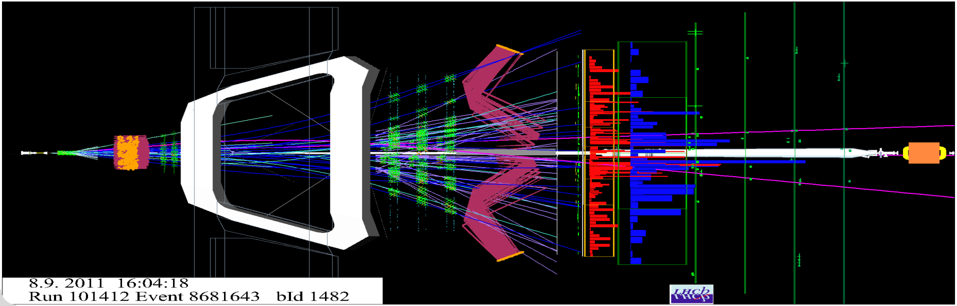
Karlsruhe Institute of Technology

# Rare B meson decays to NNLO

Matthias Steinhauser | TTP Karlsruhe

Loops and Legs, Weimar, April 30, 2014

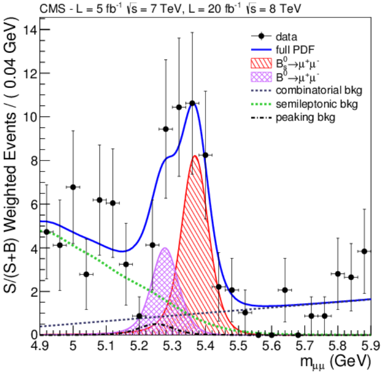


# Outline

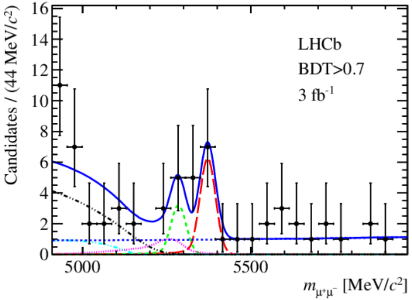
- $B_s \to \mu^+\mu^-$ [Bobeth,Gorbahn,Hermann,Misiak,Stamou,Steinhauser'13]
  - NNLO QCD
  - NLO EW
- $\bar{B} \to X_s\gamma$ [Misiak et al.]

# Experiment: invariant $(\mu^+\mu^-)$ mass

CMS




LHCb




$$\overline{\mathcal{B}}_{s\mu} = (2.9 \pm 0.7) \times 10^{-9}$$

$\geq$ 2015: uncertainty below 10% within the next 10 years

# Theory status before October 2013

- NLO QCD corrections [Buchalla,Buras'93'99; Misiak,Urban'99]
- leading-$m_t$ NLO electroweak corrections [Buchalla,Buras'98]
- uncertainty (from higher orders): $\approx 7\%$

# $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$: why interesting?

- SM prediction for $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$: $\mathcal{O}(10^{-9})$
  nevertheless: experimentally accessible
- ➪ "new physics" contribuition can be sizeable
- hope: easy to see deviations
- but: $\overline{\mathcal{B}}^{\rm exp} \approx \overline{\mathcal{B}}^{\rm SM}$
- ➪ constraints on "new physics"
  ➪ precision test of SM

# $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2} \sum_n C_n Q_n + \text{h.c.}$$

$$Q_A = (\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu) \qquad Q_S = (\bar{b}\gamma_5 s)(\bar{\mu}\mu) \qquad Q_P = (\bar{b}\gamma_5 s)(\bar{\mu}\gamma_5\mu)$$

SM: only $Q_A$

BSM: $Q_S$ and $Q_P$ important

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\,\Gamma_H^S}\beta\left[|rC_A - uC_P|^2 F_P + |u\beta C_S|^2 F_S\right] + \mathcal{O}(\alpha_{em})$$

(time-integrated, CP-averaged, [De Bruyn et al.'12])

SM: $F_P = 1$, $F_S = 1 - \Delta\Gamma^s/\Gamma_L^s$

$$r = \frac{2m_\mu}{M_{B_s}}, \beta = \sqrt{1-r^2}, u = \frac{M_{B_s}}{m_b + m_s}$$

**Aim: compute $C_A$ to 3 loops in QCD + NLO EW**

## $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2} \sum_n C_n Q_n \; + \; \text{h.c.}$$

$$Q_A = (\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu) \qquad Q_S = (\bar{b}\gamma_5 s)(\bar{\mu}\mu) \qquad Q_P = (\bar{b}\gamma_5 s)(\bar{\mu}\gamma_5\mu)$$

SM: only $Q_A$ BSM: $Q_S$ and $Q_P$ important

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\,\Gamma_H^S} \beta\, r^2 |C_A(\mu_b)|^2 \; + \; \mathcal{O}(\alpha_{em})$$

(time-integrated, CP-averaged, [De Bruyn et al.'12])

$$N = V_{tb}^\star V_{tq}\, G_F^2 M_W^2/\pi^2 \qquad r = \frac{2m_\mu}{M_{B_s}}, \beta = \sqrt{1-r^2}, u = \frac{M_{B_s}}{m_b+m_s}$$

Aim: compute $C_A$ to 3 loops in QCD + NLO EW

# $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$: missing $\mathcal{O}(\alpha_{em})$

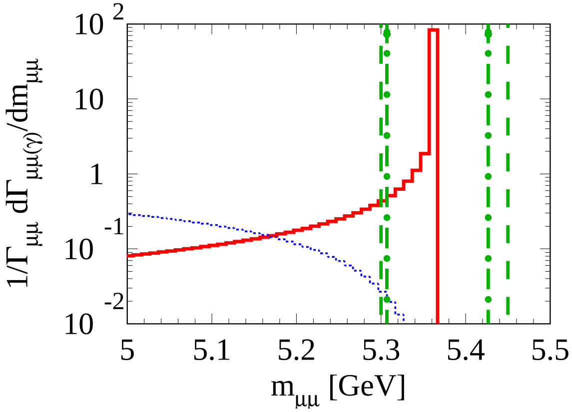
$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\,\Gamma_H^S} \beta\, r^2 |C_A(\mu_b)|^2 + \mathcal{O}(\alpha_{em})$$

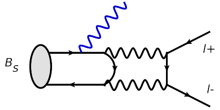
- no enhancement factor (like $\frac{1}{\sin^2\theta_W}$, $\frac{m_t^2}{M_W^2}$ or $\ln^2 \frac{M_W^2}{\mu_b^2}$)
- contribution from other operators
  e.g., $(\bar{b}\gamma_\alpha\gamma_5 q)(\bar{\ell}\gamma^\alpha\ell)$ or $(\bar{b}\gamma_\alpha P_L c)(\bar{c}\gamma^\alpha P_L s)$
  with $\gamma$ connecting quark and lepton lines
- non-perturbative effects beyond $f_{B_s}$
- compensates $\mu_b$ dependence of $C_A(\mu_b)$ (➪ $\approx 0.3\%$)
- soft Bremsstrahlung

# $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$: missing $\mathcal{O}(\alpha_{em})$

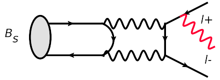
$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\, \Gamma_H^S} \beta\, r^2 |C_A(\mu_b)|^2 + \mathcal{O}(\alpha_{em})$$

- soft Bremsstrahlung: $B_s \to \mu^+\mu^- + (n\gamma)$ ($n = 0, 1, 2, \ldots$)



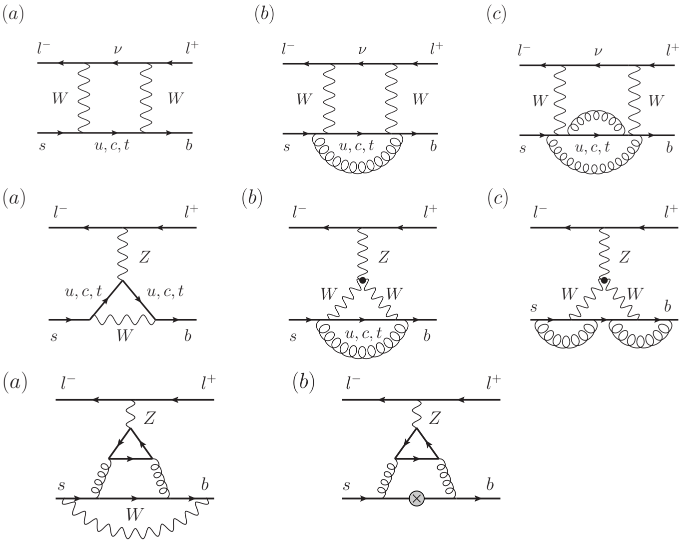




bkg for theory and exp.

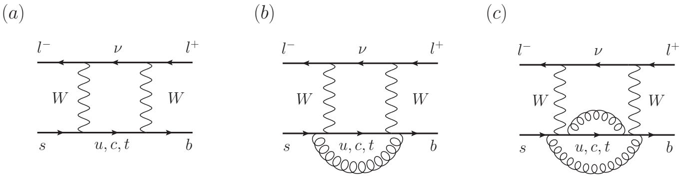



taken into account by LHCb and CMS

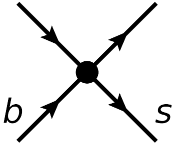
# $C_A$ to 3 loops: Feynman diagrams



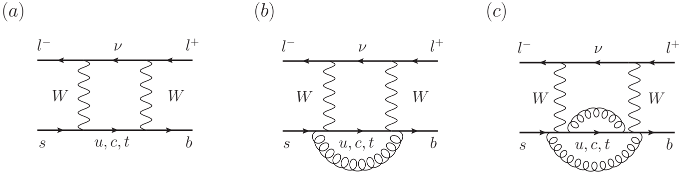
# $W$ box



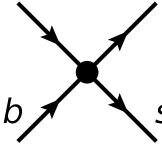

- matching in $d = 4 - 2\epsilon$ dimensions
  - effective theory: only tree-level contribution
  - evanescent operators
  - vacuum integrals, 2 masses: $M_W$ and $m_t$
    ➪ consider expansions: $M_W \approx m_t$; $M_W \ll m_t$



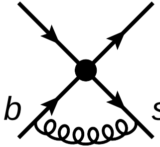
# $W$ box



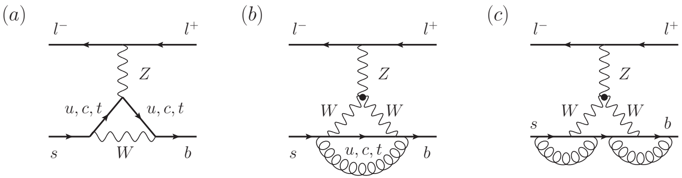

- matching in $d = 4 - 2\epsilon$ dimensions
  - effective theory: only tree-level contribution
  - evanescent operators
  - vacuum integrals, 2 masses: $M_W$ and $m_t$
    ➪ consider expansions: $M_W \approx m_t$; $M_W \ll m_t$




- matching in 4 dimensions
  - NO evanescent operators
  - loop diagrams in effective theory
  - IR regulator needed: $m_s \ll m_b \ll M_W, m_t$
    ➪ nested asymptotic expansion



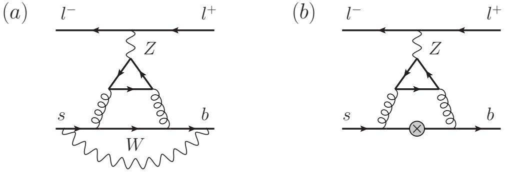
# $Z$ penguin




- matching in $d = 4 - 2\epsilon$ dimensions
- NO evanescent operators
- effective theory: only tree-level contribution
- triangle contributions

# $Z$ penguin

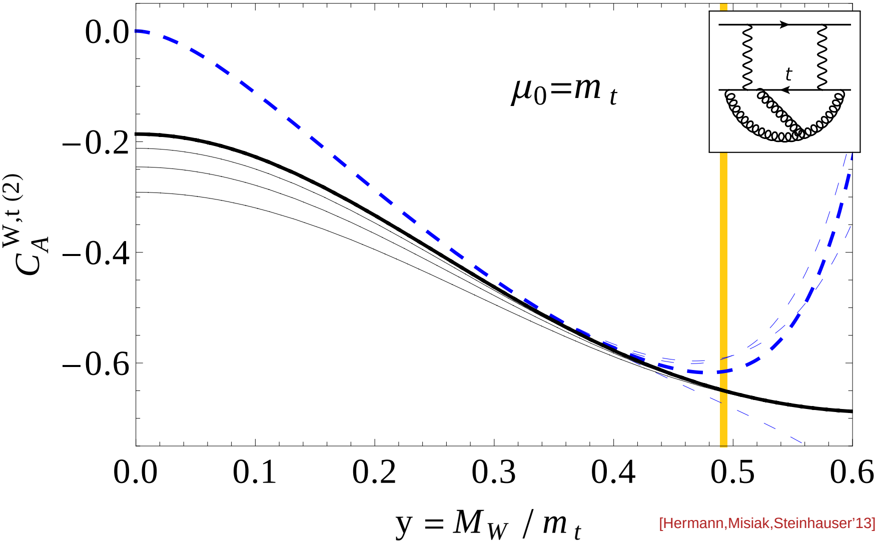
- matching in $d = 4 - 2\epsilon$ dimensions
- NO evanescent operators
- effective theory: only tree-level contribution
- triangle contributions




- $u - d$ and $c - s$ ➪ cancel
- non-zero contribution from $t - b$
- non-naive treatment of $\gamma_5$ (e.g. [Larin'93])

# $C_A$

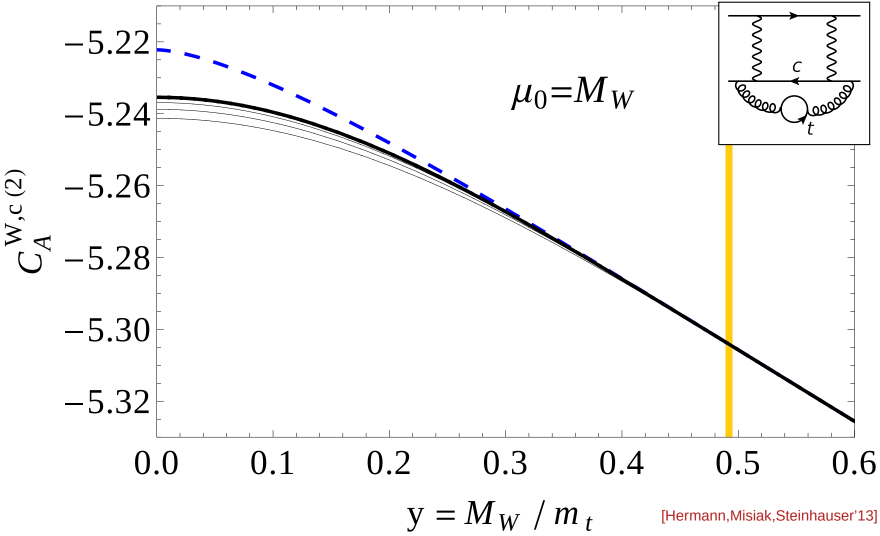
## $W$ box: top




[Hermann,Misiak,Steinhauser'13]

$C_A$

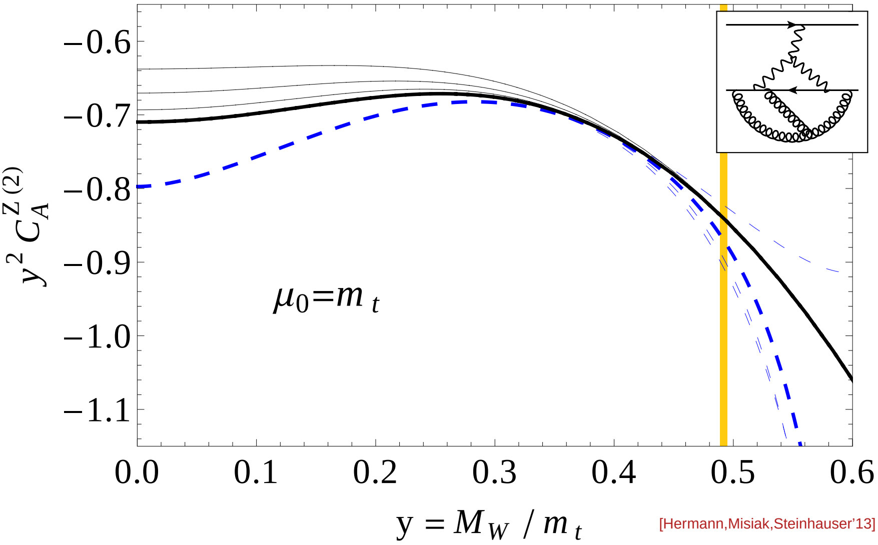
*W* box: charm




[Hermann,Misiak,Steinhauser'13]

$C_A$

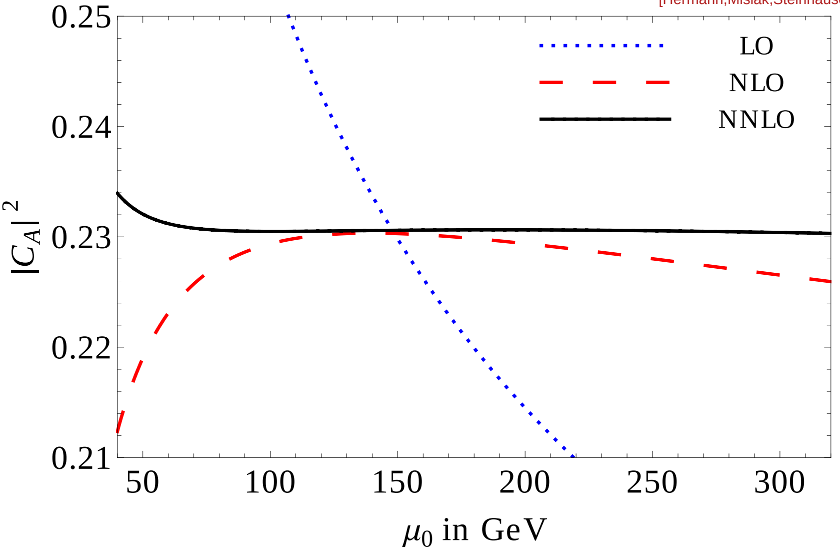
# $Z$ penguin




[Hermann,Misiak,Steinhauser'13]

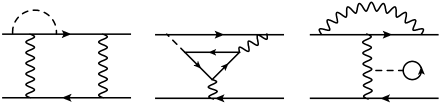
# Results: $\mu$ dependence

[Hermann,Misiak,Steinhauser'13]

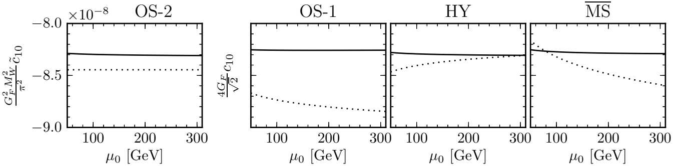


# NLO electroweak corrections

[Bobeth,Gorbahn,Stamou'13]



- different renormalization schemes ($G_F \alpha_{em}$ vs. $G_F^2 M_W^2$; OS, $\overline{\mathrm{MS}}$, hybrid)




- RGE running (not in QCD!): $C_A(\mu_0 \sim M_W, m_t) \to C_A(\mu_b \sim m_b)$
- uncertainty: 7% $\implies$ $< 1\%$

# Results: parametrization of $\mathcal{B}$

[Bobeth,Gorbahn,Hermann,Misiak,Stamou,Steinhauser'13]

$$\overline{\mathcal{B}}_{s\mu} = (3.65 \pm 0.06)\, R_{t\alpha}\, R_s \times 10^{-9} = 3.65 \pm 0.23 \times 10^{-9}$$

$$R_{t\alpha} = R_t^{3.06}\, R_\alpha^{-0.18} = \widetilde{R}_t^{3.02}\, R_\alpha^{0.032}$$

$$R_s = \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{1.615}$$

$$R_\alpha = \frac{\alpha_s(M_Z)}{0.1184} \qquad R_t = \frac{M_t^{\mathrm{OS}}}{(173.1\,\mathrm{GeV})} \qquad \widetilde{R}_t = \frac{m_t^{\overline{\mathrm{MS}}}}{(163.5\,\mathrm{GeV})}$$

# Results: uncertainties

$$\overline{\mathcal{B}}_{s\mu} = (3.65 \pm 0.06)\, R_{t\alpha}\, R_s \times 10^{-9} = 3.65 \pm 0.23 \times 10^{-9}$$

$$R_s = \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{1.615}$$

$f_{B_s}$: [FLAG], $V_{cb}$: [Gambino,Schwanda'13], $|V_{tb}^\star V_{ts}/V_{cb}|$: [CKMfitter,UTfit], $\tau_H^s$: [HFAG],

| | $f_{B_s}$ | CKM | $\tau_H^s$ | $M_t$ | $\alpha_s$ | other param. | non-param. | $\sum$ |
|---|---|---|---|---|---|---|---|---|
| $\overline{\mathcal{B}}_{s\ell}$ | 4.0% | 4.3% | 1.3% | 1.6% | 0.1% | $< 0.1\%$ | 1.5% | 6.4% |

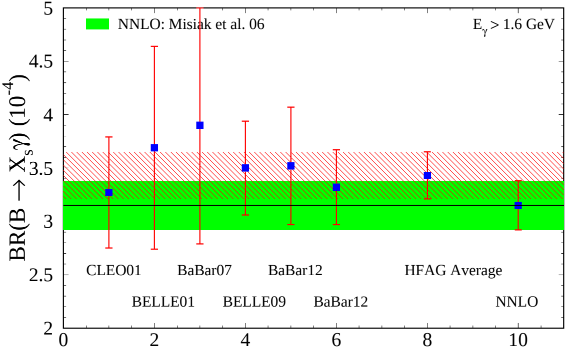
## non-parametric uncertainties

- $\mu_b$ variation
- $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_{em}\alpha_s)$ to $C_A$
- $M_B^2/M_W^2$
- $M_t^{\mathrm{OS}} \to m_t^{\overline{\mathrm{MS}}}$ transition

# Status:

$$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak,Steinhauser'06; Misiak et al.'06]




5% non-perturbative

3% parametric

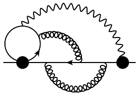
$\alpha_s, m_c, \ldots$

3% $m_c$ interpolation

3% higher order

# Updates [Czakon,Fiedler,Huber,Misiak,Schutzmeier,Steinhauser]

- update input parameters; fit to semi-leptonic $B$ decay data in kinetic scheme [Gambino,Schwanda'13]
- $m_c$ interpolation: $m_c = 0$ result for $\longrightarrow$



  [Czakon,Fiedler,Huber,Misiak,Schutzmeier,Steinhauser]
- 3- and 4-body final state contributions to the NNLO interferences among $Q_1$, $Q_2$ and $Q_8$ (BLM approximation) [Ligeti,Luke,Manohar,Wise'99; Ferroglia,Haisch'10; Misiak,Poradziński'11]
- Four-loop $Q_1, \ldots Q_6 \to Q_8$ anomalous dimensions [Czakon,Haisch,Misiak'07]
- LO $b \to sq\bar{q}\gamma$ from $Q_{1,\ldots,6}$ [Kamiński,Misiak,Poradziński'12]
- non-pert. $\mathcal{O}(\alpha_s \Lambda^2/m_b^2)$ to $(Q_7, Q_7)$ interference [Ewerth,Gambino,Nandi'10]
- non-perturbative contributions [Benzke,Lee,Neubert,Paz'10]
- ...

➪ shifts of order $\leq \pm 2\%$

Updated result for $\mathcal{B}(\bar{B} \to X_s\gamma)$ soon

# Conclusions

Karlsruhe Institute of Technology

- $\mathcal{B}(B_s \to \mu^+\mu^-)$ to NNLO QCD and NLO EW
- theory uncertainty negligible
- main uncertainty source: $f_{B_s}$ and CKM
- $\overline{\mathcal{B}}_{s\mu}{}^{\rm exp} = (2.9 \pm 0.7) \times 10^{-9}$
  $\overline{\mathcal{B}}_{s\mu}{}^{\rm th} = (3.65 \pm 0.23) \times 10^{-9}$

  $\overline{\mathcal{B}}_{d\mu}{}^{\rm exp} = \left(3.6^{+1.6}_{-1.4}\right) \times 10^{-10}$
  $\overline{\mathcal{B}}_{d\mu}{}^{\rm th} = (1.06 \pm 0.09) \times 10^{-10}$
- also predictions for $\overline{\mathcal{B}}_{se}$, $\overline{\mathcal{B}}_{s\tau}$, $\overline{\mathcal{B}}_{de}$, $\overline{\mathcal{B}}_{d\tau}$