



New applications of carlomat

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Loops and Legs in Quantum Field Theory

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- **generation of a single phase space parameterization for the Feynman diagrams of the same topology,**
- **an interface to parton density functions,**
- **improvement of the color matrix computation,**



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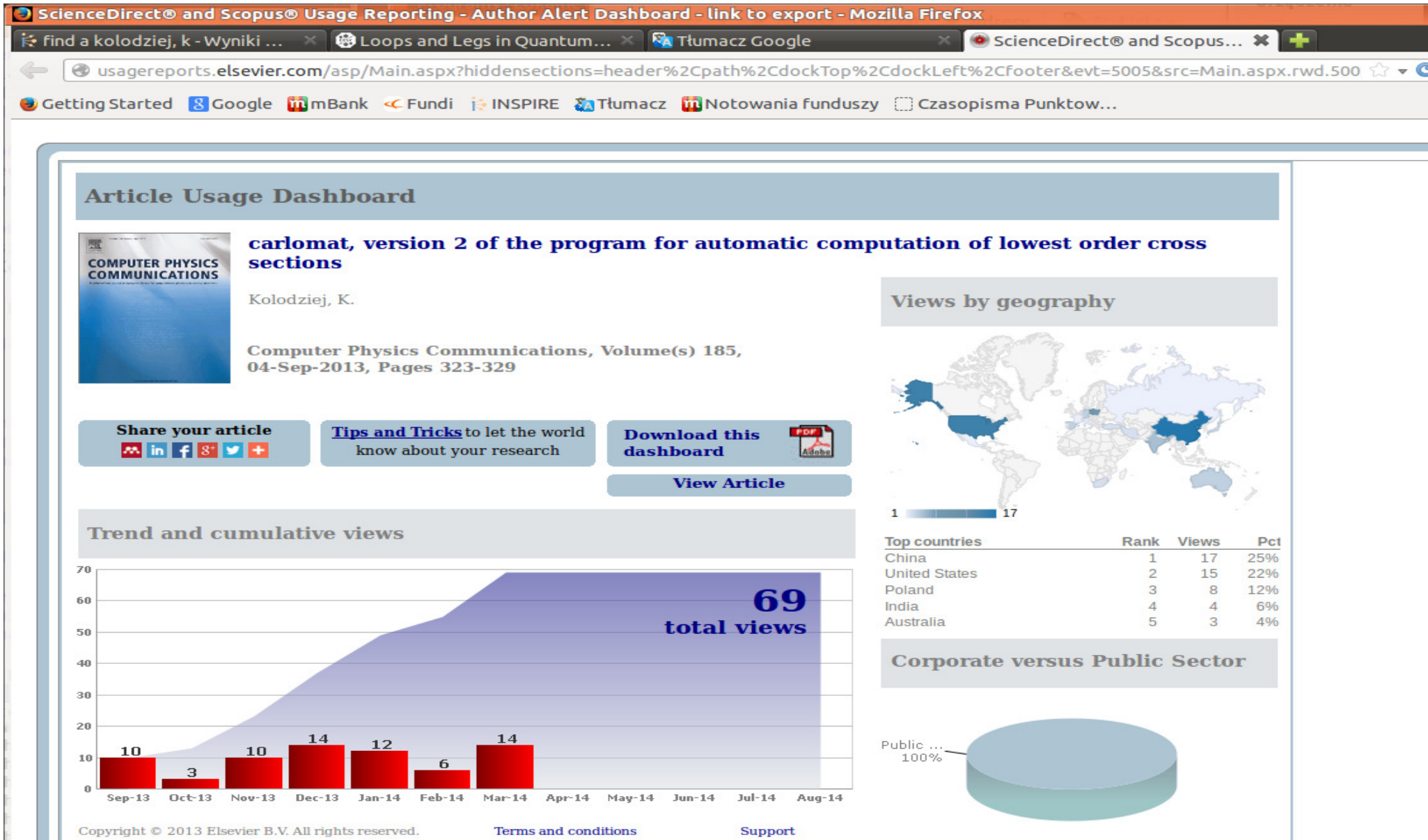
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Version 2 of carlomat was released in summer 2013 and the paper: [KK, Comput. Phys. Commun. **185** \(2014\) 323, \[arXiv:1305.5096\]](#),

was published in the beginning of 2014.

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The parameterizations are automatically combined in a single multichannel phase space integration routine.



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The phase space integration routine in `carlomat_v2.0` becomes shorter and a compilation time is reduced, by a factor of 4–5 for multiparticle reactions, compared to the previous version of the program.



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Computation of the color matrix is performed as a separate stage, that is automatically executed just after the code generation and only the nonzero elements are transferred to the MC program.



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If the CKM mixing is included then the numbers Feynman diagrams of hadronic reactions grows substantially.



Anomalous Wtb coupling

The effective Lagrangian of the Wtb interaction containing operators of dimension four and five that is implemented in the current version of the program has the following form:

$$\begin{aligned} L_{Wtb} = & \frac{g}{\sqrt{2}} V_{tb} \left[W_{\mu}^{-} \bar{b} \gamma^{\mu} (f_1^L P_L + f_1^R P_R) t \right. \\ & \left. - \frac{1}{m_W} \partial_{\nu} W_{\mu}^{-} \bar{b} \sigma^{\mu\nu} (f_2^L P_L + f_2^R P_R) t \right] \\ + & \frac{g}{\sqrt{2}} V_{tb}^{*} \left[W_{\mu}^{+} \bar{t} \gamma^{\mu} (\bar{f}_1^L P_L + \bar{f}_1^R P_R) b \right. \\ & \left. - \frac{1}{m_W} \partial_{\nu} W_{\mu}^{+} \bar{t} \sigma^{\mu\nu} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) b \right], \end{aligned}$$



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Couplings $f_i^L, f_i^R, \bar{f}_i^L, \bar{f}_i^R, i = 1, 2$, can be complex in general.

In the SM, $f_1^L = \bar{f}_1^L = 1$ and other couplings are 0.



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If CP is conserved then the following relationships hold:

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The rule does not work for the propagators in t - or u -channels, but the actual value of the top quark width should not play much of a role in them.



Anomalous top–Higgs Yukawa coupling

The most general Lagrangian of $t\bar{t}h$ interaction including corrections from dimension-six operators that has been implemented in the program has the following form [J.A. Aguilar-Saavedra, NPB821 (2009) 215]:

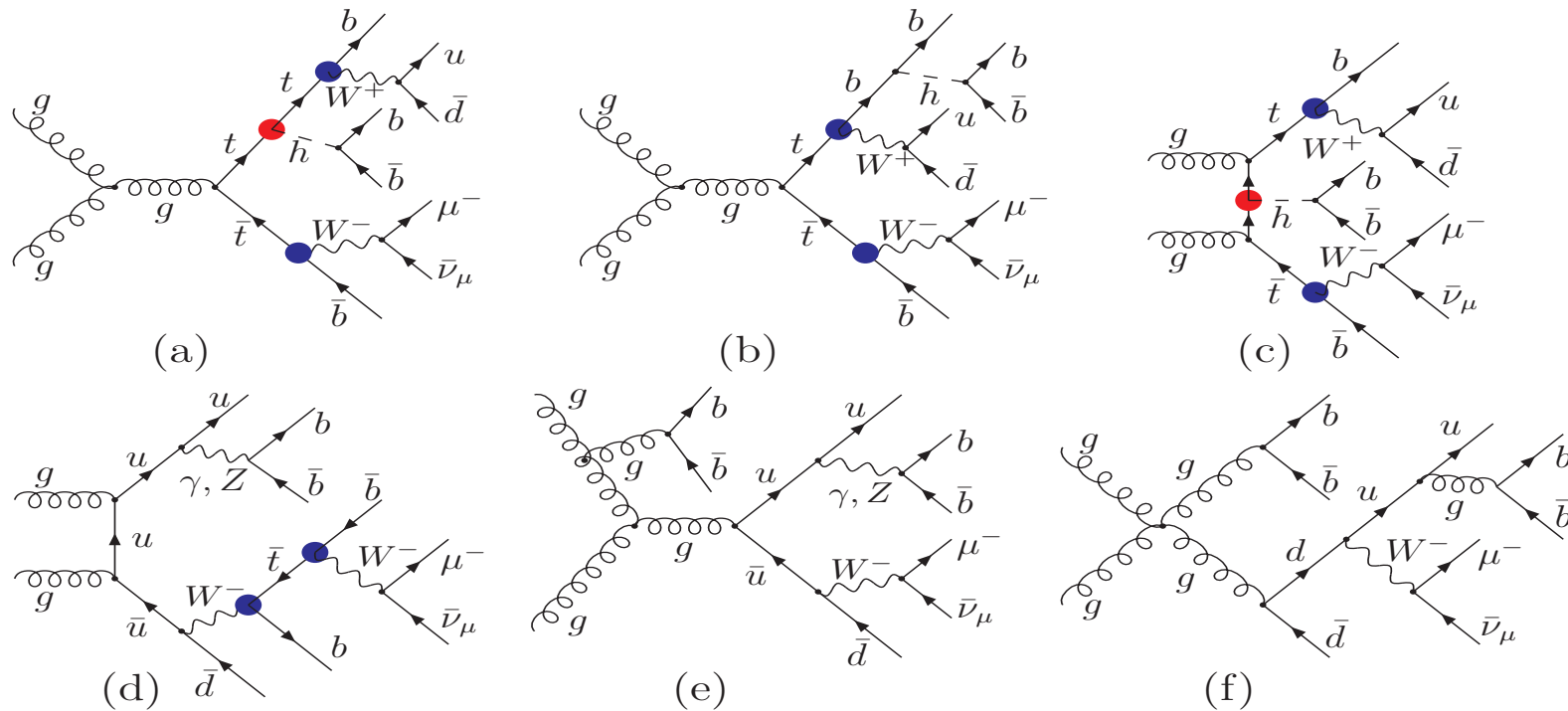
$$\mathcal{L}_{t\bar{t}h} = -g_{t\bar{t}h}\bar{t} (f + if'\gamma_5) th,$$

f and f' that describe the **scalar** and pseudoscalar **departures**, respectively, from a purely scalar top–Higgs Yukawa coupling $g_{t\bar{t}h}$ of SM are assumed to be real.

The top–Higgs Yukawa coupling of SM is reproduced for $f = 1$ and $f' = 0$.

Associated $t\bar{t}H$ production at LHC

Examples of the Feynman diagrams of $gg \rightarrow bu\bar{d}\bar{b}\mu^-\bar{\nu}_\mu\bar{b}\bar{b}$:

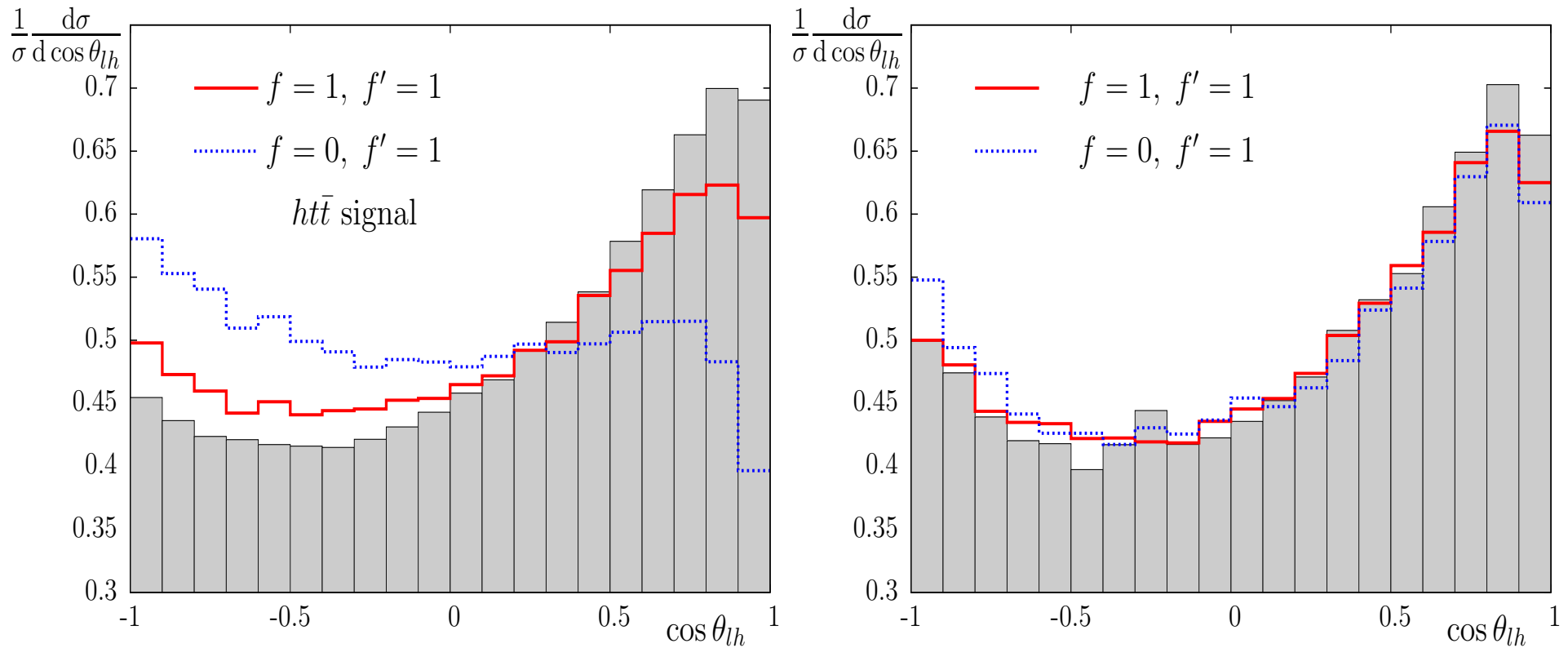


Red (blue) blobs indicate the top-Higgs (Wtb) coupling.

67 300 diagrams in the leading order of the SM, in the unitary gauge, neglecting masses smaller than m_b and the CKM mixing.

$pp \rightarrow b\bar{b}b\bar{u}d\bar{b}\mu^- \bar{\nu}_\mu$ at $\sqrt{s} = 14$ TeV

Distributions in cosine of the μ^- -Higgs boson angle with signal (left) and all (right) Feynman diagrams (CTEQ6L PDFs with $Q = 2m_t + m_h$):



From KK, JHEP **07** (2013) 083.



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At low energies, the hadronic final states consist mostly of pions, accompanied by one or more photons.



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$$\begin{aligned}\mathcal{L}_\pi^{\text{sQED}} = & \partial_\mu \varphi (\partial^\mu \varphi)^* - m_\pi^2 \varphi \varphi^* - ie (\varphi^* \partial_\mu \varphi - \varphi \partial_\mu \varphi^*) A^\mu \\ & + e^2 g_{\mu\nu} \varphi \varphi^* A^\mu A^\nu.\end{aligned}$$



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The bound state nature of the charged pion can be taken into account by the substitutions:

$$e \rightarrow eF_\pi(q^2), \quad e^2 \rightarrow e^2 |F_\pi(q^2)|^2,$$

where $F_\pi(q^2)$ is the charged pion form factor (not implemented).



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Simulation of processes involving the EM interaction of nucleons.



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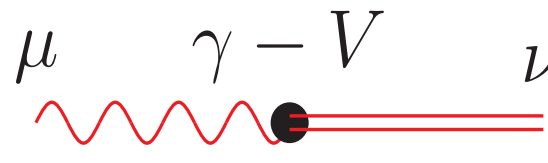


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Implementation of the particle mixing is more challenging.

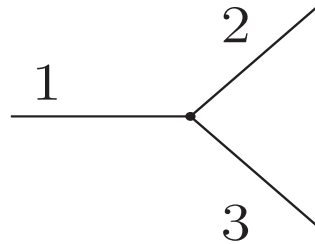

$$\mu \quad \gamma - V \quad \nu \quad \equiv \quad -ef_{\gamma V} g^{\mu\nu}$$

Substantial changes in the code generating part of the program are required.



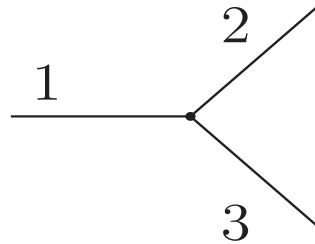
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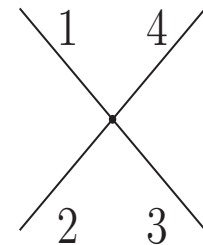
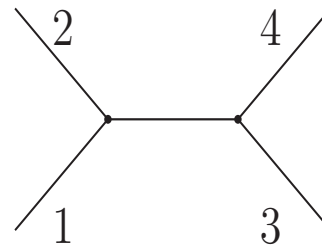
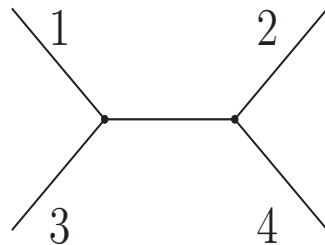
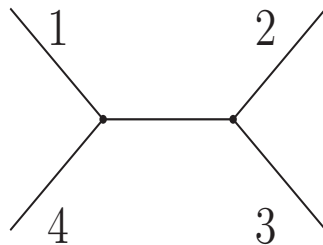


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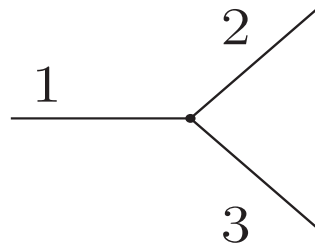


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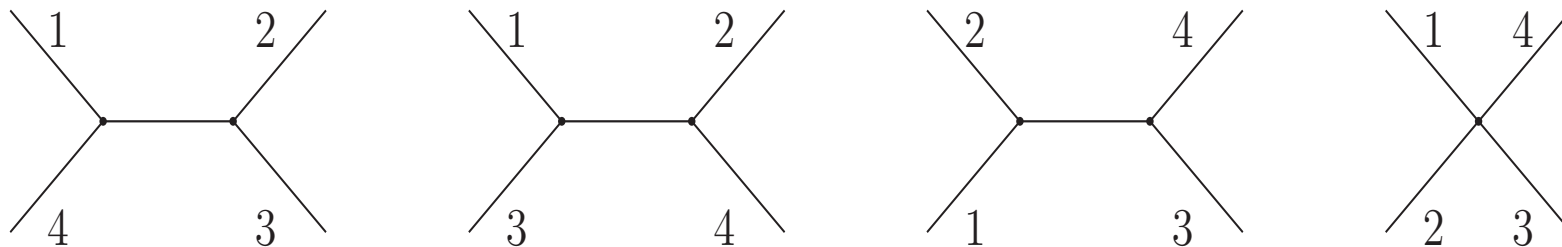


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Line 5 is attached to **each line**, including the internal ones, and to each **triple vertex** \Rightarrow **25 topologies** of a **5 particle** process.



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The particle mixing is added just at this stage.



Thank you for your attention