# GRACE for ILC

J. Fujimoto (KEK),
29-04 Loops&Legs2014 @ Weimar

on behalf of the collaboration with

P.H. Khiem(SOKENDAI Univ. and KEK),
Y. Kurihara, T.Kaneko, T.Ishikawa, Y. Shimizu(KEK),
Y.Yasui(TMC), K.Kato, N. Nakazawa, K. Tobimatsu
(Kogakuin Univ.), M. Igarashi(Tokai Univ.) ,
J.A.M. Vermaseren(NIKHEF) and T. Ueda(KIT)

# Introduction

(1) LHC discovered Higgs boson [1,2]

(2) ILC is expected to measure properties of Higgs boson very precisely: mass, spin, CP and Gauge/Yukawa couplings.

(3) ILC is also expected to measure properties of top-quark very precisely: mass, the coupling to the Higgs boson and gauge bosons.

(4) Standard Model should provide the reference values of the cross sections, branching ratios, etc. as much as precise to explore the beyond SM.

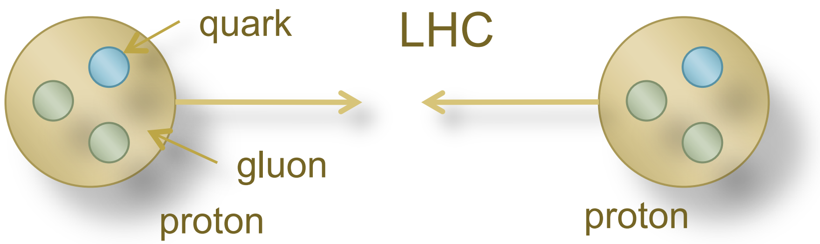
[1] PLB 716 (2012) 1

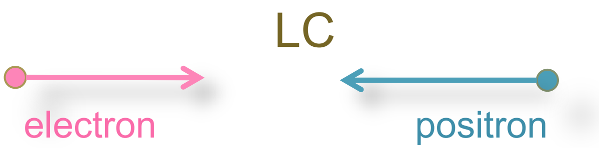
[2] PLB 716 (2012) 30

# ILC features : cleanliness

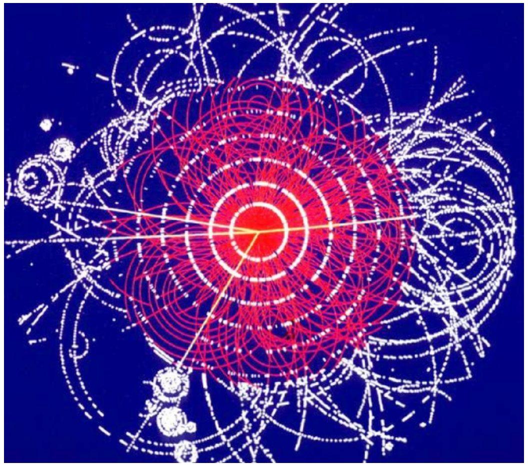
- Collision of two elementary particles

→ Theoretically clean
(less theoretical uncertainties)

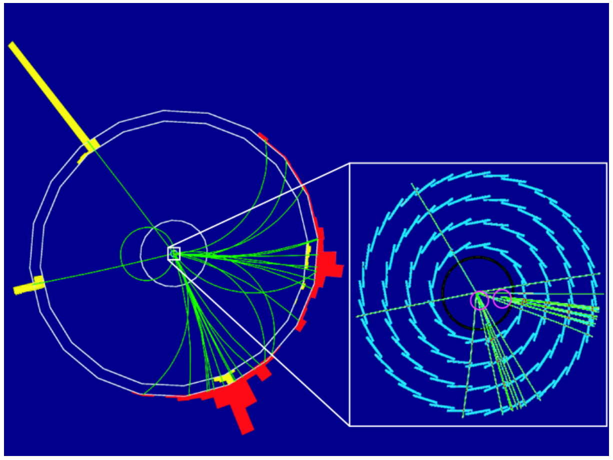




LHC



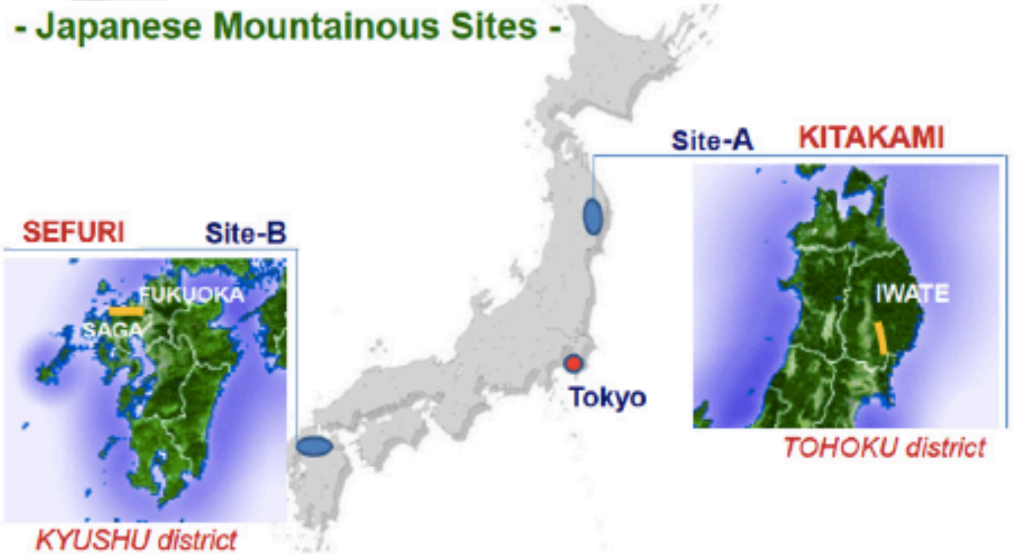
LC



LINEAR COLLIDER COLLABORATION

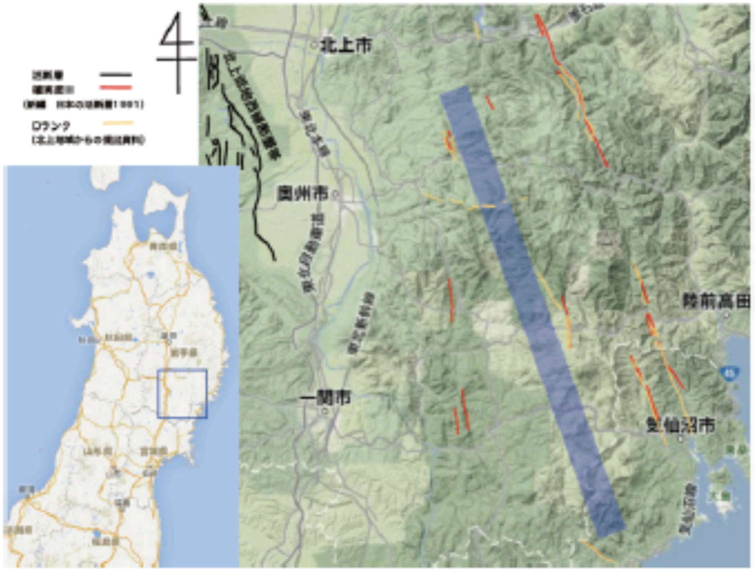
# Japan – Preferred Site selection

"Issues that could lead to particularly serious difficulties for the Sefuri site are that the route passes under or near a dam lake, and that the route passes under a city zone.
Also, the lengths of access tunnels are longer for the Sefuri site than for the Kitakami site leading to a large merit for the latter in terms of cost, schedule, and drainage"

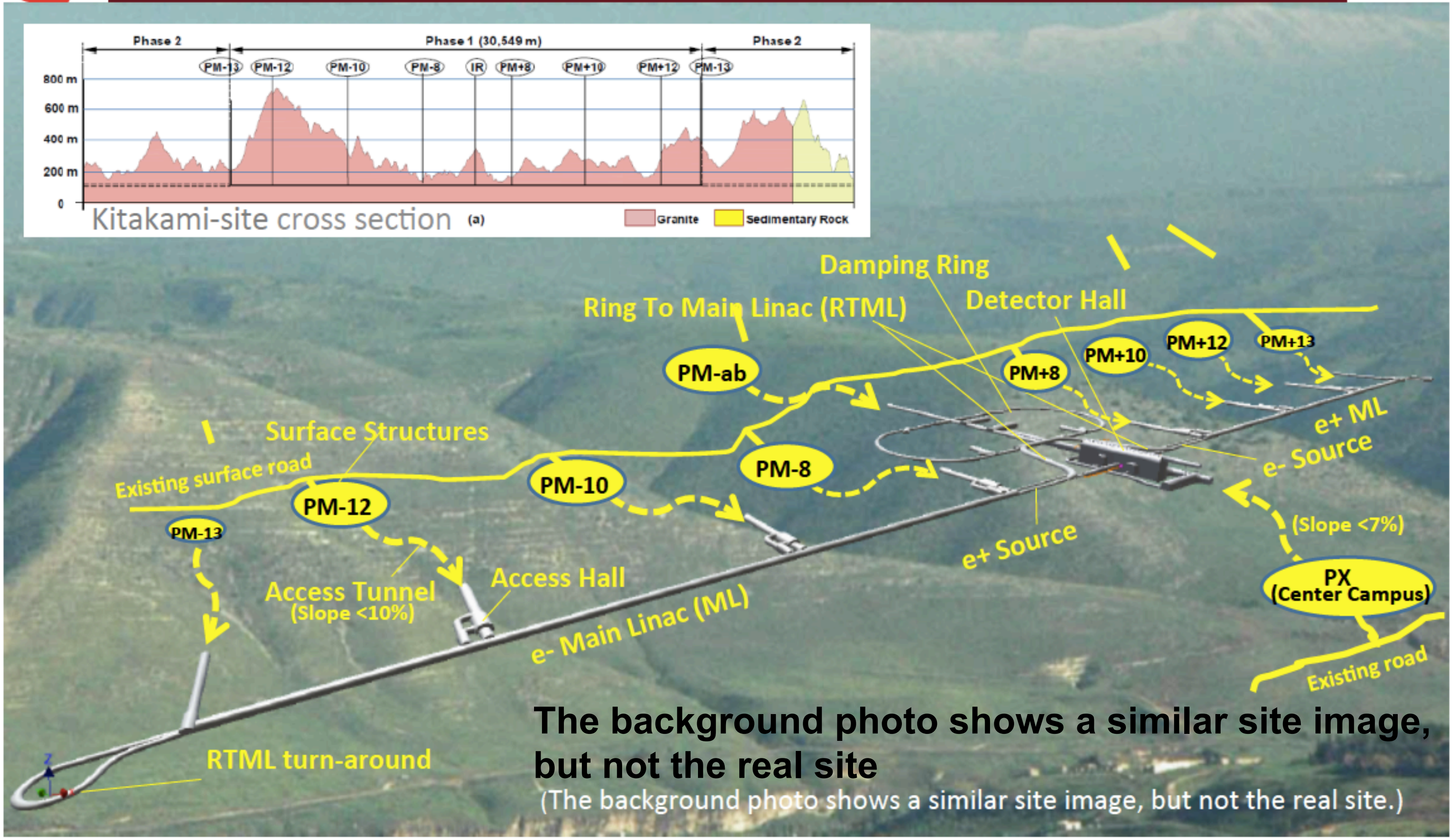


Three slides from Mike Harrison
@LCWS13 at Tokyo

# Preferred Site selected



# Site Specific Design



**The background photo shows a similar site image, but not the real site**

(The background photo shows a similar site image, but not the real site.)

Need to establish the IP and linac orientation
Then the access points and IR infrastructure
Then linac length and timing

# Time scale and ILC Upgrade Options

- Assume an optimistic scenario:
  - International agreement reached in 2~3 years
  - Then, the real LC lab will be established
  - Experiments will start ~10 years later from now
    - 250 GeV CM (Higgs factory)
      - x4 luminosity @ 3E34/cm$^2$s
      - x2 Nbunch, x2 rep rate; 120 → 200 MW wall plug
    - 500 GeV CM
      - x2 luminosity @3.6E34/cm$^2$s
      - x2 Nbunch; 160 → 200 MW wall plug
    - 1 TeV CM
      - x1.4 luminosity @5E34/cm$^2$s
      - Aggressive beam params;
      - Same wall plug power

# Bhabha and Radiative Bhabha scattering as key-processes in ILC

- When ILC pursues precise measurements with a few % errors, the luminosity measurement should be done with a few permill precision
- The luminosity measurements of ILC may be carried out based on the Bhabha scattering or the radiative Bhabha scattering
- Here we discuss full O($\alpha$) electroweak corrections to the radiative Bhabha process by means of GRACE, based on

P.H. Khiem et.al. arXiv:1403.6557

# Former studies on the radiative Bhabha

- Analytical expressions of tree level

  K. Tobimatsu and M. Igarashi, CPC 136(2001) 105

- Available generators(examples)

  BHAGEN-1PH

  M. Caffo and H. Czyz, CPC 100 (1997) 99–118

  κκMC

  A. Yost and B Ward, Conf. Proc. C060726(2006) 697

- Full QED corrections:

  S. Actis et,al. Phys. Lett. B682:419–427, 2010

# GRACE : the generator of event generators

- Feynman rules based on SM and MSSM
- Any orders of Feynman diagrams can be generated automatically
  - 1-loop diagrams of SM and MSSM can be evaluated
  - 1-loop integrals up to 4-point functions are equipped
  - 1-loop integrals up to 6-point functions are evaluated with the reduction method
  - 2-loop integrals up to 4-point functions are evaluated numerically
- GRACE is originally designed for e+e- collisions
  - It produced grc4f event generators for LEP-II
  - Several processes for ILC had been evaluated

# O($\alpha$) corrections calculated by GRACE for ILC

- 2 → 2

  $e^-e^+ \rightarrow$ $t\bar{t}$、$W^+W^-$、ZZ

  G. Belanger et.al. Phys. Rept. 430 (2006) 117–209

- 2 → 3

  $e^-e^+ \rightarrow$ $\nu\bar{\nu}H$ G. Belanger et.al. Phys. Lett. B559 (2003) 252–262

  $e^-e^+ \rightarrow$ $t\bar{t}H$ G. Belanger et.al. Phys. Lett. B571 (2003) 163–172

  $e^-e^+ \rightarrow$ ZHH F. Belanger et.al, Phys. Lett. B576 (2003) 152–164

  $e^-e^+ \rightarrow$ $\nu\bar{\nu}\gamma$

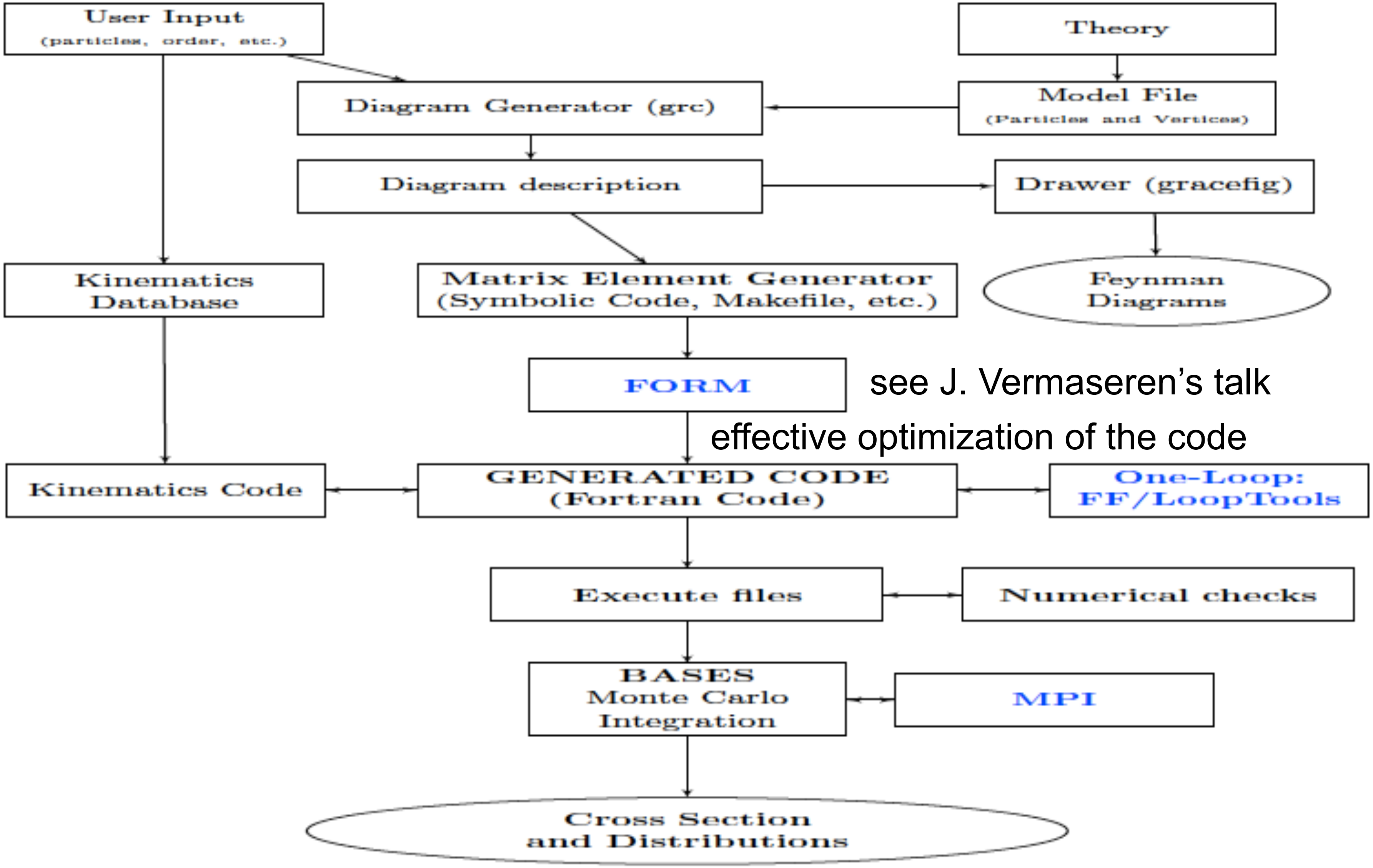
  F. Boudjema et.al Nucl. Instrum. Meth. A534 (2004) 334–338

  $e^-e^+ \rightarrow$ $t\bar{t}\gamma$ P.K. Khiem et.al, Eur. Phys. J. C73 (2013) 2400

- 2 → 4

  $e^-e^+ \rightarrow$ $\nu\bar{\nu}HH$ K. Kato et.al, PoS HEP 2005 (2006) 312

# GRACE scheme



User Input
(particles, order, etc.)

Theory

Model File
(Particles and Vertices)

Diagram Generator (grc)

Diagram description

Drawer (gracefig)

Kinematics Database

Matrix Element Generator
(Symbolic Code, Makefile, etc.)

Feynman Diagrams

FORM

see J. Vermaseren’s talk

effective optimization of the code

Kinematics Code

GENERATED CODE
(Fortran Code)

One-Loop:
FF/LoopTools

Execute files

Numerical checks

BASES
Monte Carlo
Integration

MPI

Cross Section
and Distributions

The non-linear gauge fixing Lagrangian condition[4]

$$\begin{aligned}\mathcal{L}_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\ &\quad +\xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\ &\quad -\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2 .\end{aligned}$$

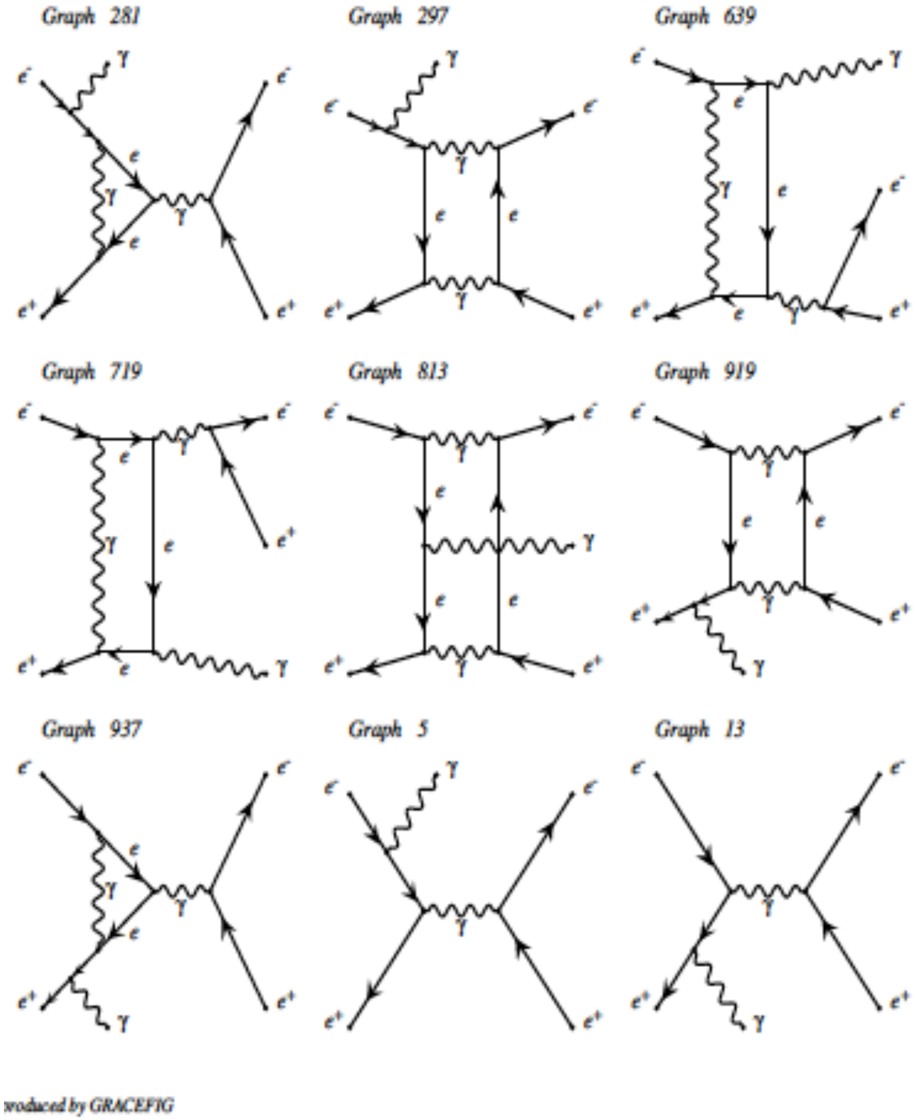
- $\xi_W = \xi_Z = \xi_A = 1$: 'tHoof-Feynman gauge

$$\frac{1}{k^2-M_W^2}\left[g_{\mu\nu}-(1-\xi_W)\frac{k^\mu k^\nu}{k^2-\xi_W^2 M_W^2}\right]$$

- ↪ the result must be independence of non-linear gauge parameters

---

[4] Phys. Rept. **430**, 117 (2006)



## The process: $e^+e^- \to e^+e^-\gamma$

Model = "nlg2301.mdl";

Process;

ELWK = {5, 3};

Initial = {electron, positron} ;

Final = {photon, electron, positron}

Expand = Yes;

OPI = No;

Kinem = "2302";

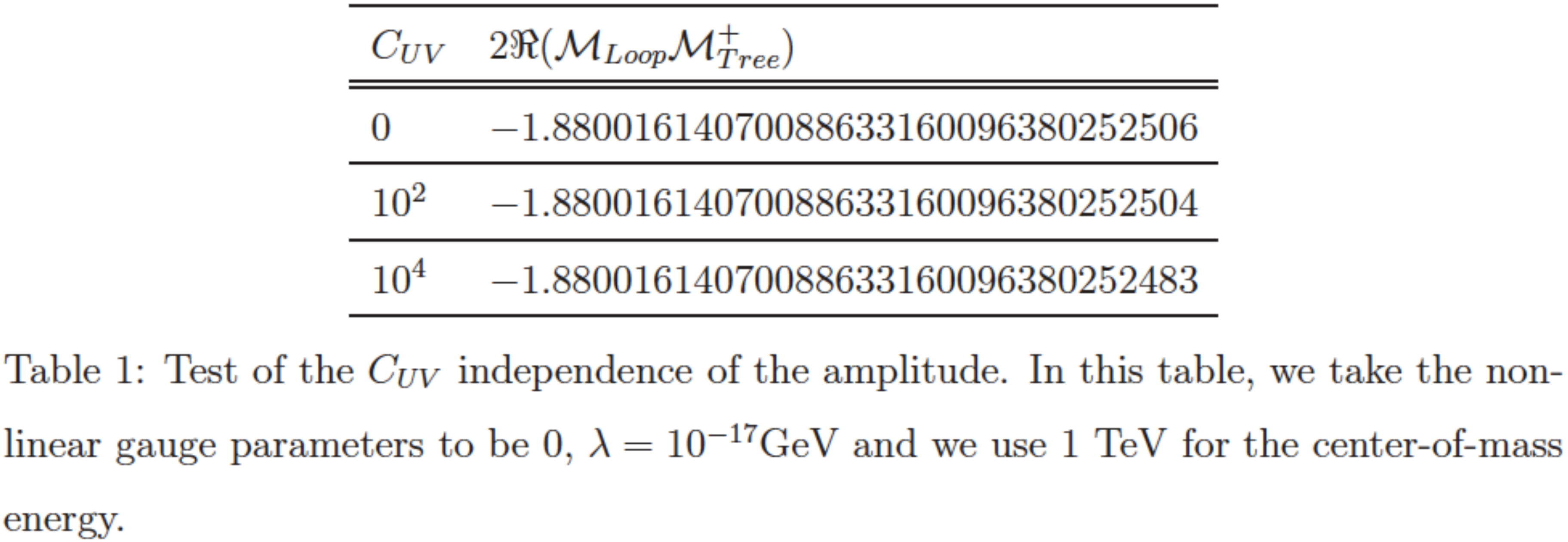
Pend;

- 32 tree diagrams,
- 3456 one-loop diagrams.

# Check of calculations（1）

$$\begin{aligned}\sigma_{\mathbf{tot}}^{e^-e^+\gamma_H} &= \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H} + \int d\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}(C_{UV},\{\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\epsilon},\tilde{\kappa}\},\lambda) \\ &+ \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H}\delta_{\mathbf{soft}}(\lambda \le E_{\gamma_S} < k_c) + \int d\sigma_{\mathbf{H}}^{e^-e^+\gamma_H\gamma_S}(E_{\gamma_S} \ge k_c).\end{aligned}$$

- Test of the Cuv independence of the amplitudes



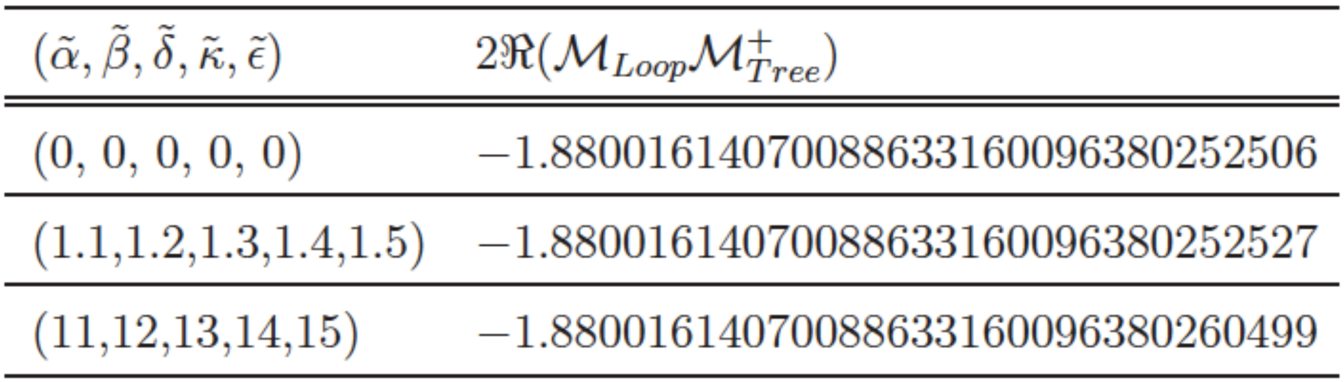
| $C_{UV}$ | $2\Re(\mathcal{M}_{Loop}\mathcal{M}^{+}_{Tree})$ |
|---|---|
| 0 | $-1.88001614070088633160096380252506$ |
| $10^2$ | $-1.88001614070088633160096380252504$ |
| $10^4$ | $-1.88001614070088633160096380252483$ |

Table 1: Test of the $C_{UV}$ independence of the amplitude. In this table, we take the non-linear gauge parameters to be 0, $\lambda = 10^{-17}$GeV and we use 1 TeV for the center-of-mass energy.

# Check of calculations (2)

$$\sigma_{\mathbf{tot}}^{e^-e^+\gamma_H} = \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H} + \int d\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) + \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H}\delta_{\mathbf{soft}}(\lambda \le E_{\gamma_S} < k_c) + \int d\sigma_{\mathbf{H}}^{e^-e^+\gamma_H\gamma_S}(E_{\gamma_S} \ge k_c).$$

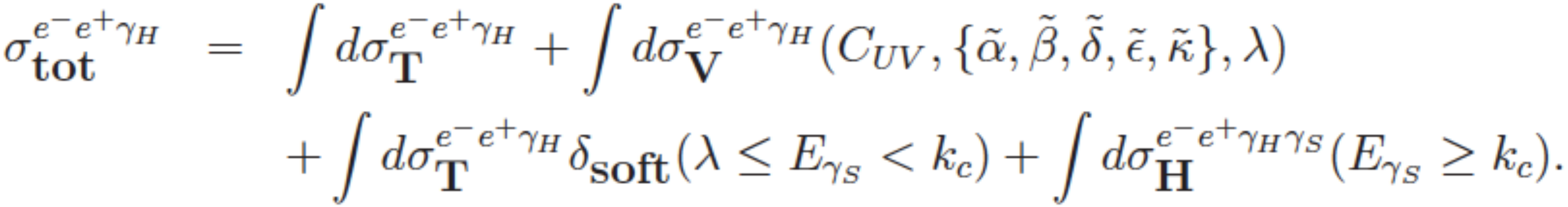
- Gauge invariance of the amplitudes



| $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon})$ | $2\Re(\mathcal{M}_{Loop}\mathcal{M}^{+}_{Tree})$ |
|---|---|
| (0, 0, 0, 0, 0) | −1.88001614070088633160096380252506 |
| (1.1,1.2,1.3,1.4,1.5) | −1.88001614070088633160096380252527 |
| (11,12,13,14,15) | −1.88001614070088633160096380260499 |

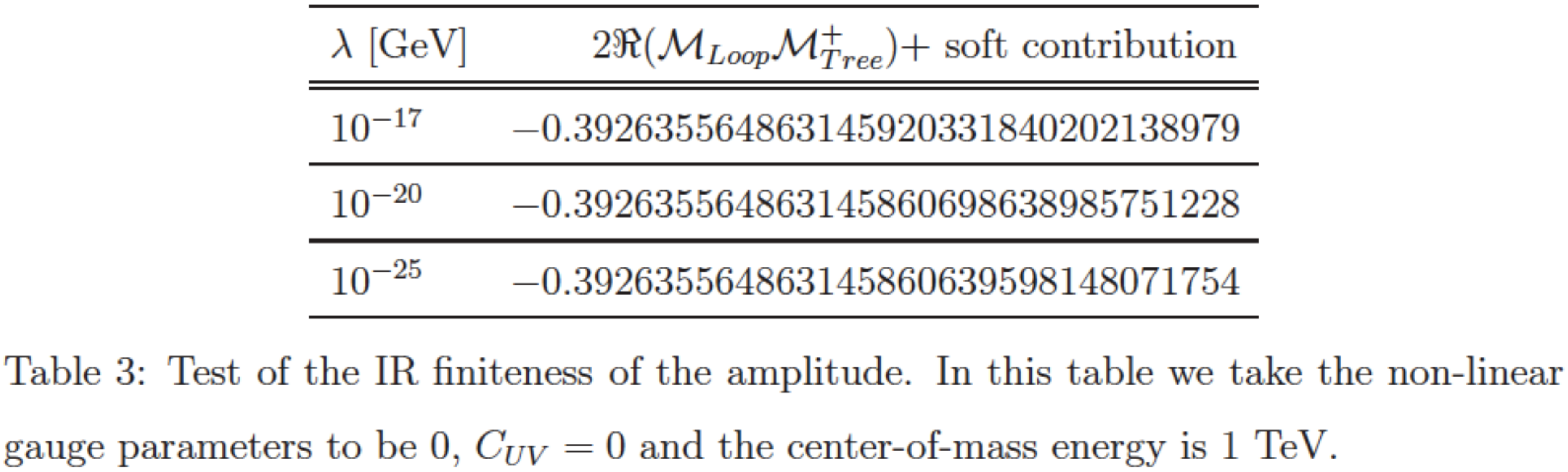
Table 2: Gauge invariance of the amplitude. In this table, we set $C_{UV} = 0$, the photon mass is $10^{-17}$GeV and a 1 TeV center-of-mass energy.

# Check of calculations (3)



$$\sigma_{\mathbf{tot}}^{e^- e^+ \gamma_H} = \int d\sigma_{\mathbf{T}}^{e^- e^+ \gamma_H} + \int d\sigma_{\mathbf{V}}^{e^- e^+ \gamma_H}(C_{UV}, \{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\epsilon}, \tilde{\kappa}\}, \lambda) + \int d\sigma_{\mathbf{T}}^{e^- e^+ \gamma_H} \delta_{\mathbf{soft}}(\lambda \leq E_{\gamma_S} < k_c) + \int d\sigma_{\mathbf{H}}^{e^- e^+ \gamma_H \gamma_S}(E_{\gamma_S} \geq k_c).$$

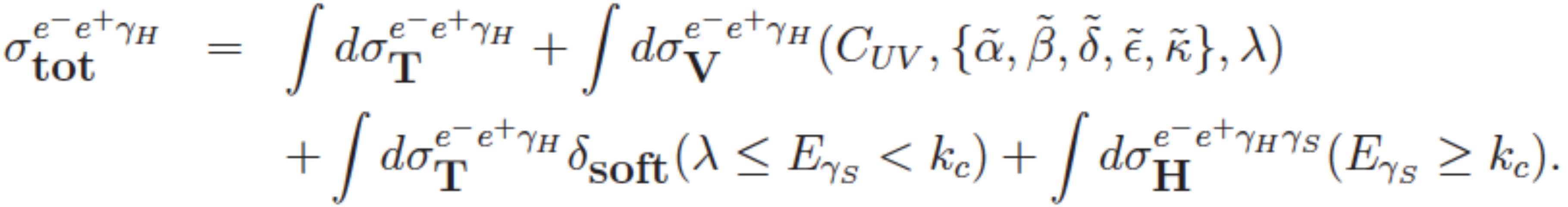
- Test of infrared finiteness



| $\lambda$ [GeV] | $2\Re(\mathcal{M}_{Loop}\mathcal{M}^{+}_{Tree})$+ soft contribution |
|---|---|
| $10^{-17}$ | $-0.392635564863145920331840202138979$ |
| $10^{-20}$ | $-0.392635564863145860698638985751228$ |
| $10^{-25}$ | $-0.392635564863145860639598148071754$ |

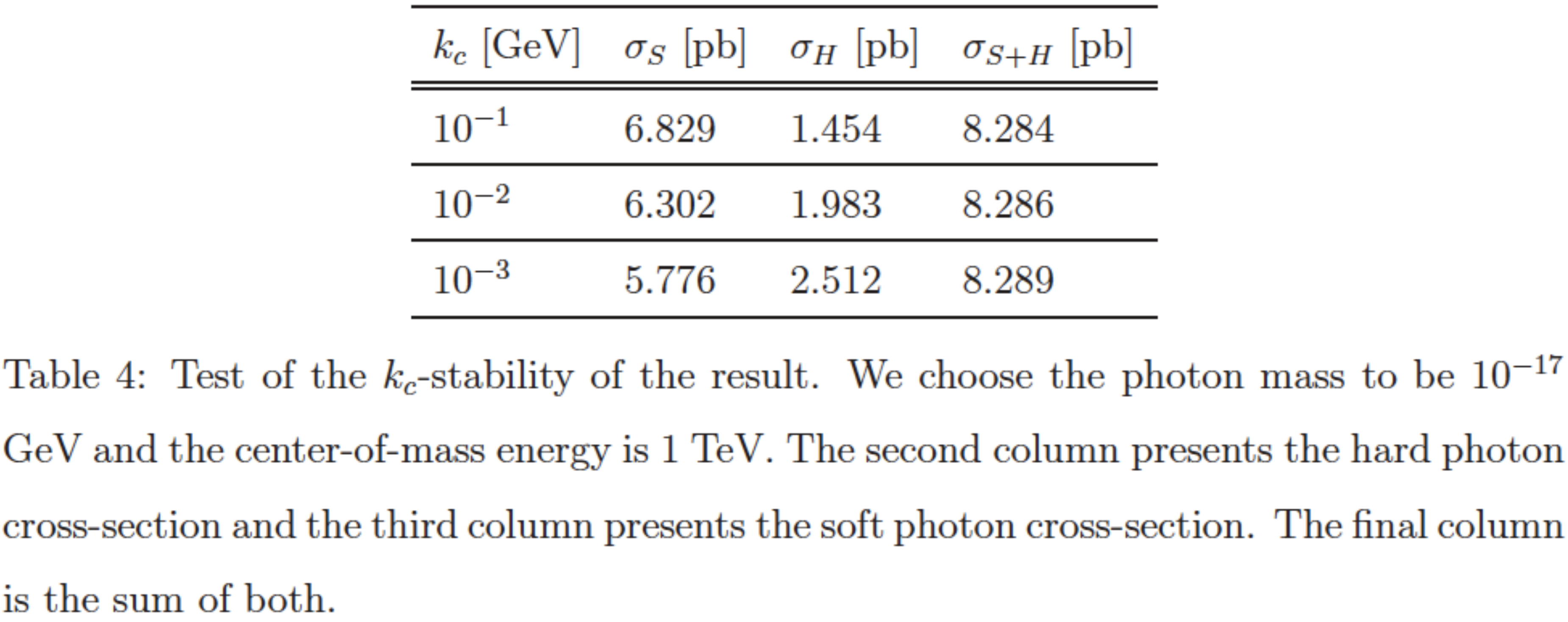
Table 3: Test of the IR finiteness of the amplitude. In this table we take the non-linear gauge parameters to be 0, $C_{UV} = 0$ and the center-of-mass energy is 1 TeV.

# Check of calculations (4)



$$
\begin{aligned}
\sigma_{\mathbf{tot}}^{e^-e^+\gamma_H} &= \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H} + \int d\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}(C_{UV},\{\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\epsilon},\tilde{\kappa}\},\lambda) \\
&+ \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H}\,\delta_{\mathbf{soft}}(\lambda \le E_{\gamma_S} < k_c) + \int d\sigma_{\mathbf{H}}^{e^-e^+\gamma_H\gamma_S}(E_{\gamma_S} \ge k_c).
\end{aligned}
$$

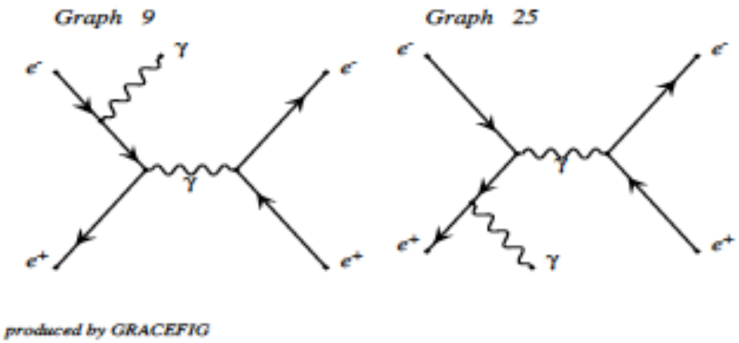
- Test of kc stability



| $k_c$ [GeV] | $\sigma_S$ [pb] | $\sigma_H$ [pb] | $\sigma_{S+H}$ [pb] |
|---|---|---|---|
| $10^{-1}$ | 6.829 | 1.454 | 8.284 |
| $10^{-2}$ | 6.302 | 1.983 | 8.286 |
| $10^{-3}$ | 5.776 | 2.512 | 8.289 |

Table 4: Test of the $k_c$-stability of the result. We choose the photon mass to be $10^{-17}$ GeV and the center-of-mass energy is 1 TeV. The second column presents the hard photon cross-section and the third column presents the soft photon cross-section. The final column is the sum of both.

## ❶ The large numerical cancellation problem.



$$\sum_{\lambda=0}^{3} \epsilon^{\mu}_{\lambda}(q)\epsilon^{\nu}_{\lambda}(q) \to -g^{\mu\nu} + \frac{q^{\mu}n^{\nu} + q^{\nu}n^{\mu}}{n.q} - n^2\frac{q^{\mu}q^{\nu}}{(n.q)^2}$$

| Amplitude | Non-Axial Gauge | Axial Gauge |
|---|---|---|
| $\mathcal{M}_1^2 + \mathcal{M}_2^2$ | $0.1116212357 \cdot 10^{+13}$ | $0.3644158264 \cdot 10^{+02}$ |
| $2\mathcal{M}_1^*\mathcal{M}_2$ | $-0.1116212356 \cdot 10^{+13}$ | $0.1546482734 \cdot 10^{+03}$ |
| $\lvert\mathcal{M}_1 + \mathcal{M}_2\rvert^2$ | $0.1910871582 \cdot 10^{+03}$ | $0.1910898560 \cdot 10^{+03}$ |

## ❷ The Monte-Carlo integration step costs much in CPU time.

The process: $e^+e^- \to e^+e^-\gamma$

| CPU | Memory | CPU time |
|---|---|---|
| Intel(R) Xeon(R), X5660@2.80GHz | 49 GB | $\geq$3 months @$\sqrt{s}$. |

$\Longrightarrow$ **BASES with MPI**[5]

10days @ $\sqrt{s}$. w/ 10 CPUs

---

[5] **The Message Passing Interface:** *http://www.mcs.anl.gov/research/projects/mpi*

$$\delta_{EW} = \frac{\sigma(\alpha)}{\sigma_{Tree}} - 1. \quad \delta_{QED} = \frac{\sigma^{QED}(\alpha) - \sigma_0^{QED}}{\sigma_{Tree}}. \quad \delta_W = \delta_{EW} - \delta_{QED}.$$

$E^{\mathbf{cut}}_{\gamma,e^+,e^-} \geq 10$ GeV, $\theta^{\mathbf{cut}}_{\gamma,e^+,e^-} = 10°$;

$\theta^{\mathbf{cut}}_{\gamma/\{e^+,e^-\}} = 10°$, $\theta^{\mathbf{cut}}_{e^+/e^-} = 10°$.

$M_Z = 91.1876$ GeV

$M_W = 80.3759$ GeV

$M_H = 126$ GeV

$m_t = 173.5$ GeV

$\Gamma_Z = 2.3549$ GeV

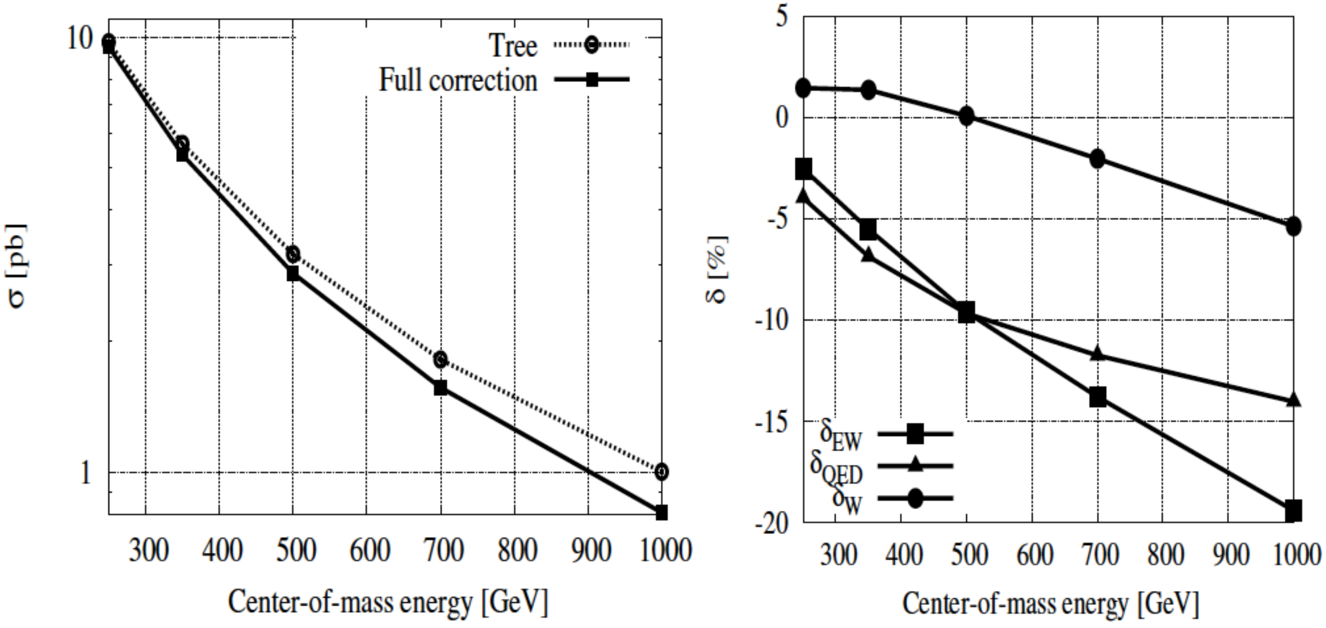


Figure 2: In this figure, the cross-section (left) and full electroweak corrections (right) are presented as a function of the center-of-mass energy.

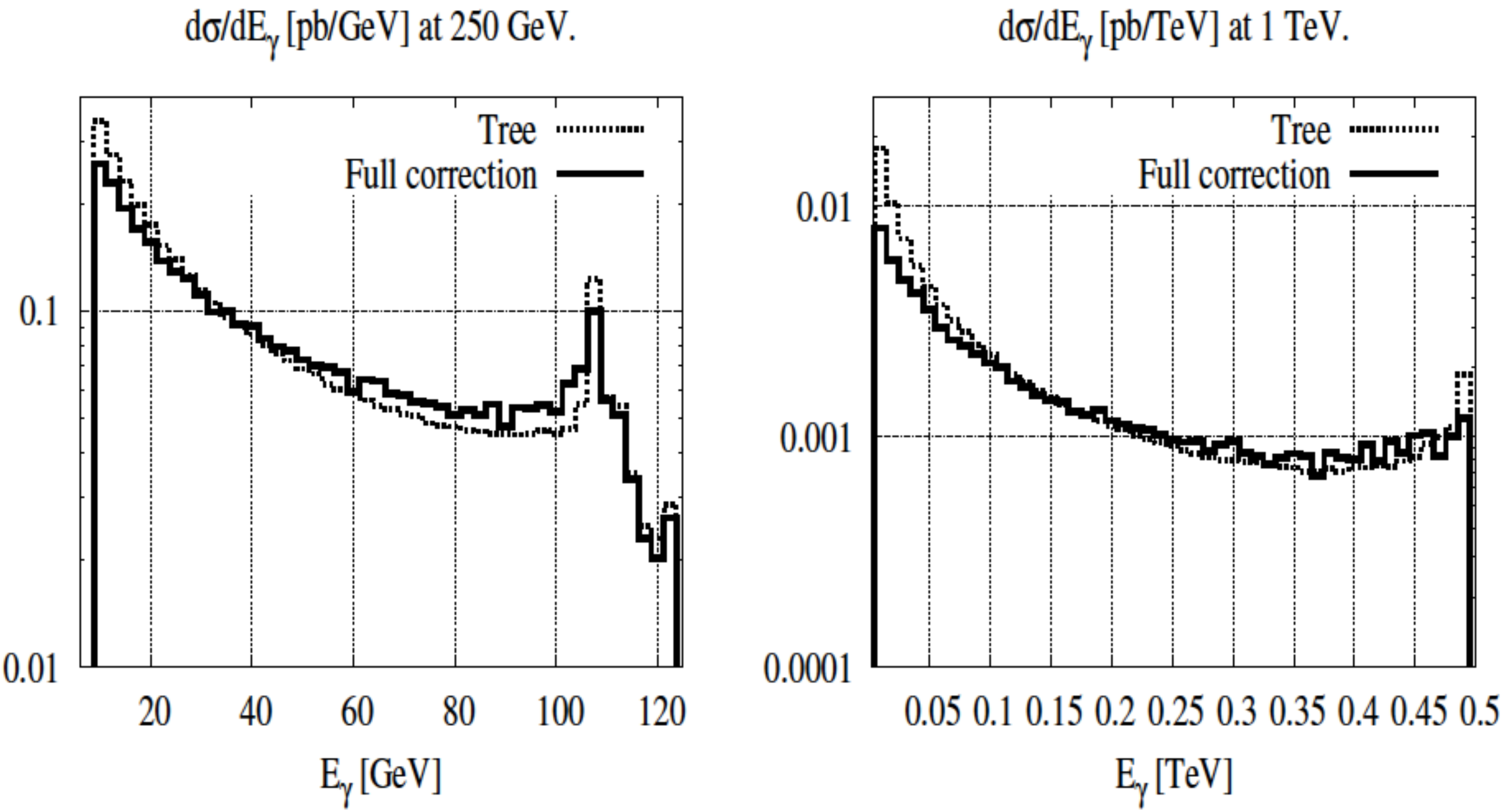


Figure 3: The differential cross-section as a function of the photon energy at $\sqrt{s} = 250$ GeV and $\sqrt{s} = 1$ TeV.

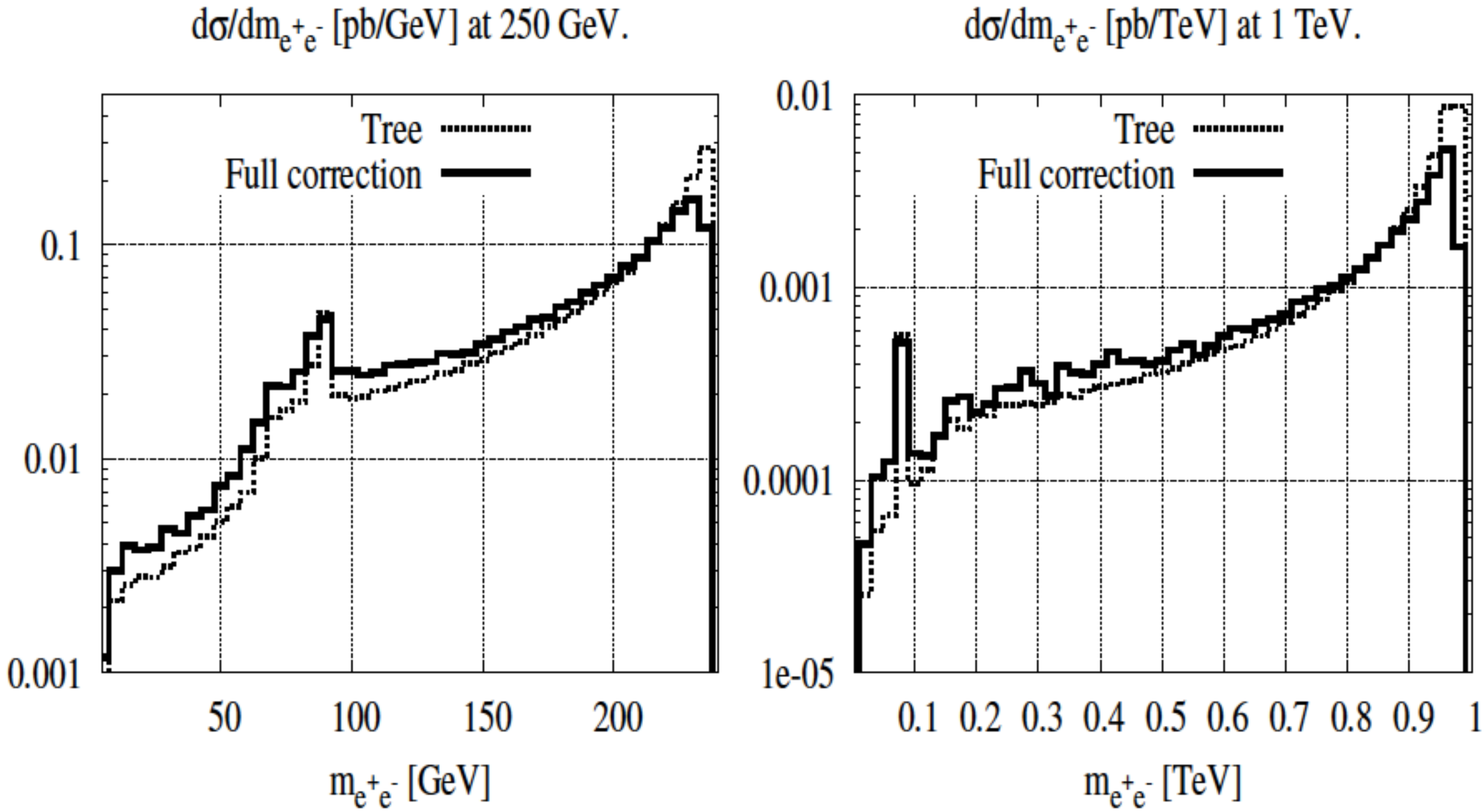


Figure 4: The differential cross-section as a function of the invariant mass of the $e^-$, $e^+$ pair. At the left $\sqrt{s} = 250$ GeV and at the right $\sqrt{s} = 1$ TeV.

# The physical results of the process $e^+e^- \to e^+e^-\gamma$

- We find that the numerical value of the full electroweak radiative corrections varies from $-2\%$ to $-20\%$ in the range of center-of-mass energy from 250 GeV to 1TeV.
- This contribution is sizable. The full electroweak correction to the process play important role for the determination luminosity at ILC in the future.

# Summary and Outlook

- According to an optimistic scenario, ILC is coming ~10 years later from now.
- The duty of ILC is the precise measurements.
- The role of the SM is provision of the reference values to explore the beyond SM.
- Here we discussed full O($\alpha$) electroweak corrections to the radiative Bhabha process by means of GRACE.
- We think full O($\alpha^2$) electroweak corrections should be accomplished at least for Bhabha and top-quark pair production.