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Mass-Corrections to Double-Higgs Production & TopoID

based on [Nucl. Phys. B **875**, 1 (2013); arXiv:1305.7340 [hep-ph]]

Jens Hoff in collaboration with Jonathan Grigo, Kirill Melnikov and Matthias Steinhauser | Loops and Legs, Weimar 2014

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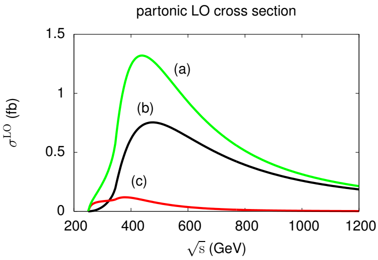
Motivation

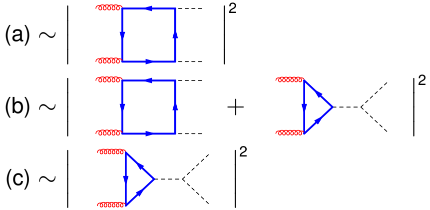
Higgs discovered, now measure & verify:

- couplings to particles ($\sim$ masses?)
- self-couplings

Higgs Potential in the Standard Model:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda H^4, \quad \lambda^{\text{SM}} = \frac{m_H^2}{2v^2} \approx 0.13, \quad v\text{: Higgs vev.}$$





Measurement at the (14 TeV) LHC

- $b\bar{b}\gamma\gamma$ channel, 600 fb^{-1} [Baur, Plehn, Rainwater; '04]
  $\Rightarrow \lambda \neq 0$
- $b\bar{b}\gamma\gamma$, $b\bar{b}\tau^+\tau^-$ channels [Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira; '13]
  $\Rightarrow$ **"promising"**,
  $b\bar{b}W^+W^-$ channel
  $\Rightarrow$ **"not promising"**
- ratio double-Higgs/single-Higgs cross sections; [Goertz, Papaefstathiou, Yang, Zurita; '13]
  600 fb^{-1} $\Rightarrow \lambda > 0$,
  3000 fb^{-1} $\Rightarrow \lambda^{+30\%}_{-20\%}$
- $b\bar{b}\gamma\gamma$ channel, 3000 fb^{-1} [Barger, Everett, Jackson, Shaughnessy; '14]
  $\Rightarrow \lambda \pm 40\%$
- many others, e.g. [Dolan, Englert, Spannowsky; '12]
  [Papaefstathiou, Yang, Zurita; '13]
  [Barr, Dolan, Englert, Spannowsky; '13]

Theory Status

- LO (exact M_t dependence) [Glover, van der Bij; '88] [Plehn, Spira, Zerwas; '98]
- NLO ($M_t \to \infty$ limit) $\approx$ LO + 100% [Dawson, Dittmaier, Spira; '98]

just recently:

- NLO + NNLL ($M_t \to \infty$) $\approx$ NLO + 20% [Shao, Li^2, Wang; '13]
- NNLO soft-virt. approx. ($M_t \to \infty$) $\approx$ NLO + 20% [de Florian, Mazzitelli; '13]
- NNLO ($M_t \to \infty$) $\approx$ NLO + 20% [de Florian, Mazzitelli; '13]

$$\sigma_{\text{tot., hadr.}} \approx \left(20^{\text{LO}} + 20^{\text{NLO, } M_t\to\infty} + 8^{\text{NNLO, } M_t\to\infty}\right) \text{ fb}$$

$$\text{for } \sqrt{s_{\text{hadr.}}} = 14 \text{ TeV}, \mu = 2m_H$$

this talk:

- NLO $\mathcal{O}(1/M_t)$ corrections [Grigo, Hoff, Melnikov, Steinhauser; '13]
- ...

Why $\mathcal{O}(1/M_t)$ corrections?

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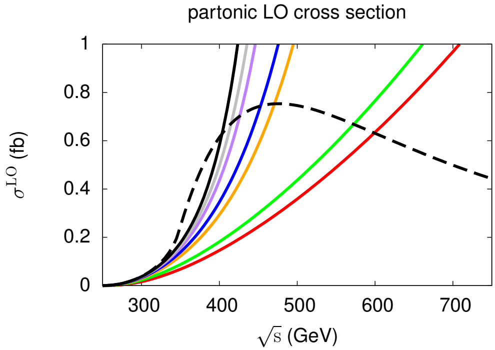


$M_t = 173$ GeV

$m_H = 126$ GeV

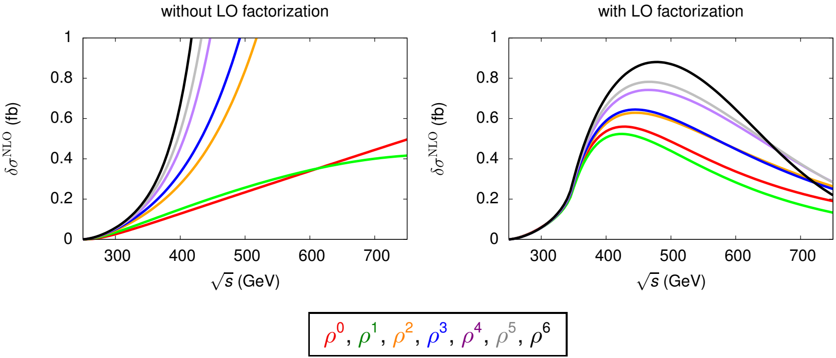
$\sqrt{s} \geq 252$ GeV

$\rho = \frac{m_H^2}{M_t^2} \approx 0.5$

$\rho^0, \rho^1, \rho^2, \rho^3, \rho^4, \rho^5, \rho^6$

- dashed: exact in M_t, solid: expansions in ρ
- hadronic cross section: 50% smaller for $M_t \to \infty$

Partonic Cross Sections

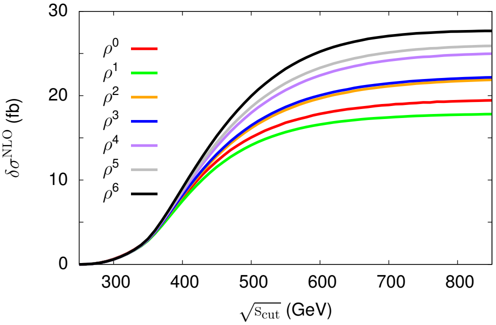


Poor convergence $\Rightarrow$ factorize exact LO cross section

$$\sigma^{\text{NLO}}_{\text{expanded}} \rightarrow \frac{\sigma^{\text{LO}}_{\text{exact}}}{\sigma^{\text{LO}}_{\text{expanded}}}\sigma^{\text{NLO}}_{\text{expanded}} = \sigma^{\text{LO}}_{\text{exact}}\frac{\sigma^{\text{NLO}}_{\text{expanded}}}{\sigma^{\text{LO}}_{\text{expanded}}}$$

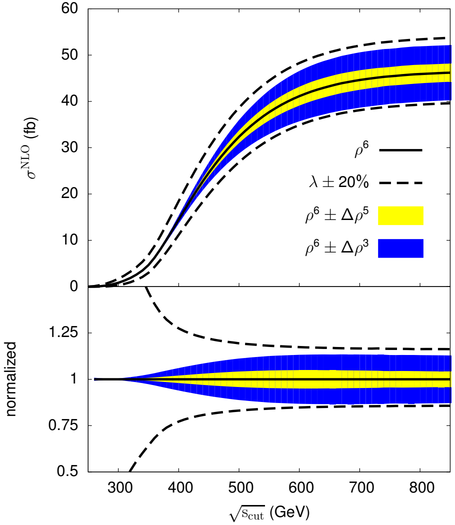
Hadronic Cross Sections

- behavior for low s: estimation of Q^2 distribution (virt.: $\sqrt{s} = \sqrt{Q^2}$, soft real: $\sqrt{s} \approx \sqrt{Q^2}$)
- $\sqrt{s_{\text{cut}}}$ = cut on partonic $\sqrt{s} \approx$ invariant mass of Higgs pair



- enhancement of low-s contributions by gluon luminosity
- observation: cross section dominated by soft and virtual corrections

Triple Higgs Coupling

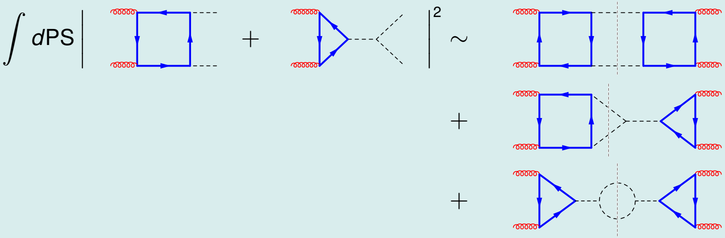


$\Rightarrow$ w/ $\mathcal{O}(1/M_t)$ corrections sensitive for $\mathcal{O}(10\%)$ deviations in λ

Methods

Forward Scattering & Optical Theorem:

$$\sigma_{\text{tot.}}(gg \to HH) \quad \sim \quad \text{Disc.}(\mathcal{M}(gg \to gg))$$



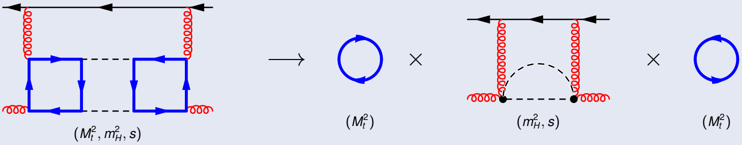
- pro
  - forward scattering $\Rightarrow$ simplified kinematics
  - loop and phase space integration at once
  - calculating Disc.$(\ldots)$ just for master integrals
- con
  - more loops & diagrams
  - only total cross section

Methods

Asymptotic Expansion:

- expand at diagram level $\equiv$ series expansion in analytic result
- hierarchy: $M_t^2 \gg s, m_H^2 \;\Rightarrow$ series in $\rho = m_H^2/M_t^2$
- effectively reduce number of loops & scales

Example: NLO real 4-loop 3-scale diagram



(note: in general more than 1 sub-diagram)

Toolchain

At NLO:

1.) create diagrams: `QGRAF` [Nogueira; '93]

2.) select appropriate cuts [Hoff, Pak; (unpublished)]

3.) asymptotic expansion: `q2e` and `exp` [Harlander, Seidensticker, Steinhauser; '98]

4.) reduction to scalar integrals: `(T)FORM` [Vermaseren; '90] [Tentyukov, Vermaseren; '10] [Kuipers, Ueda, Vermaseren, Vollinga; '13]

5.) reduction to master integrals (MIs): `FIRE` [Smirnov; '08] [Smirnov^2; '13]

Bottleneck: 4.) reduction to scalar integrals

- limited in the gg-channel $\mathcal{O}\left(\rho^{n\geq 6}\right)$: $\mathcal{O}$(4 weeks) runtime & $\mathcal{O}$(5 TB) disk space

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☛ `FORM` code

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☛ topology definition

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Toolchain

Towards NNLO:

1.) create diagrams: `QGRAF` [Nogueira; '93]

2.) select appropriate cuts [Hoff, Pak; (unpublished)]

➥ `TopoID`: graph information

3.) asymptotic expansion: `q2e` and `exp` [Harlander, Seidensticker, Steinhauser; '98]

➥ `TopoID`: `FORM` code

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➥ `TopoID`: topology definition

5.) reduction to MIs, identification of MIs:

- `rows` [Hoff, Pak; (unpublished)]
- `FIRE` [Smirnov; '08] [Smirnov^2; '13]

TopoID

```
             -------                      -   ----
--->---+--- |__   __| ------------------ | | |  _ \  ---+--->---
        \      | |   __    ----     __   | | | | \ \    |
         +     | |  /  \  |  _ \   /  \  | | | |  \ \   |
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         |   \            | |                        / \
--->-----+----+---------- |_| ----------------------+---+--->---
```

`Mathematica` package `TopoID` – **Topo**logy **ID**entification [Hoff, Pak; unpublished]

- topology construction
  (identification, minimization, partial fractioning, factorization, . . .)
- access to properties
  (completeness, linear dependence; sub-topologies, scalelessness, symmetries; graphs, unitarity cuts, . . .)
- `FORM` code generation
  (diagram mapping, topology processing, Laporta reduction, . . .)
- master integral (MI) identification
  (base changes, non-trivial relations, . . .)

Definition of a Topology

"**Topology**" = Diagram Class/Family T

set of N scalar propagators $\{d_i\}$ w/ arbitrary powers $\{a_i\}$ = "**Indices**"

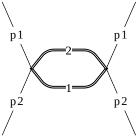
$$T(a_1,\ldots,a_N) = \left\{\Pi_{i=1}^{I} \int dk_i\right\} \left\{\Pi_{j=1}^{N} \frac{1}{[m_j^2 + q_j^2]^{a_j}}\right\}$$

- E external momenta $\{p_i\}$, I internal momenta $\{k_i\}$;
  N masses $\{m_i\}$ and line momenta $\{q_i\}$

$$q_i = \sum_{j=1}^{E} c_{ij} p_j + \sum_{j=1}^{I} d_{ij} k_k$$

- diagrammatic representation(s), e.g. at $\mathrm{LO}_{gg\to hh}$

$$T(a_1,a_2) = \int dk \frac{1}{[m_H^2 + k^2]^{a_1}} \frac{1}{[m_H^2 + (k+p_1+p_2)^2]^{a_2}} =$$



Different Types of Topologies

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- Invariants, Scalar Products for the process:

$$x_{p_i p_j} = p_i \cdot p_j, \quad s_{p_i k_j} = p_i \cdot k_j, \quad s_{k_i k_j} = k_l \cdot k_j$$

- e.g. in $\mathrm{LO}_{gg \to hh}$

$$p_1^2 = p_2^2 = 0,\ p_1 \cdot p_2 = -\frac{s}{2}, \quad p_1 \cdot k,\ p_2 \cdot k, \quad k^2$$

- completeness: all s_{ij} expressible via d_i

Diagram-Topologies (generic)

$\Rightarrow$ constructed from scalar diagram propagators (1-to-1)

$\Leftarrow$ mapping-pattern for Feynman diagrams (N-to-1)

- in general: linearly dependent, not complete

"**Laporta**"/**Reduction**-Topologies (basic)

- suitable for reduction: linearly independent, complete

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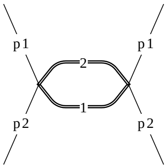

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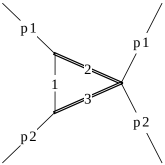

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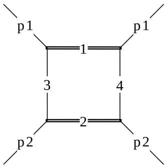

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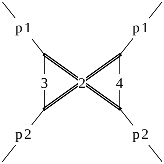

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Topology Identification

α-Representation + Canonical Ordering

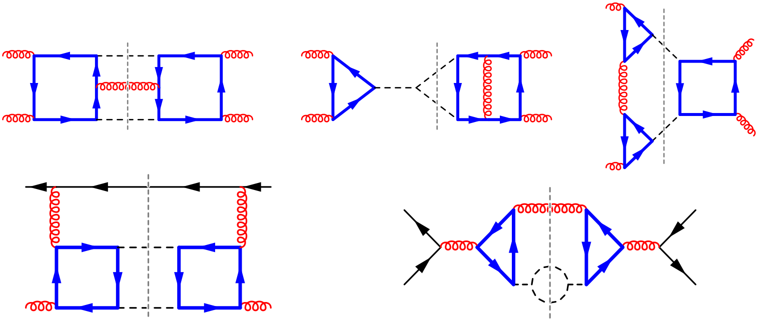
- unique identifier for topologies, Feynman integrals
- sub-topologies, scalelessness, symmetries, factorization, ...

based on [Pak; '11]

- Higher-order calculations: large complexity
  (more loops, legs, scales; number of diagrams)
- Automatization/Optimization:
  minimize number of handled objects/terms in each step
  - $\rightarrow$ Feynman diagrams (`QGRAF`)
  - $\rightarrow$ `TopoID` (+ reduction algorithm)
  - $\rightarrow$ unrenormalized result expressed via MIs (minimal set)

Diagrams at NLO

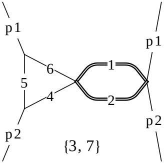
- virtual corrections:
  - $gg \to HH$: 126 two-loop diagrams
  - $gg \to gg$: 1052 four-loop diagrams (cross check)
- real corrections:
  - $gg \to gg$: 1530 four-loop diagrams (2 indep. calcs.)
  - $qg \to qg$: 34 four-loop diagrams
  - $q\bar{q} \to q\bar{q}$: 34 four-loop diagrams

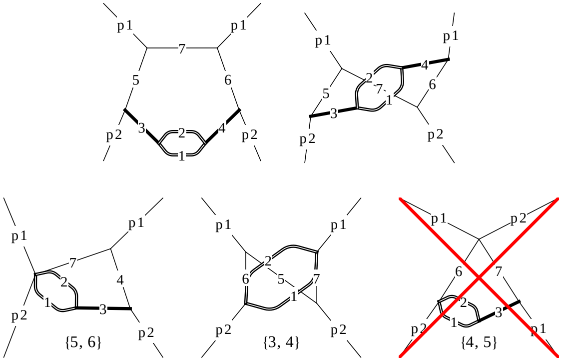


Topologies at NLO

real

virtual



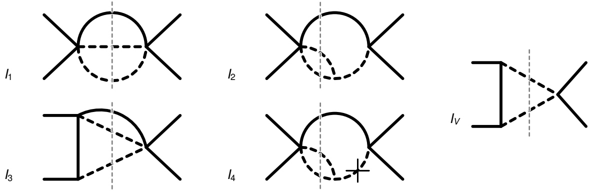


Master Integrals at NLO

- NLO: 4 real and 1 virtual (+ 2-loop tadpoles)
- phase space integrals depend on $s = (q_1 + q_2)^2$ and m_H
- derive 1-dimensional integral representation: e.g.

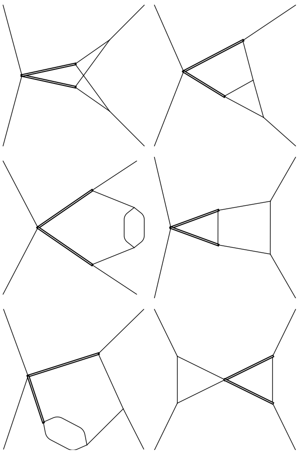
$$I_1 = \mathcal{N} s^{1-2\epsilon} \delta^{5/2-3\epsilon} \int_0^1 \frac{d\mu}{\sqrt{1-\mu\delta}} (1-\mu)^{1/2-\epsilon} \mu^{1-2\epsilon}, \quad \delta = 1 - \frac{4m_H^2}{s}$$

- simplification: expand up to $\mathcal{O}\left(\delta^{100}\right)$
  $\Rightarrow$ very good convergence, small impact on numerics
  $\Rightarrow$ analytic results for partonic cross sections

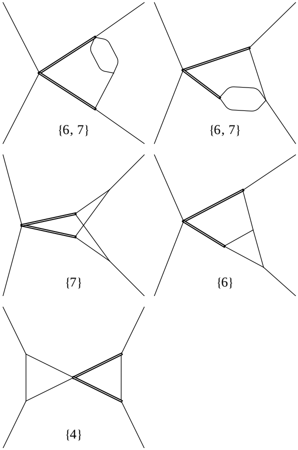


Topologies at NNLO: virtual 2-loop

generic

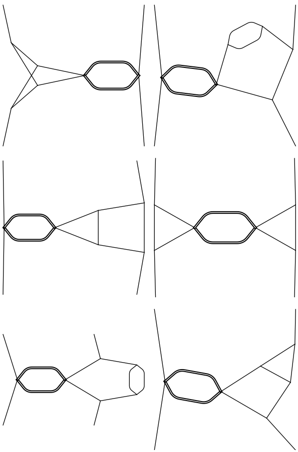


basic

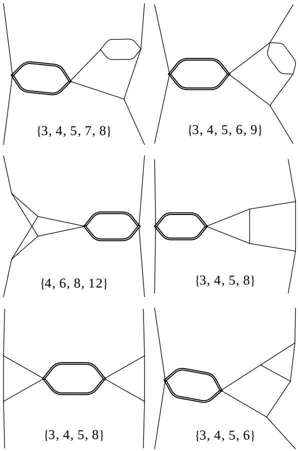


Topologies at NNLO: virtual 3-loop

generic

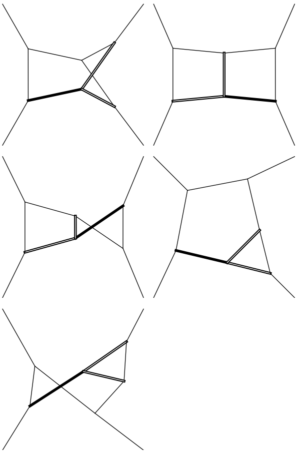


basic

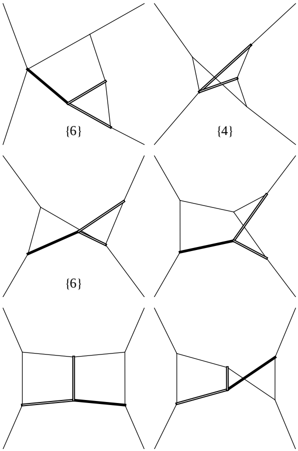


Topologies at NNLO: real 2-loop

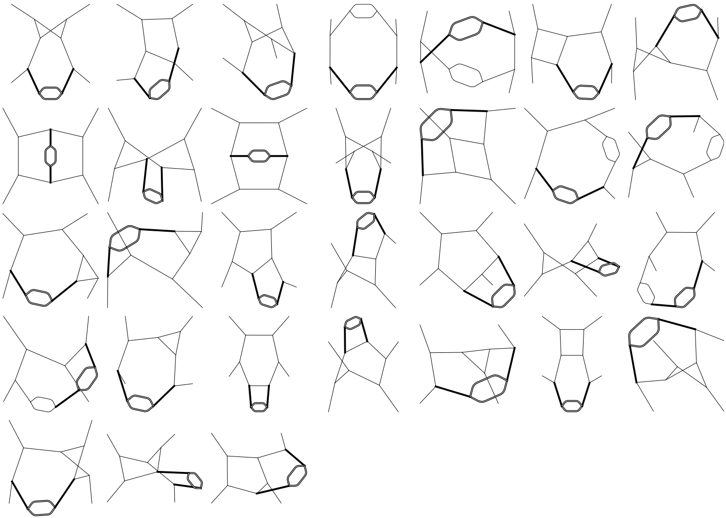
generic



basic



Topologies at NNLO: real 3-loop



Conclusion

$\sigma^{\text{NLO}}(pp \to HH)$: top-mass corrected, 14 TeV, $\mu = 2m_H$, w/o cut

$$19.7^{\text{LO}} + 19.0^{\text{NLO}, M_t \to \infty} \text{ fb} \quad \to \quad 19.7^{\text{LO}} + (27.3 \pm 5.9)^{\text{NLO}, 1/M_t^{12}} \text{ fb}$$

summary:

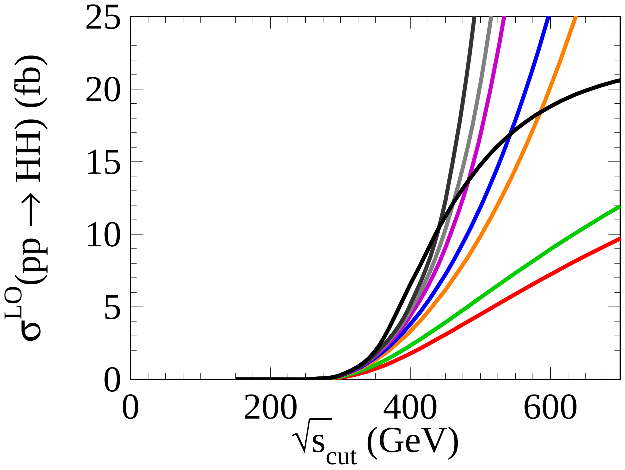
- 1st independent check of $M_t \to \infty$ result [Dawson, Dittmaier, Spira; '98]
- analytic results for partonic cross sections
- top mass corrections at NLO up to $\mathcal{O}(1/M_t^{12})$
  - $\Rightarrow$ numerically important corrections (+ 20%)
  - $\Rightarrow$ reliable estimate for uncertainties

`TopoID`:

- organize loop calculations
- completely generic, process independent
- automatized/optimized

Backup . . .

Hadronic LO Cross Section



Partonic Cross Sections

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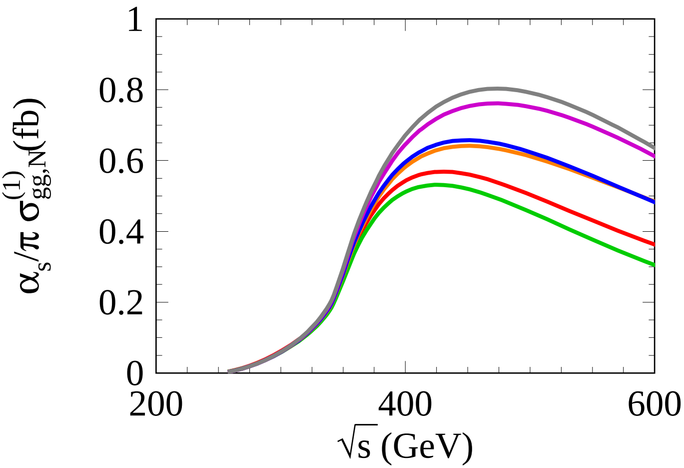
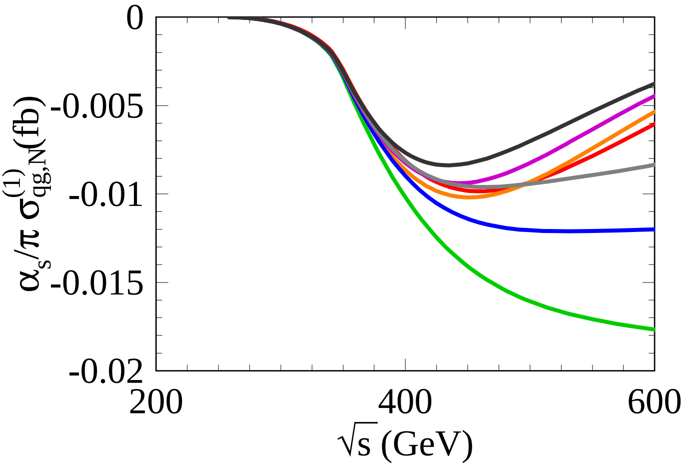
Gluon-Gluon Channel:



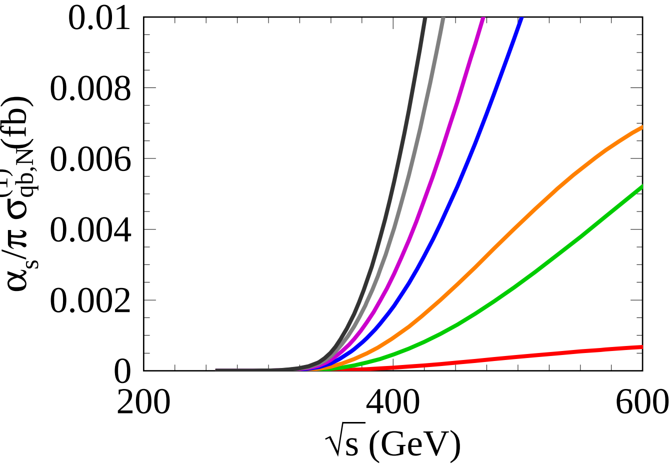
Partonic Cross Sections

Quark-Gluon Channel:

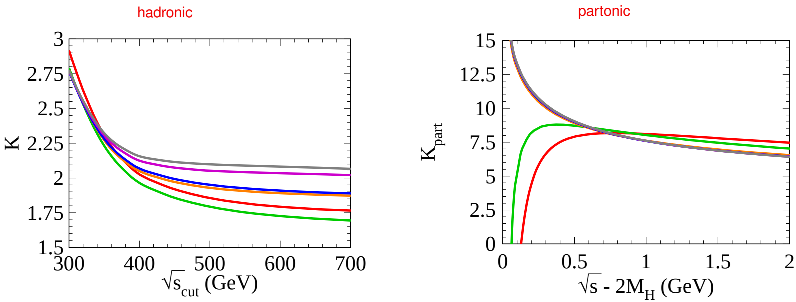


Partonic Cross Sections

Quark-Anti-Quark Channel:

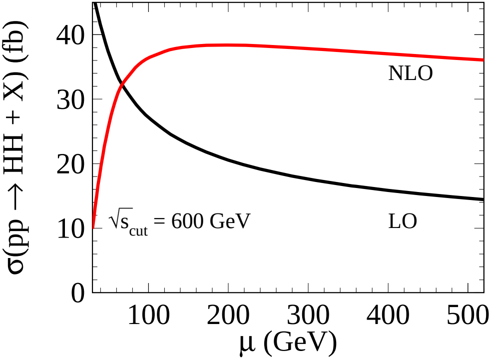


K-Factors



- large K-factors ($K = \sigma_{\text{NLO}}/\sigma_{\text{LO}} \approx 2 - 3$)
- strong dependence on $\sqrt{s_{\text{cut}}} \lesssim 400$ GeV
- close to threshold $\sqrt{s_{\text{cut}}} \approx 2m_H$ large enhancement; highly dependent on $\mathcal{O}(\rho)$ (cf. partonic plot)
- note: LO cross section suppressed at threshold

Scale Dependence



$$\mu = \mu_F = \mu_R$$
$$\mu_{\text{central}} = 2m_H$$
$$\sigma^{\text{LO}} = 18^{+6}_{-4}\ \text{fb}$$
$$\sigma^{\text{NLO}} = 38^{+0}_{-2}\ \text{fb}$$

- NLO curve almost μ independent
- NLO corrections of the same size as LO

$\Rightarrow$ weak μ dependence: misleading error estimate

Hadronic Cross Section

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$\sigma^{\text{NLO}}(pp \to HH)[fb]$, MSTW2008 PDFs, $\sqrt{s_{\text{had}}} = 14$ TeV, $\mu = 2m_H$

LO	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5	ρ^6	LO + δNLO
19.7 (22.4*)	19.0	16.4	21.5	21.4	24.5	25.3	27.3	47.0

* with LO pdfs