VFN Scheme for Event Shapes and Final State Jets

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Loops and Legs in Quantum Field Theory, April 27 - Mai 2, 2014

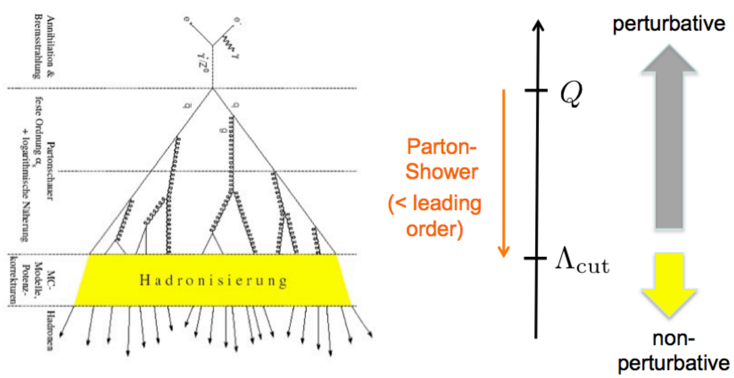
Outline

- Motivation and Aims
- Factorization theorem for massless quarks
- Secondary massive quark effects
- Factorization & renormalization & consistency conditions
- Rapidity logarithms
- Conclusions & Outlook

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
More papers to come

Why complete mass dependence?

- Measurement of α_s from eventshapes (Q=14,22 GeV)
- Top quark production
- Continuous description:
  - Validity range: bHQET vs. SCET (ttbar, bbbar, etc.)
  - Different EFT scenarios
  - Merge with initial state mass effects (DIS, pp)
- Complete systematics: $Q \leftrightarrow m \leftrightarrow \Lambda$
  - Systematics of masses in MCs
  - Consistent implementation of short-distance masses



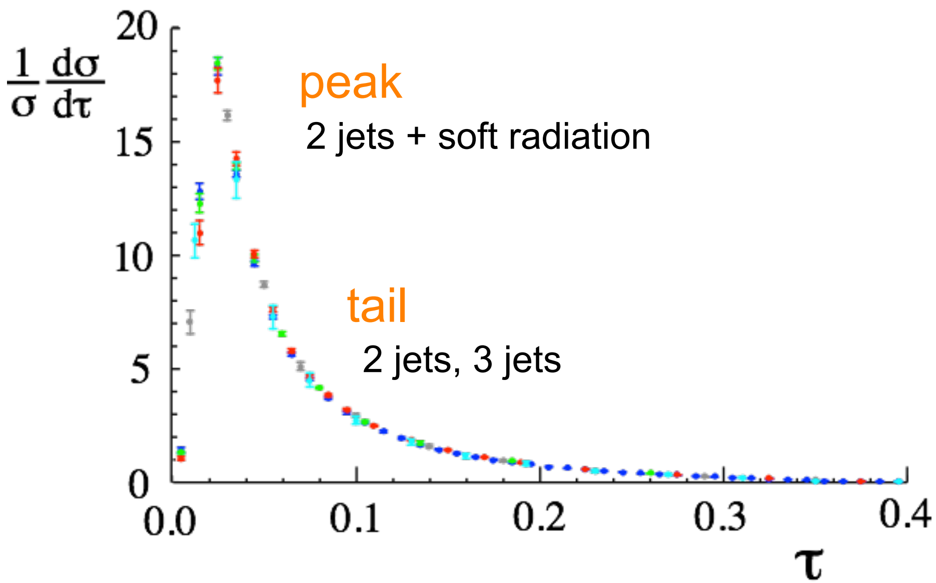
Thrust

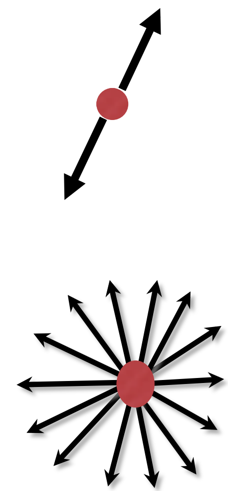
→ consider: dijet in e^+e^- annihilation

e.g. Thrust:

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{Q} \qquad \tau = 1 - T$$

ALEPH, DELPHI, L3, OPAL, SLD





$\tau = 0$

$\tau = 0.5$

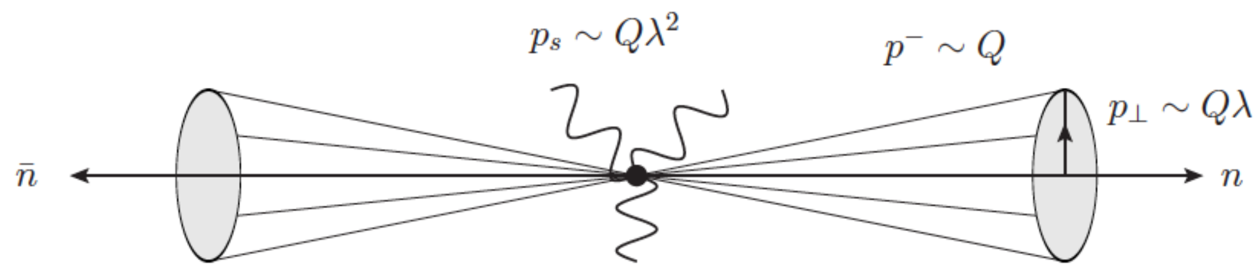
→ Mass mode treatment of this talk applicable to any SCET-1-type observable

→ We use thrust to be definite and as a first important application.

Massless Quark SCET

Bauer, Fleming, Luke
Bauer, Fleming, Pirjol, Stewart

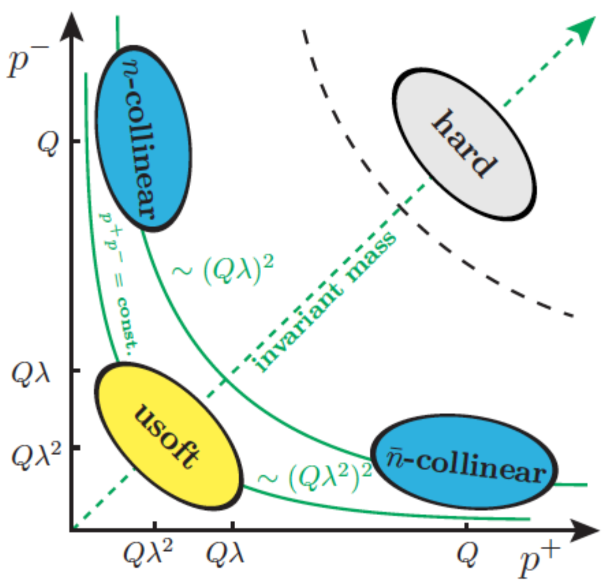
→ consider: dijet in e^+e^- annihilation, all quarks are light ($m_q < \Lambda$)



$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp$$

$$p^2 = p^- p^+ + p_\perp^2$$



mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	–
n-collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

$$d\sigma = \mathcal{H} \cdot \mathcal{J} \otimes \mathcal{S}$$

Korchemsky, Sterman

Massless Quark Thrust in FO

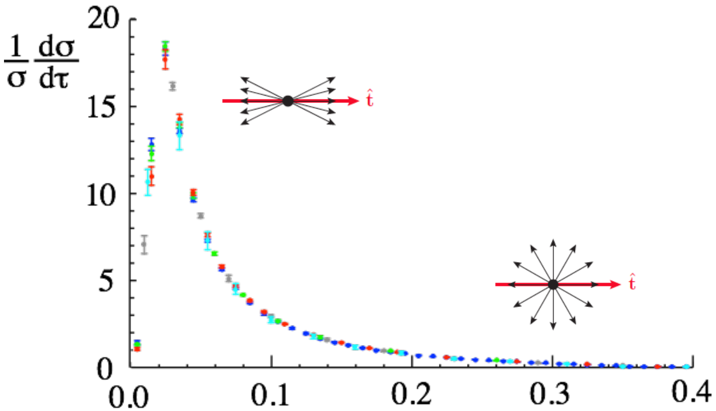
$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[(\frac{\pi^2}{6}-\frac{1}{2})\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\Big(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\Big)_+\Big]$$

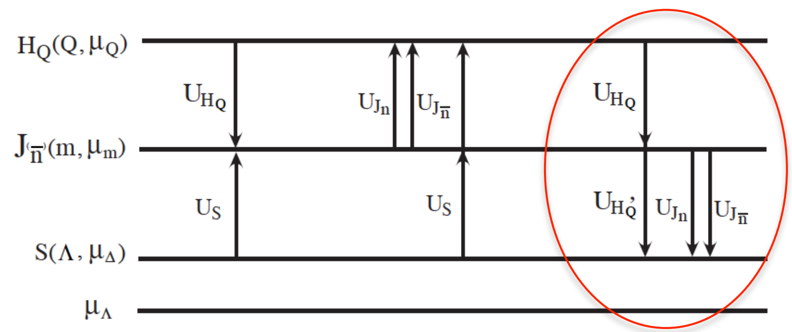
$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\Big[(\frac{\pi^2}{6}-\frac{1}{2})\delta(\tau) - \frac{3}{2}(\frac{1}{\tau})_+ - 2(\frac{\ln(\tau)}{\tau})_+} + \{\text{non-sing. terms}\}\Big]$$

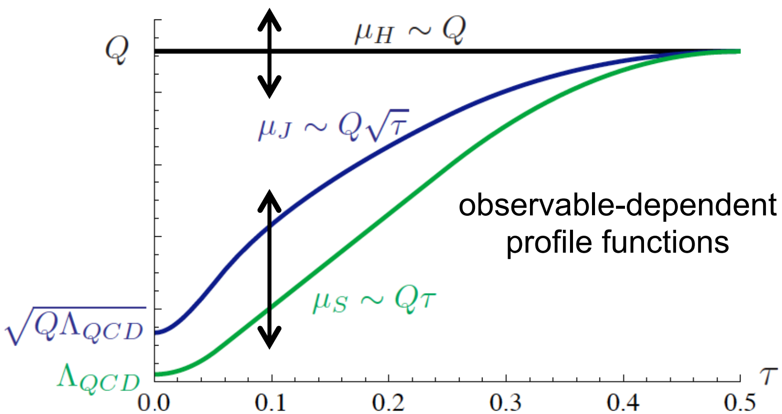
Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\, J_0(Q\ell,\mu)\, S_0\left(Q\tau-\ell,\mu\right)$$

Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman







→ evolution with n_l light quark flavors

→ consistency conditions w.r. to different evolution choices

→ top-down evolution considered in the following

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s)\int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

VFN Scheme for Final State Jets

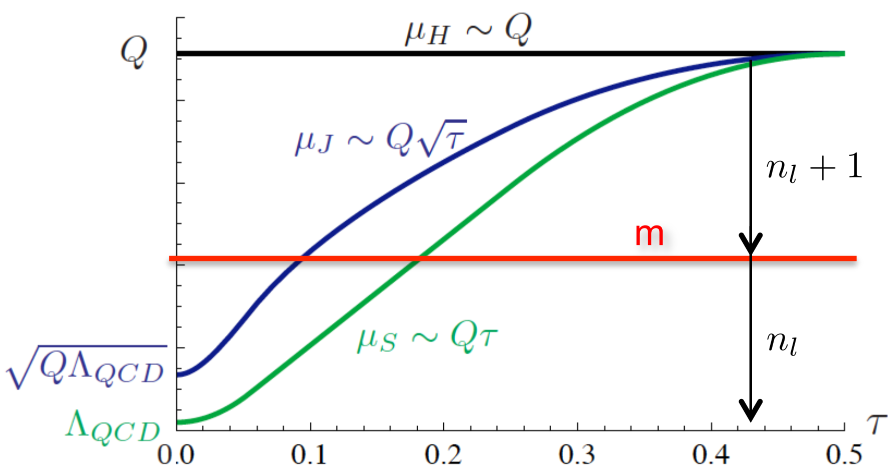
Gritschacher, AH,
Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



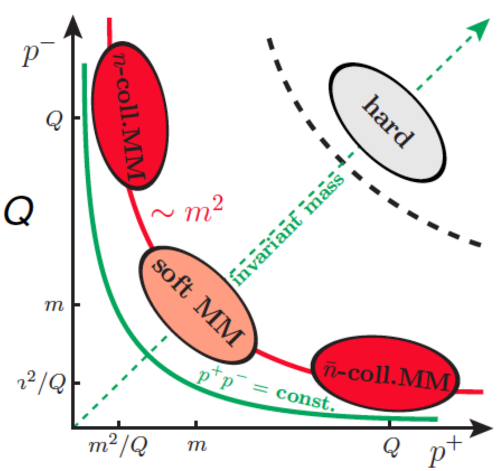
Aims:

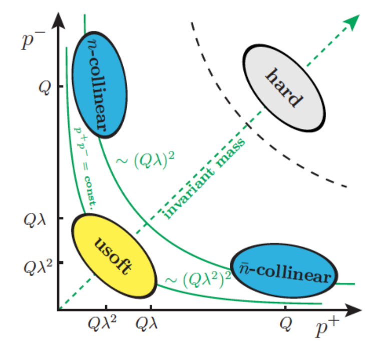
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+,-,\perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2





VFNS for the R-ratio

R-ratio for massless quarks: → valid up to term $O(m^2_{light}/s)$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \text{Im}\left[-i \int dx\, e^{ix.q} \left\langle 0 \middle| T j^\mu(x) j_\mu(0) \middle| 0 \right\rangle \right]$$

→ vector current conserved: not renormalized

→ UV divergences only related to strong coupling + field renorm.

→ MSbar result for any scale μ_0

$$= N_c \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s(\mu_0)}{\pi} + \frac{\alpha_s^2(\mu_0)}{\pi^2} \left[f_3 - \frac{\beta_0}{4} \ln\left(\frac{s}{\mu_0^2}\right) \right] + \ldots \right\}$$

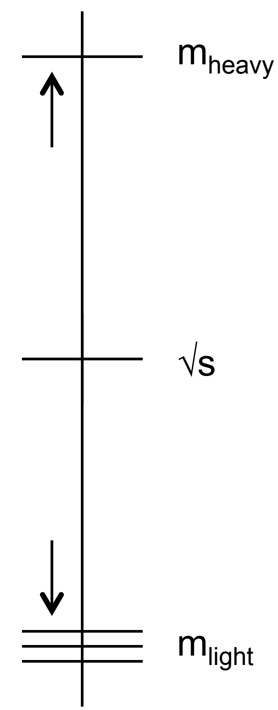
$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} = -\beta_0 \frac{\alpha_s^2(\mu)}{(4\pi)} + \ldots$$

$$\beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$

→ no large logarithms for $\mu_0 \sim \sqrt{s}$

→ $\sqrt{s}$ characteristic scale

$$= N_c \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s(\sqrt{s})}{\pi} + \frac{\alpha_s^2(\sqrt{s})}{\pi^2} f_3 + \ldots \right\}$$



→ Same calculation applies also if there is an ultramassive quark with $m_{heavy} \gg \sqrt{s}$ (up to terms $O(s/m^2_{heavy})$)

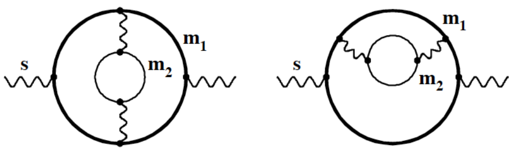
→ Decoupling of very heavy degrees of freedom

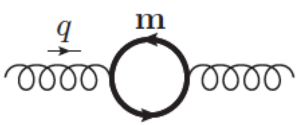
VFNS for the R-ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

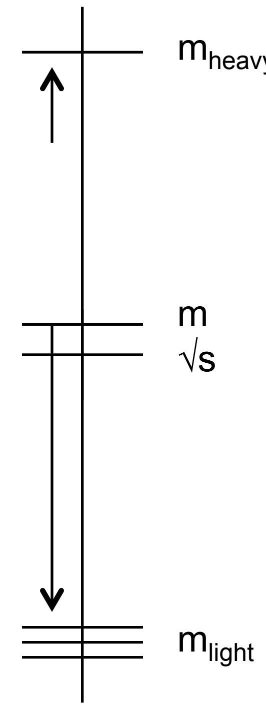
Virtual quarks:





$$= i(q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(q^2)$$

$$= \frac{T_f \alpha_s}{\pi}\left[\frac{1}{3\epsilon} - 2\int_0^1 dx\, x(1-x)\ln\frac{m_2^2 - x(1-x)(q^2+i\epsilon)}{\mu^2}\right]$$



Choice 1: → MSbar for n_{light} quarks and massive quark
→ strong coupling $\alpha_s^{(n_l+1)}(\mu)$ with $\beta_0 = 11 - 2/3(n_l+1)$

- R calculation stable for m~√s but also for m≪√s (calculation smoothly approaches the massless result)
- R calculation with large logarithm for m≫√s

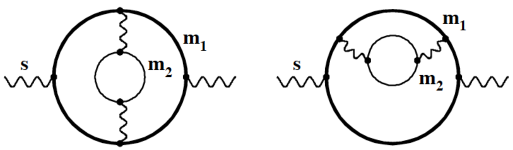
$$R(s) = R^{(0)} + \frac{\alpha_s^{(n_l+1)}(s)}{\pi} R^{(1)} + \left(\frac{\alpha_s^{(n_l+1)}(s)}{\pi}\right)^2 \left[R^{(2)} + \frac{T_f R^{(1)}}{3}\ln\frac{m^2}{s}\right] + \ldots$$

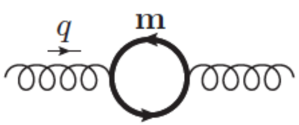
VFNS for the R-ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

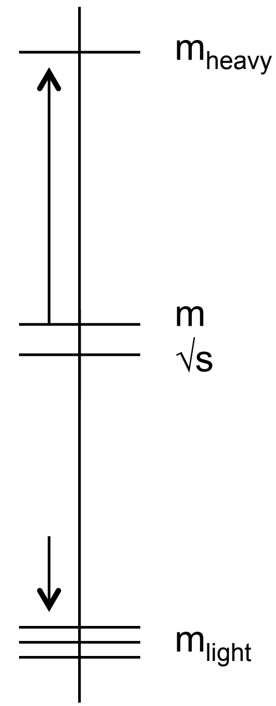
Virtual quarks:





$$= i(q_\mu q_\nu - g_{\mu\nu}q^2)\,\Pi(q^2)$$

$$= \frac{T_f \alpha_s}{\pi}\left[\frac{1}{3\epsilon} - 2\int_0^1 dx\, x(1-x)\ln\frac{m_2^2 - x(1-x)(q^2+i\epsilon)}{\mu^2}\right]$$



Choice 2: → MSbar for n_{light} quarks, on-shell for massive quark (subtract $\Pi(0)$)
→ strong coupling $\alpha_s^{(n_l)}(\mu)$ with $\beta_0 = 11 - 2/3n_l$

- R calculation stable for m~√s but also for m≫√s
- (calculation smoothly approaches the decoupling result)
- R calculation with large logarithm for m≪√s

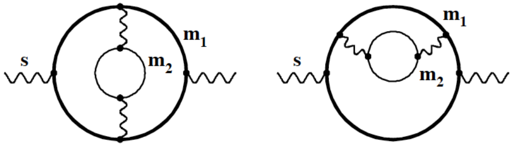
$$R(s) = R^{(0)} + \frac{\alpha_s^{(n_l\quad)}(s)}{\pi}R^{(1)} + \left(\frac{\alpha_s^{(n_l\quad)}(s)}{\pi}\right)^2\left[\tilde{R}^{(2)} - \frac{T_f R^{(1)}}{3}\ln\frac{m^2}{s}\right] + \ldots$$

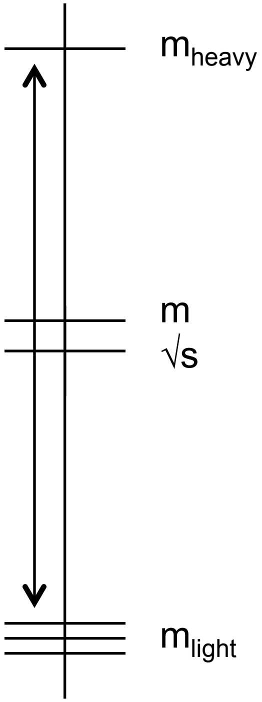
VFNS for the R-ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

Virtual quarks:





→ Choice 1 and choice 2 are equally good for μ ~ √s ~ m

→ Scheme relation for the strong coupling:

$$\alpha_s^{(n_l)}(\mu) = \alpha_s^{(n_l+1)}(\mu)\left(1 + \frac{T_f\alpha_s^{(n_l+1)}(\mu)}{3\pi}\ln\frac{m^2}{\mu^2} + \ldots\right)$$

→ <u>Variable flavor number scheme:</u> (VFN)
Choice 1 for μ ~ √s ≳ m
Choice 2 for μ ~ √s ≲ m
Swap 1↔2 at √s ~ μ_m ~ m

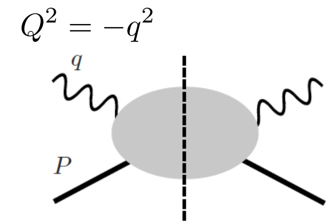
→ Full m^2/s dependence without approximations and w.o. any large logarithms

Collins - Wilczek - Zee (CWZ) scheme

→ comes at the cost of additional μ_m-dependence

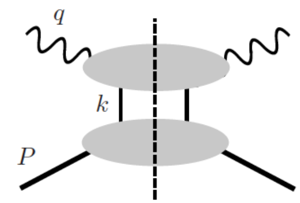
VFNS for Hadron Collisions

e.g. Deep Inelastic Scattering: $\frac{d\sigma(e^- p \rightarrow e^- + X)}{dQ\, dx}$



→ consider all quarks as as light ($m_q < \Lambda$)

→ quark number operators with an anomalous dimension between proton states → DGLAP equations



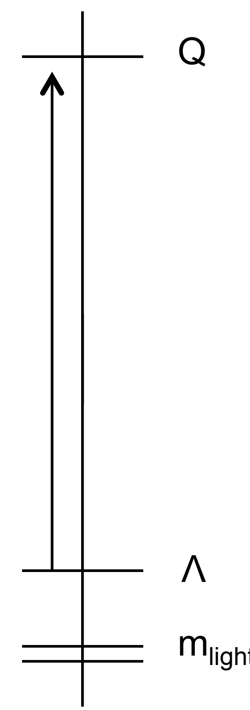
→ Hadronic tensor:

$$W_{\mu\nu}(Q,x) \sim \sum_{\text{partons a}} f_a(\mu) \otimes w_{\mu\nu}(Q,x,\mu)$$

→ μ-dependence with DGLAP equations for (light) parton distribution functions

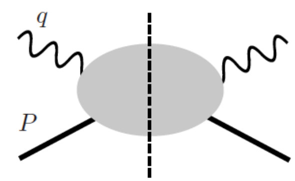
$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_x^1 \frac{d\xi}{\xi} \times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{q_i g}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \\ P_{g q_j}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) & P_{gg}\left(\frac{x}{\xi}, \alpha_s(Q^2)\right) \end{pmatrix} \begin{pmatrix} q_j(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}, \quad (11)$$

$$\frac{d\alpha_s(Q)}{d\ln Q^2} = -\beta_0 \frac{\alpha_s^2(Q)}{(4\pi)} + \ldots \qquad \beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$



VFNS for Hadron Collisions

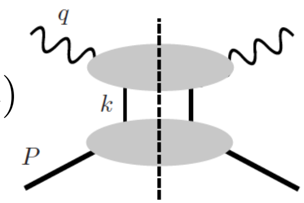
e.g. Deep Inelastic Scattering: $\frac{d\sigma(e^- p \rightarrow e^- + X)}{dQ\, dx}$



→ realistic case: massive quarks with Q > m > Λ
(charm, bottom [top])

→ Hadronic tensor:

$$W_{\mu\nu}(m,Q,x) \sim \sum_{a=q,g,Q} f_a^{(n_l+1)}(\mu) \otimes w_{\mu\nu}(m,Q,x,\mu)$$

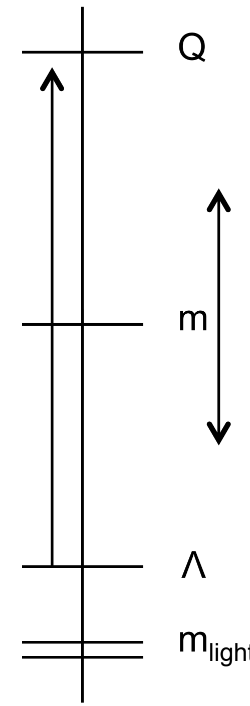


ACOT-VFN scheme:

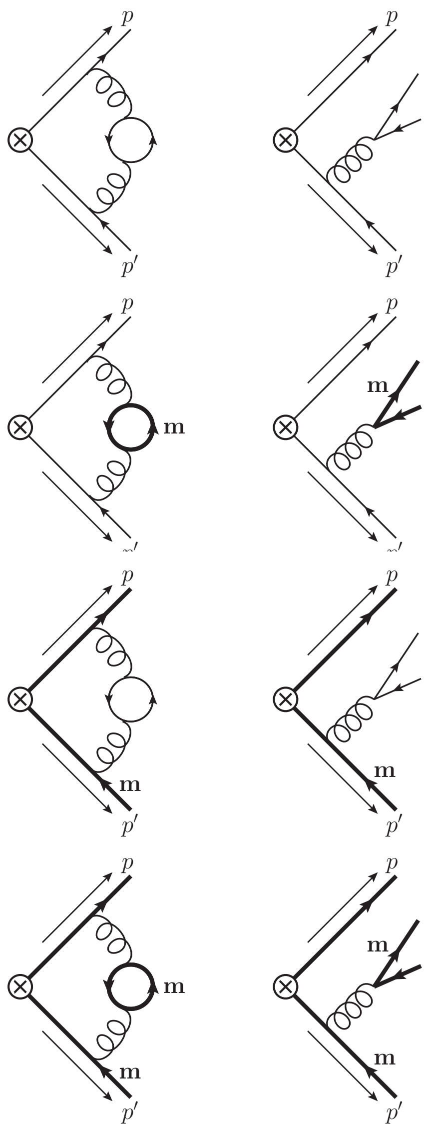
- DGLAP evolution for n_l flavors for $\mu \lesssim m$ (only light quarks)
- DGLAP evolution for n_l+1 flavors for $\mu \gtrsim m$ (light quarks + massive quark)
- Flavor matching for α_s and the pdfs at $\mu_m \sim m$

$$f_{q,g,Q}^{(n_l+1)}(\mu_m) = \sum_{a=q,g} F_{q,g,Q|a}(m,\mu_m) \otimes f_a^{(n_l)}(\mu_m)$$

→ hard coefficient $w_{\mu\nu}(m,Q,x)$ approaches massless $w_{\mu\nu}(Q,x)$ for m→0
→ calculations of $w_{\mu\nu}(m,Q,x)$ involves subtraction of pdf IR mass singularities
→ full dependence on m/Q without any large logarithms



Fully Massive Thrust



→ fully massless
- Full N^{3}LL' (u.t. 4-loop cusp)+ 3-loop non-singular
- Gap scheme for soft function

SCET authors: Becher, Schwartz,
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman
Fixed-order authors: Gehrmann etal, Weinzierl

→ <u>secondary</u> massive
- Full N^{2}LL'/N^{3}LL
- Four different physical situations

Pietrulewicz, AH, Gritschacher, Jemos 2013+2014
→ paper with all details very soon

→ <u>primary</u> massive
- Full N^{2}LL'/N^{3}LL on the way
- Three different physical situations

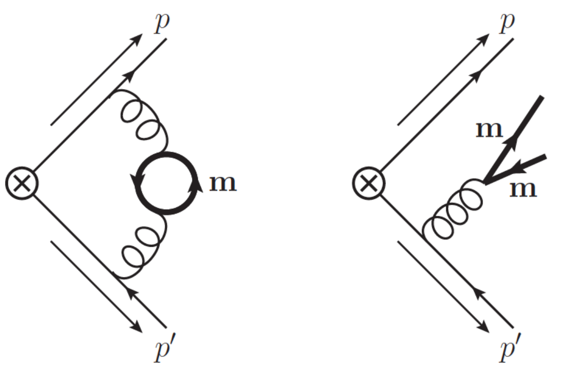
→ <u>primary</u> massive
<u>secondary</u> massive

No details in this talk!

FO Result: Secondary Massive Quarks

Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production



→ fixed order singular terms

$$r \equiv \sqrt{1+\frac{4m^2}{Q^2}}\,,\ b \equiv \sqrt{1-\frac{4m^2}{Q^2\tau}}\,,\ w \equiv \sqrt{1-\frac{4m^2}{Q^2\tau^2}}$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\bigg|_{\mathcal{O}(\alpha_s^2 C_F T_F)} = \frac{\left(\alpha_s^{(n_l)}(\mu)\right)^2 C_F T_F}{(4\pi)^2}$$

$$\times\Bigg\{\delta(\tau)\Bigg[\left(-r^4+2r^2+\frac{5}{3}\right)\left(4\mathrm{Li}_3\left(\frac{r-1}{r+1}\right)+\frac{1}{3}\ln^3\left(\frac{r-1}{r+1}\right)-\frac{2\pi^2}{3}\ln\left(\frac{r-1}{r+1}\right)-4\zeta(3)\right)$$

$$+\left(\frac{46}{9}r^3+\frac{10}{3}r\right)\left(4\mathrm{Li}_2\left(\frac{r-1}{r+1}\right)+\ln^2\left(\frac{r-1}{r+1}\right)-\frac{2\pi^2}{3}\right)+\left(\frac{220}{9}+\frac{400}{27}r^2\right)\ln\left(\frac{1-r^2}{4}\right)+\frac{476}{9}r^2+\frac{2426}{81}\Bigg]$$

$$+\frac{1}{\tau}\theta\left(\tau-\frac{4m^2}{Q^2}\right)\Bigg[-\frac{64}{3}\mathrm{Li}_2\left(-\frac{1-b}{1+b}\right)-\frac{32}{3}\ln^2\left(\frac{1+b}{2}\right)+\frac{32}{3}\ln^2\left(\frac{1-b}{2}\right)-\frac{16}{3}\ln^2\left(\frac{1-b}{1+b}\right)$$

$$+\ln\left(\frac{1-b}{1+b}\right)\left(b^4-2b^2+\frac{241}{9}\right)-\frac{10}{27}b^3+\frac{482}{9}b-\frac{16\pi^2}{9}\Bigg]$$

$$\tau \sim \frac{m^2}{Q^2} \ll 1 \quad \text{(collinear)}$$

$$+\frac{1}{\tau}\theta\left(\tau-\frac{2m}{Q}\right)\Bigg[\frac{64}{3}\mathrm{Li}_2\left(-\frac{1-w}{1+w}\right)+\frac{32}{3}\ln^2\left(\frac{1+w}{2}\right)-\frac{32}{3}\ln^2\left(\frac{1-w}{2}\right)+\frac{16}{3}\ln^2\left(\frac{1-w}{1+w}\right)-\frac{160}{9}\ln\left(\frac{1-w}{1+w}\right)$$

$$+\frac{64}{27}w^3-\frac{320}{9}w+\frac{16\pi^2}{9}\Bigg]\Bigg\} \qquad +\,\delta S_m^{\mathrm{real},\Delta}\left(\frac{Q\tau}{m}\right)$$

$$\tau \sim \frac{m}{Q} \ll 1 \quad \text{(soft)}$$

FO Result: Secondary Massive Quarks

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\bigg|_{\mathcal{O}(\alpha_s^2 C_F T_F)} = \frac{\left(\alpha_s^{(n_l)}(\mu)\right)^2 C_F T_F}{(4\pi)^2}$$

$$\times \Bigg\{ \delta(\tau) \Bigg[\left(-r^4 + 2r^2 + \frac{5}{3}\right)\left(4\mathrm{Li}_3\left(\frac{r-1}{r+1}\right) + \frac{1}{3}\ln^3\left(\frac{r-1}{r+1}\right) - \frac{2\pi^2}{3}\ln\left(\frac{r-1}{r+1}\right) - 4\zeta(3)\right)$$

$$+ \left(\frac{46}{9}r^3 + \frac{10}{3}r\right)\left(4\mathrm{Li}_2\left(\frac{r-1}{r+1}\right) + \ln^2\left(\frac{r-1}{r+1}\right) - \frac{2\pi^2}{3}\right) + \left(\frac{220}{9} + \frac{400}{27}r^2\right)\ln\left(\frac{1-r^2}{4}\right) + \frac{476}{9}r^2 + \frac{2426}{81}\Bigg]$$

$$+ \frac{1}{\tau}\theta\left(\tau - \frac{4m^2}{Q^2}\right)\Bigg[-\frac{64}{3}\mathrm{Li}_2\left(-\frac{1-b}{1+b}\right) - \frac{32}{3}\ln^2\left(\frac{1+b}{2}\right) + \frac{32}{3}\ln^2\left(\frac{1-b}{2}\right) - \frac{16}{3}\ln^2\left(\frac{1-b}{1+b}\right)$$

$$+ \ln\left(\frac{1-b}{1+b}\right)\left(b^4 - 2b^2 + \frac{241}{9}\right) - \frac{10}{27}b^3 + \frac{482}{9}b - \frac{16\pi^2}{9}\Bigg]$$

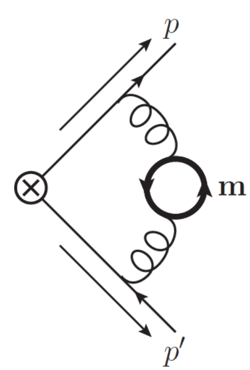
$$\tau \sim \frac{m^2}{Q^2} \ll 1 \quad \text{(collinear)}$$

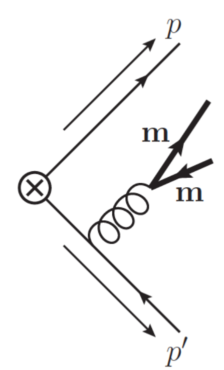
$$+ \frac{1}{\tau}\theta\left(\tau - \frac{2m}{Q}\right)\Bigg[\frac{64}{3}\mathrm{Li}_2\left(-\frac{1-w}{1+w}\right) + \frac{32}{3}\ln^2\left(\frac{1+w}{2}\right) - \frac{32}{3}\ln^2\left(\frac{1-w}{2}\right) + \frac{16}{3}\ln^2\left(\frac{1-w}{1+w}\right) - \frac{160}{9}\ln\left(\frac{1-w}{1+w}\right)$$

$$+ \frac{64}{27}w^3 - \frac{320}{9}w + \frac{16\pi^2}{9}\Bigg]\Bigg\} + \delta S_m^{\mathrm{real},\Delta}\left(\frac{Q\tau}{m}\right)$$

$$\tau \sim \frac{m}{Q} \ll 1 \quad \text{(soft)}$$

$$r \equiv \sqrt{1 + \frac{4m^2}{Q^2}}\,,\; b \equiv \sqrt{1 - \frac{4m^2}{Q^2\tau}}\,,\; w \equiv \sqrt{1 - \frac{4m^2}{Q^2\tau^2}}$$

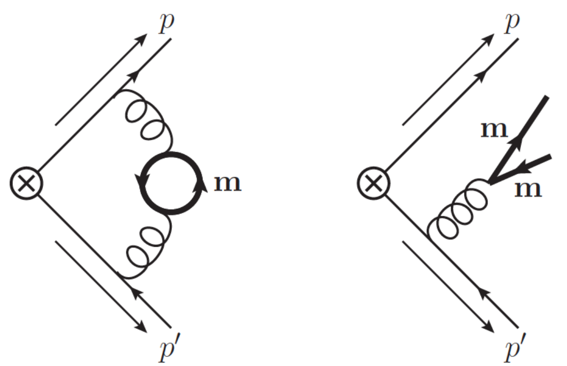




VFN Scheme: Secondary Massive Quarks

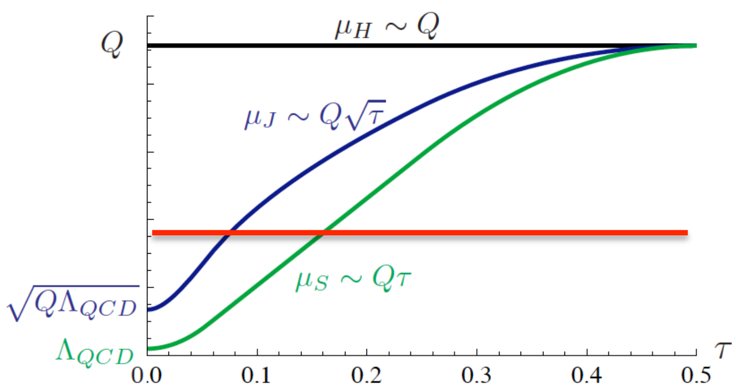
Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production

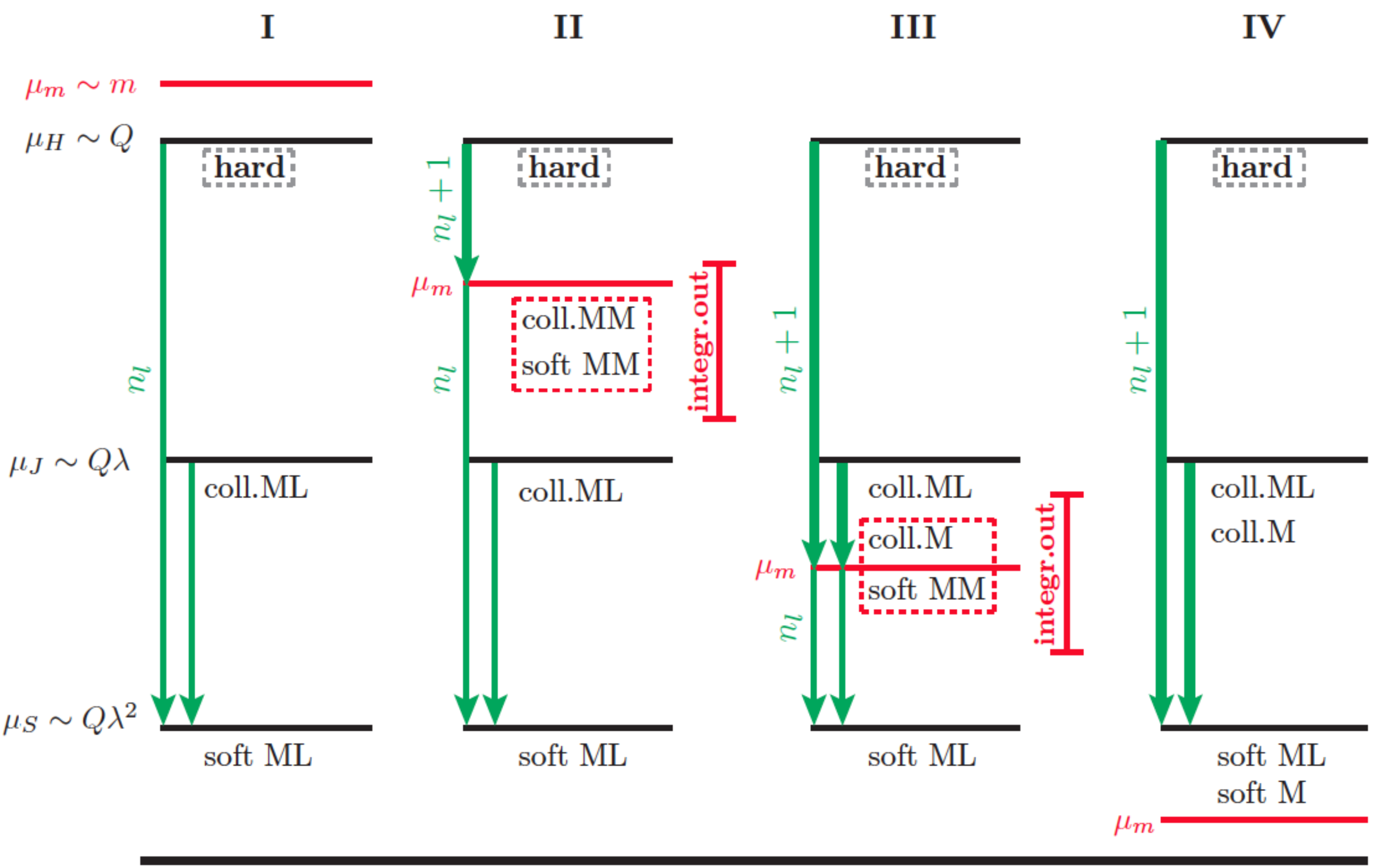


→ does not lead to bHQET-type theory when the jet scale approaches the quark mass

→ only SCET-type theories



VFN Scheme: Secondary Massive Quarks



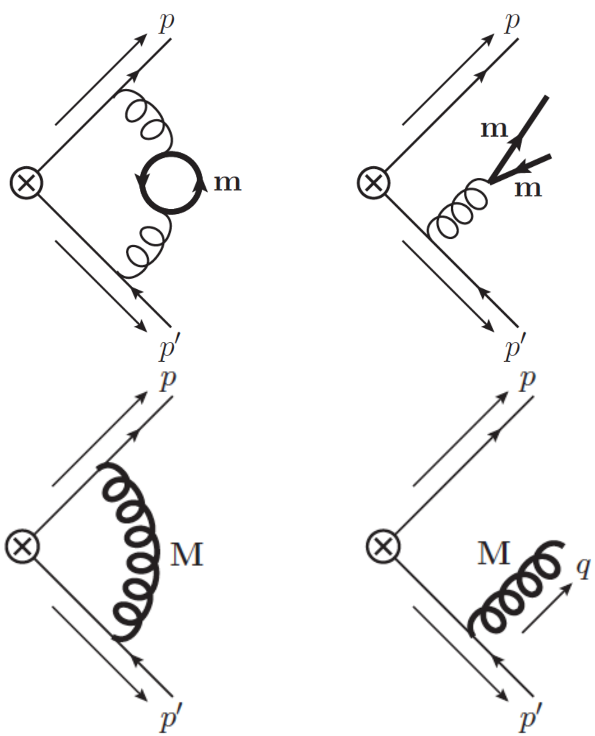
MM = mass-mode, ML = massless, M = massive

→ See Piotr's talk.

VFN Scheme: Secondary Massive Quarks

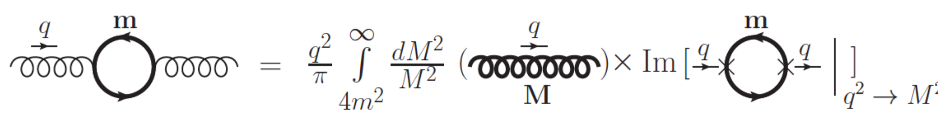
Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production



→ field theory: close relation to the problem of massive gauge boson radiation

→ dispersion relation: massive quark results can be obtained directly from massive gluon calculations when quark pair treated inclusively (e.g. hard coefficient, jet function)



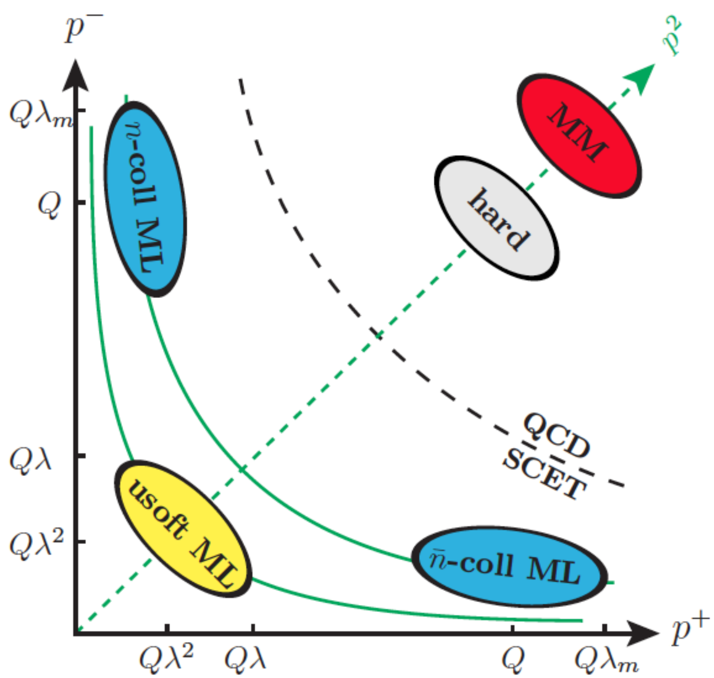
→ separation of conceptual issues to be resolved and calculations issues related to gluon splitting.

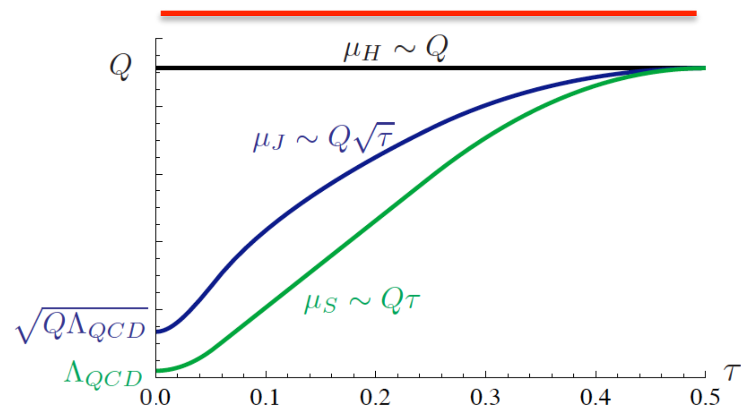
→ explicit two-loop calculation needed when quarks are treated exclusively (e.g. soft function → hemisphere prescription)

Gritschacher, AH, Jemos, Pietrulewicz 2013

VFN Scheme: Secondary Massive Quarks

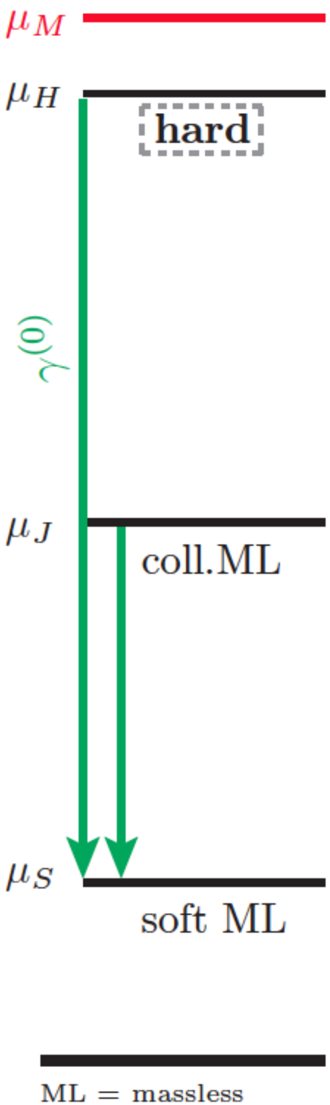
Scenario 1: $\lambda_m > 1 > \lambda > \lambda^2$ (m > Q > J > S)





- EFT only contains light quarks
- Massive quark only in current matching coeff.
- Decoupling for m/Q → ∞

VFN Scheme: Secondary Massive Quarks



integrate out mass modes at QCD level

$$\frac{d\sigma}{d\tau} \sim \left|C^{I}(\mu_H)\right|^2 U_H^{(0)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0(Q\tau - \ell, \mu_S)$$

$U_H^{(0)}$, $U_J^{(0)}$: massless evolution factors

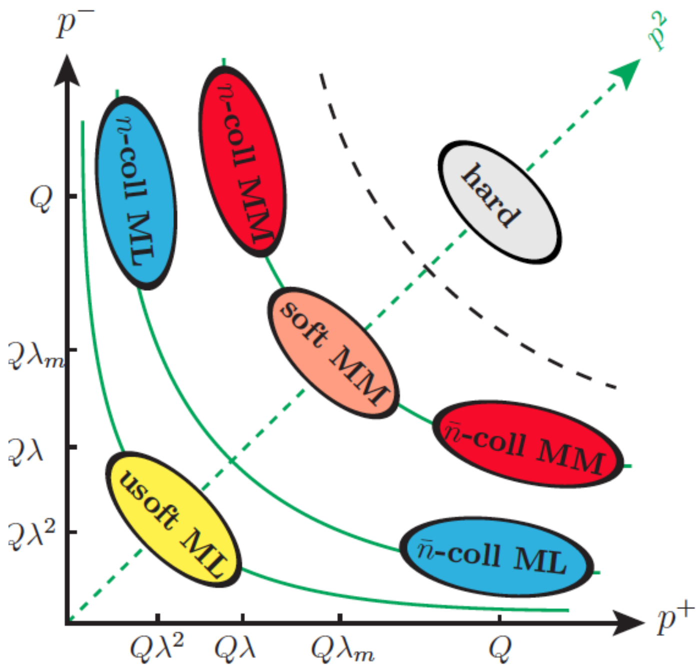
$C^{I}(\mu_H) = C_0(\mu_H) + \delta F_m^{\mathrm{QCD}}$

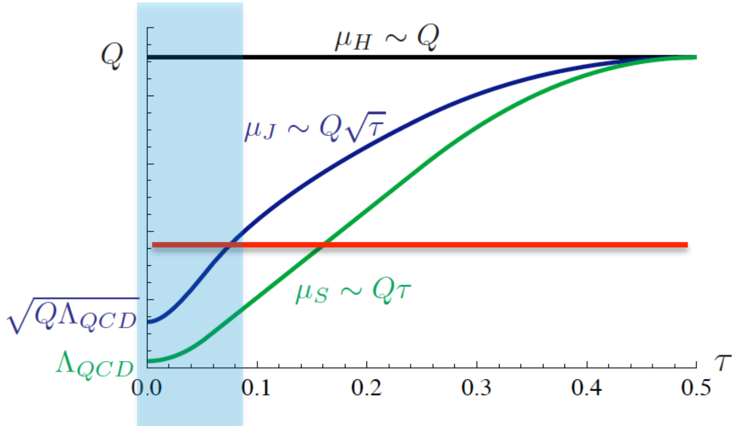
C_0: massless matching coefficient
$\delta F_m^{\mathrm{QCD}}$: massive full theory contribution (OS)
$\rightarrow$ decoupling for $M/Q \rightarrow \infty$
$\rightarrow$ IR-divergent expression for $M/Q \rightarrow 0$

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

VFN Scheme: Secondary Massive Quarks

Scenario 2: $1 > \lambda_m > \lambda > \lambda^2$ ($Q > m > J > S$)

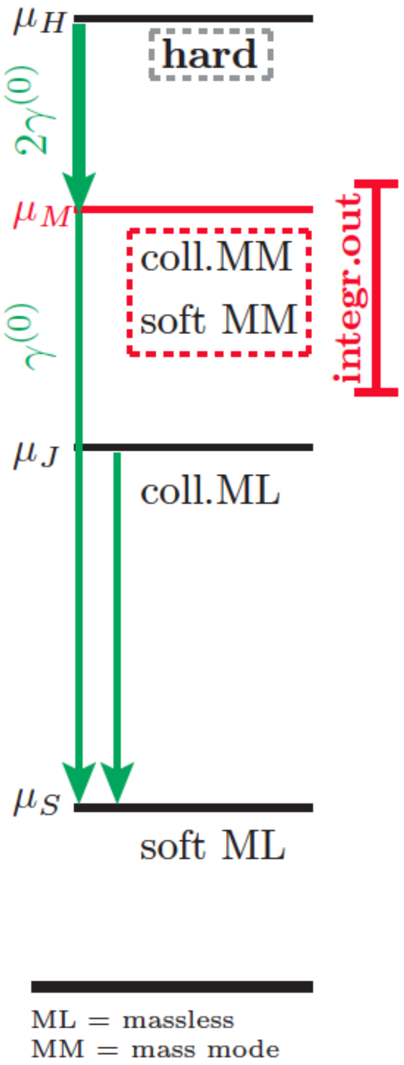




- Massive modes only virtual
- Jet and soft function as in massless case
- Hard coefficient must have massless limit
- Known Sudakov problem for massive gauge boson

Chiu, Golf, Kelley, Manohar
Chiu, Führer, Hoang, Kelley

VFN Scheme: Secondary Massive Quarks



mass modes enter SCET, but integrated out before the jet scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_J,\mu_S)\, S_0(Q\tau - \ell,\mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\mathrm{eff}}(\mu_H)$ ⟵ Contains all mass-singularities

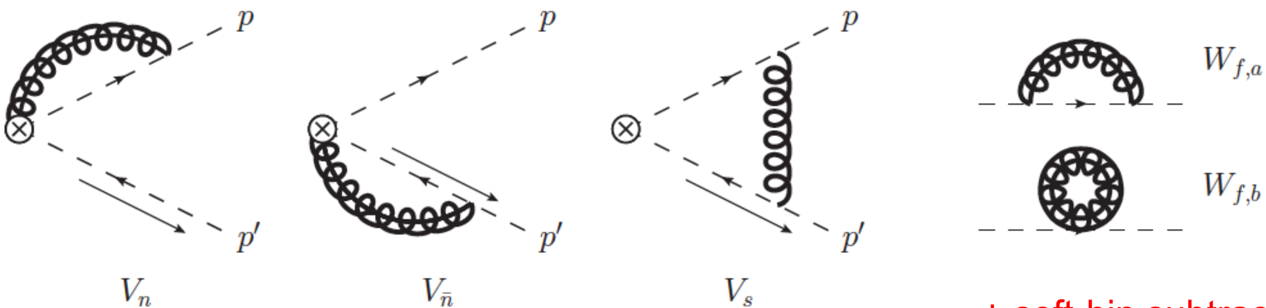
$\delta F_m^{\mathrm{eff}}$: massive SCET contribution

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

$U_i^{(1)}$ stands for: (a) massive gluon dynamical
(b) (n_l+1)-evolution

VFN Scheme: Secondary Massive Quarks

Scenario 2: mass mode SCET calculation



+ soft-bin subtractions

$$\delta F_m^{\text{eff}}(Q,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\ln\left(\frac{M^2}{\mu^2}\right)\left[2\ln\left(\frac{-Q^2}{\mu^2}\right) - \ln\left(\frac{M^2}{\mu^2}\right) - 3\right] - \frac{5\pi^2}{6} + \frac{9}{2}\right\}$$

Chiu, Golf, Kelley, Manohar (2008)

Chiu, Fuhrer, Hoang, Kelley, Manohar (2009)

rapidity logarithms

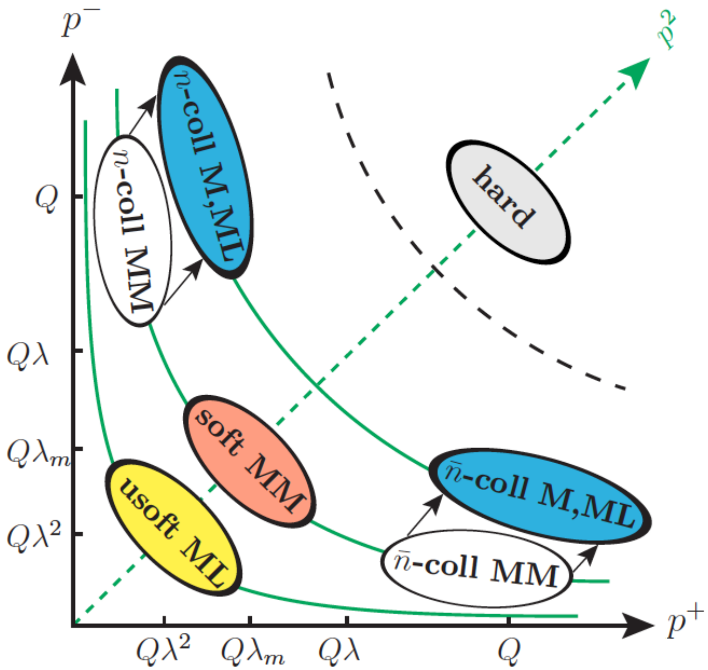
large logarithm $\ln\left(\frac{M^2}{\mu_H^2}\right)$ cancels between $\mathcal{C}^I$ and δF_m^{eff}

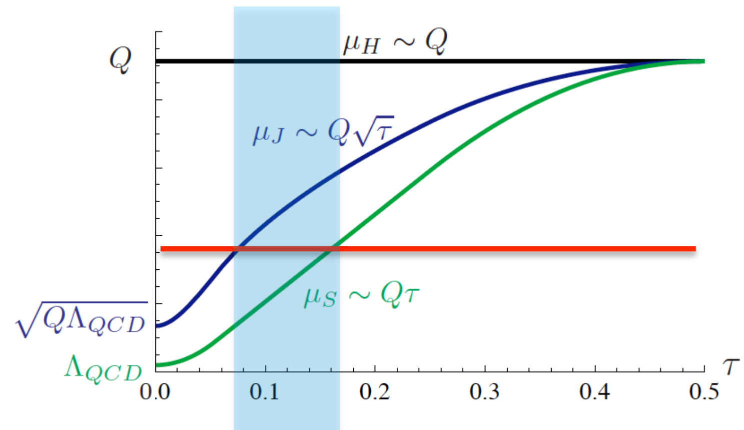
correct massless limit for $\mathcal{C}^{II}(\mu_H)$:

$$\mathcal{C}^{II}(Q,M,\mu_H) = \mathcal{C}^I(Q,M,\mu_H) - \delta F_m^{\text{eff}}(Q,M,\mu_H) \overset{M\to 0}{\longrightarrow} 2C_0(Q,\mu_H)$$

VFN Scheme: Secondary Massive Quarks

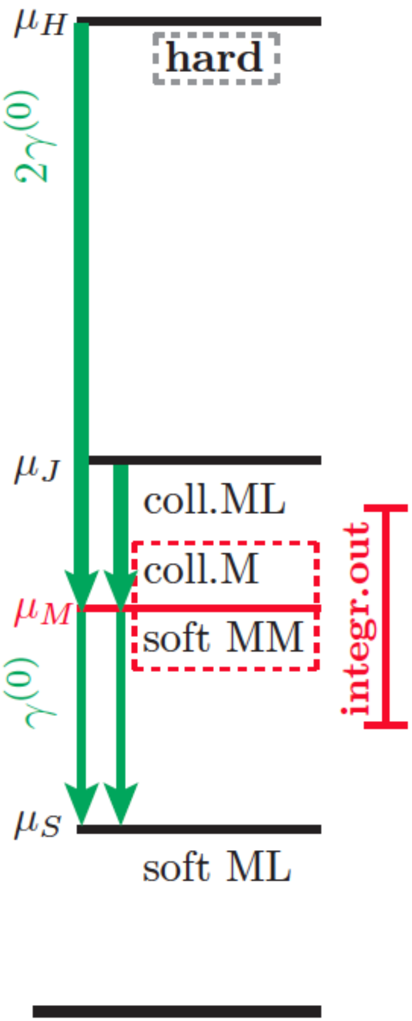
Scenario 3: $1 > \lambda > \lambda_m > \lambda^2$ (Q > J > m > S)





- Current evolution unchanged w.r. to Scen. 2
- Hard coefficient must have massless limit
- Jet function has massless limit
- Massive and massless collinear in same sector
- Collinear mass modes integrated out at m

VFN Scheme: Secondary Massive Quarks



ML = massless
MM = mass mode
M = massive

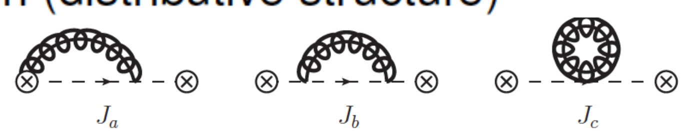
mass modes enter jet sector,
but integrated out before the soft scale

$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S)$$
$$\times \int d\ell \int ds \int ds' \int ds'' J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_J,\mu_M)$$
$$\times \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_M,\mu_S)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\rm virt}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\rm real}(s)$$

$\delta J_m^{\rm virt}$: virtual piece of jet function (distributive structure)

- Soft-bin subtraction
- Rapidity singularities cancel
- UV divergences agree with massless case

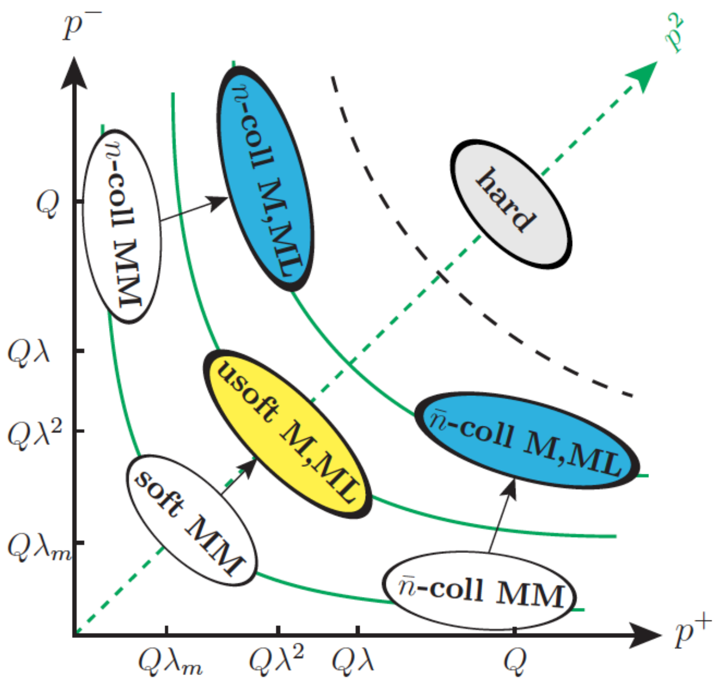


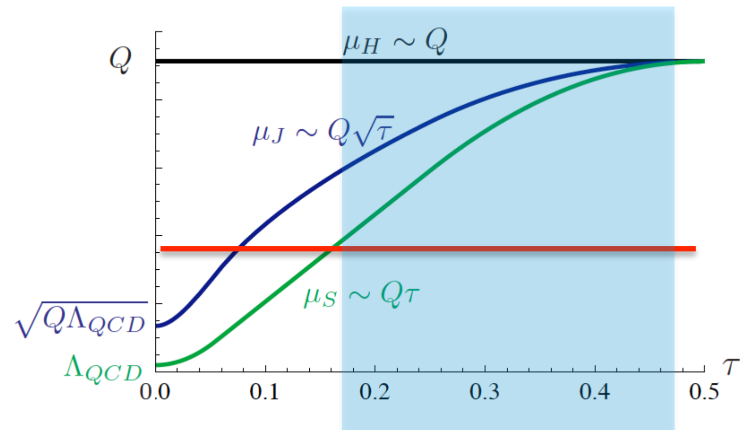
$\delta J_m^{\rm real}$: real radiation piece of jet function (function)

- finite
- sum of virtual and real: rapidity logs cancel
- sum of virtual and real: approaches massless jet function for m $\to$ 0

VFN Scheme: Secondary Massive Quarks

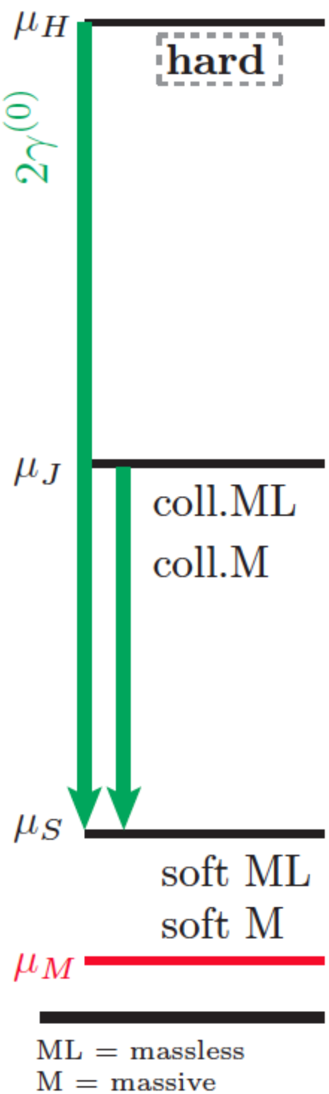
Scenario 4: $1 > \lambda > \lambda^2 > \lambda_m$ (Q > J > S > m)





- Current evolution unchanged w.r. to Scen. 2
- Jet function and evolution as in Scen. 2
- Massive and massless coll. modes same sector
- Massive and massless soft modes same sector
- Hard coefficient, jet and soft function must have massless limit
- All RG-evolution for (n_l+1) flavors

VFN Scheme: Secondary Massive Quarks



mass modes enter all sectors

$$\frac{d\sigma}{d\tau} \sim \left|C^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_J, \mu_S)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\rm virt}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)

- Rapidity singularities cancel between contributions from both hemispheres (+,-)
- UV divergences agree with massless case

$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

- finite
- sum of virtual and real: rapidity logs cancel
- sum of virtual and real: approaches massless soft function for m $\rightarrow$ 0

VFN Scheme: MM Threshold Corrections

The calculation of the mass mode matching corrections for current, jet and soft function can be carried out by matching the factorization theorem to a full QCD calculation.

But there is a more efficient method based on the fact that current, jet and soft functions are gauge-invariant quantities that can be renormalized separately.

- Evolution with VFN and matching can be related to the use of different renormalization conditions within a single effective theory.
- Use scenario 4 effective theory where the massive quark is contained in hard, collinear and soft sectors.

Example: Jet function

$$J^{\rm bare} = Z_J^{OS} \otimes J^{\rm OS} = Z_J^{\overline{\rm MS}} \otimes J^{\overline{\rm MS}}$$

On-shell condition: decoupling for m→∞

$$J^{\rm OS}(s,m,\mu) = J^{(n_l)}(s,\mu) + \theta(s-4m^2)\,\delta J_m^{\rm real}(s,m) \stackrel{m\gg s}{\longrightarrow} J^{(n_l)}(s,\mu)$$

$\overline{\rm MS}$ condition: massless limit for m→0

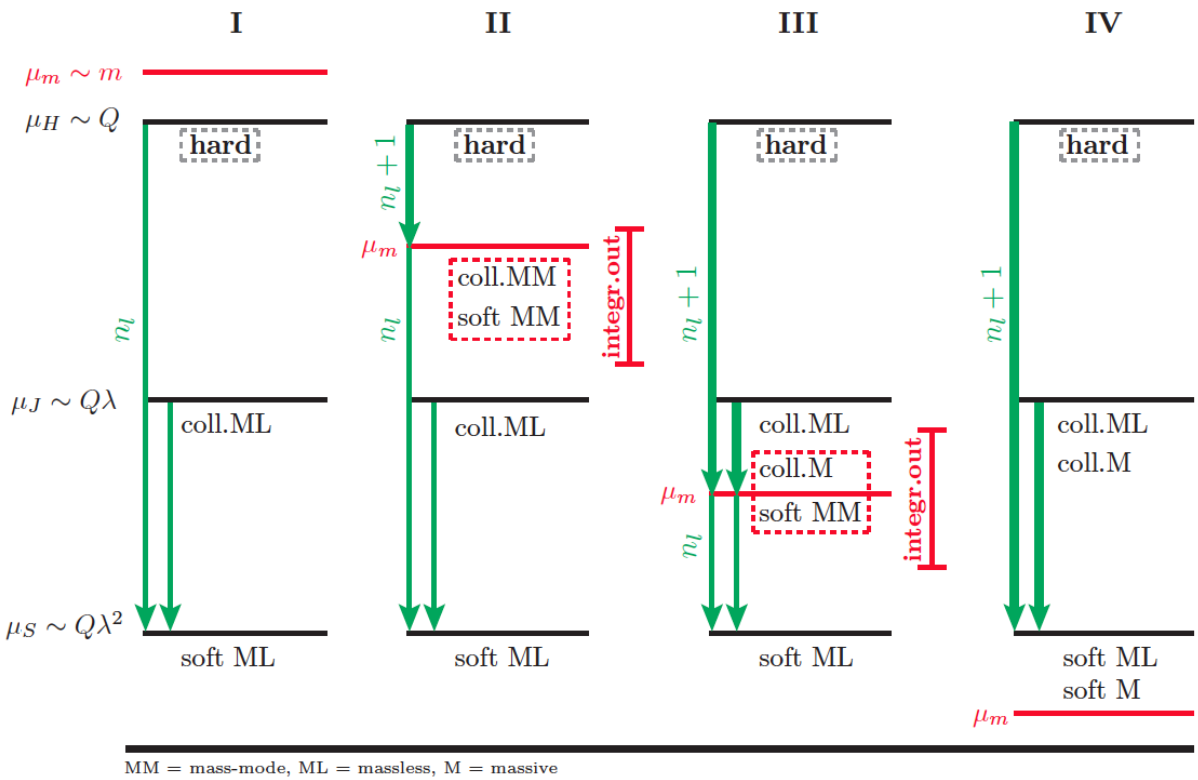
$$J^{\overline{\rm MS}}(s,m,\mu) = J^{(n_l+1)}(s,\mu) + \delta J_m^{\rm dist}(s,m,\mu) + \theta(s-4m^2)\,\delta J_m^{\rm real}(s,m) \stackrel{m\ll s}{\longrightarrow} J^{(n_l+1)}(s,\mu)$$

$$\Rightarrow \quad \boxed{\mathcal{M}_J(s,m,\mu) = J^{\overline{\rm MS}}(s,m,\mu) \otimes \left(J^{\rm OS}(s,m,\mu)\right)^{-1}}$$

- Renormalization approach automatically implies (perturbative) continuity of the evolution through the MM threshold → no scale hierarchies are involved/needed anywhere!

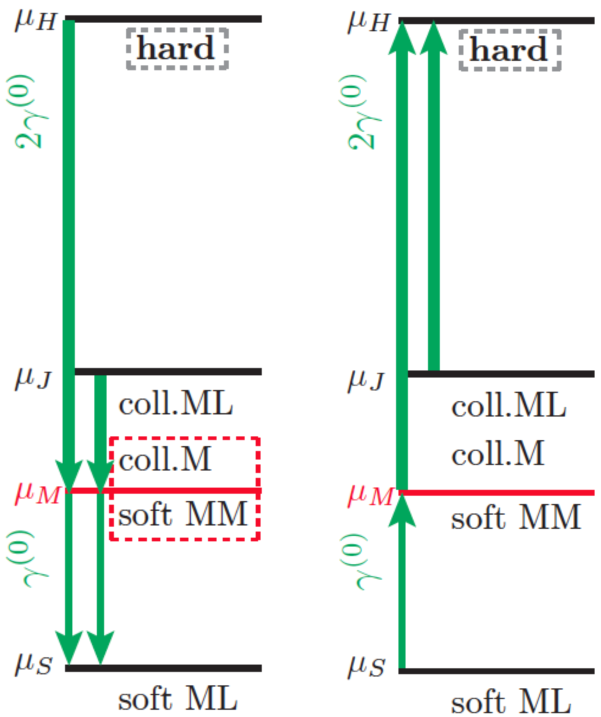
Factorized RG-evolution

- Hard coefficient, jet and soft function DO NOT CARE about the scenario.
- They care whether they are defined above or below the mass scale and whether they cross the mass threshold during the RG-evolution.
- The scenarios can be patched together from the factorized and evolved hard, jet and soft functions → universality



Consistency Conditions: Threshold Corrections

Important role of consistency relation: soft – jet – hard for scenario III



alternative description in bottom-up running ($\mu \sim \mu_H$):

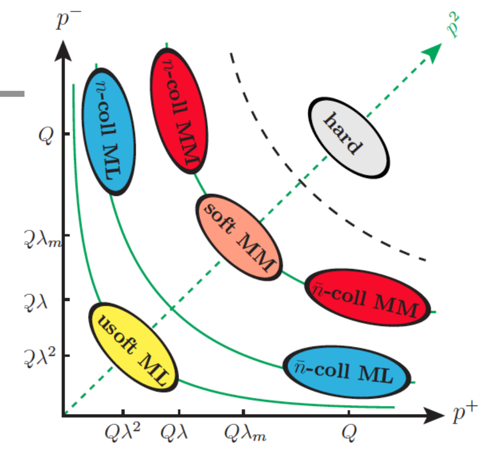
$$\frac{d\sigma}{d\tau} \sim \left|C''(\mu_H)\right|^2 \int d\ell \int d\ell' \int d\ell'' \int ds \int ds' \\ \times U_J^{(1)}(s-s',\mu_J,\mu_H)\, J_0(s',\mu_J)\, U_S^{(1)}(\ell''-s/Q,\mu_M,\mu_H) \\ \times \mathcal{M}_S(\ell'-\ell'',\mu_M)\, U_S^{(0)}(\ell-\ell',\mu_S,\mu_M)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

$$\mathcal{M}_S(\ell,\mu_M) = \delta(\ell) + \delta S_m^{\rm virt}(\ell,\mu_M)$$

consistency relation: $\mathcal{M}_S(\ell,\mu_M) = Q\,\left|\mathcal{M}_H(\mu_M)\right|^2 \mathcal{M}_J(Q\ell,\mu_M)$

similarly: $U_S^{(1)}(\ell,\mu_S,\mu_M) = Q\,U_H^{(1)}(\mu_M,\mu_S)\,U_J^{(1)}(Q\ell,\mu_M,\mu_S)$

Rapidity Logarithms



- Secondary mass effects start at $O(\alpha_s^2)$
- Counting for rapidity logs: α_s Log ~ 1
- At $O(\alpha_s^2)$:
  - No resummation to all orders needed
  - Need terms at $O(\alpha_s^3$ Log) and $O(\alpha_s^4$ Log$^2)$

$O(\alpha_s)$

$O(\alpha_s^2)$

$$\mathcal{M}_H(Q,m,\mu_m) = 1 + \frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2} \ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\frac{4}{3}L_m^2 + \frac{40}{9}L_m + \frac{112}{27}\right\}$$
$$+ \Bigg[\frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\left\{\frac{4}{9}L_m^3 + \frac{38}{9}L_m^2 + \left(\frac{242}{27} + \frac{2\pi^2}{3}\right)L_m - \frac{52}{9}\zeta(3) + \frac{875}{54} + \frac{5\pi^2}{9}\right\}$$
$$+ \frac{\left(\alpha_s^{(n_l+1)}\right)^3 C_F T_F}{(4\pi)^3}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\mathcal{M}_H^{(3)} + \sum_{n=0}^{3} a_n L_m^n\right\}$$
$$+ \frac{\left(\alpha_s^{(n_l+1)}\right)^4 C_F^2 T_F^2}{(4\pi)^4}\ln^2\left(-\frac{m^2}{Q^2}\right)\left\{\frac{8}{9}L_m^4 + \frac{160}{27}L_m^3 + \frac{416}{27}L_m^2 + \frac{4480}{243}L_m + \frac{6272}{729}\right\}\Bigg]$$

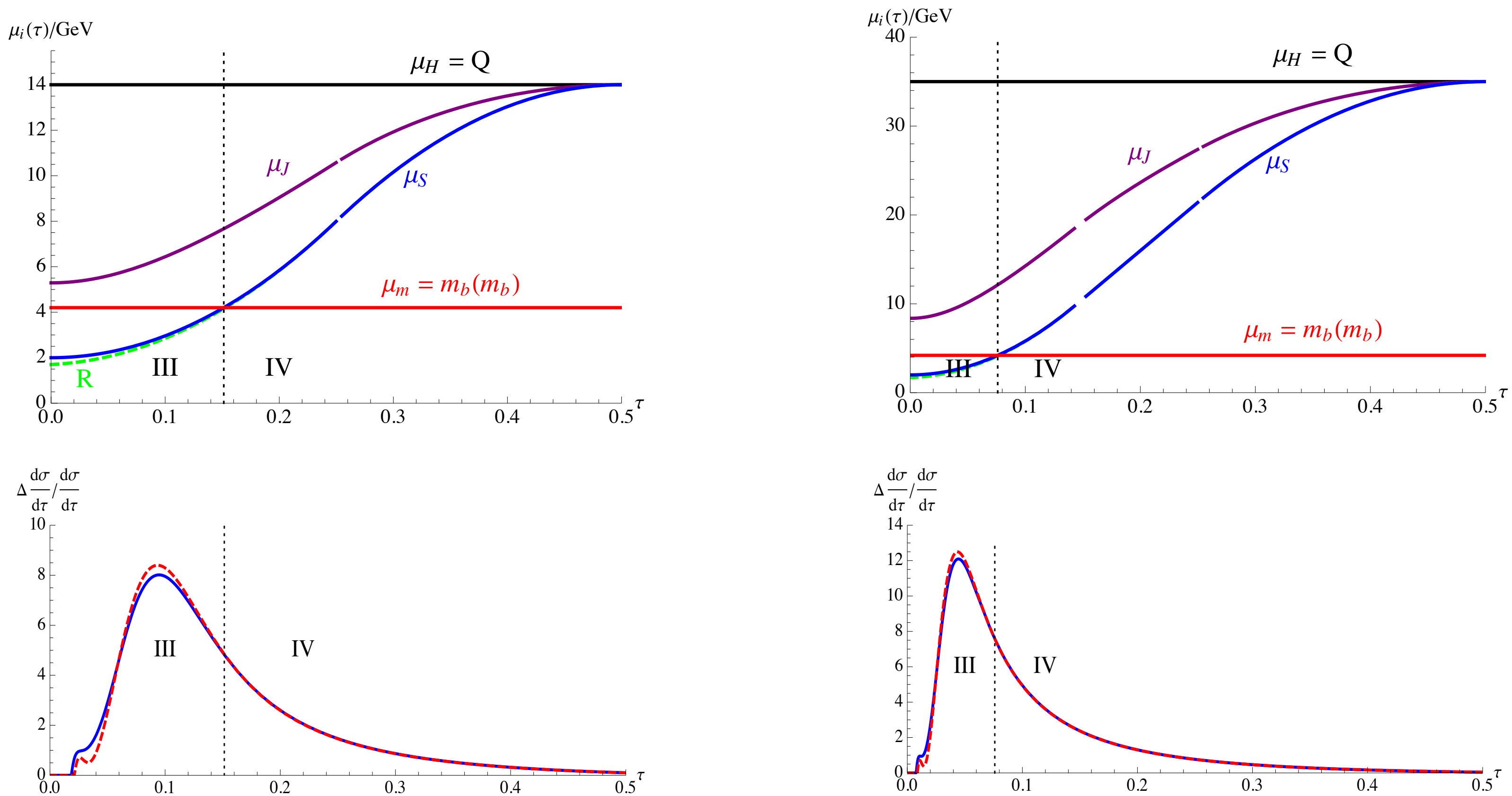
$$L_M = \ln\left(\frac{m^2}{\mu_m^2}\right)$$

VFN Scheme for Final State Jets

Numerical results: secondary bottom effects (Q=14, 35 GeV, $m_b(m_b)$=4.2 GeV)

→ O(α_s^2) fixed-order + N^{3}LL summations

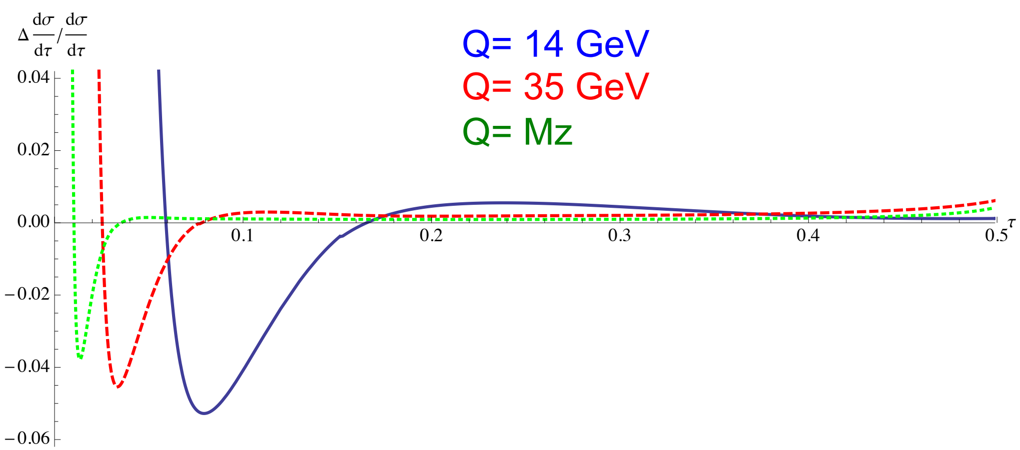
$$\alpha_s^{(5)}(M_Z) = 0.114\,, \quad \Omega_1^{(5)}(13\text{ GeV}) = 0.5\text{ GeV}$$



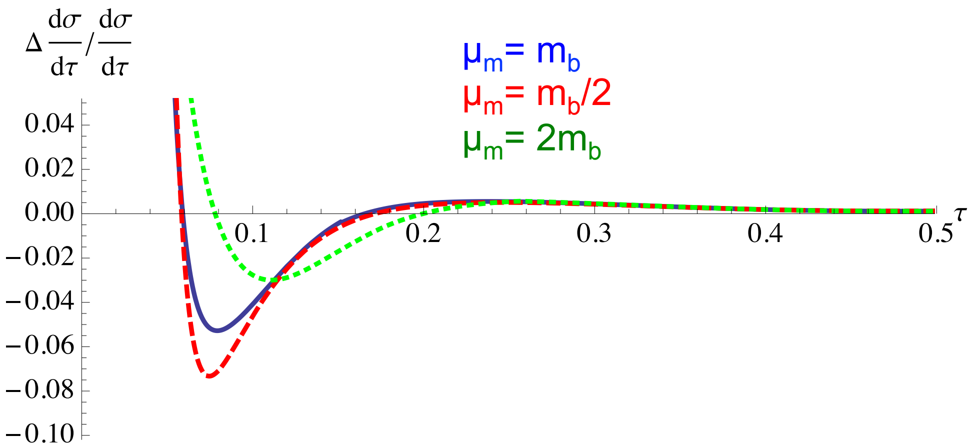
VFN Scheme for Final State Jets

Numerical results: secondary bottom effects (Q=14, 35 GeV)

$$\alpha_s^{(5)}(M_Z) = 0.114\,, \quad \Omega_1^{(5)}(13\ \mathrm{GeV}) = 0.5\ \mathrm{GeV}$$



→ size of corrections in the peak region sizeable even for large Q/m



→ matching scale dependence of mass correction sizeable since fixed-order mass corrections are $O(\alpha_s^2)$

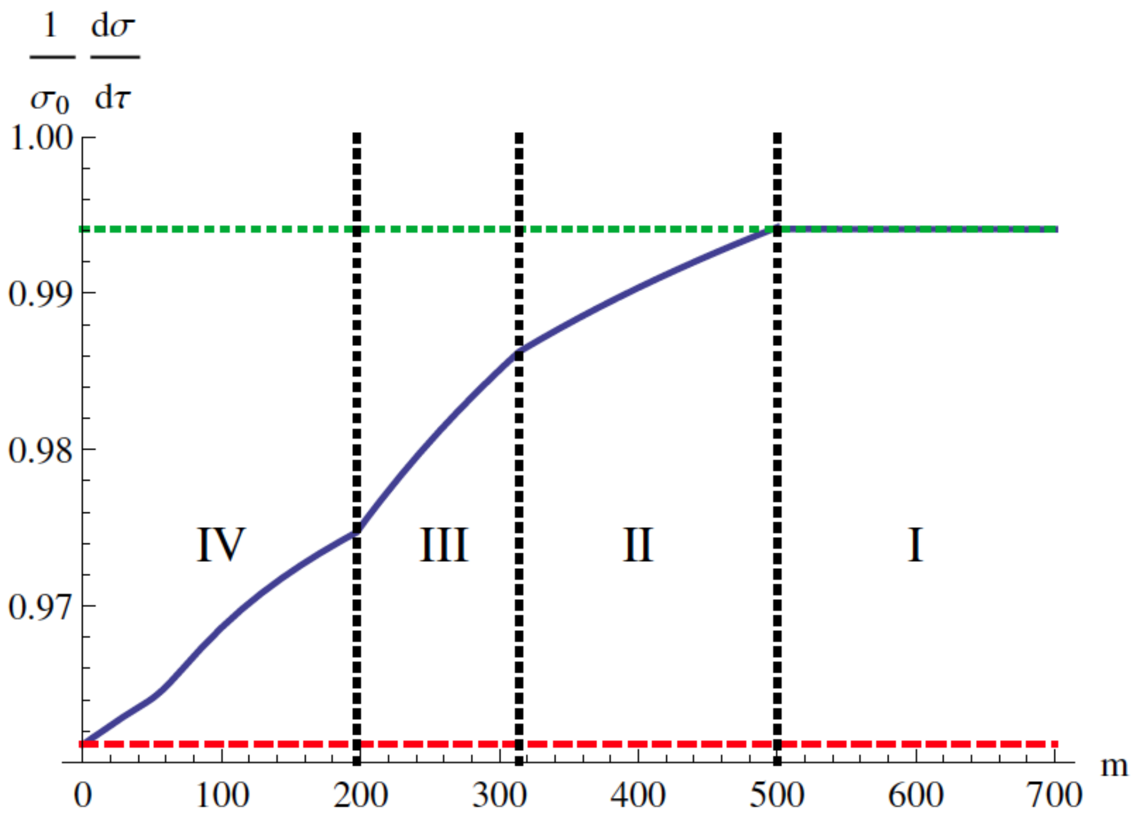
VFN Scheme for Final State Jets

Consistency check: continuous transition and correct limiting behaviour

Thrust distribution: $Q = 500$ GeV, $\tau = 0.15$ fixed, vary mass

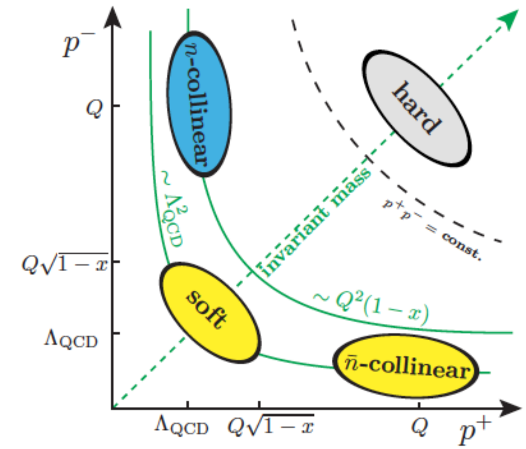
massless limit (6 flavors): dashed

decoupling limit (5 flavors): dotted



Consistency with VFNS in DIS ($x \to 1$)

- $x \to 1$: experimentally barely accessible (small pdfs!)
  but: nontrivial factorization setup $\to$ interesting as a showcase for concepts
- quite a lot of SCET literature
  `Manohar (2003), Becher, Neubert, Pecjak (2006),`
  `Chay, Kim (2006, 2010, 2013), Fleming, Zhang (2013), ...`
- here: $1 - x \sim \Lambda_{\rm QCD}/Q$, conveniently: Breit frame



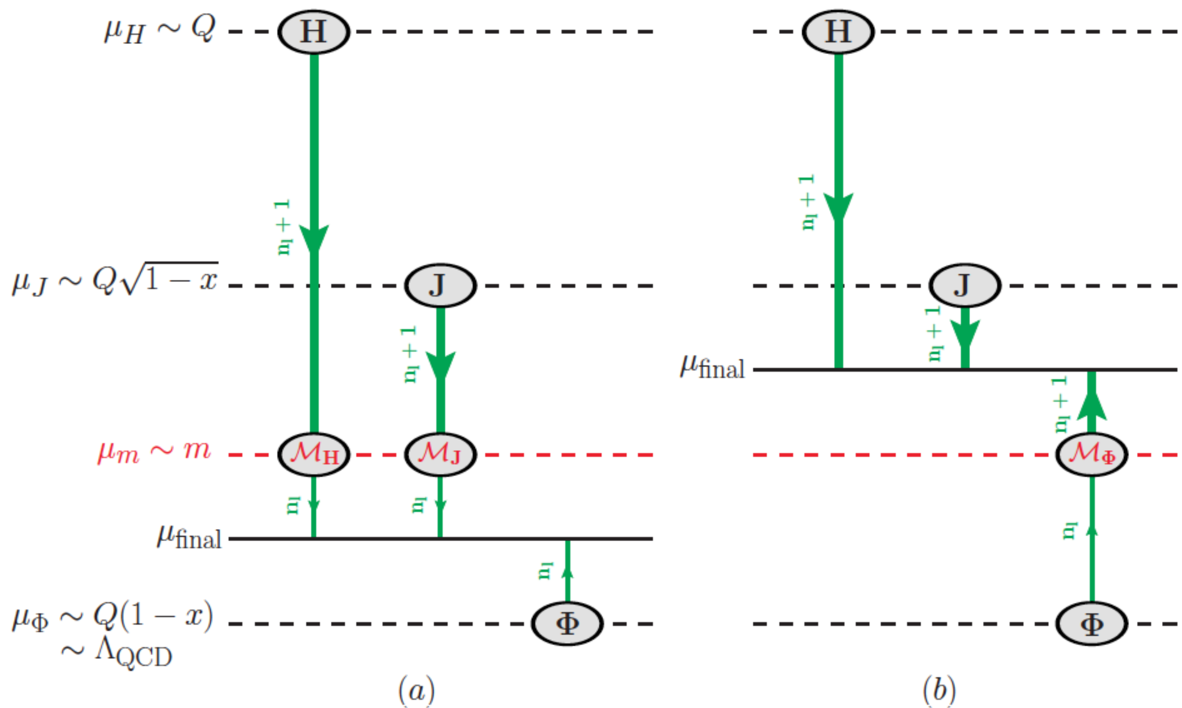
Factorization theorem:

$$F_1 \sim \sum_{i=q} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \underbrace{S_{\rm DIS}(\mu_\Phi) \otimes f_{i/P}(\mu_\Phi)}_{=\Phi_{i/P}(\mu_\Phi)}$$

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\rm DIS}(\mu_H) = |C(\mu_H)|^2$
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\rm DIS}(\mu_J)$
- at $\mu_\Phi \sim \Lambda_{\rm QCD}$: pdf $\Phi_{q/P}(\mu_\Phi)$
  $\leftrightarrow$ in SCET II: collinear initial state function $f_{q/P}(\mu_\Phi) \otimes$ soft function $S_{\rm DIS}(\mu_\Phi)$

Consistency with VFNS in DIS (x→1)



physical cross section independent of μ_{final} → (a) and (b) equivalent

→ relation between evolution factors

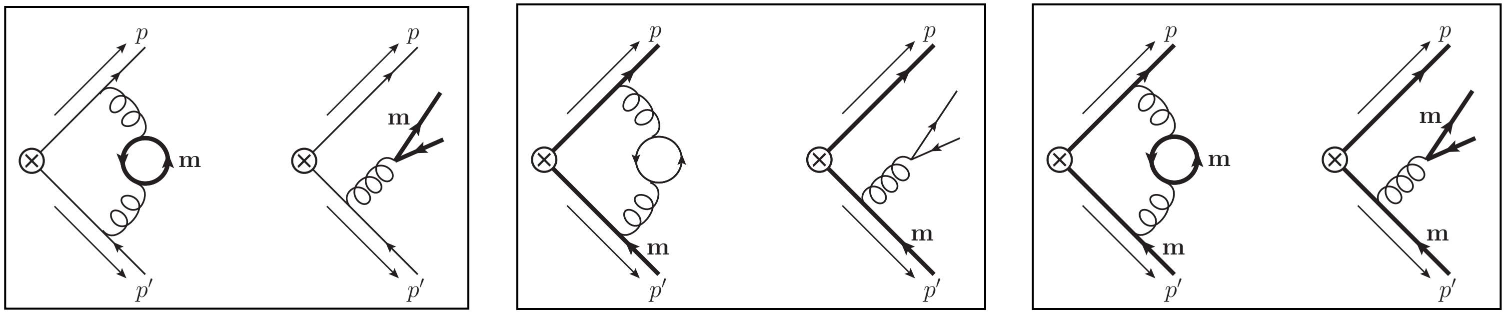
$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l + 1$$

→ relation between matching conditions

$$\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi$$

Outlook & Conclusion

→ VFN Scheme for final state jets with massive quarks



→ Sums all large logarithms involving m (if they exist)

→ Keeps full mass dependence of singular terms

$$Q \gg J \gg S$$
$$\leftarrow\leftarrow \quad m \quad \rightarrow\rightarrow$$

→ Fully consistent and integrable with VFNS scheme for PDFs, beam fcts, …

→ Allows ZVNS applications for "minimalistic" quark mass implementation

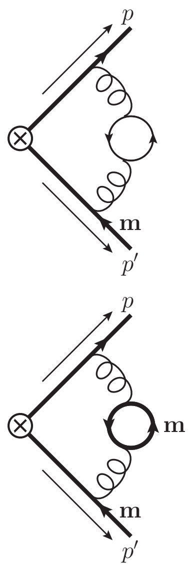
(ONLY in case if large mass logs exist !)

→ Needs non-trivial mass-dependent ME calculations if mass is of order of another scale

→ Treatment for pp collisions very soon….

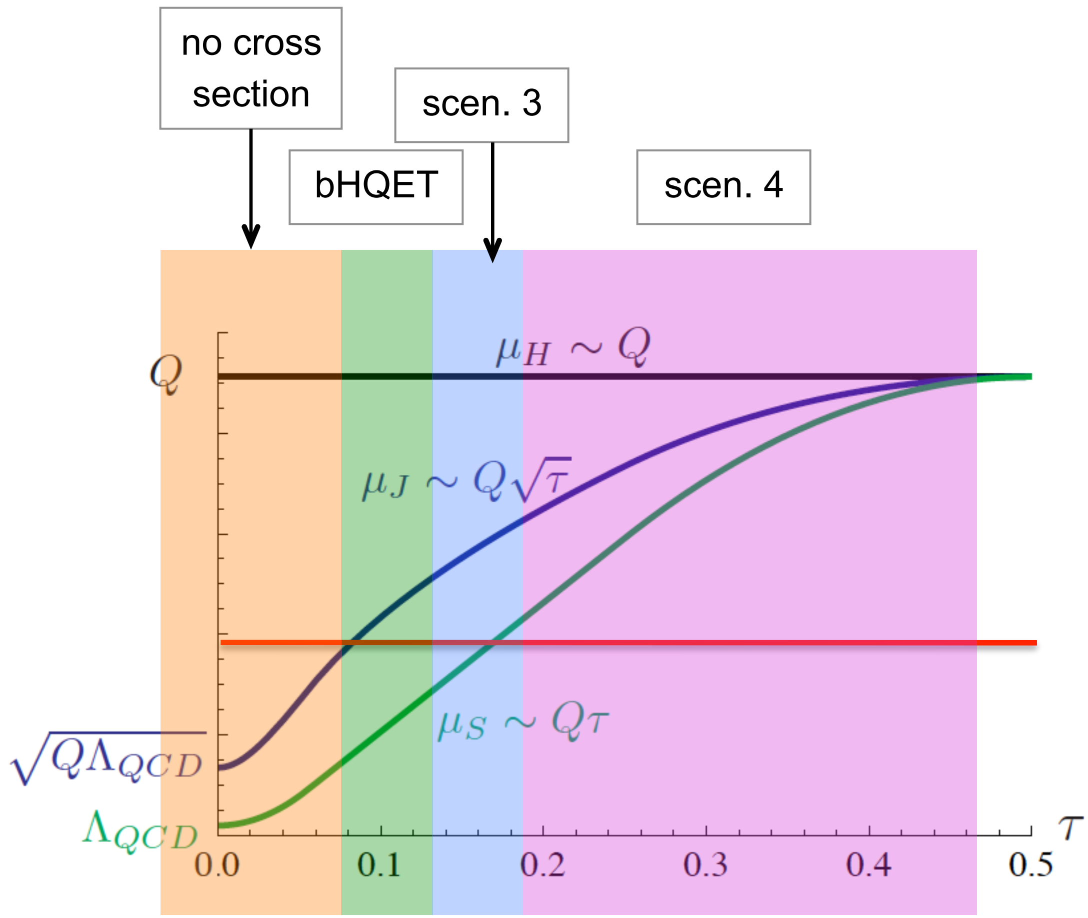
Backup Slides

VFN Scheme: Primary Massive Quarks



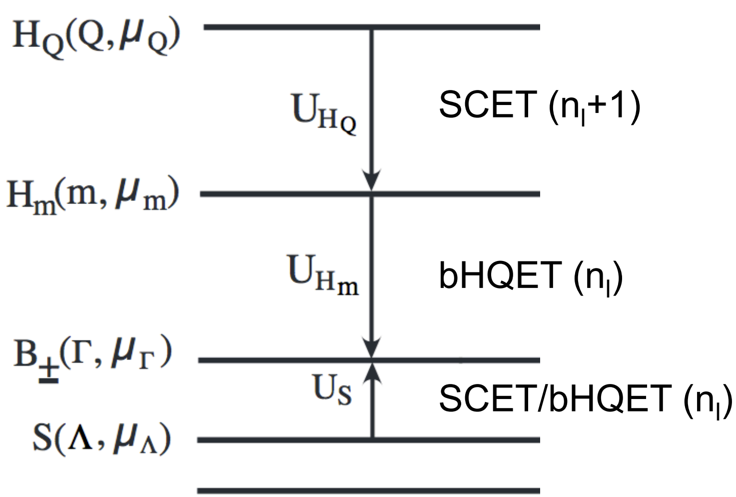
→ bHQET-type theory when
the jet scale approaches the quark mass

→ two SCET-type theories



VFN Scheme: Primary Massive Quarks

SCET/bHQET: $Q >> J \sim m > \Delta m > m/Q\, \Delta m$



- Small components of massive quark integrated out at $\mu_m \sim m$
- bHQET current evolution for $\mu < m$
- SCET current evolution for $\mu > m$
- Soft function identical to primary massless case (boosted massive quarks)

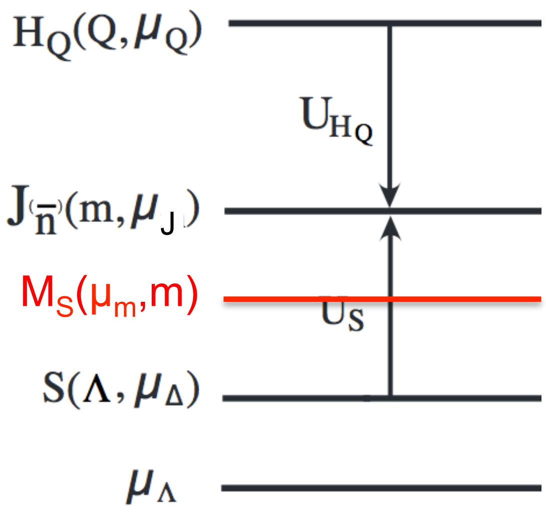
All two-loop FO input now known! N^2LL'/N^3LL

$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\text{bHQET}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)\, U_{H_m}^{(n_l)}\left(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\right)$$

$$\int ds \int dk\, B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}},\mu_B,m_J^{(n_l)}\right) U_S^{(n_l)}(k,\mu_B,\mu_S)\, S_{\text{part}}^{(n_l)}\left(Q\,\tau - Q\,\tau_{\text{MIN}} - \frac{s}{Q} - k,\mu_S\right)$$

VFN Scheme: Primary Massive Quarks

SCET scen. 3: Q >> J > m > S



- Same as scenario 3 for primary massless, but with massive jet function

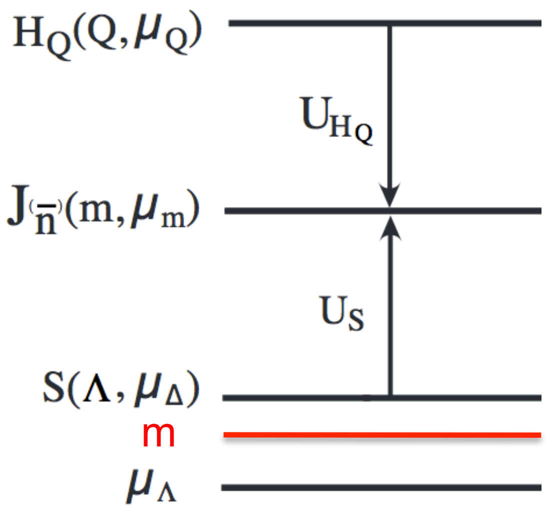
N^2LL'/N^3LL up to two-loop massive SCET jet function.

$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\text{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int ds \int dk\, dk'\, dk''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$
$$\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\text{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

$$n_f = n_\ell + 1$$

VFN Scheme: Primary Massive Quarks

SCET scen. 4: Q >> J > S > m



- Same as scenario 4 for primary massless, but with massive jet function

N²LL'/N³LL up to two-loop massive SCET jet function.

✓ Consistency relations: Evolution factors and mass mode threshold corrections

✓ Perturbative continuity

$$\left|\frac{1}{\sigma_0}\frac{d\hat{\sigma}(\tau)}{d\tau}\right|^{\mathrm{SCET-IV}} = Q\,H_Q^{(n_f)}(Q,\mu_Q)\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s\int \mathrm{d}k\,J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\,S_{\rm part}^{(n_f)}\left(Q\,\tau - Q\,\tau_{\rm min} - \frac{s}{Q} - k,\mu_S\right)$$

$$n_f = n_\ell + 1$$

Short-Distance Masses

- Mass dependence in all FO components of all factorization theorems
- Most relevant quark mass dependence contains in the jet functions (SCET & bHQET)
- Mass definition must be close with the scale of the respective functions (→profile functions)

$\mu \geq m$: MSbar mass (n_l+1)

$$\bar{m}(\mu) = m_{\rm pole} - \bar{m}(\mu) \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \ln^k \frac{\mu}{\bar{m}}$$

→ usual MSbar RG-evolution

$\mu < m$: R-scale short-distance mass (n_l)

- Jet mass: from bHQET jet function
- MSR mass: derived from MSbar mass coefficients
- Many others possible

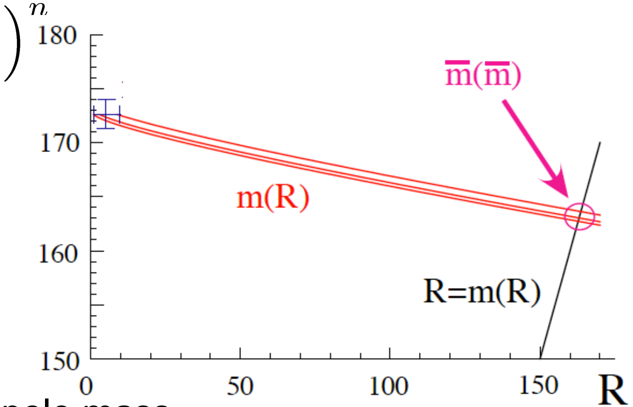
Jain, Scimemi, Stewart 08

Jain, Scimemi, Stewart, AH 08

$$m(R) = m_{\rm pole} - \delta m(R) \qquad \delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n$$

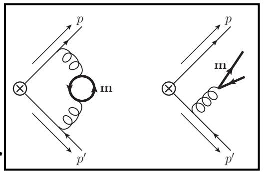
$$R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}$$

$$m(R_1) - m(R_0) = \int_{R_0}^{R_1} \frac{dR}{R} R\,\gamma^R[\alpha_s(R)]$$



$\mu_m \sim m$: matching: → pert. renormalons-free relation through pole mass

Gap Parameter



- Remove O(Λ) renormalon in partonic soft function
- Gap matching in R-evolution at mass scale
- Subtraction for finite mass not strictly needed, but included to have smooth behavior for massless limit
- R-evolution mass dependent at O(α_s^2)

contains renormalon

$$S(\ell,\mu) = \int d\ell'\, S_{\rm part}(\ell-\ell',\mu)\, S_{\rm model}(\ell-\Delta)$$

$$\Delta = \bar\Delta(R,\mu) + \delta(R,\alpha_s,\mu)$$

renormalon-free

$$S(\ell,\mu) = \int d\ell'\, S_{\rm part}(\ell-\ell'+\delta,\mu)\, S_{\rm model}(\ell-\bar\Delta)$$

$$\delta(R,\mu) = \frac{Re^{\gamma_E}}{2}\frac{d}{d\ln(ix)}\Big[\ln \tilde S_{\tau,\rm part}(x,\mu)\Big]\Big|_{x=(iRe^{\gamma_E})^{-1}}$$

Kluth, AH 10

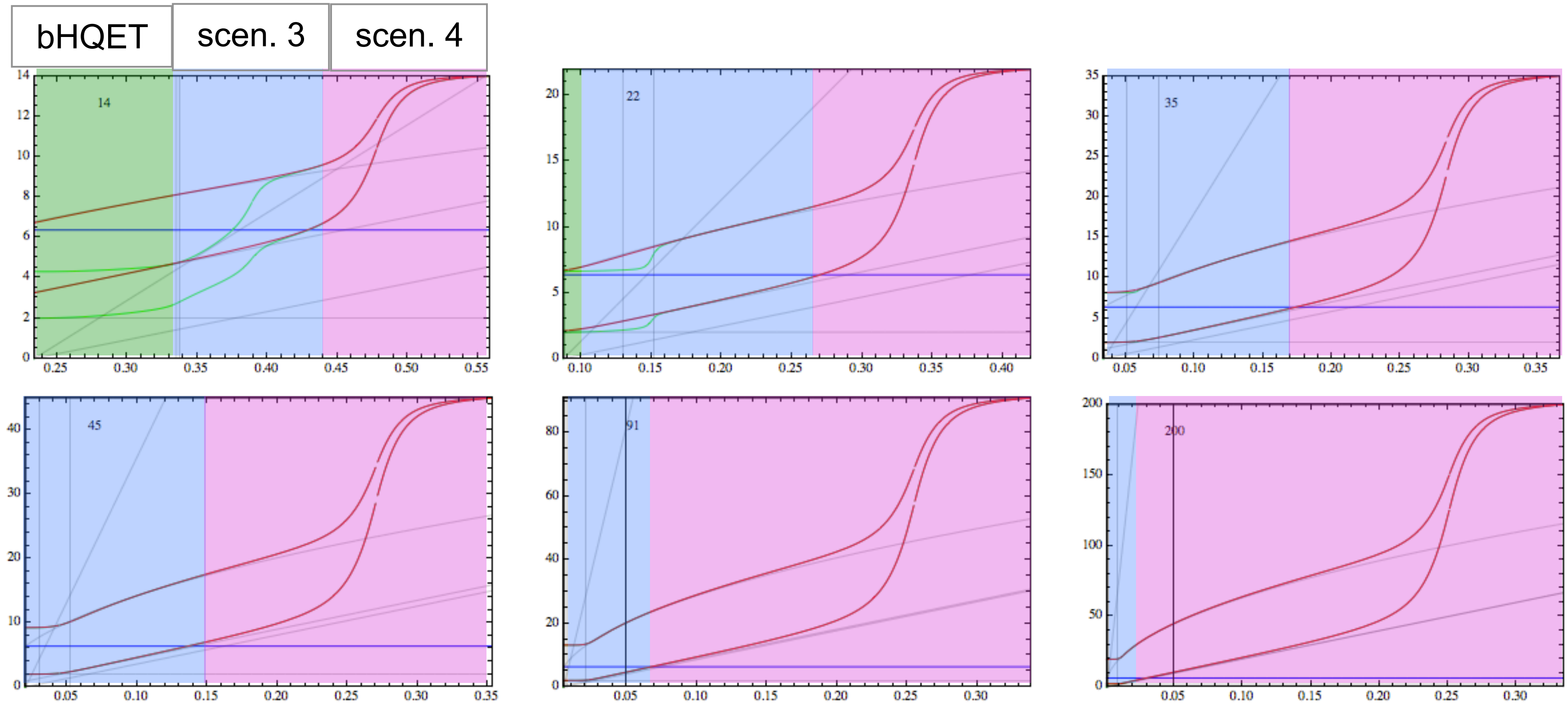
$\mu_m \sim m$: matching:

Gritschacher, AH, Jemos, Pietrulewicz 2013

$$\bar\Delta^{(n_\ell)}(R,\mu) - \bar\Delta^{(n_\ell+1)}(R,m,\mu) = e^{\gamma_E}\, R\left[\left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left(\delta_{2,m}(R,m,\mu) + \frac{4}{3}T_F\,\delta_1\,\ln\frac{\mu^2}{m^2}\right)\right]$$

VFN Scheme: Primary Massive Quarks

Profile functions: m=4.5, Q= 14, 22, 35, 45, 91, 200 GeV



→ Scenario 4 was used in our current thrust analysis based on data Q ≥ 35 GeV

VFN Scheme: Primary Massive Quarks

First prelim. analysis: m=4.5, Q= 14, 22, 35, 91 GeV ($NNLL_{resum}$ + $NLO_{fixed\text{-}order}$)