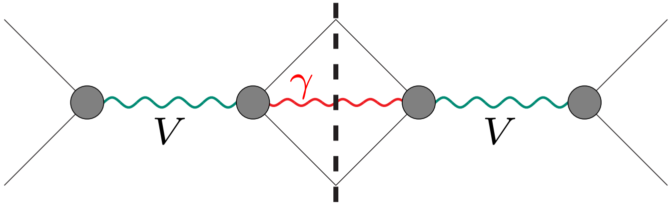
● $=$ on-shell tree-like subamplitudes

Classification of real photonic corrections in PA

Factorizable contributions to $|\mathcal{M}|^2$:

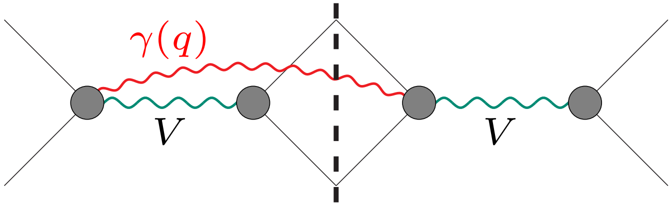


Initial-state radiation



Final-state radiation

Non-factorizable contributions to $|\mathcal{M}|^2$:



$+$ c.c.

Only $q = \mathcal{O}(\Gamma_V)$ relevant !

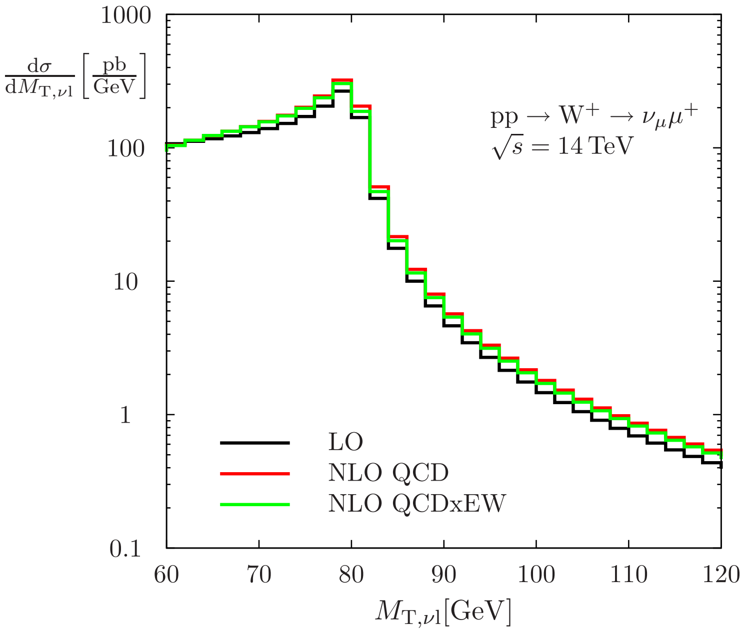
calculable from modified eikonal currents:

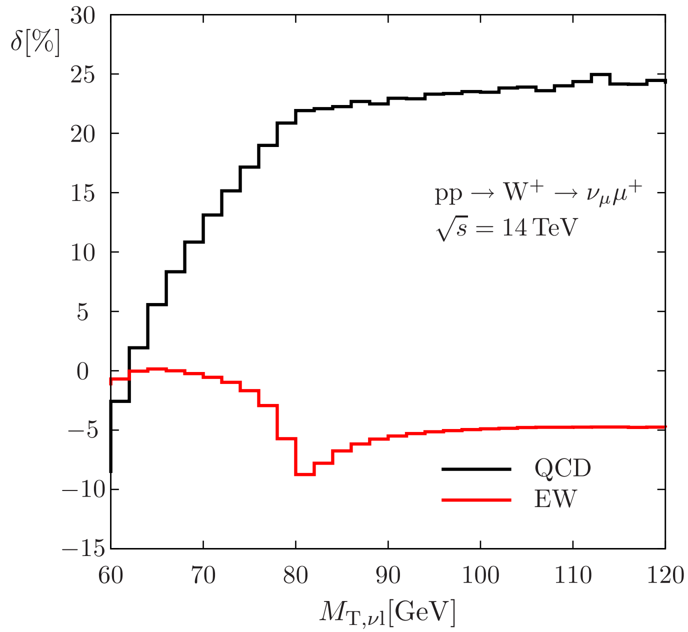
$$\mathrm{d}\sigma_{\mathrm{non-fact}} = \mathrm{d}\sigma_0\, \delta^{\mathrm{real}}_{\mathrm{non-fact}}, \qquad \delta^{\mathrm{real}}_{\mathrm{non-fact}} = \frac{\alpha}{2\pi^2}\int \frac{\mathrm{d}^3\mathbf{q}}{q^0}\,\mathrm{Re}\{\mathcal{J}^{\mu}_{\mathrm{prod}}\mathcal{J}^{*}_{\mathrm{dec},\mu}\},$$

$$\mathcal{J}^{\mu}_{\mathrm{prod}} = Q_1\frac{p_1^\mu}{p_1 q} - Q_2\frac{p_2^\mu}{p_2 q} - (Q_1 - Q_2)\frac{(p_1+p_2)^\mu}{p_1 q + p_2 q},$$

$$\mathcal{J}^{\mu}_{\mathrm{dec}} = \left[-Q_1'\frac{k_1^\mu}{k_1 q} + Q_2'\frac{k_2^\mu}{k_2 q} + (Q_1' - Q_2')\frac{(k_1+k_2)^\mu}{k_1 q + k_2 q}\right]\frac{(k_1+k_2)^2 - \mu_V^2}{(k_1+k_2+q)^2 - \mu_V^2}$$

Transverse-mass distribution for W production





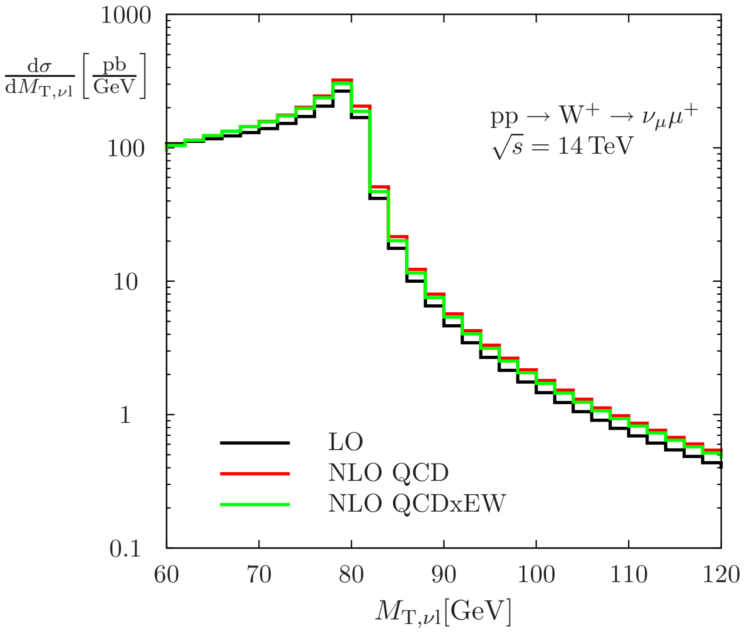
Features of $M_{\mathrm{T},\nu l}$:

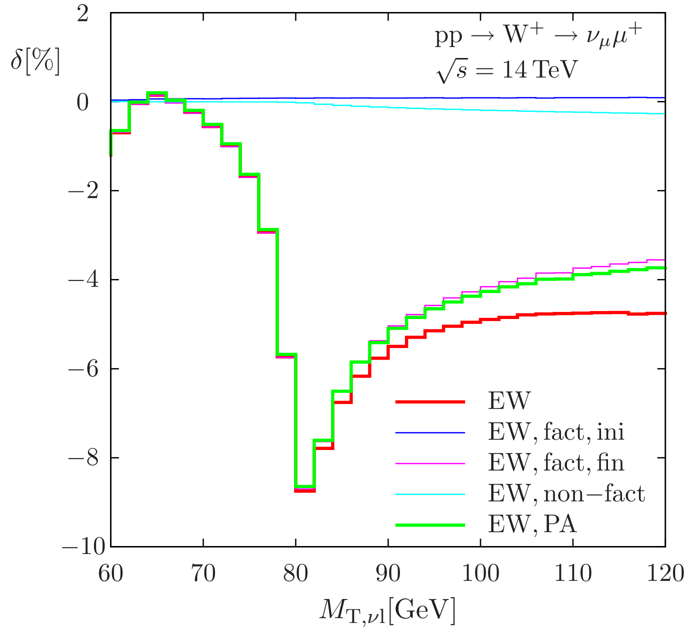
- most important observable for M_{W} det.
- stability wrt QCD corrs/uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_{\mathrm{T}}$

Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

Transverse-mass distribution for W production





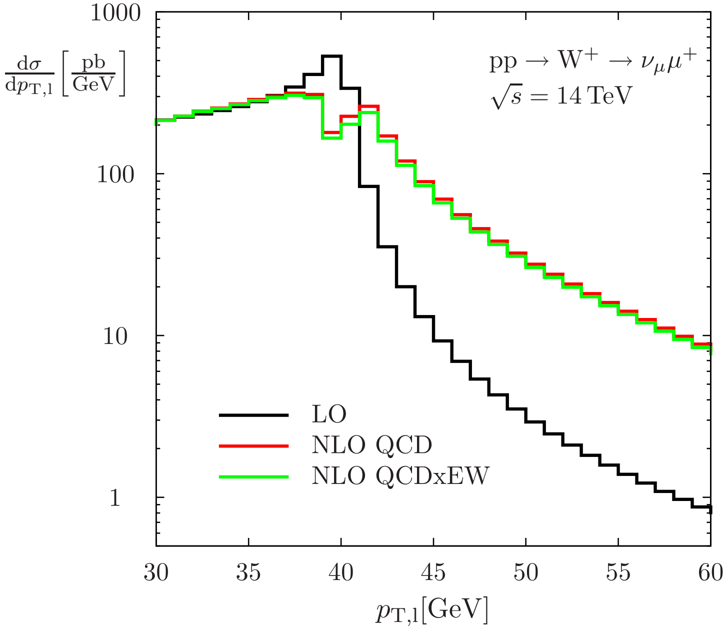
Features of $M_{T,\nu l}$:

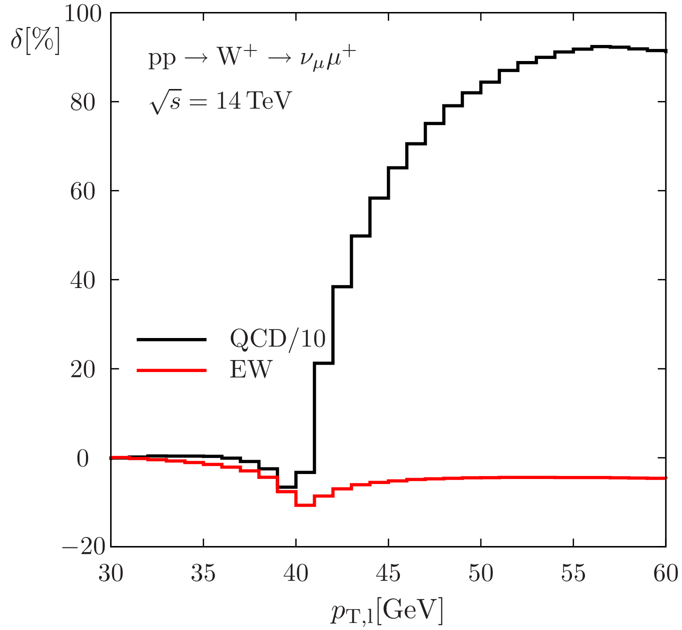
- most important observable for M_W det.
- stability wrt QCD corrs/uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_T$

Pole approximation (PA):

- PA reproduces EW corr near resonance
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

Transverse-momentum distribution for W production





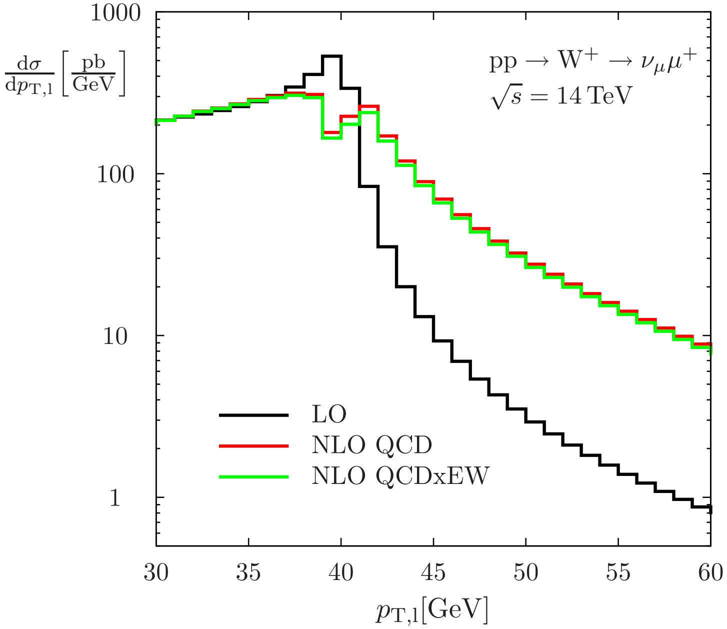
Features of $p_{\mathrm{T},l}$:

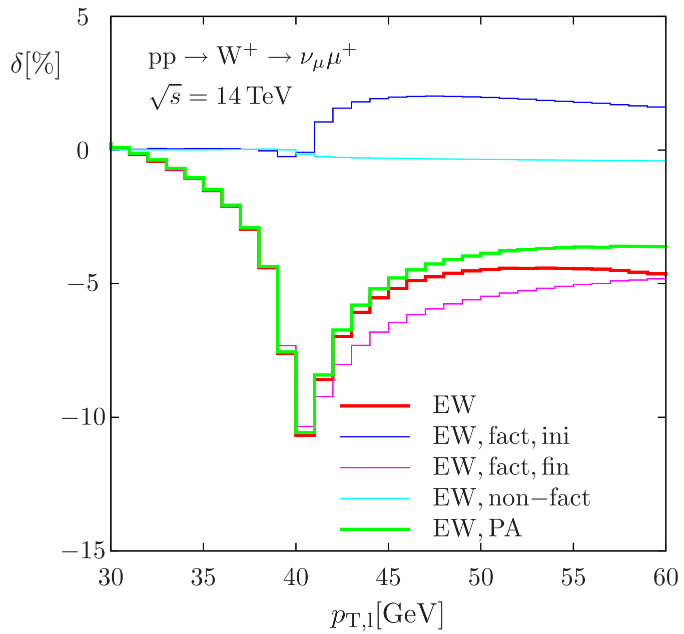
- also relevant for M_W measurement
- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties

Corrections:

- QCD corrections huge above resonance (jet recoil)
- EW corrections distort resonance shape as well

Transverse-momentum distribution for W production





Features of $p_{T,l}$:

- also relevant for M_W measurement
- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties

PA works well:

- EW corr reproduced near resonance
- factorizable FS corrs distort resonance shape
- factorizable IS corrs overwhelmed by QCD
- non-fact. corrs flat and negligible

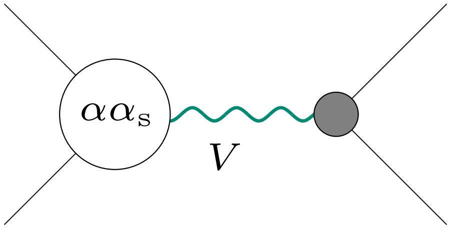
Pole expansion @ $\mathcal{O}(\alpha\alpha_\mathrm{s})$

and

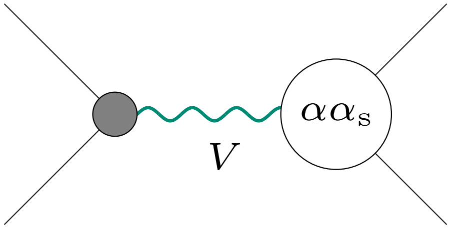
non-factorizable corrections

Classification of $\mathcal{O}(\alpha\alpha_s)$ corrections in PA

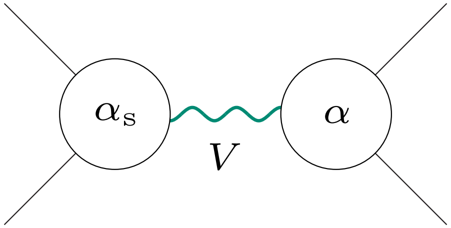
Factorizable contributions: (only virtual contributions indicated)



- no significant resonance distortion expected
- no PDFs with $\mathcal{O}(\alpha\alpha_s)$ corrections

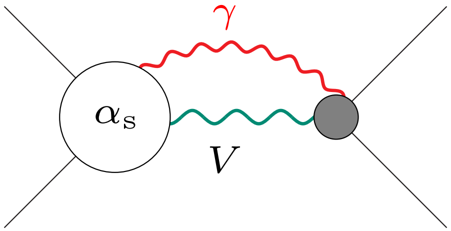


- only $Vl\bar{l}'$ counterterm contributions
  $\hookrightarrow$ uniform rescaling, no distortions



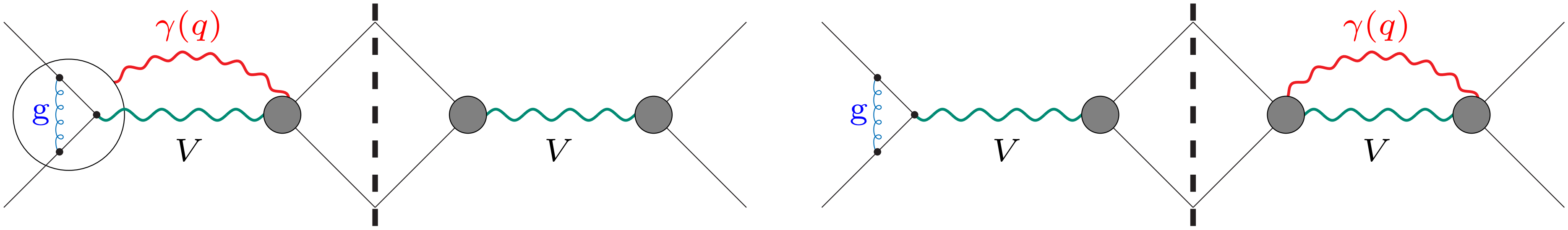
- significant resonance distortions from FSR

Non-factorizable contributions: (only virtual contributions indicated)



- small @ $\mathcal{O}(\alpha)$, but could be enhanced by large $\mathcal{O}(\alpha_s)$ corrections (jet recoil)
- calculated and discussed in the following

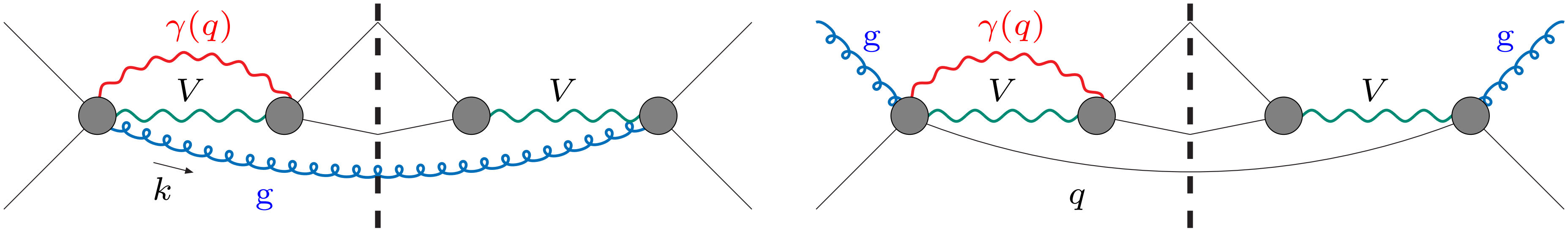
Virtual–virtual contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Result: $$|\mathcal{M}|^2\Big|_{\substack{\mathrm{non-fact}\\ \mathrm{virt-virt}}} = 4\,\mathrm{Re}\{\delta^{(\alpha_s,\mathrm{virt})}_{q\bar{q}'V}\}\,\mathrm{Re}\{\delta^{(\alpha,\mathrm{virt})}_{\mathrm{non-fact}}\}\,|\mathcal{M}_0|^2$$

- factorized structure (1-loop) $\times$ (1-loop) after non-trivial cancellations
- expansion of all loops in $q^\mu \sim \Gamma_V \sim (p^2-\mu_V^2)/M_V \to 0$
- issue of overlapping IR singularities
- different methods applied $\rightarrow$ results agree
  - ◇ diagrammatic calculation (expansion via Mellin–Barnes technique)
  - ◇ gauge-invariance argument à la Yennie/Frauschti/Suura '61
    (even holds to any order $\alpha\alpha_s^n$, $n=1,2,\dots$)
  - ◇ effective field theory for unstable particles Beneke et al. '03,'04

Virtual–real contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Result: $$\left.|\mathcal{M}|^2\right|_{\substack{\text{non-fact}\\ \text{virt-real}}} = 2\,\mathrm{Re}\{\delta^{(\alpha,\text{virt})}_{\text{non-fact},q\bar{q}'\to l\bar{l}'\mathrm{g}}\}\,|\mathcal{M}_{0,q\bar{q}\to l\bar{l}'\mathrm{g}}|^2, \qquad \text{etc.}$$

- From explicit diagrammatic calculation analogous to NLO $\mathcal{O}(\alpha)$ calculation
- New feature in qg channels: γ exchange between final-state particles
  Structure different from initial–final interferences $\;\rightarrow\;$ enhancement ?

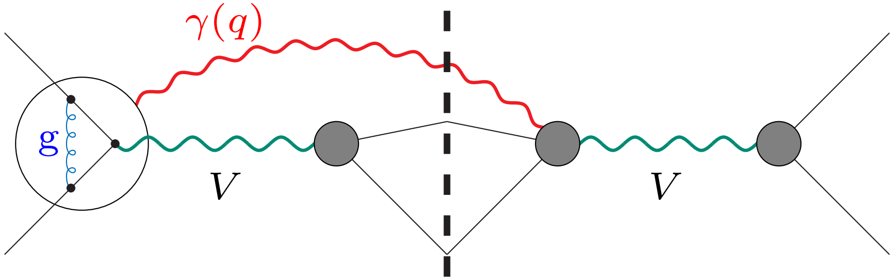
Example: W production $u\bar{d} \to \mathrm{W} \to \nu_l l^+ \mathrm{g}$

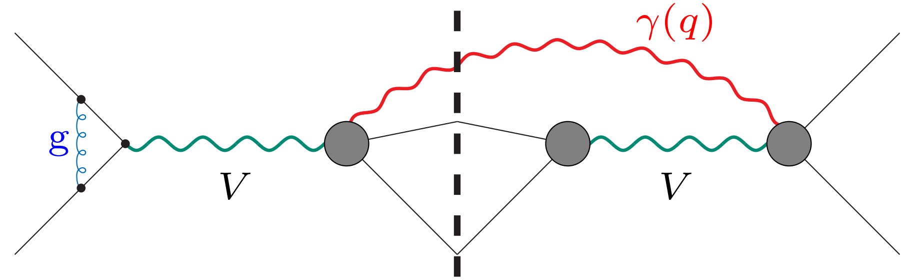
$$\delta^{(\alpha,\text{virt})}_{\text{non-fact},u\bar{d}\to\nu_l l^+\mathrm{g}} = -\frac{\alpha}{2\pi}\Biggl\{-2 + Q_d\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2-\hat{t}_{ug}}{\hat{t}_{dl}}\right) - Q_u\,\mathrm{Li}\left(1+\frac{M_\mathrm{W}^2-\hat{t}_{dg}}{\hat{t}_{ul}}\right)$$
$$-\left[\frac{c_\epsilon}{\epsilon} - 2\ln\left(\frac{\mu_\mathrm{W}^2-\hat{s}}{\mu M_\mathrm{W}}\right)\right]\left[1 + Q_d\ln\left(\frac{M_\mathrm{W}^2-\hat{t}_{ug}}{-\hat{t}_{dl}}\right) - Q_u\ln\left(\frac{M_\mathrm{W}^2-\hat{t}_{dg}}{-\hat{t}_{ul}}\right)\right]\Biggr\}$$

($\hat{t}_{qj} = (p_q - k_j)^2$, on-shell projection for W !)

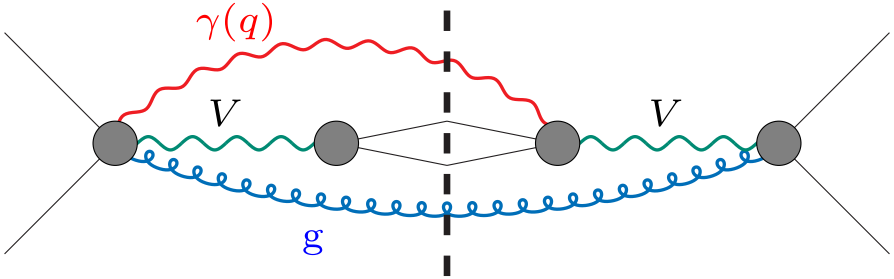
$$\xrightarrow[k\to 0]{} \delta^{(\alpha,\text{virt})}_{\text{non-fact},u\bar{d}\to\nu_l l^+}$$

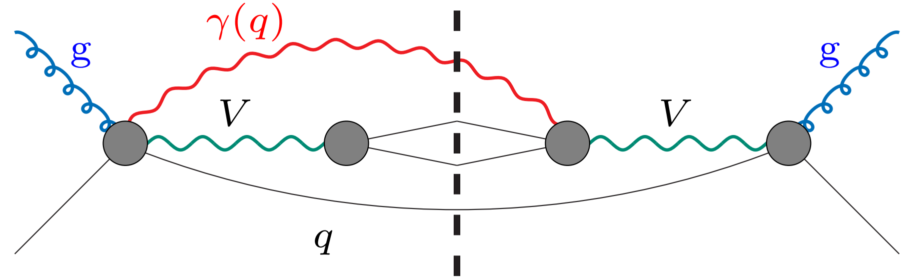
Real–virtual and real–real contributions to non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections





Result: $$|\mathcal{M}|^2\Big|_{\substack{\text{non-fact}\\ \text{real-virt}}} = 2\,\mathrm{Re}\{\delta_{q\bar{q}'V}^{(\alpha_s,\text{virt})}\}\,\delta_{\text{non-fact}}^{(\alpha,\text{real})}\,|\mathcal{M}_0|^2$$





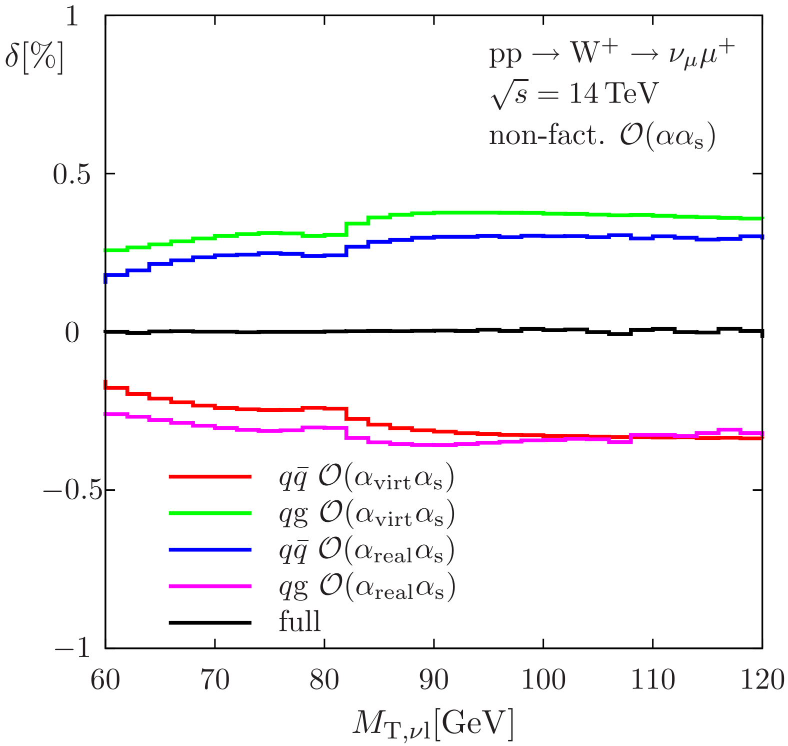
Result: $$|\mathcal{M}|^2\Big|_{\substack{\text{non-fact}\\ \text{real-real}}} = \delta_{\text{non-fact}}^{(\alpha,\text{real})}\,|\mathcal{M}_{0,q\bar{q}\to l\bar{l}'\text{g}}|^2, \qquad \text{etc.}$$

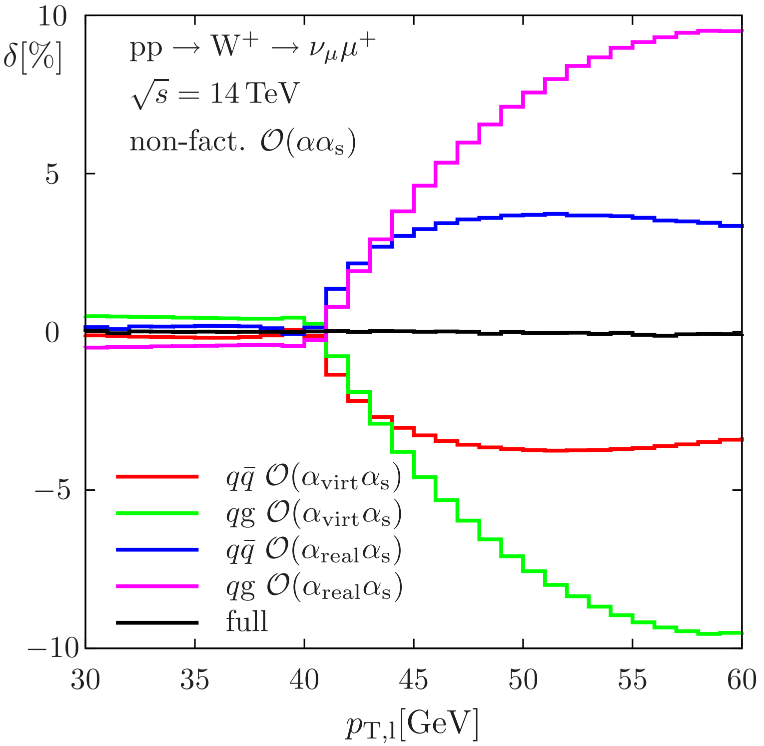
$$\delta_{\text{non-fact}}^{(\alpha,\text{real})} = \frac{\alpha}{2\pi^2}\int\frac{\mathrm{d}^3\mathbf{q}}{q^0}\,\mathrm{Re}\{\mathcal{J}^{\mu}_{\text{prod}}\mathcal{J}^{*}_{\text{dec},\mu}\}$$

Note: factorization, e.g., justified by YFS argument as in virtual–virtual case

Non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

W production:





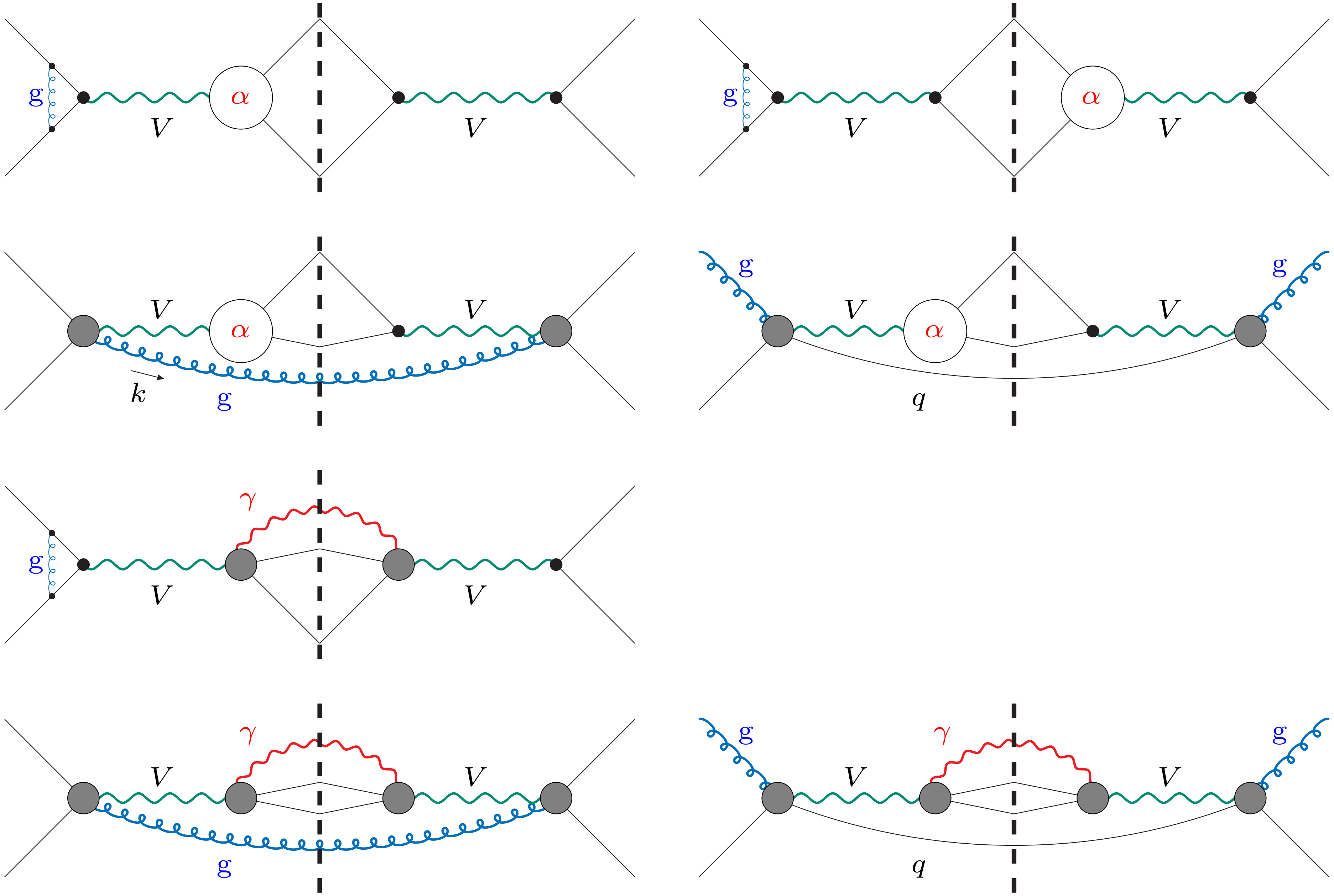
- $\delta \;=\; \delta_{\text{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- Full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections tiny
  due to complete cancellation between virtual and real corrections

Dominant factorizable corrections

—

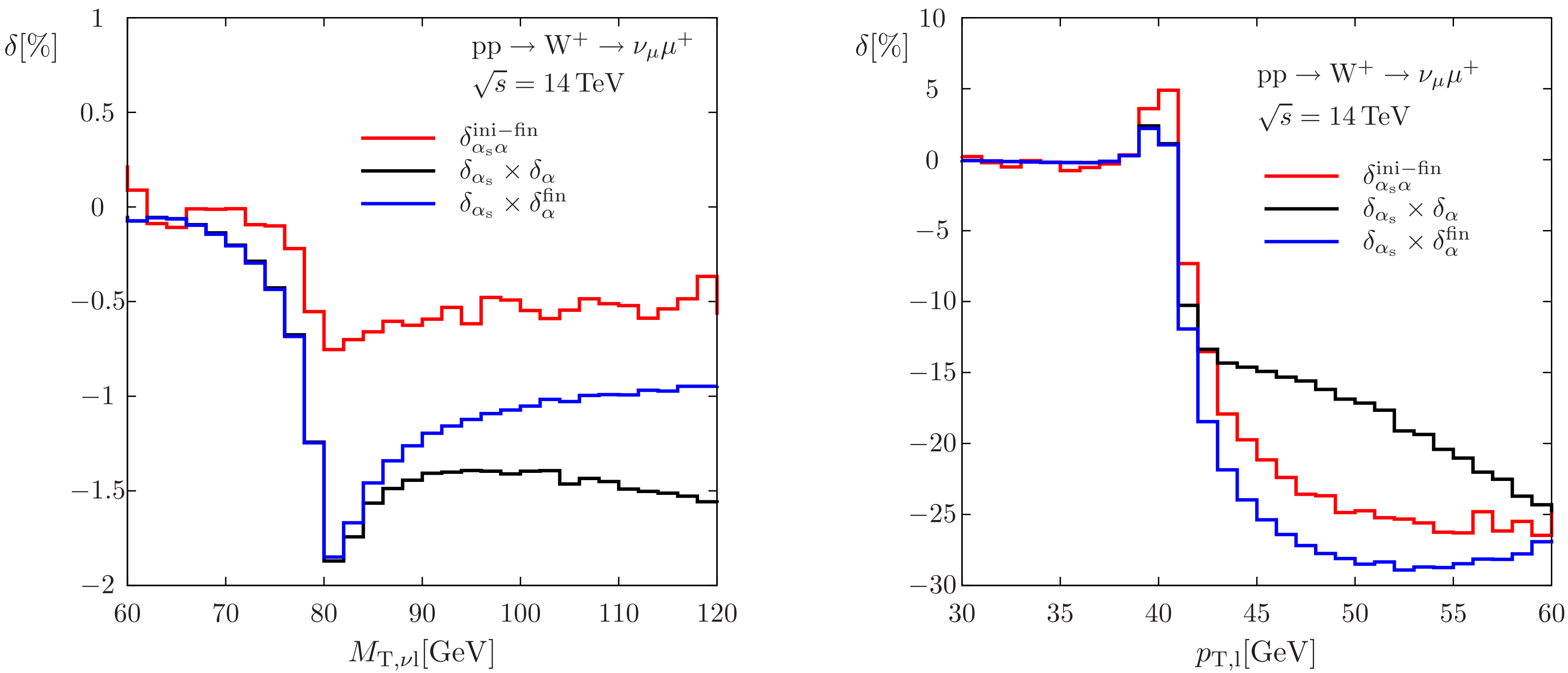
preliminary results

Contributions to initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Numerical results (preliminary!) on initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

W production: (γ recombination applied, “dressed leptons”)



- No naive factorization: $\delta^{\mathrm{ini-fin}}_{\alpha_s\alpha} \neq \delta_{\alpha_s} \times \delta^{\mathrm{fin}}_\alpha$
- Homework:
  - comparison of $\delta^{\mathrm{ini-fin}}_{\alpha_s\alpha}$ with MC approach $\mathrm{d}\sigma_{\alpha_s} \otimes (\gamma$ shower$)$
  - estimate shifts in M_W, M_Z by $\delta^{\mathrm{ini-fin}}_{\alpha_s\alpha}$
  - procedure for employing new correction in state-of-the-art predictions

Conclusions

High-precision Drell–Yan physics @ LHC

- promises M_{W} with accuracy $\Delta M_{\mathrm{W}} < 10\,\mathrm{MeV}$ and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ with $\mathcal{O}$(LEP precision)
- requires highest possible theoretical precision near resonances
  NNLO QCD + NLO EW + QCD resummations etc. known
  $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ is biggest unknown correction

$\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ in pole approximation

- non-factorizable corrections calculated $\to$ negligible
  $\hookrightarrow$ only factorizable corrections to $2 \to 1$ and/or $1 \to 2$ processes relevant
- $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ corrections to $q\bar{q}' \to V$ production
  $\hookrightarrow$ no significant resonance distortion expected
- $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ corrections to $V' \to l\bar{l}'$ decay
  $\hookrightarrow$ only irrelevant rescaling of distributions (only from counterterms)
- $\left[\mathcal{O}(\alpha_{\mathrm{s}})\text{ to } q\bar{q}' \to V\right] \otimes \left[\mathcal{O}(\alpha)\text{ to } V' \to l\bar{l}'\right]$
  $\hookrightarrow$ significant resonance distortions expected, ... preliminary results shown

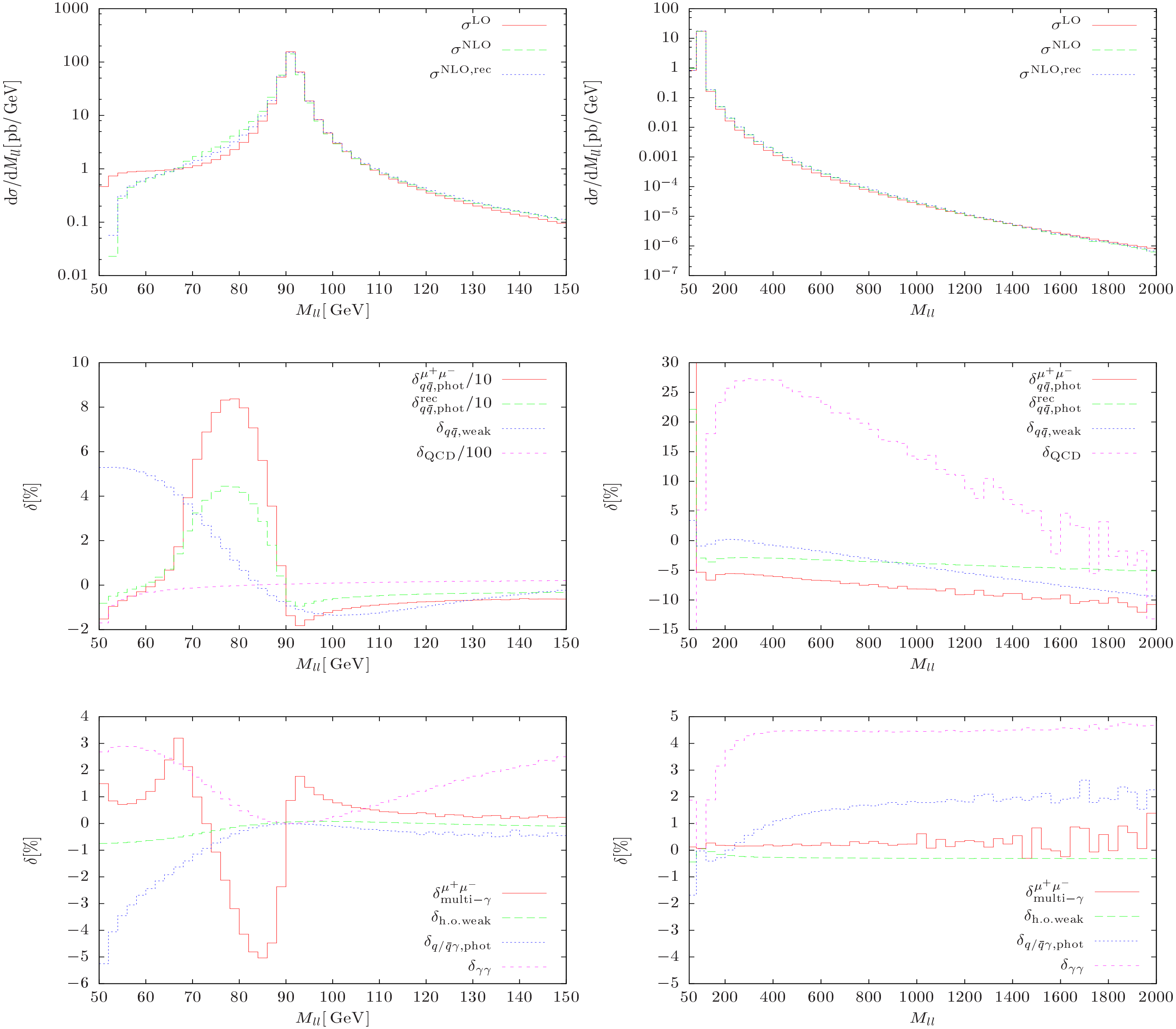
Backup slides

QCD and EW corrections to W/Z production:

• NNLO QCD corrections	Hamberg et al. '91; Harlander, Kilgore '02; Anastasiou et al. '03; Melnikov, Petriello '06; Catani et al. '09
• soft + virtual N^{3}LO QCD	Moch, Vogt '05; Laenen, Magnea '05; Idilbi et al. '05; Ravindran, Smith '07
• QCD resummations	Arnold, Kauffman '91; Balazs et al. '95,'97; R.K.Ellis et al. '97; Qiu, Zhang '00; Kulesza et al. '01,'02; Landry et al. '02; Berge et al. '05; Bozzi et al. '08
• MC@NLO matching	Frixione, Webber '06
• NLO EW correction to W production	S.D., Krämer '01; Zykunov '01; Baur, Wackeroth '04; Arbuzov et al. '05 Carloni Calame et al. '06; Brensing et al. '07
• NLO EW correction to Z production	Baur, Keller, Sakumoto '97; Baur, Wackeroth '99 Brein, Hollik, Schappacher '99; Zykunov '05; Arbuzov et al. '06; Carloni Calame et al. '07; S.D., Huber '09
• multi-photon radiation via leading logs	Baur, Stelzer '99; Carloni Calame et al. '03 Placzek, Jadach '04; Brensing et al. '07; S.D., Huber '09
• photon-induced processes	Arbuzov, Sadykov '07; Brensing et al. '07; Carloni Calame et al. '07; S.D., Huber '09
• POWHEG matching of QCD/EW corrs.	Bernaciak, Wackeroth '12; Barze et al. '13
• NLO SUSY corrections in the MSSM	Brensing et al. '07; S.D., Huber '09

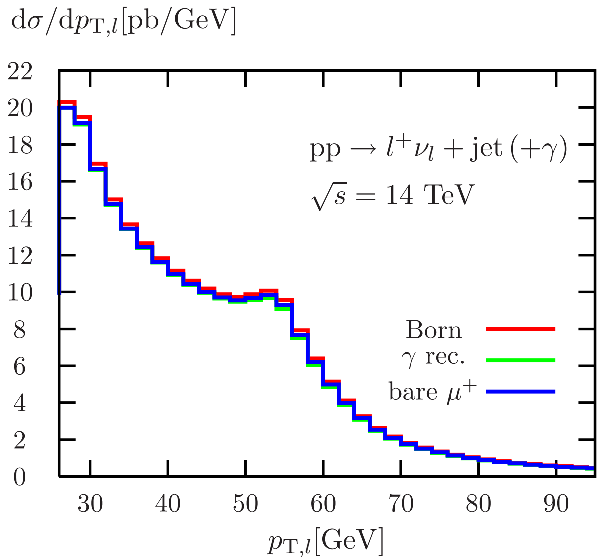
Corrections to Z production – overview

S.D., Huber '09

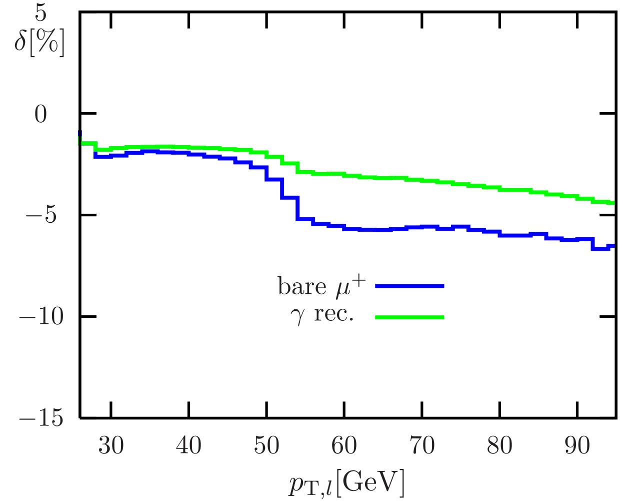


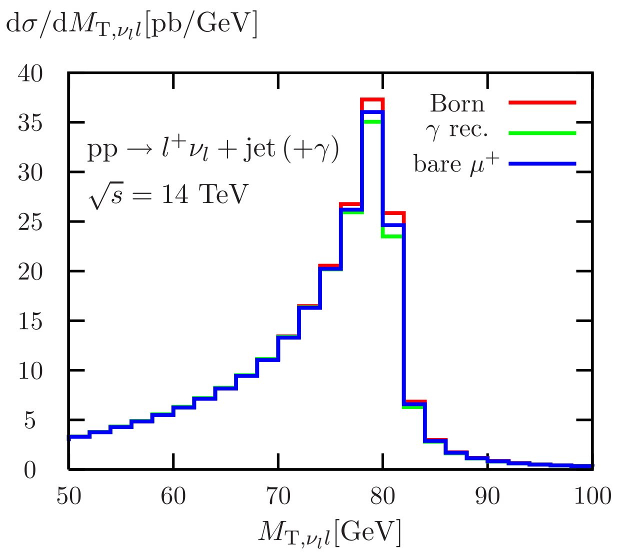
Comparison of EW corrections to W+jet and single (jet-inclusive) W production

$\hookrightarrow$ argument for factorization QCD$\times$EW if EW corrections coincide

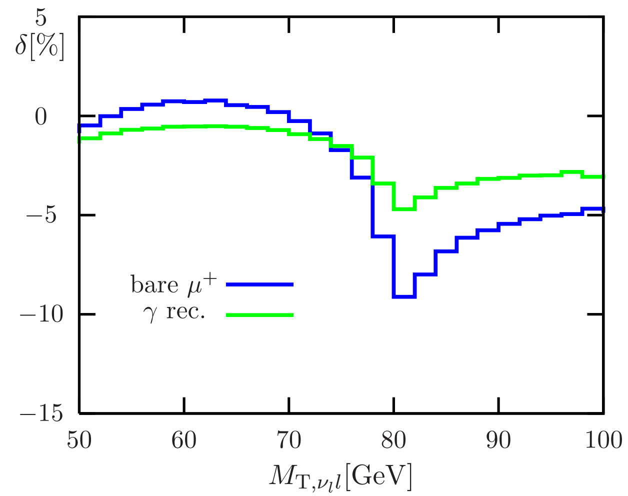


Denner et al. '09





Denner et al. '09



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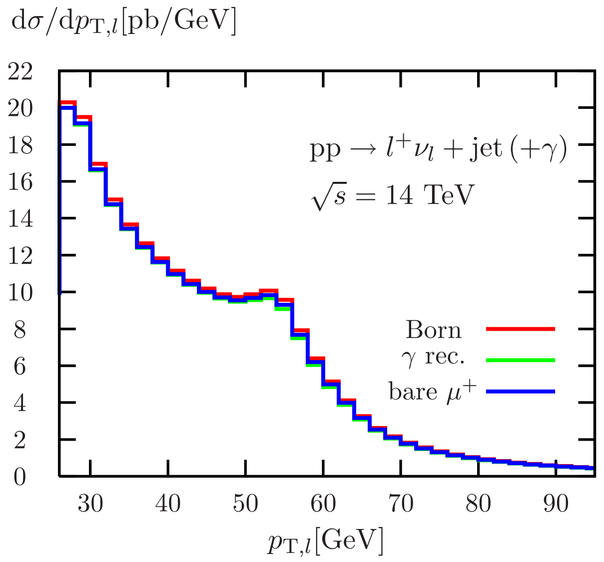
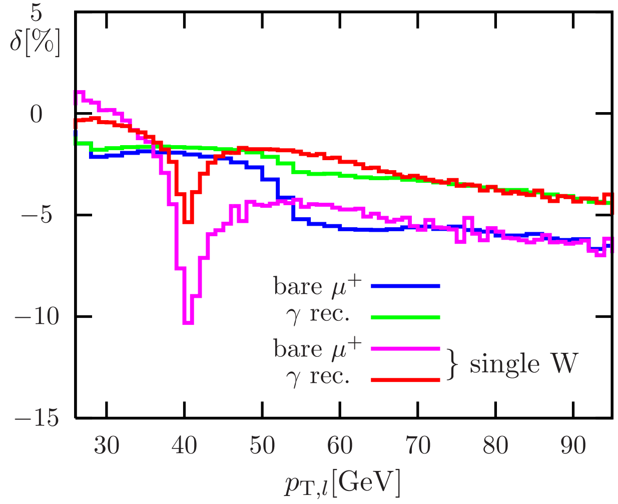
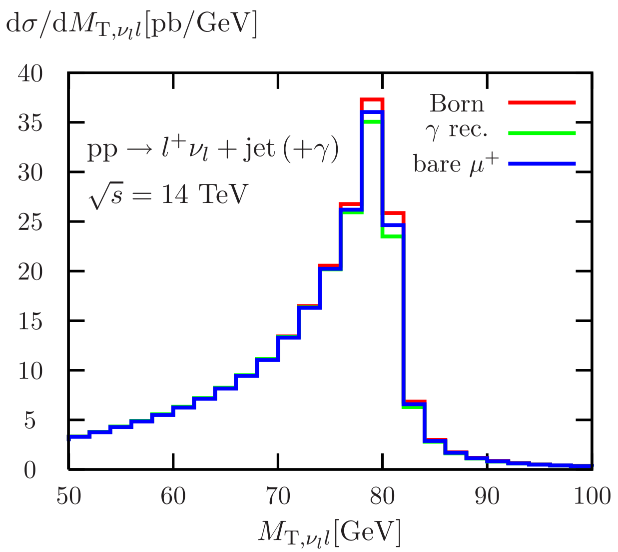


Denner et al. ’09

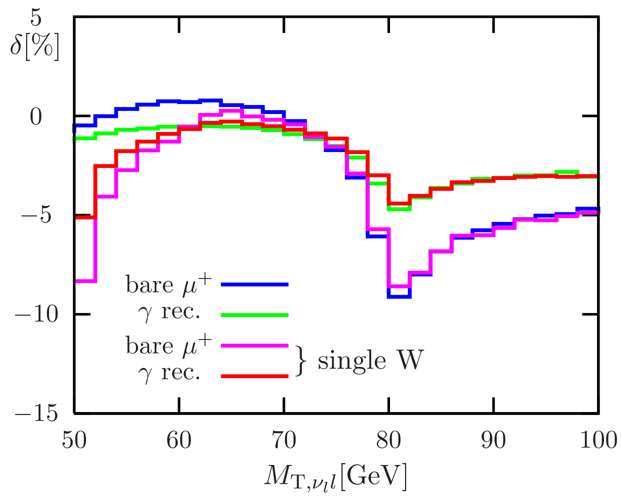


Jet recoil destroys simple factorization !

Single-W results from
S.D./Krämer ’01; Brensing et al. ’07



Denner et al. ’09



EW corrections factorize from hard gluon emission near Jacobian peak !

Perturbative evaluation of leading pole approximation (PA)

Expansion of matrix element:

($A^{(n)} \equiv n$-loop contribution to A)

$$\mathcal{M} = \mathcal{M}^{(0)}$$

} LO: complete leading order

$$+\frac{W^{(1)}(M_V^2)}{p^2-\mu_V^2} - \frac{W^{(0)}(M_V^2)\Sigma^{(1)\prime}(M_V^2)}{p^2-\mu_V^2} + \mathcal{M}^{(1)}_{\mathrm{non-fact}}$$

} NLO: correction to residue and non-factorizable corrections

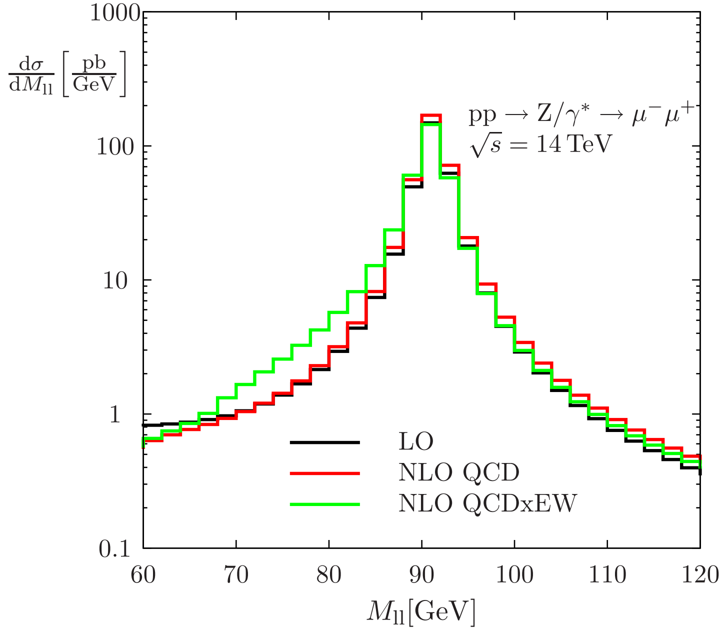
$+$ higher orders

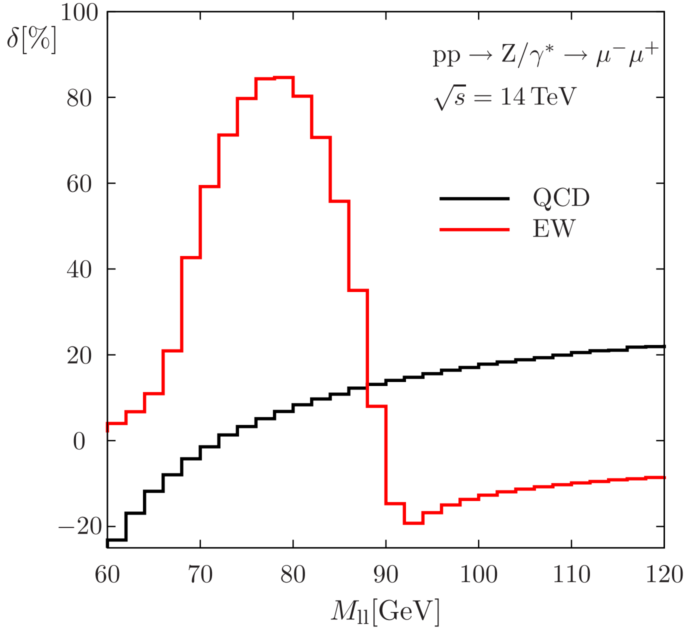
Comments:

- inclusion of $\mathcal{M}^{(0)}$ is usually easier $+$ better than its expansion
- naive estimate of relative theoretical uncertainty (TU) in NLO:

$$\mathrm{TU} \quad \sim \quad \begin{cases} \frac{\alpha}{\pi} \times \frac{\Gamma_V}{M_V} \times \mathrm{const.} & \text{in resonance region } |p^2 - M_V^2| \lesssim M_V\Gamma_V \\ \frac{\alpha}{\pi} \times \mathrm{const.} & \text{off resonance } |p^2 - M_V^2| \gg M_V\Gamma_V \end{cases}$$

PA for NLO corrections to the invariant-mass distribution for Z production



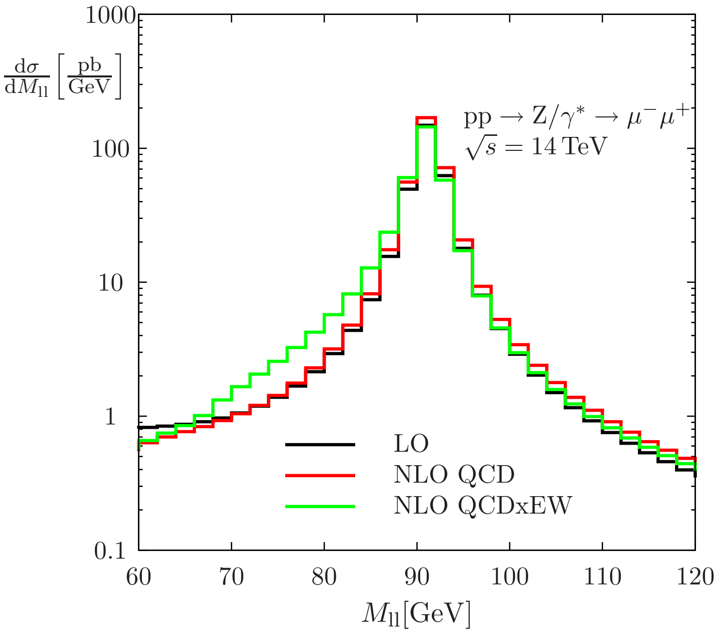


Reference process for M_{W} measurement

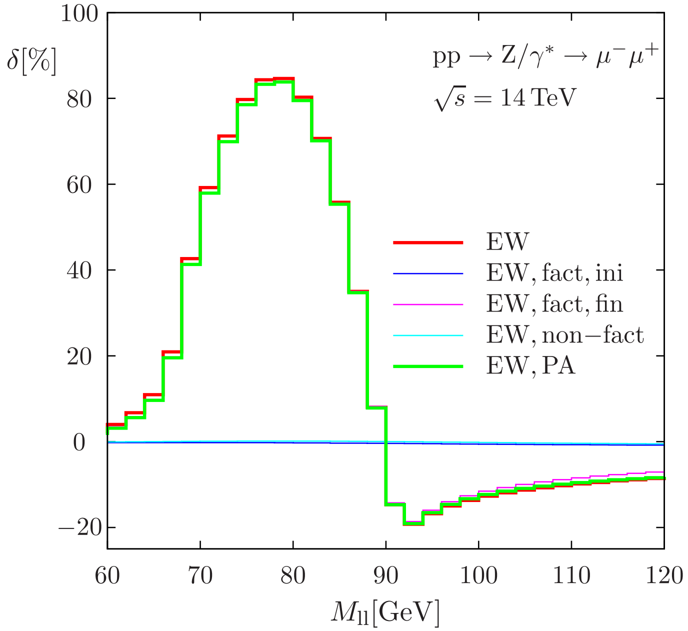
Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

PA for NLO corrections to the invariant-mass distribution for Z production



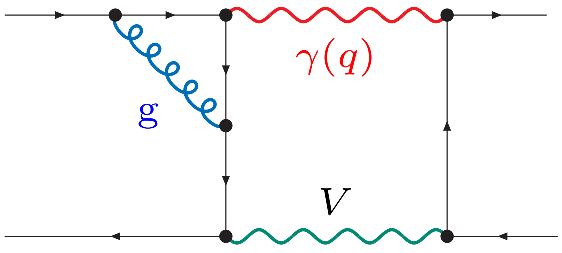
Reference process for M_W measurement



Behaviour of PA analogous to $M_{T,\nu l}$:

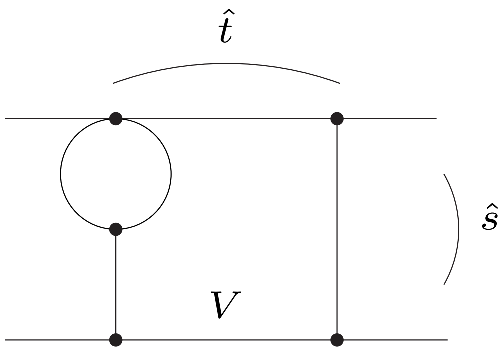
- PA reproduces EW corr near resonance
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

Example: Two-loop box graph



$$\sim \quad -\frac{C_\mathrm{F}\alpha_\mathrm{s}}{4\pi}\,\frac{Q_q Q_l \alpha}{2\pi}\,\mathcal{M}_0\,(1-\epsilon)\,(-\hat t)\,(\mu_V^2-\hat s)\,I(\hat s,\hat t)$$

Master integral:



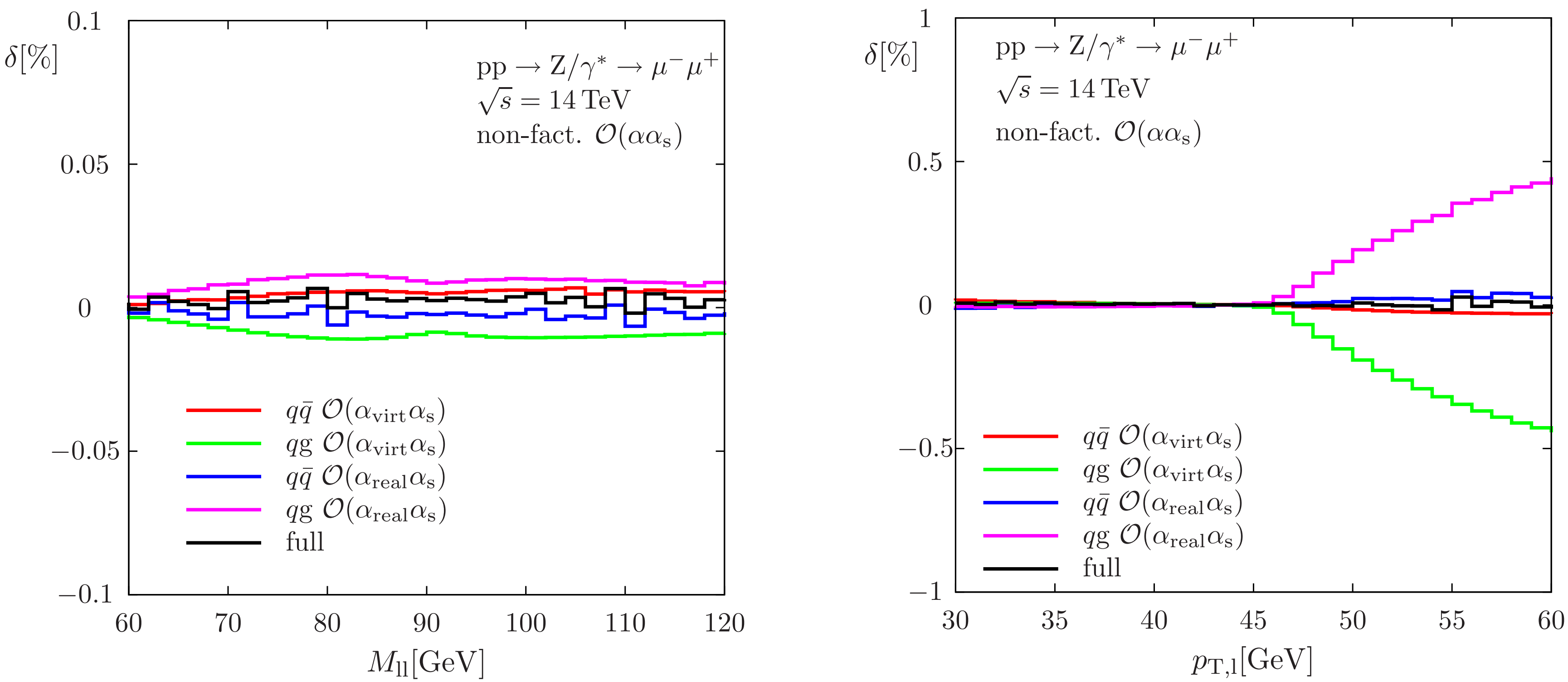
$$= \quad I(\hat s,\hat t) \quad = \quad \left(\frac{(2\pi\mu)^{2\epsilon}}{\mathrm{i}\pi^2}\right)^2 \int \mathrm{d}^D q \int \mathrm{d}^D q' \,\frac{1}{q^2\cdots}$$

$$= \frac{c_\epsilon^2}{(-\hat t)(\mu_V^2-\hat s)}\left(\frac{\mu_V^2-\hat s}{M_V^2}\right)^{-3\epsilon}\left(\frac{-\hat t}{\mu^2}\right)^{-2\epsilon}\left\{\frac{1}{2\epsilon^3}+\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left[\mathrm{Li}_2\left(1+\frac{\hat t}{M_V^2}\right)+\frac{5\pi^2}{12}+2\right]\right.$$
$$+\,2\,\mathrm{Li}_3\left(\frac{-\hat t}{M_V^2}\right)+\mathrm{Li}_3\left(1+\frac{\hat t}{M_V^2}\right)-6\zeta(3)+\ln^2\left(\frac{-\hat t}{M_V^2}\right)\ln\left(1+\frac{\hat t}{M_V^2}\right)$$
$$\left.-\,2\ln\left(\frac{-\hat t}{M_V^2}\right)\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{\hat t}{M_V^2}\right)\right]+\frac{5\pi^2}{6}+2\,\mathrm{Li}_2\left(1+\frac{\hat t}{M_V^2}\right)+4+\mathcal{O}(\hat s-\mu_V^2)+\mathcal{O}(\epsilon)\right\}$$

Note: many cancellations in sum over all contributions $(1/\epsilon^4, \mathrm{Li}_3, \zeta(3), \ldots)$

Non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

Z production:



- $\delta = \delta_{\mathrm{non-fact,real}\gamma}(E_\gamma > \Delta E), \quad \Delta E = 10^{-4}\sqrt{\hat{s}}/2 \ll \Gamma_V$
- Full non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections tiny
  due to complete cancellation between virtual and real corrections