NLO corrections to Z production in association with several jets

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I: **The numerical method**

II: **General improvements**

III: **First results for $pp \to Z + 5$ jets**

Part I

The numerical method

Z plus jet production at the LHC

Experimental status:

- The LHC experiments have measured Z production in association with up to 7 jets.

Theoretical status:

- NLO corrections to $Z + 0$ jets, $Z + 1$ jet, $Z + 2$ jets known for a long time.
- NLO corrections to $Z + 3$ jets and $Z + 4$ jets calculated by Blackhat collaboration.

Challenge:

- Can one calculate the NLO corrections to $Z + 5$ jets, $Z + 6$ jets and $Z + 7$ jets ?

Scaling behaviour with the number of jets

As the number of jets increases, the scaling behaviour with the number of jets is the relevant quantity.

- Bad: Factorial or exponential growth.
- Better: Polynomial growth.

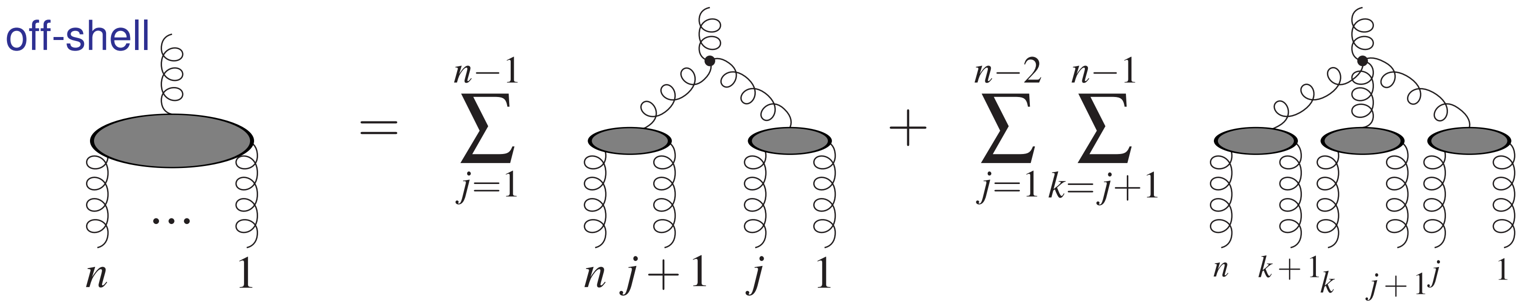
Using recurrence relations, we can achieve n^3-behaviour at LO.

What about loops ?

- Unitarity methods: n^9
- Numerical methods: n^3

Recurrence relations

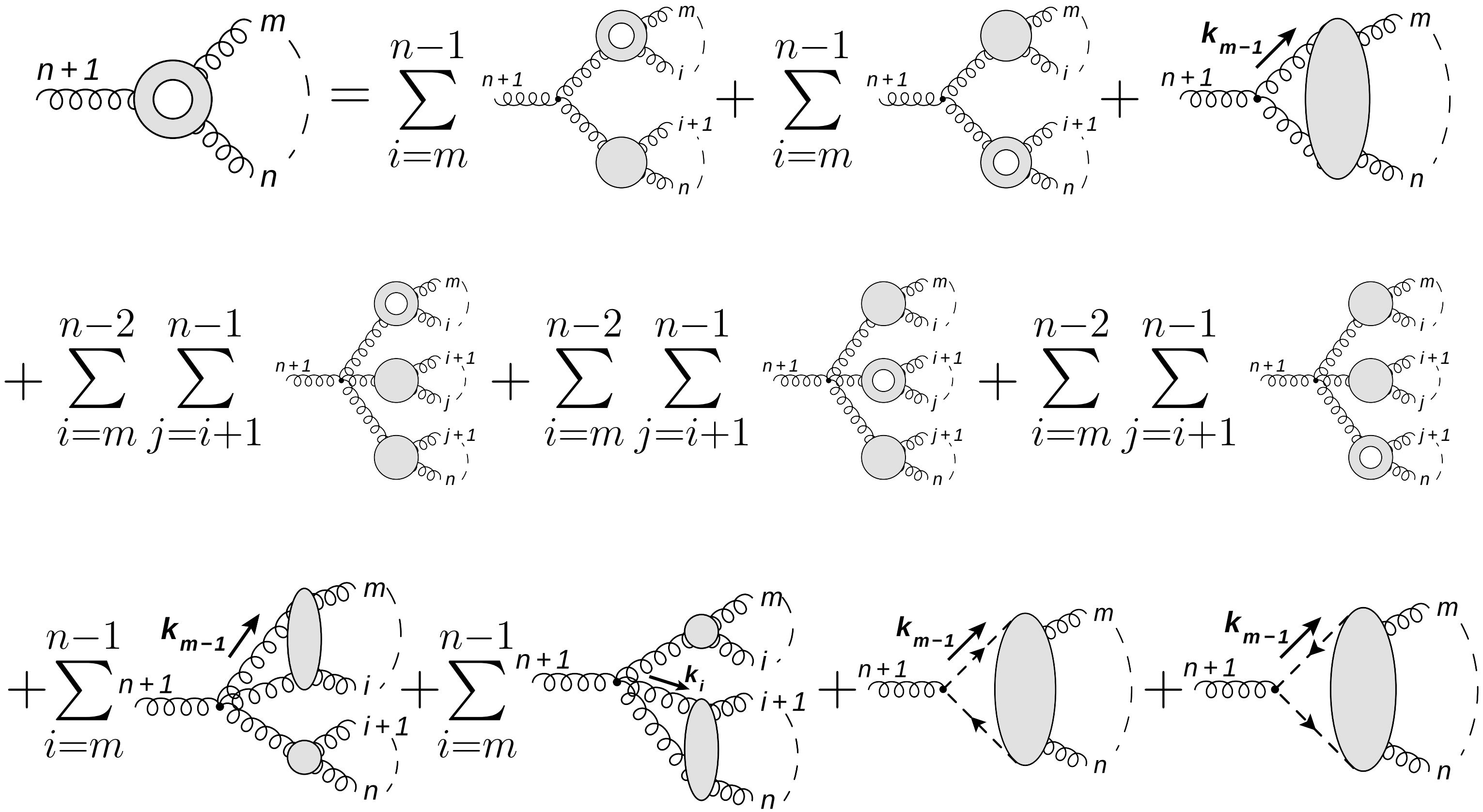
Off-shell currents provide an efficient way to calculate amplitudes:



No Feynman diagrams are calculated in this approach !

F. A. Berends and W. T. Giele

The one-loop recurrence relations



Draggiotis et al., ’06; van Hameren, ’09; Becker, Reuschle, S.W., ’10; Cascioli, Maierhöfer, Pozzorini, ’11

Numerical NLO QCD calculations

Proceed through the following steps:

1. Local subtraction terms for soft, collinear and ultraviolet singular part of the integrand of one-loop amplitudes
2. Contour deformation for the 4-dimensional loop integral.
3. Numerical Monte Carlo integration over phase space and loop momentum.

Not a new idea: Nagy and Soper proposed in ’03 this method, working graph by graph.

(see also: Soper; Krämer, Soper; Catani et al.; Kilian, Kleinschmidt)

What is new: The IR-subtraction terms can be formulated at the level of amplitudes, no need to work graph by graph.

The IR-subtraction terms are universal and amasingly simple.

Subtraction method for loop integrals

Use subtraction also for the virtual part:

$$\int_{n+1} d\sigma^R + \int_n d\sigma^V \quad = \quad \underbrace{\int_{n+1} \left(d\sigma^R - d\sigma^A\right)}_{\text{convergent}} + \underbrace{\int_n \left(\mathbf{I}+\mathbf{L}\right) \otimes d\sigma^B}_{\text{finite}} + \underbrace{\int_n \left(d\sigma^V - d\sigma^{A'}\right)}_{\text{convergent}}$$

- In the last term $d\sigma^V - d\sigma^{A'}$ the Monte Carlo integration is over a phase space integral of n final state particles plus a 4-dimensional loop integral.

- All explicit poles cancel in the combination $\mathbf{I}+\mathbf{L}$.

- Divergences of one-loop amplitudes related to IR-divergences (soft and collinear) and to UV-divergences.

M. Assadsolimani, S. Becker, D. Götz, Ch. Reuschle, Ch. Schwan, S.W.

The infrared subtraction terms for the virtual corrections

Local unintegrated form:

$$G^{(1)}_{\text{soft+coll}} = -4\pi\alpha_s i \sum_{i\in I_g} \left(\frac{4p_i p_{i+1}}{k_{i-1}^2 k_i^2 k_{i+1}^2} - 2\frac{S_i g^{UV}_{i-1,i}}{k_{i-1}^2 k_i^2} - 2\frac{S_{i+1} g^{UV}_{i,i+1}}{k_i^2 k_{i+1}^2} \right) A_i^{(0)}.$$

with $S_q = 1$, $S_g = 1/2$. The function $g^{UV}_{i,j}$ provides damping in the UV-region:

$$\lim_{k\to\infty} g^{UV}_{i,j} = O\left(k^{-2}\right), \qquad \lim_{k_i || k_j} g^{UV}_{i,j} = 1.$$

Integrated form:

$$S_\varepsilon^{-1} \mu^{2\varepsilon} \int \frac{d^D k}{(2\pi)^D} G^{(1)}_{\text{soft+coll}} = \frac{\alpha_s}{4\pi} \frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)} \sum_{i\in I_g} \left[\frac{2}{\varepsilon^2} \left(\frac{-2p_i \cdot p_{i+1}}{\mu^2} \right)^{-\varepsilon} + \frac{2}{\varepsilon} \left(S_i + S_{i+1} \right) \left(\frac{\mu_{\text{UV}}^2}{\mu^2} \right)^{-\varepsilon} \right] A_i^{(0)}$$

$$+ O(\varepsilon),$$

UV-subtraction terms

In a fixed direction in loop momentum space the amplitude has up to quadratic UV-divergences.

Only the integration over the angles reduces this to a logarithmic divergence.

For a local subtraction term we have to match the quadratic, linear and logarithmic divergence.

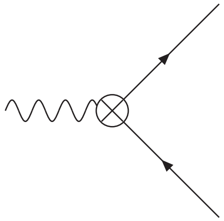
The subtraction terms have the form of counter-terms for propagators and vertices.

The complete UV-subtraction term can be calculated recursively.

UV-subtraction terms

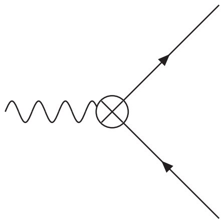
Example: The quark-gluon vertex.

Local unintegrated form:



$$= \quad i g^3 S_\varepsilon^{-1} \mu^{4-D} \int \frac{d^D k}{(2\pi)^D i} \frac{2\,(1-\varepsilon)\,\bar{k}\!\!\!/\,\gamma^\mu \bar{k}\!\!\!/ + 4\mu_{UV}^2 \gamma^\mu}{\left(\bar{k}^2 - \mu_{UV}^2\right)^3}$$

Integrated form:



$$= \quad i \frac{g^3}{(4\pi)^3} \gamma^\mu (-1) \left(\frac{1}{\varepsilon} - \ln \frac{\mu_{UV}^2}{\mu^2} \right) + O(\varepsilon)$$

We can ensure that the integrated expression is proportional to the Born.

Contour deformation

With the subtraction terms for UV- and IR-singularities one removes

- UV divergences
- Pinch singularities due to soft or collinear partons

Still remains:

- Singularities in the integrand, where a deformation into the complex plane of the contour is possible.
- Pinch singularities for exceptional configurations of the external momenta (thresholds, anomalous thresholds ...)

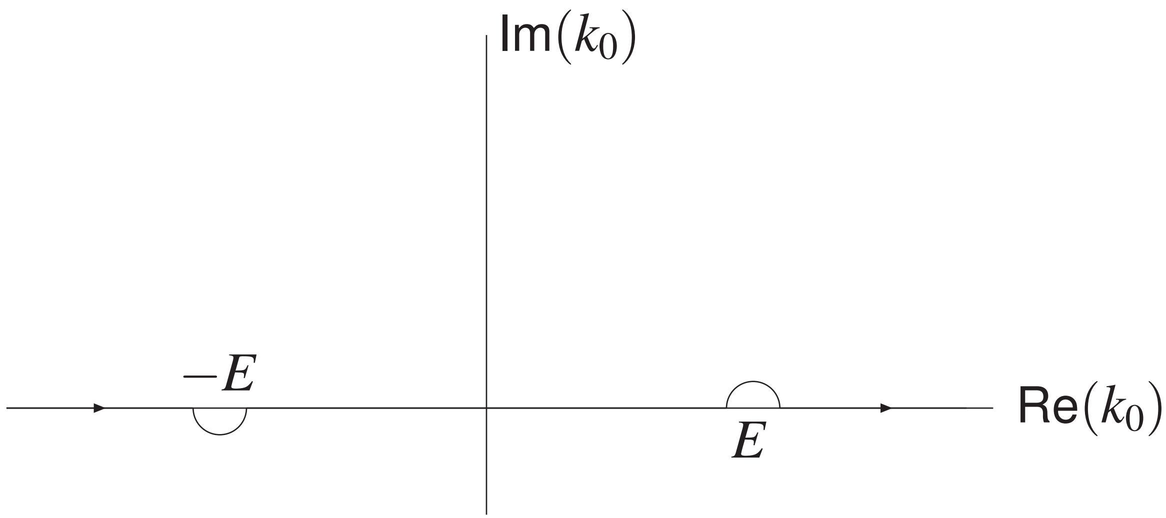
Contour deformation

$$I = \int d^4k \underbrace{[f(k) - g(k)]}_{h(k)}$$

$h(k)$ meromorphic function of four complex variables k_0, k_1, k_2, k_3.

Integration over a surface of (real) dimension 4 in $\mathbb{C}^4$.

I independent of the choice of the surface, as long as no poles are crossed.

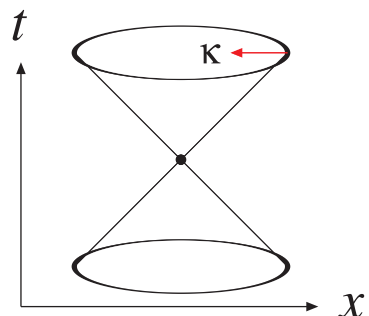


What is the best choice for the surface, in order to minimize Monte Carlo integration errors ?

Direct contour deformation

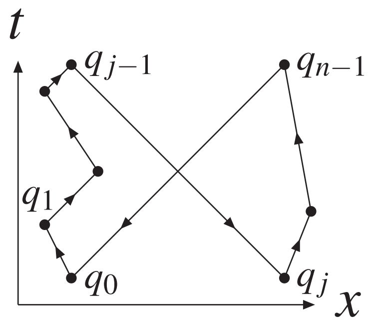
Deformation of the loop momentum:

$$k_{\mathbb{C}} = k_{\mathbb{R}} + i\kappa$$

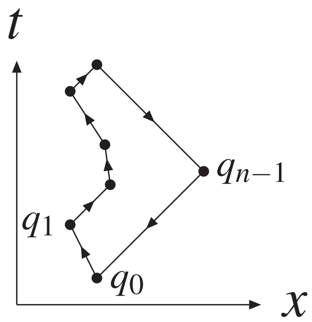


Single cone

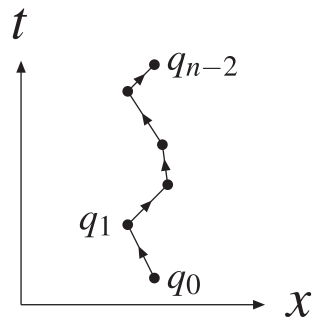
For n cones draw only the origins of the cones:



generic with 2 initial partons



initial partons adjacent



no initial partons

Gong, Nagy, Soper, ’08; Becker, Reuschle, S.W., ’12

Efficiency

With the local subtraction terms and the contour deformation we obtain an integral, where the loop integration can – in principle – be performed with Monte Carlo methods.

However, the integrand is oscillating:

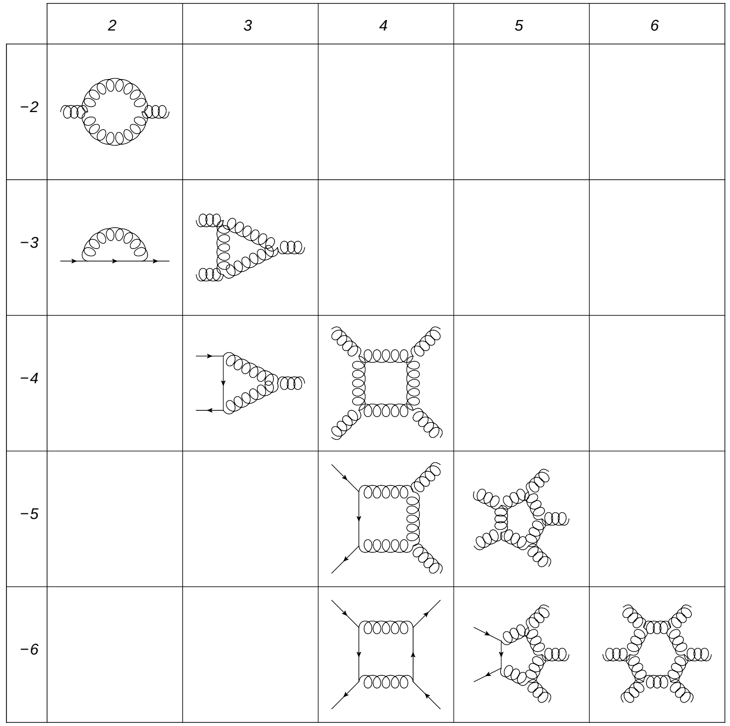
$$I = \int_0^1 dx \, \left[c + A \sin\left(2\pi x\right) \right], \qquad A \gg c$$

This leads to large Monte Carlo integration errors.

Solution: Antithetic variates: Evaluate the integrand at x and $(1-x)$.

UV improvement

Ultraviolet behaviour of some example diagrams:



To the right: number of external particles

In the vertical:
leading power of the large $|k|$-behaviour

UV-finiteness requires fall off like $|k|^{-5}$.

$|k|^{-5}$ contribution is odd under $k \to -k$ and integrates to zero.

However, $|k|^{-5}$ term gives a large contribution to the Monte Carlo error.

UV improvement

- Split the integration holomorphic into two channels:

$$1 = \left[\prod_{j=1}^{n} \frac{k_j^2 - m_j^2}{\bar{k}^2 - \mu_{\rm UV}^2} \right] + \left[1 - \prod_{j=1}^{n} \frac{k_j^2 - m_j^2}{\bar{k}^2 - \mu_{\rm UV}^2} \right]$$

First channel: simple pole structure, can be evaluated with a simple contour. Second channel: Integrand falls off with two additional powers of $|k|$ in the ultraviolet.

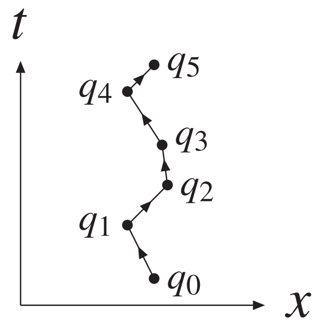
- Improvement of the counterterms for the propagators and three-valent vertices from $|k|^{-5}$ to $|k|^{-7}$.

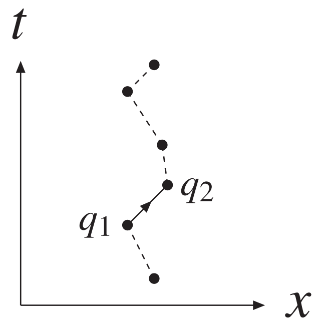
- Use antithetic Monte Carlo integration technique: Evaluate k and $(-k)$ together.

Infrared channels

Non-holomorphic splitting:

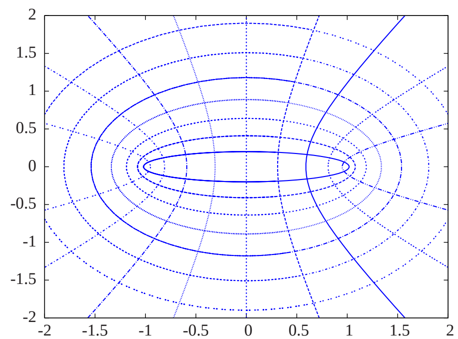
$$I_{\text{int}} = \sum_i \int \frac{d^4k}{(2\pi)^4} w_i(k) f(k),$$





Weights:

$$w_i(k) = \frac{\left(\frac{1}{|k_i^2||k_{i+1}^2|}\right)^\alpha}{\sum_j \left(\frac{1}{|k_j^2||k_{j+1}^2|}\right)^\alpha},$$



Coordinate system, where a line segment $[q_i, q_{i+1}]$ is singled out:
Generalised elliptical coordinates

Use technique of antithetic variates in these coordinates.

Part II

General improvements

Random polarisations

Matrix element with n external particles: Instead of summing over all 2^n spin states, introduce

$$\varepsilon_\mu(\phi) \quad = \quad e^{i\phi}\varepsilon_\mu^+ + e^{-i\phi}\varepsilon_\mu^- .$$

and replace the summation over the spin states by an integration over the angle ϕ: P. Draggiotis, R. Kleiss, C. Papadopoulos, ’98

$$\sum_{\lambda=\pm} \varepsilon_\mu^{\lambda^*} \varepsilon_\nu^\lambda \quad = \quad \frac{1}{2\pi} \int_0^{2\pi} d\phi \, \varepsilon_\mu(\phi)^* \varepsilon_\nu(\phi)$$

Works for Born and virtual part straightforward. For the real part the subtraction terms are usually spin-summed and thus non-local in ϕ.

Extension of the dipole formalism to random polarisations:

D. Götz, Ch. Schwan and S.W., ’12

Colour decomposition at one-loop

One-loop amplitudes (and Born amplitudes with multiple quark pairs):

Partial amplitudes can be decomposed further into primitive amplitudes (gauge-invariant, cyclic ordered, fixed routing of fermions).

Z. Bern, L. Dixon, D. Kosower, '95

For amplitudes with more than one quark-antiquark pair this decomposition is non-trivial.

- Use Feynman diagrams and solve a (large) system of linear equations.

  Ellis et al., '11; Ita, Ozeren, '11 ; Badger et al., '12

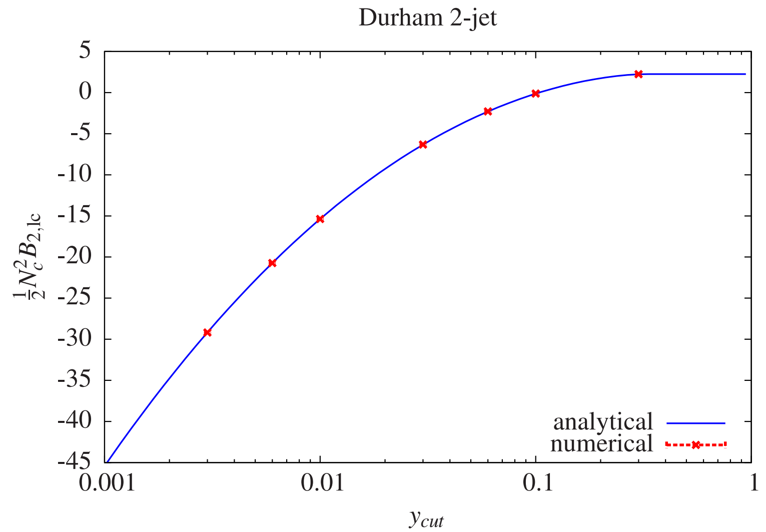
- More elegant: Obtain colour decomposition directly through shuffle relations.

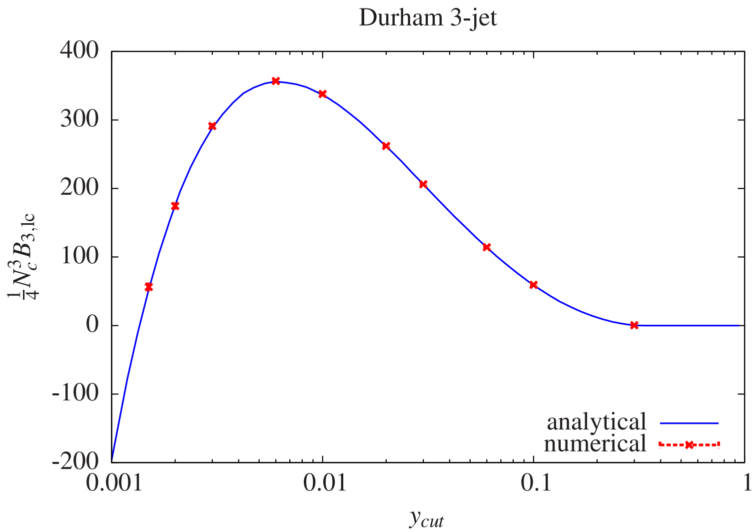
  Ch. Reuschle and S.W., '13

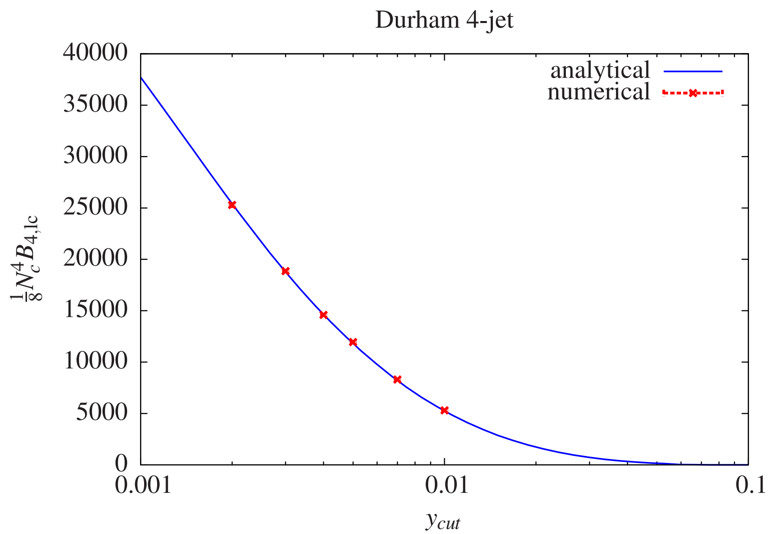
Part III

Numerical results

Jet rates in electron-positron annihilation



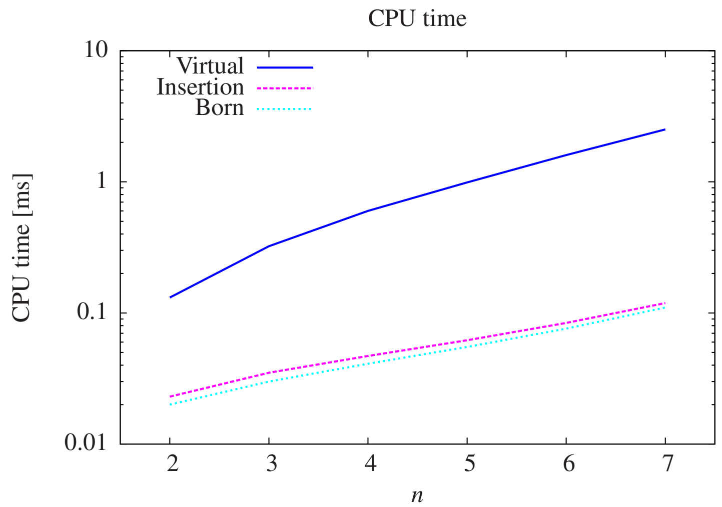




CPU scaling behaviour

Scaling of the CPU time for one evaluation of the integrand (Born, virtual, insertion) with the number of external particles:

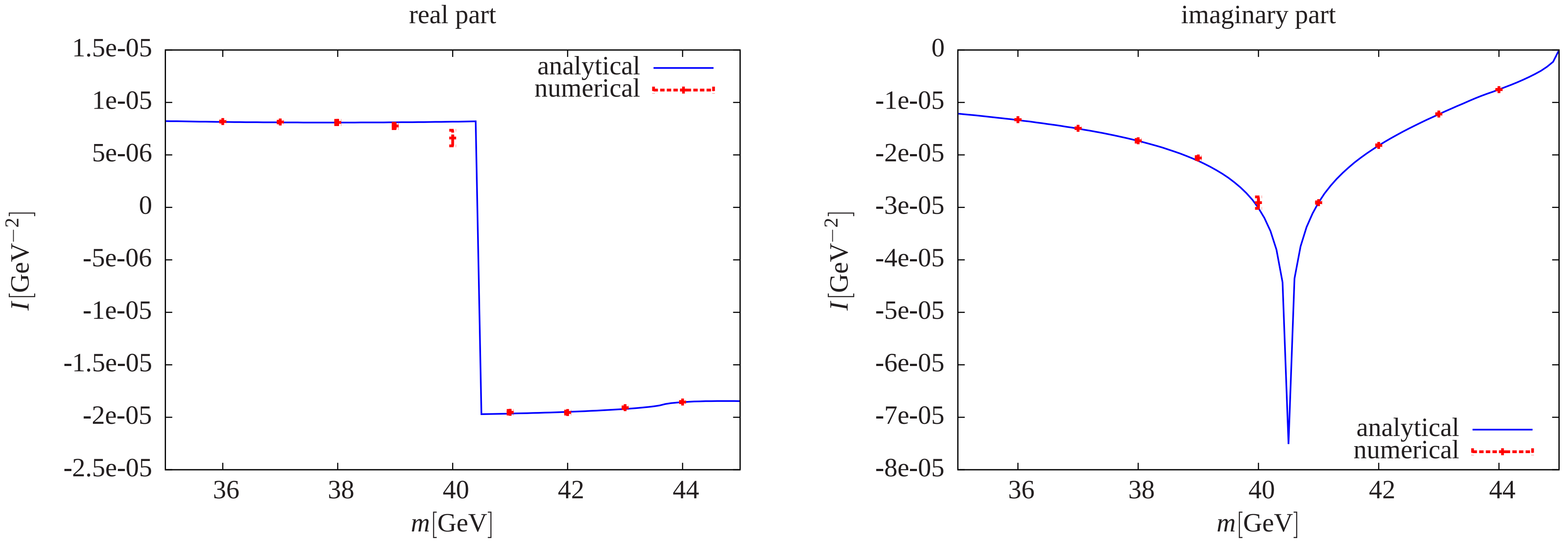
$$\text{CPU time} \sim n^4$$



- n^4-behaviour from recurrence relations
- helicity summation replaced by smooth integration over random polarisations
- Real part: Extension of the dipole formalism to random polarisations

  D. Götz, Ch. Schwan and S.W., '12

Extension to massive particles



Comparison of the results obtained by Monte Carlo integration with the analytical results in the vicinity of a threshold.

S. Becker and S.W., '12

Preliminary results on $pp \to Z + 5$ jets

Process $pp \to Z + 5$ jets $\to e^+ e + 5$ jets at $\sqrt{s} = 7\ \mathrm{TeV}$ with CTEQ6M/CTEQ6L1.
Jets defined by anti-kt-algorithm with $R = 0.5$.

Cuts:

$$p_l^\perp > 20\ \mathrm{GeV}, \quad |\eta_l| < 2.5, \quad 66\ \mathrm{GeV} < m_{l\bar{l}} < 116\ \mathrm{GeV},$$
$$p_{\mathrm{jet}}^\perp > 25\ \mathrm{GeV}, \quad |\eta_{\mathrm{jet}}| < 3.$$

Scale chosen on a per-event basis:

$$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = \frac{1}{2} H^{\perp\prime} = \frac{1}{2} \left(E_Z^\perp + \sum_j p_j^\perp \right).$$

Leading-colour approximation:

$$\sigma_{\mathrm{LO,lc}} = 0.138 \pm 0.009\ \mathrm{pb}, \qquad \sigma_{\mathrm{NLO,lc}} = 0.161 \pm 0.113\ \mathrm{pb}.$$

Conclusions

- The numerical method for the computation of NLO corrections offers a good scaling behaviour.
- First results on $pp \to Z + 5$ jets.
- Public program available soon.