The Search for the Schwinger Effect: Non-perturbative Pair Production from Vacuum

Gerald Dunne
University of Connecticut

DESY Seminar & Physics in Intense Fields: PIF 2013, July 2013

- probing the quantum vacuum
- fundamental physics and the Schwinger effect
- QFT methods, optimization and pulse shaping
- outlook : conceptual and computational issues

pre-quantum mechanics

horror vacui: nature abhors a vacuum

Aristotle, c350 BC

Naturall reason abhorreth vacuum

Cranmer, 1550

pre-quantum mechanics

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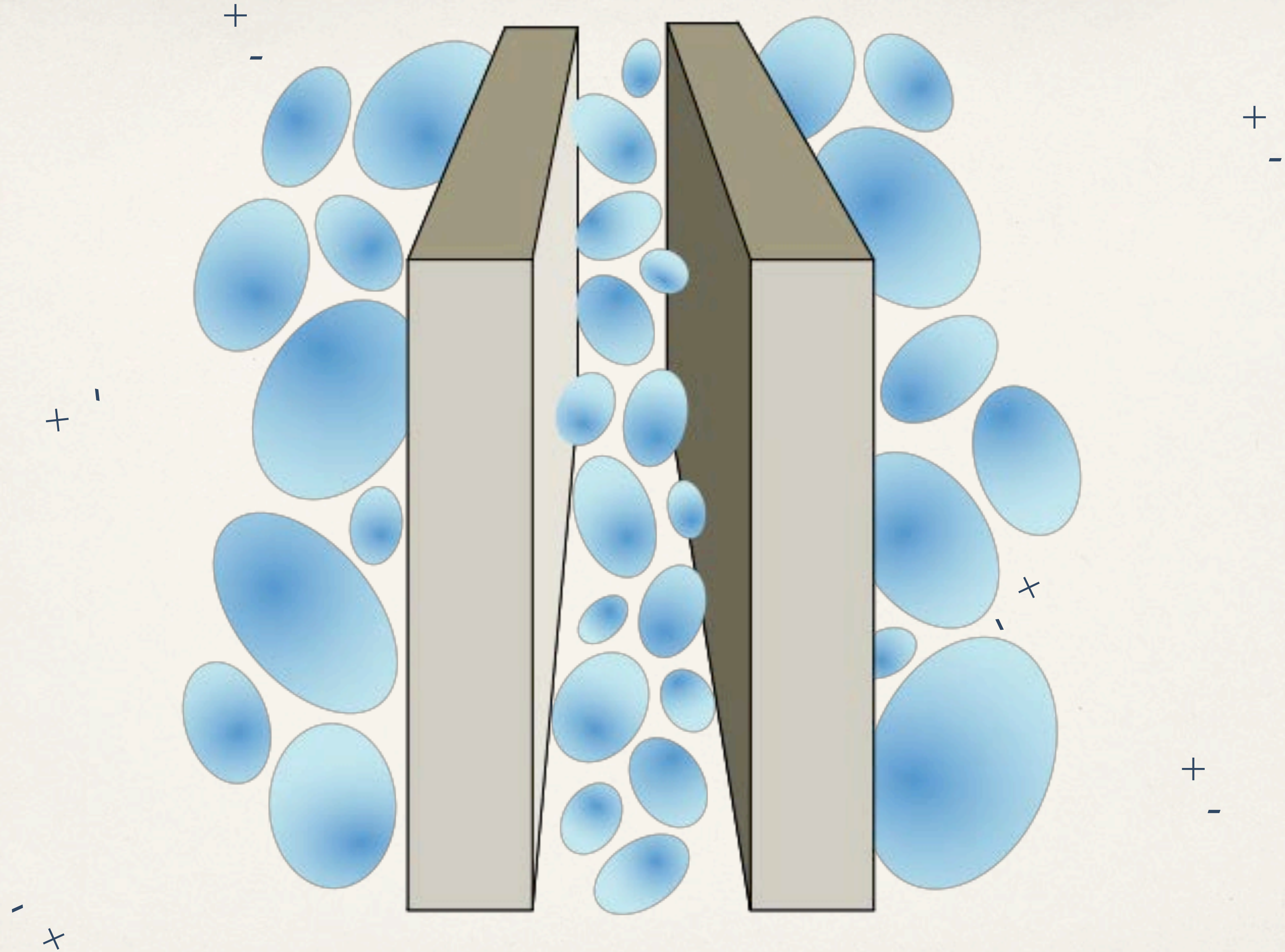
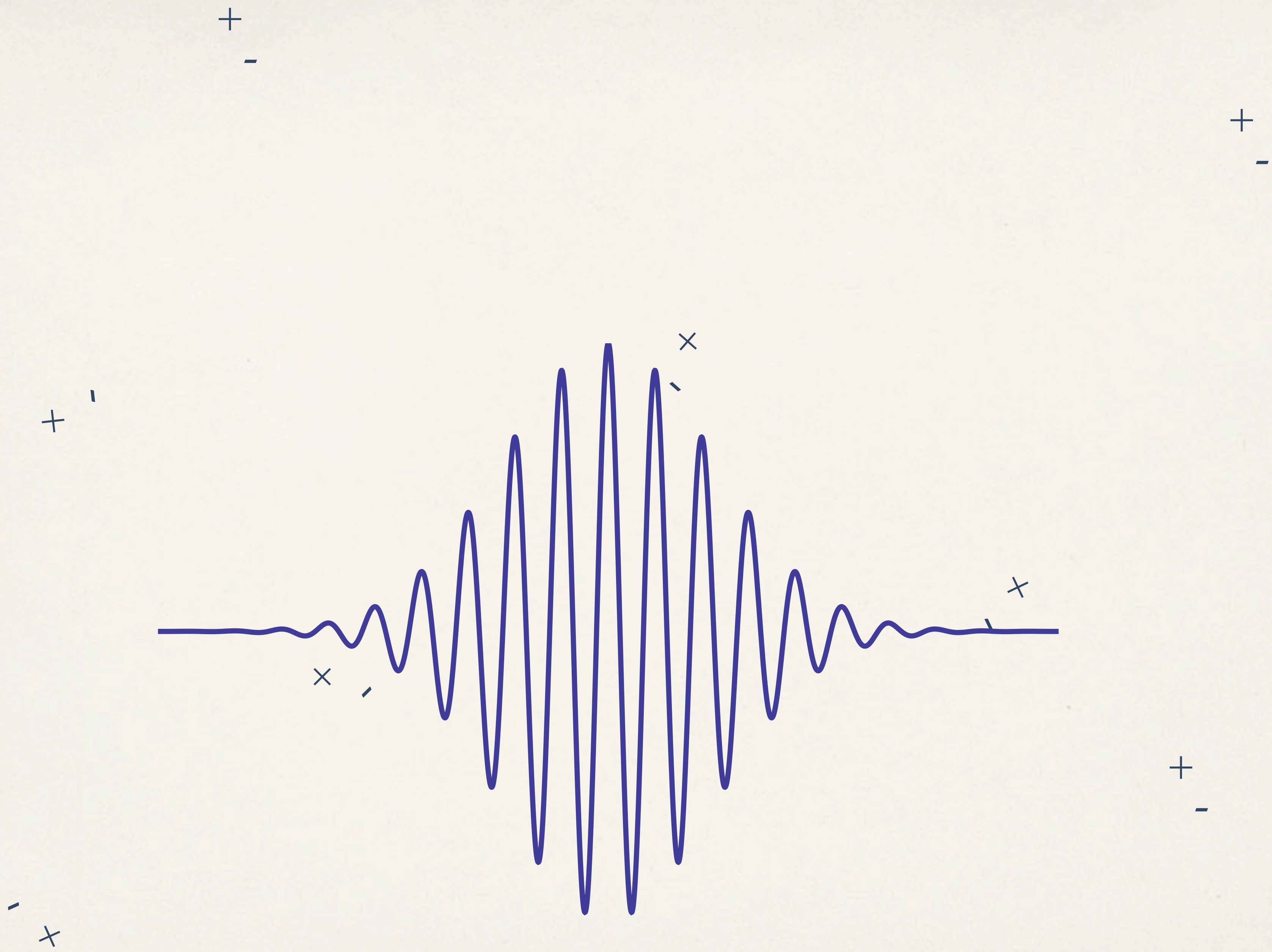
post-quantum mechanics

A vacuum is a hell of a lot better than some of the stuff that nature replaces it with

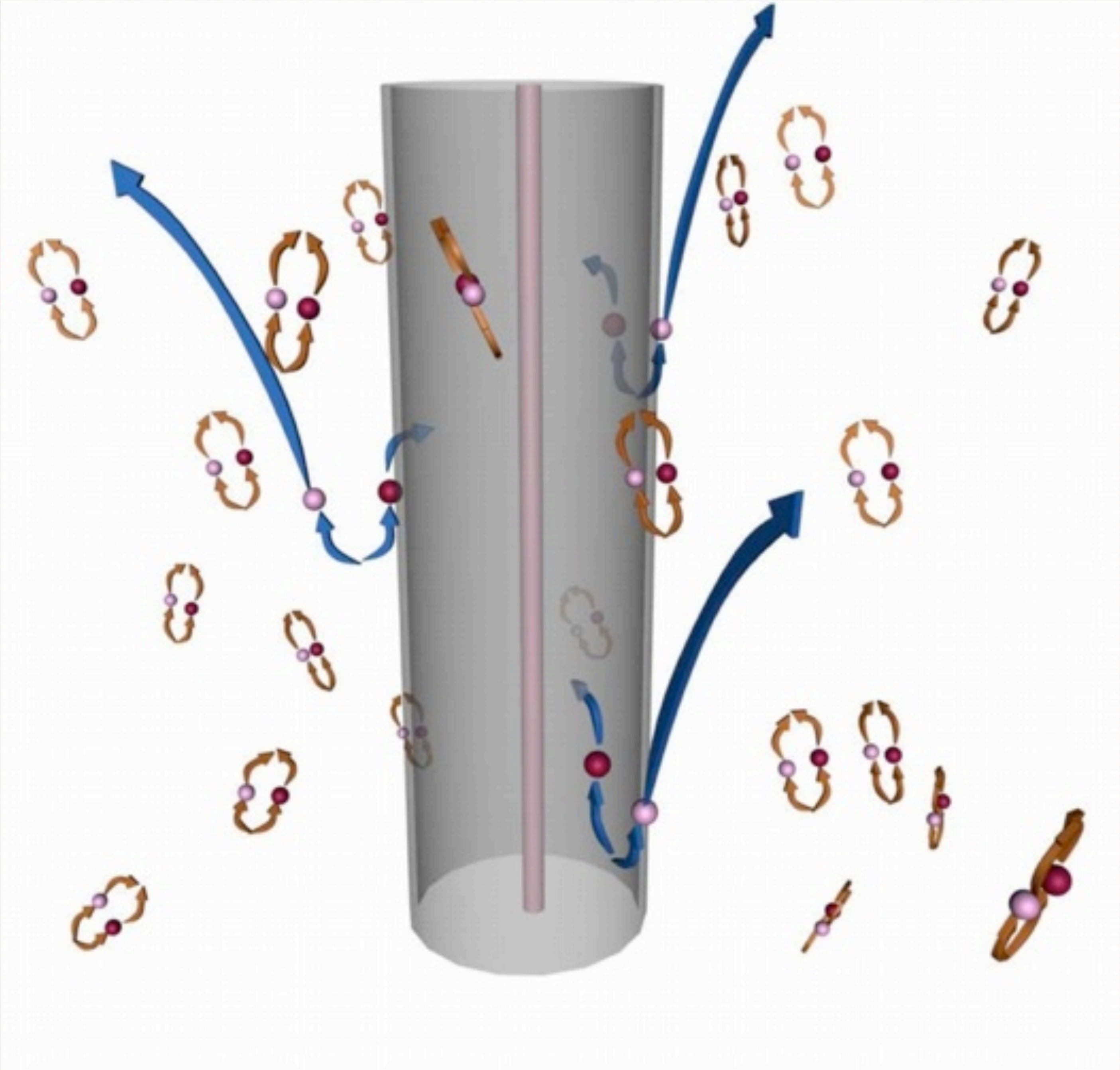
Tennessee Williams, "Cat on a Hot Tin Roof", 1955



Casimir effect



QED vacuum polarization



Hawking radiation

"Schwinger effect": e+e- pair production from vacuum

inherent instability of QED vacuum

probe with an external (laser) electric field

Sauter (Bohr), 1931
Heisenberg & Euler, 1936
Feynman, 1949
Schwinger, 1951

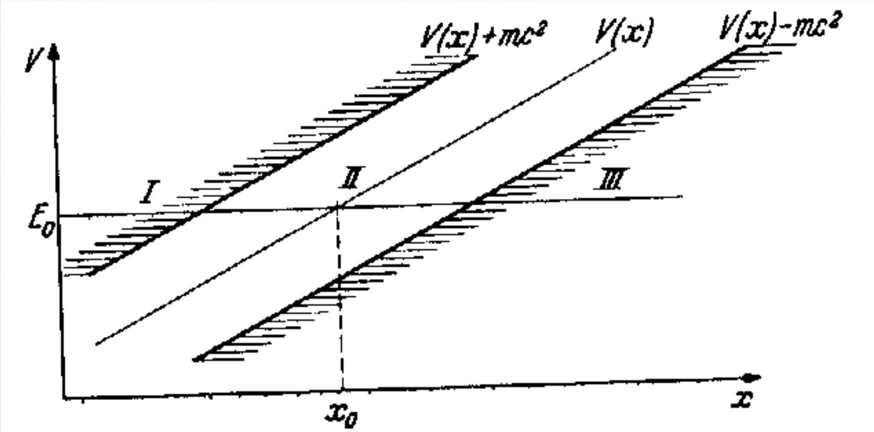
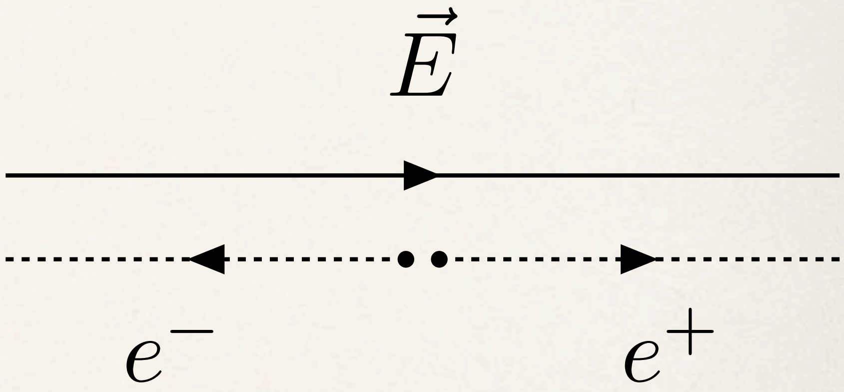


Fig. 1.



external E field accelerates apart a virtual e+e- pair

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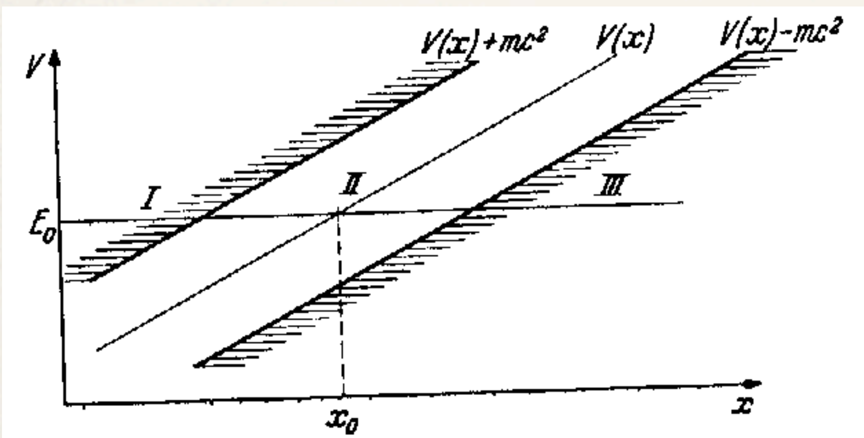

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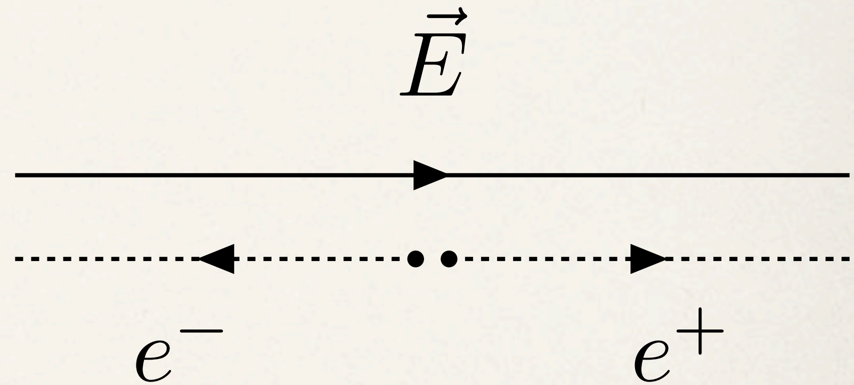

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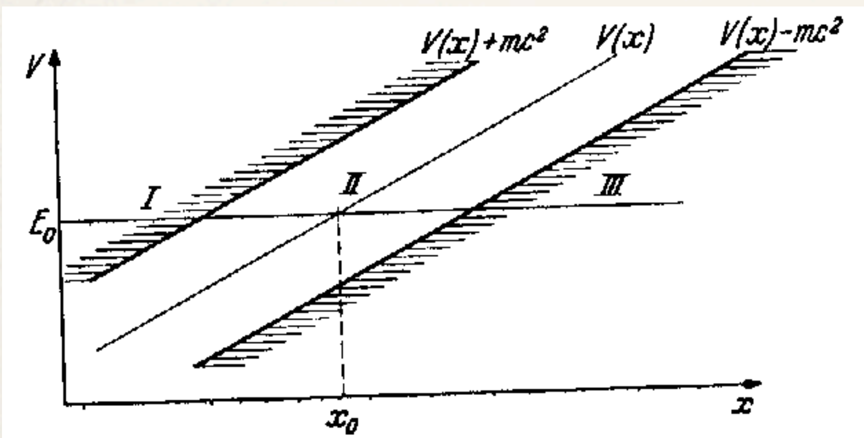

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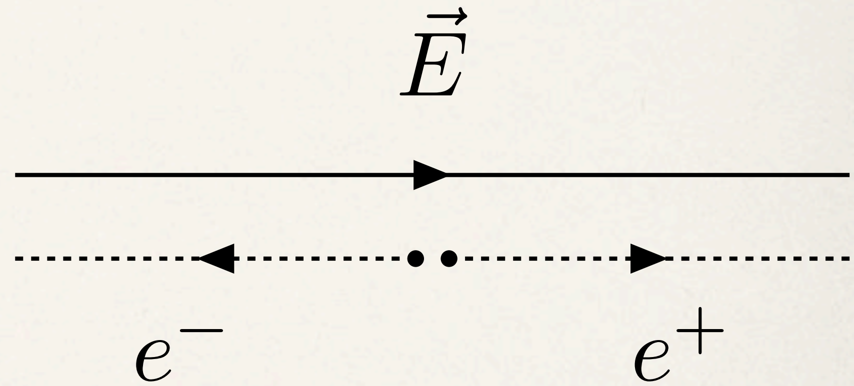

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$$2\,e\,\mathcal{E}\,\frac{\hbar}{m\,c} \sim 2\,m\,c^2 \qquad \mathcal{E}_c \sim \frac{m^2\,c^3}{e\,\hbar} \sim 10^{16}\,\mathrm{V/cm}$$

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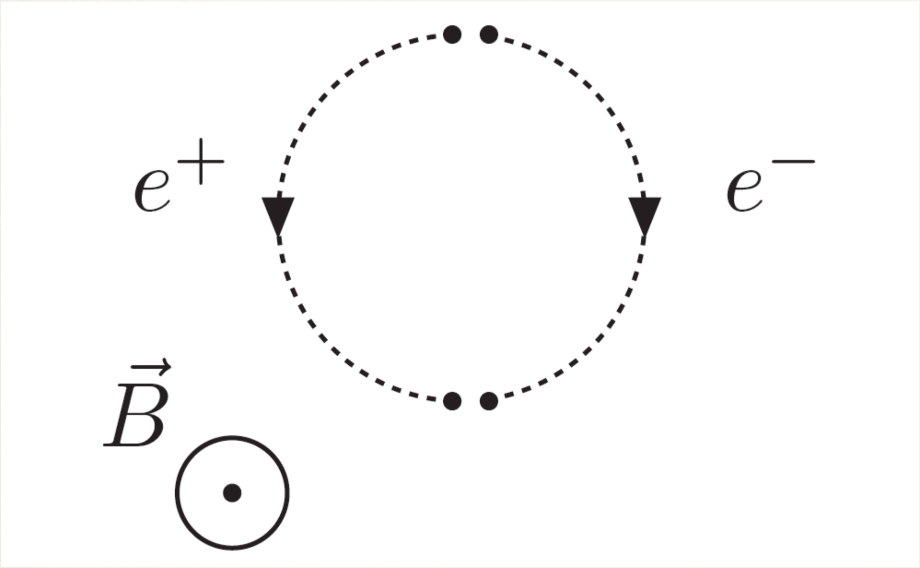

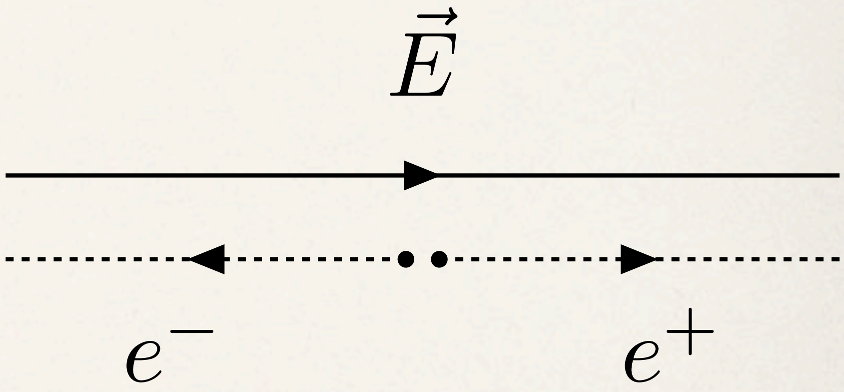

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On the behavior of an electron in a homogeneous electric field in Dirac’s relativistic theory

By **Fritz Sauter** in Munich

$$D = e^{-k^2\pi} \qquad k^2 = \frac{2\pi}{hc}\frac{(mc^2)^2}{v} \sim 1 \qquad \frac{vh}{mc} \sim mc^2$$

“This case would correspond to around 10^{16} volt / cm.”

This agrees with the conjecture of N. Bohr that was given in the introduction, that one first obtains the finite probability for the transition of an electron into the region of negative impulse when the potential ramp *vh*/*mc* over a distance of the Compton wavelength *h*/*mc* has the order of magnitude of the rest energy.

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* I would like to thank Herrn Prof. Heisenberg for the friendly tip about this hypothesis of N. Bohr.

huge field strengths & intensities suggest: lasers

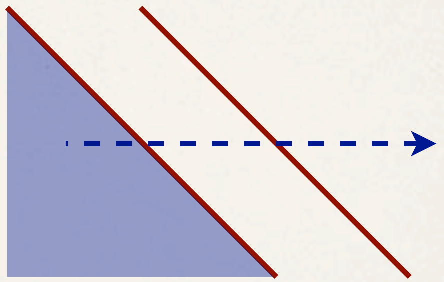
analogy to ionization

vacuum pair production

$$P_{EH} \sim \exp\left[-\pi \frac{m^2 c^3}{e\,\mathcal{E}\,\hbar}\right]$$

$$\mathcal{E}_c = \frac{m^2 c^3}{e\hbar} \approx 1.3 \times 10^{16} V/cm$$

$$I_c = \frac{c}{8\pi}\mathcal{E}_c^2 \approx 4 \times 10^{29} W/cm^2$$



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non-perturbative

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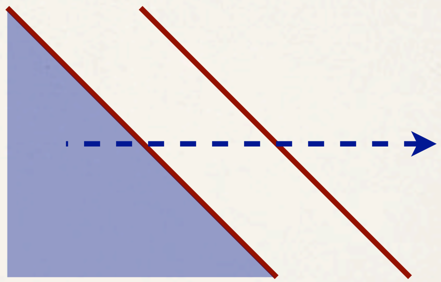

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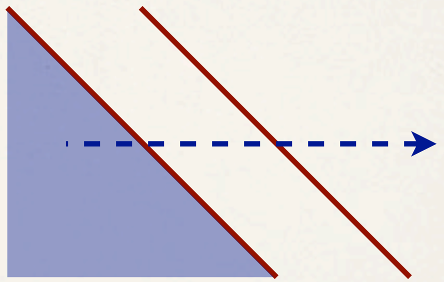

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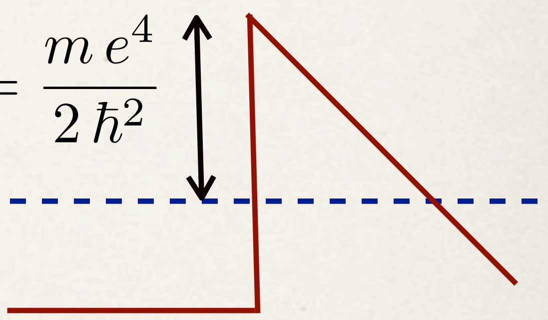
atomic ionization

$$E_b = \frac{m\,e^4}{2\,\hbar^2}$$



$$P_{\text{ionization}} \sim \exp\left[-\frac{2}{3}\frac{m^2 e^5}{\mathcal{E}\,\hbar^4}\right]$$

$$\mathcal{E}_c^{\text{ion}} = \frac{m^2 e^5}{\hbar^4} = \left(\frac{e^2}{\hbar c}\right)^3 \frac{m^2 c^3}{e\hbar} = \alpha^3 \mathcal{E}_c \approx 10^9 V/cm$$

$$I_c^{\text{ion}} = \alpha^6 I_c \approx 10^{16}\, W/cm^2$$

analogy to ionization

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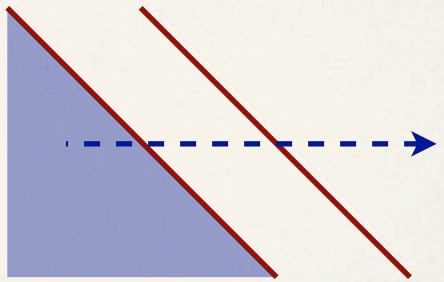

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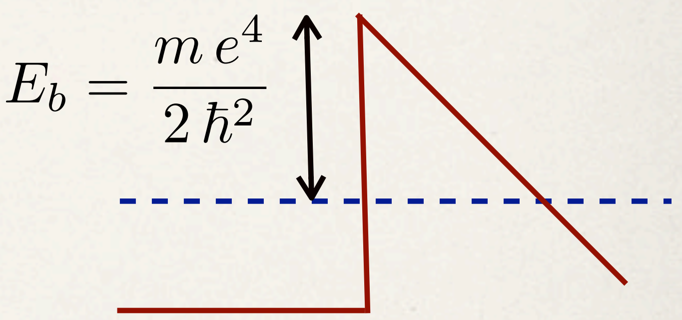

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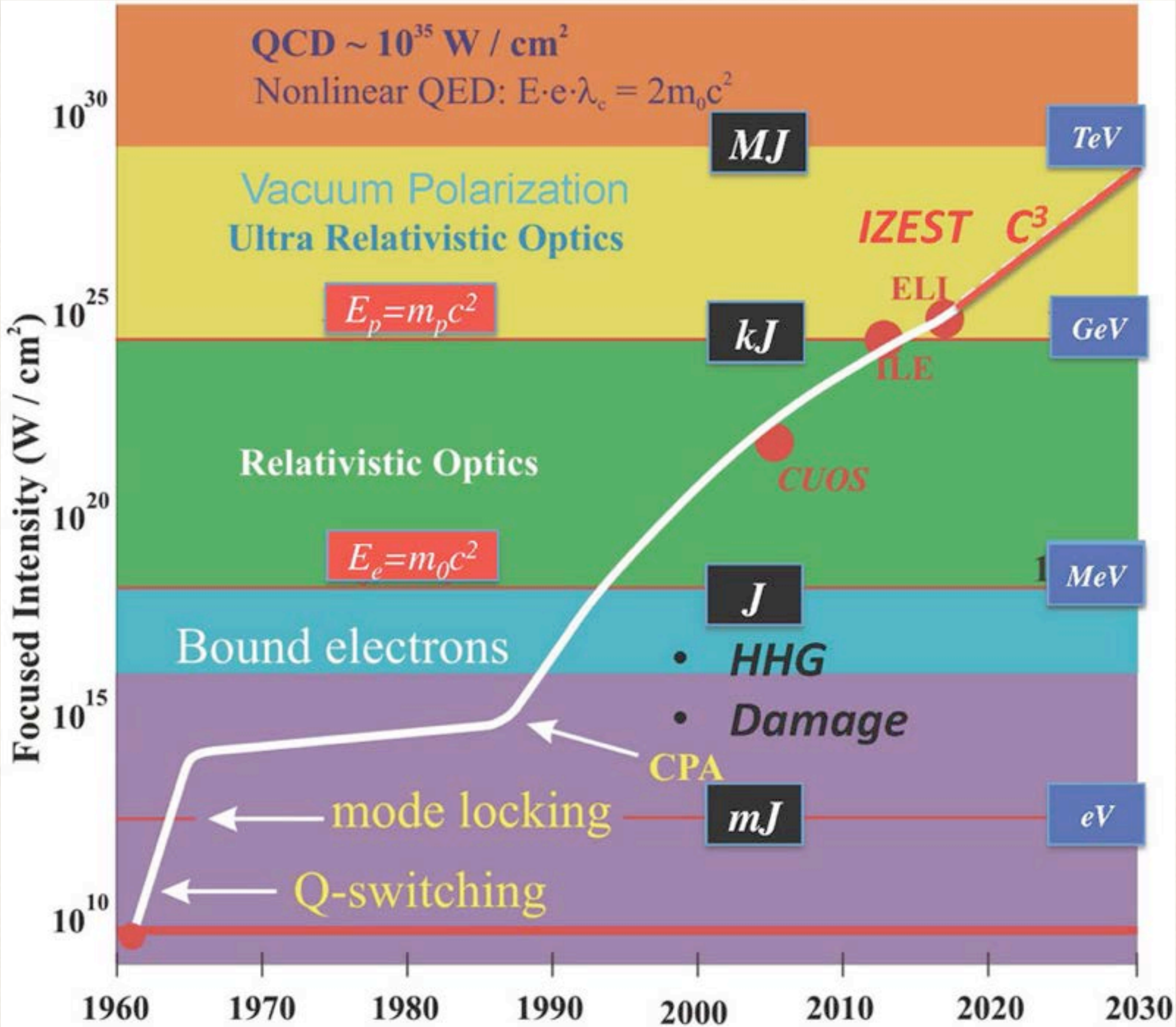
huge energy & intensity scale difference

why should particle physicists be interested in physics in ultra-intense laser fields ?

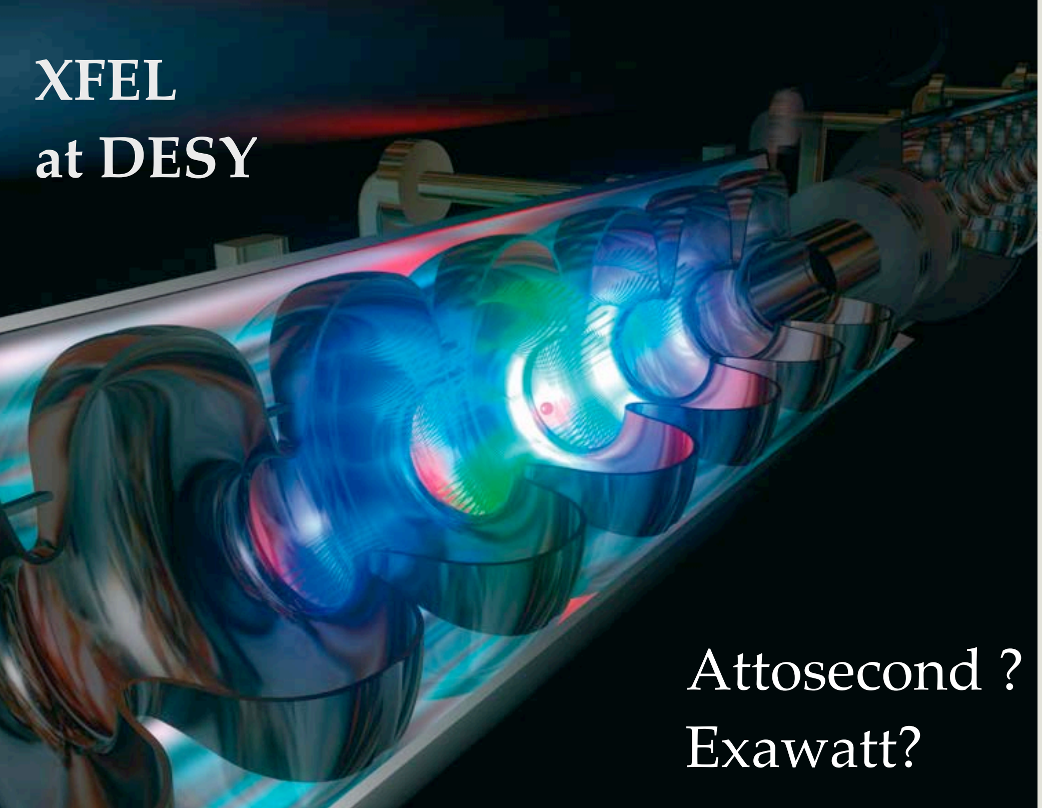
- direct, controllable, experimental access to matter in extreme environments
- direct access to nonlinear and nonperturbative region of QFTs
- novel experiments/regimes to search for new physics

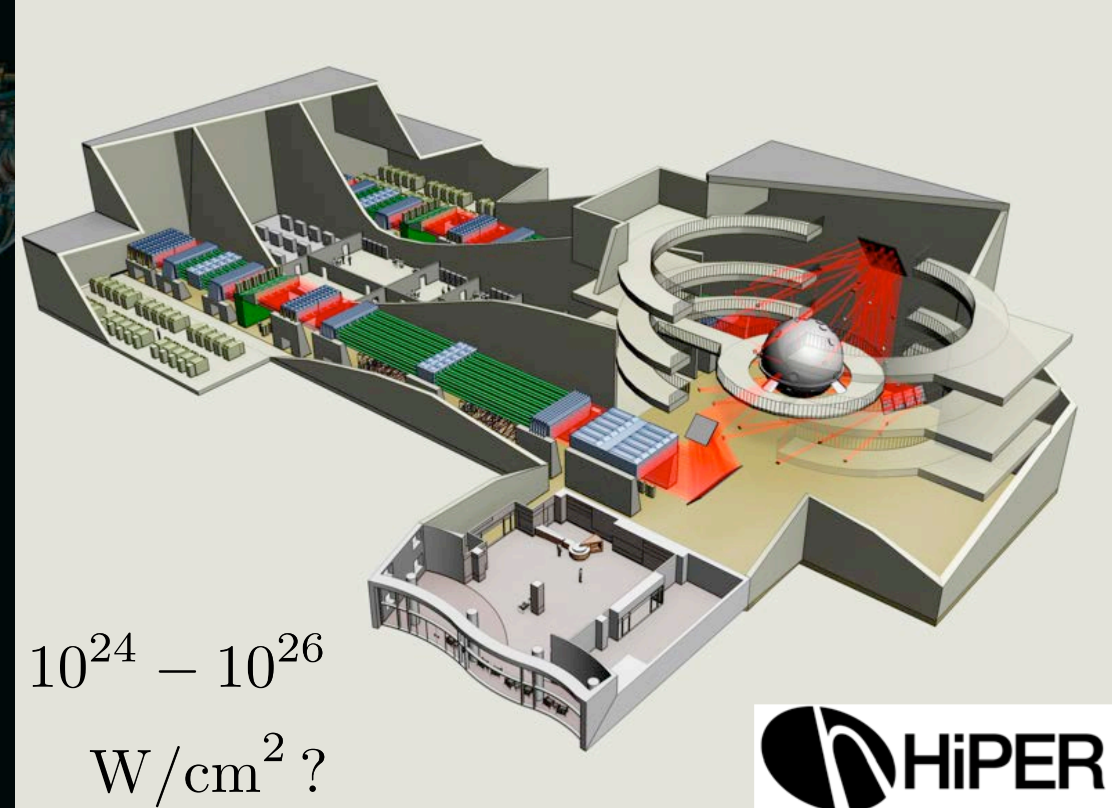
  - ✦ vacuum energy: mass generation; dark energy
  - ✦ physics beyond the standard model
  - ✦ axion and ALP searches; dark matter
  - ✦ QED and QFT at ultra-high intensity and in strong E & B fields
  - ✦ non-equilibrium QFT: e.g. quark-gluon-plasma, chiral magnetic effect
  - ✦ back-reaction, cascading
  - ✦ astrophysical applications: neutron stars, magnetars, black holes
  - ✦ cosmological particle production (Parker, Zeldovich)
  - ✦ Hawking radiation

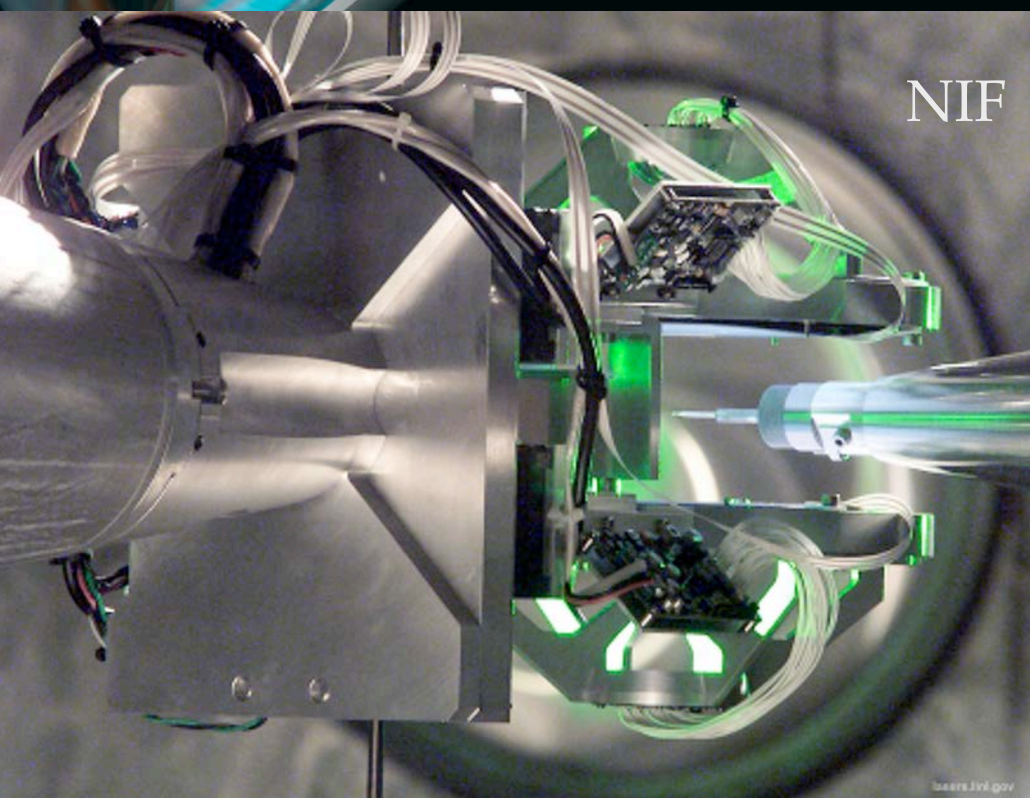


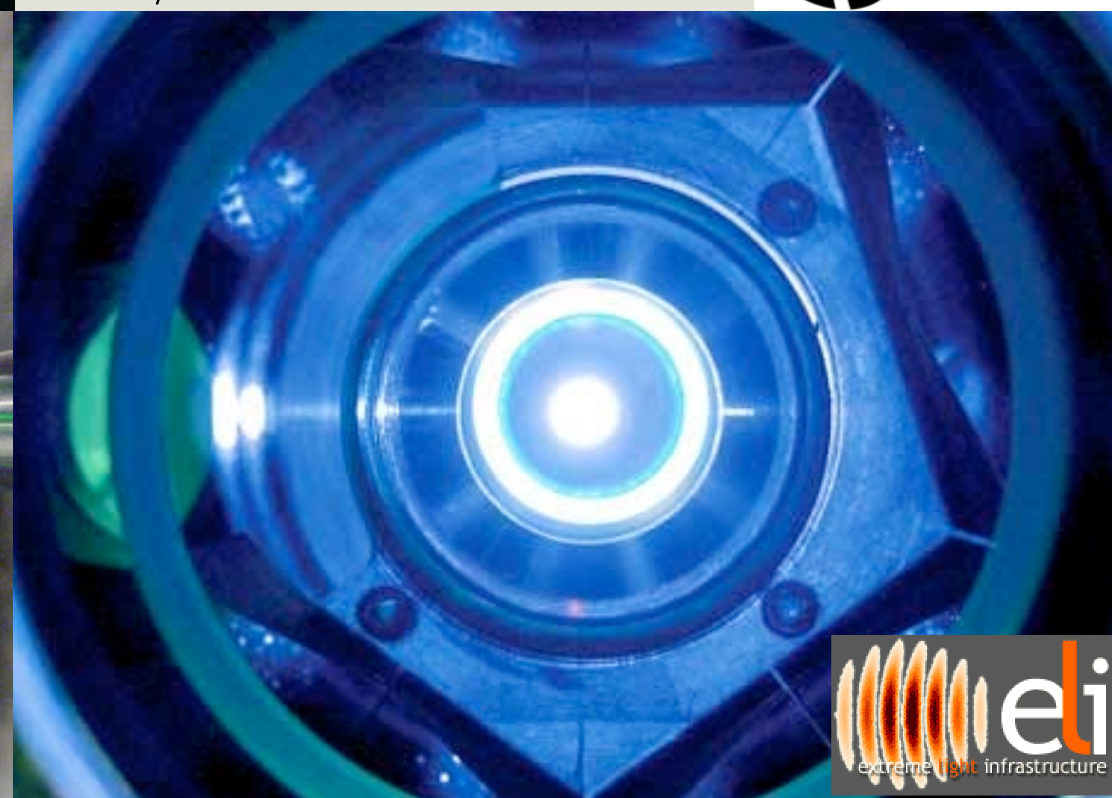
IZEST, ELI, XCELS, HiPER, XFEL, NIF, GEKKO-EXA, POLARIS, ...

Mourou, Tajima


XFEL
at DESY
Attosecond ?
Exawatt?


$10^{24} - 10^{26}$
W/cm^2 ?
HiPER


NIF


eli
extreme light infrastructure

a new field of high-intensity laser/particle physics is forming

input from:
particle physics, laser physics, accelerator physics, plasma physics, ...

some laser-based fundamental physics experiments

PVLAS: Polarizzazione del Vuoto con LASer

ALPS
Any Light Particle Search

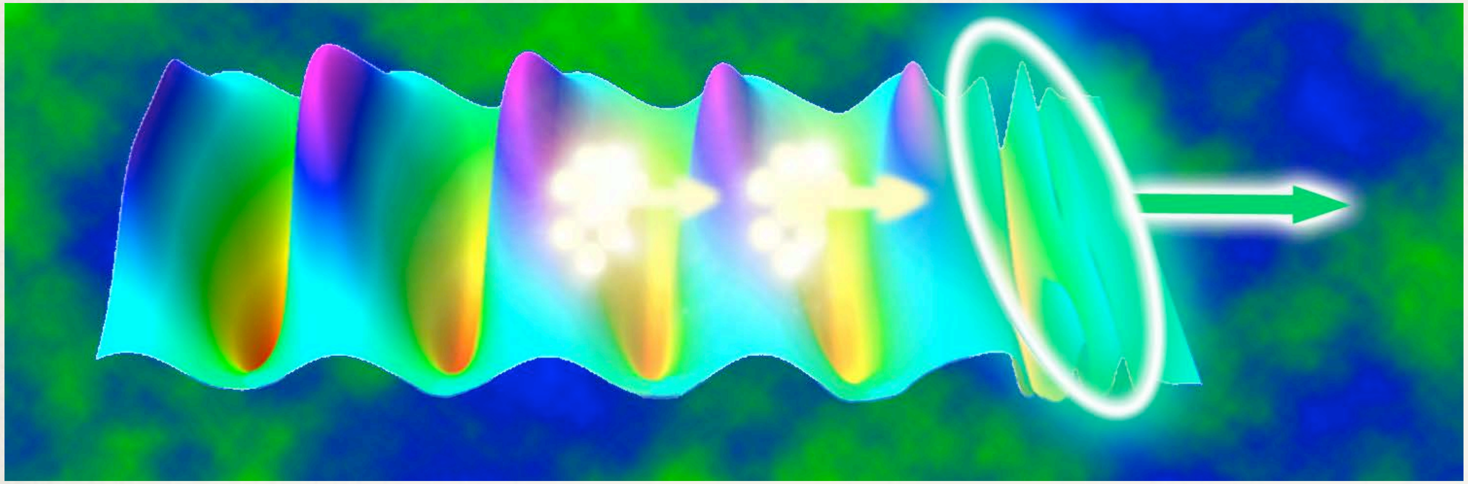
Fermilab GammeV

Biréfringence Magnétique du Vide (BMV)

LIPSS: Light Pseudoscalar and Scalar Search

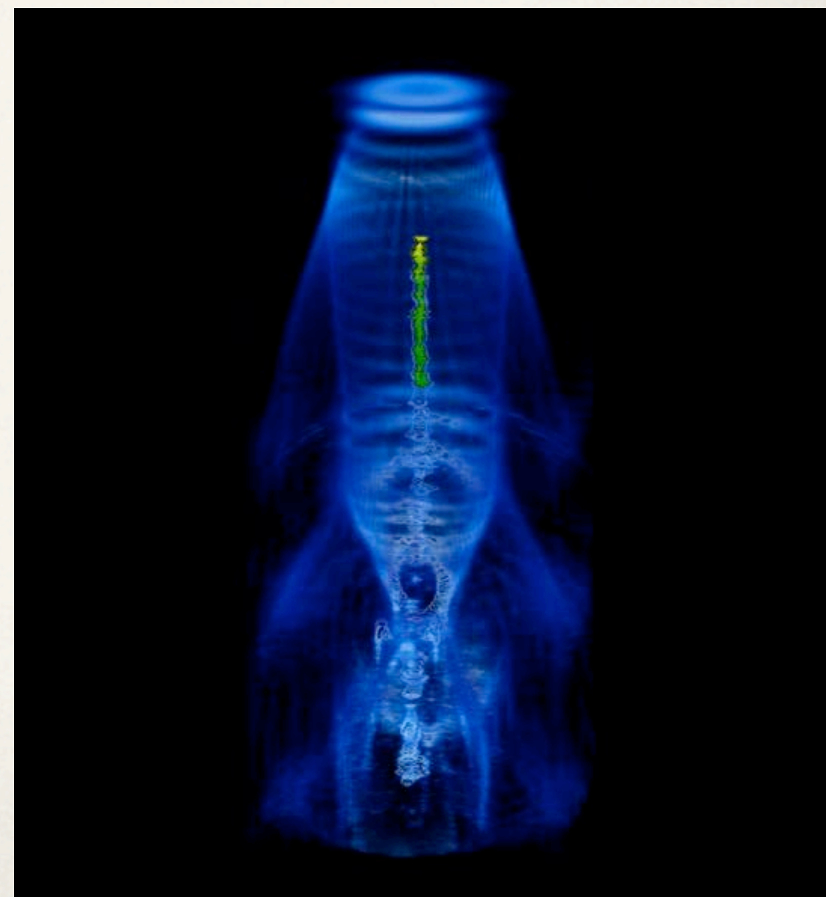
OSQAR: Optical Search for QED vacuum magnetic birefringence, Axions and photon Regeneration

laser wakefield acceleration

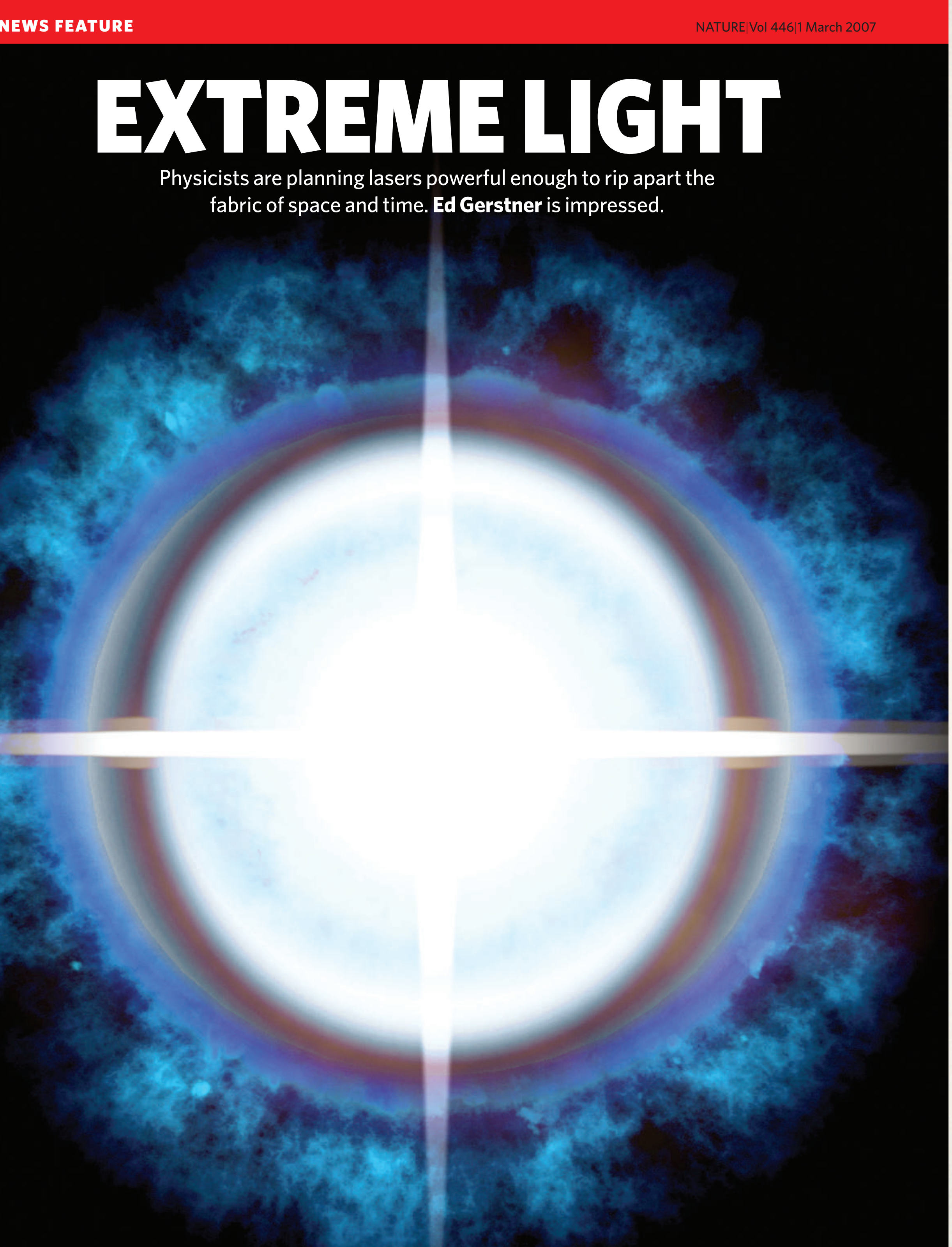


BELLA laser at LBNL

1 GeV in < 1m; goal: 10 GeV in 10 cm



the Schwinger effect captures the public imagination ...

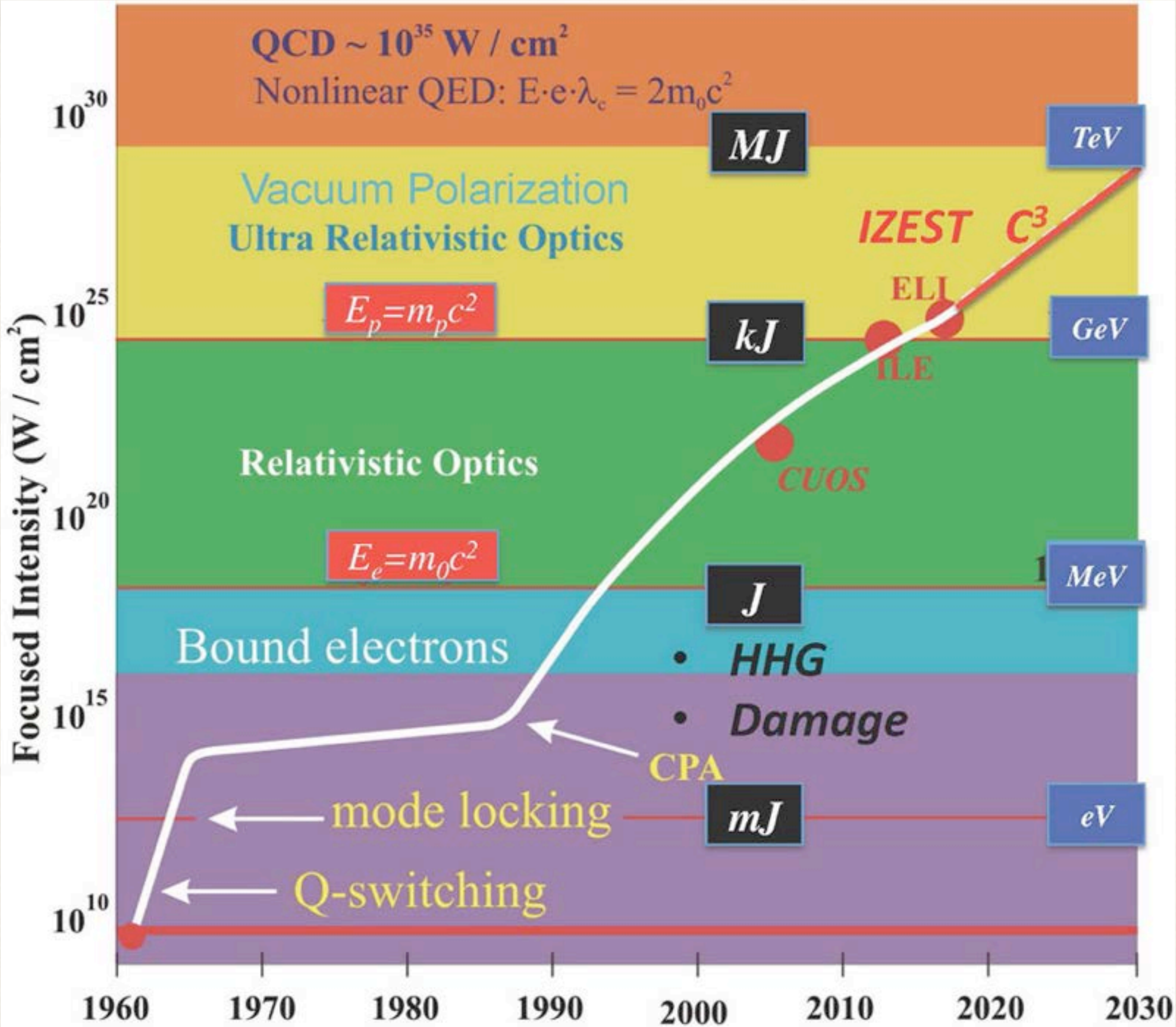


EXTREME LIGHT

Physicists are planning lasers powerful enough to rip apart the fabric of space and time. **Ed Gerstner** is impressed.

``Physicists are planning lasers powerful enough to rip apart the fabric of space and time”

eli
extreme light infrastructure



IZEST, ELI, XCELS, HiPER, XFEL, NIF, GEKKO-EXA, POLARIS, ...

Mourou, Tajima

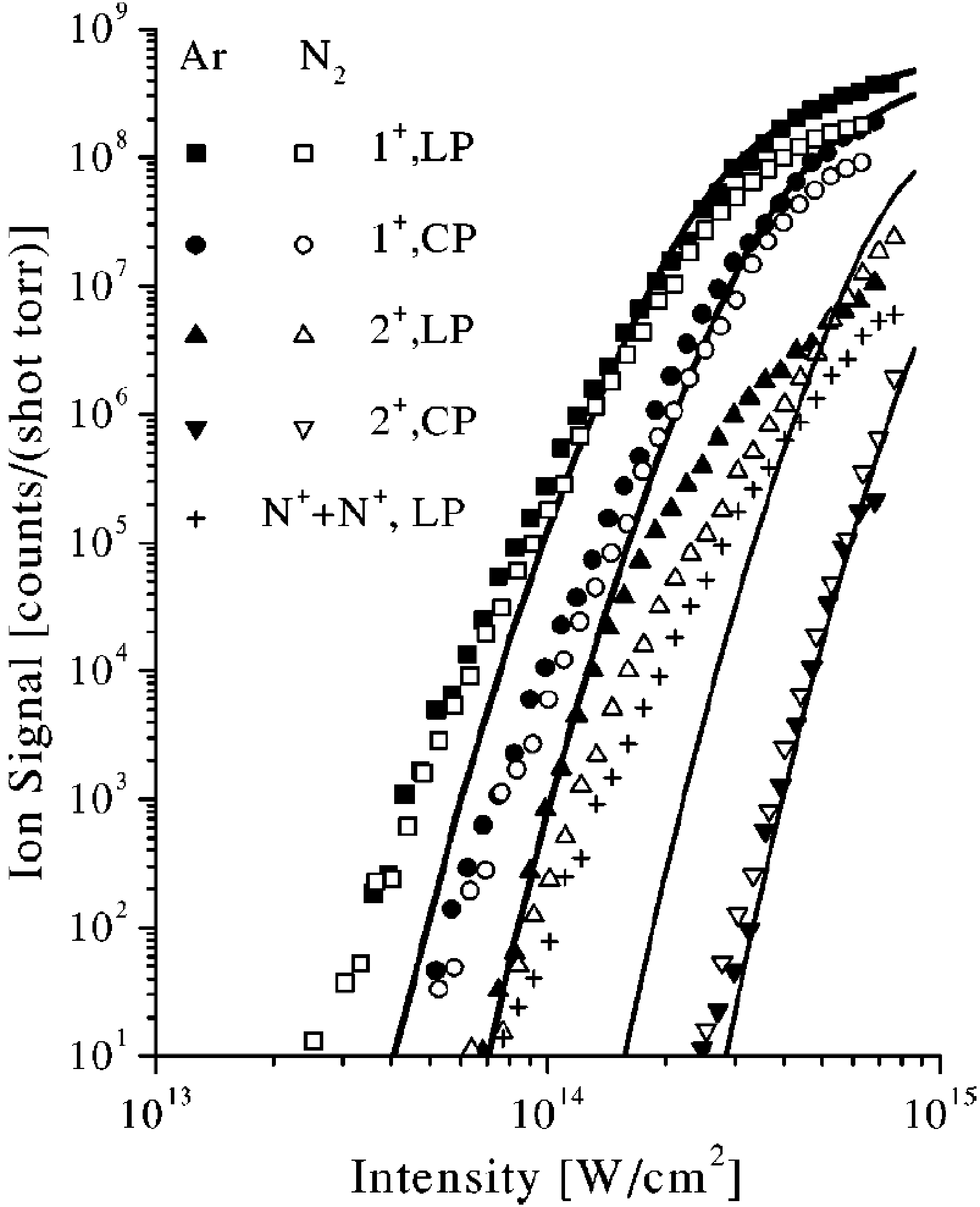
how critical is the critical field?

do we really need $10^{29} \mathrm{W/cm^2}$?

recall: constant field approximation:

$$I_c^{\mathrm{Schwinger}} \approx 10^{29} \mathrm{W/cm}^2$$

$$I_c^{\mathrm{Ionization}} \approx 10^{16} \mathrm{W/cm}^2$$



E_b~15 eV

atomic ionization

ionization is seen well below the sharp cutoff critical field

$$I_c \sim 10^{16}\, W/cm^2$$

G. Gibson et al, 1998

how critical is the critical field?

do we really need $10^{29}\mathrm{W/cm^2}$?

the constant field approximation only gives a rough estimate

there is a lot of interesting physics in going beyond the constant field approximation

experimentally necessary and theoretically challenging

Keldysh approach in QED

Keldysh, 1964; Brézin/Itzykson, 1970; Popov, 1971

monochromatic sinusoidal field :

$$\mathcal{E}(t) = \mathcal{E}\, \cos(\omega\, t)$$

$$\mathcal{A}(t) = -\frac{\mathcal{E}}{\omega}\, \sin(\omega\, t)$$

$$\omega_t \sim \frac{c}{\frac{mc^2}{e\mathcal{E}}} = \frac{e\mathcal{E}}{mc}$$

new scale : ω

"Keldysh" adiabaticity parameter :

$$\gamma \equiv \frac{\omega}{\omega_t} = \frac{m\, c\, \omega}{e\, \mathcal{E}} \equiv \frac{1}{a_0}$$

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time-dependent WKB:

$$P_{\rm QED} \sim \begin{cases} \exp\left[-\pi\, \frac{m^2\, c^3}{e\, \hbar\, \mathcal{E}}\right] & , \quad \gamma \ll 1 \quad \text{(nonperturbative)} \\ \left(\frac{e\, \mathcal{E}}{\omega\, m\, c}\right)^{4mc^2/\hbar\omega} & , \quad \gamma \gg 1 \quad \text{(perturbative)} \end{cases}$$

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tunnelling

multiphoton

Positron Production in Multiphoton Light-by-Light Scattering

SLAC E-144

D. L. Burke, R. C. Field, G. Horton-Smith, J. E. Spencer, and D. Walz
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

S. C. Berridge, W. M. Bugg, K. Shmakov, and A. W. Weidemann
Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996

C. Bula, K. T. McDonald, and E. J. Prebys
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C. Bamber,* S. J. Boege,† T. Koffas, T. Kotseroglou,‡ A. C. Melissinos, D. D. Meyerhofer,§ D. A. Reis, and W. Ragg‖
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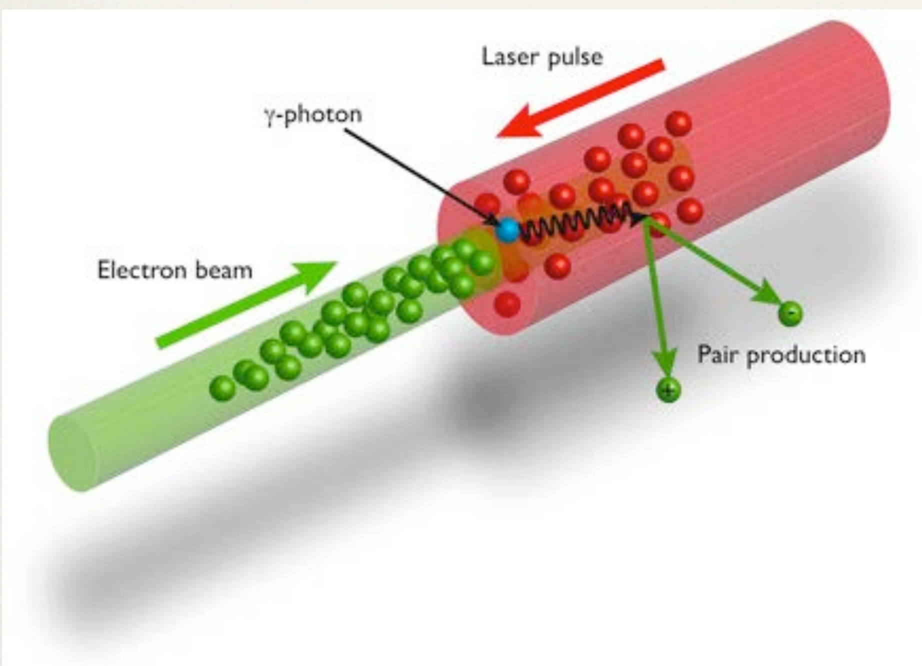

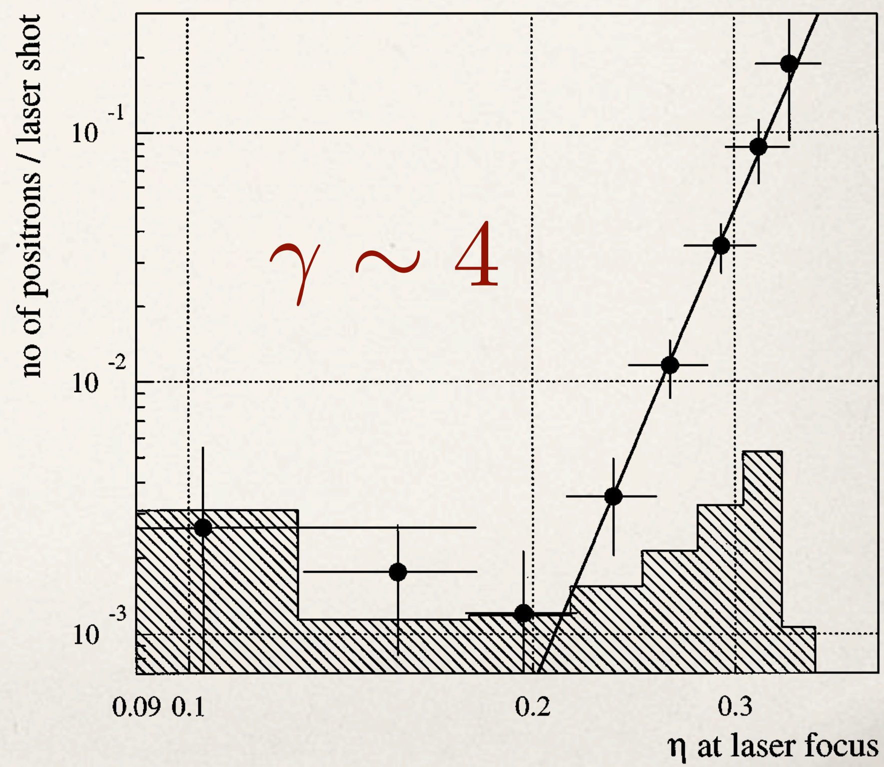

trident process could probe across perturbative and non-perturbative regime

opportunity for precision tests

PRL **105,** 080401 (2010) PHYSICAL REVIEW LETTERS week ending 20 AUGUST 2010

Complete QED Theory of Multiphoton Trident Pair Production in Strong Laser Fields

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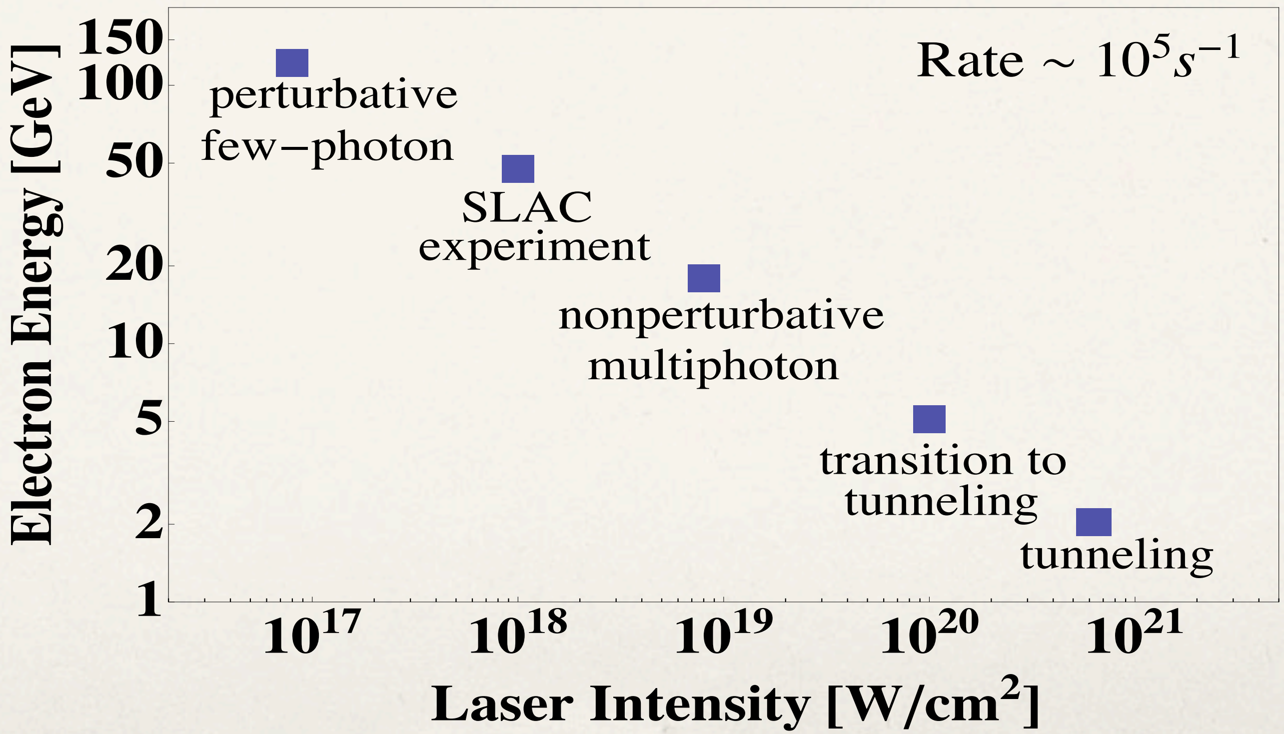
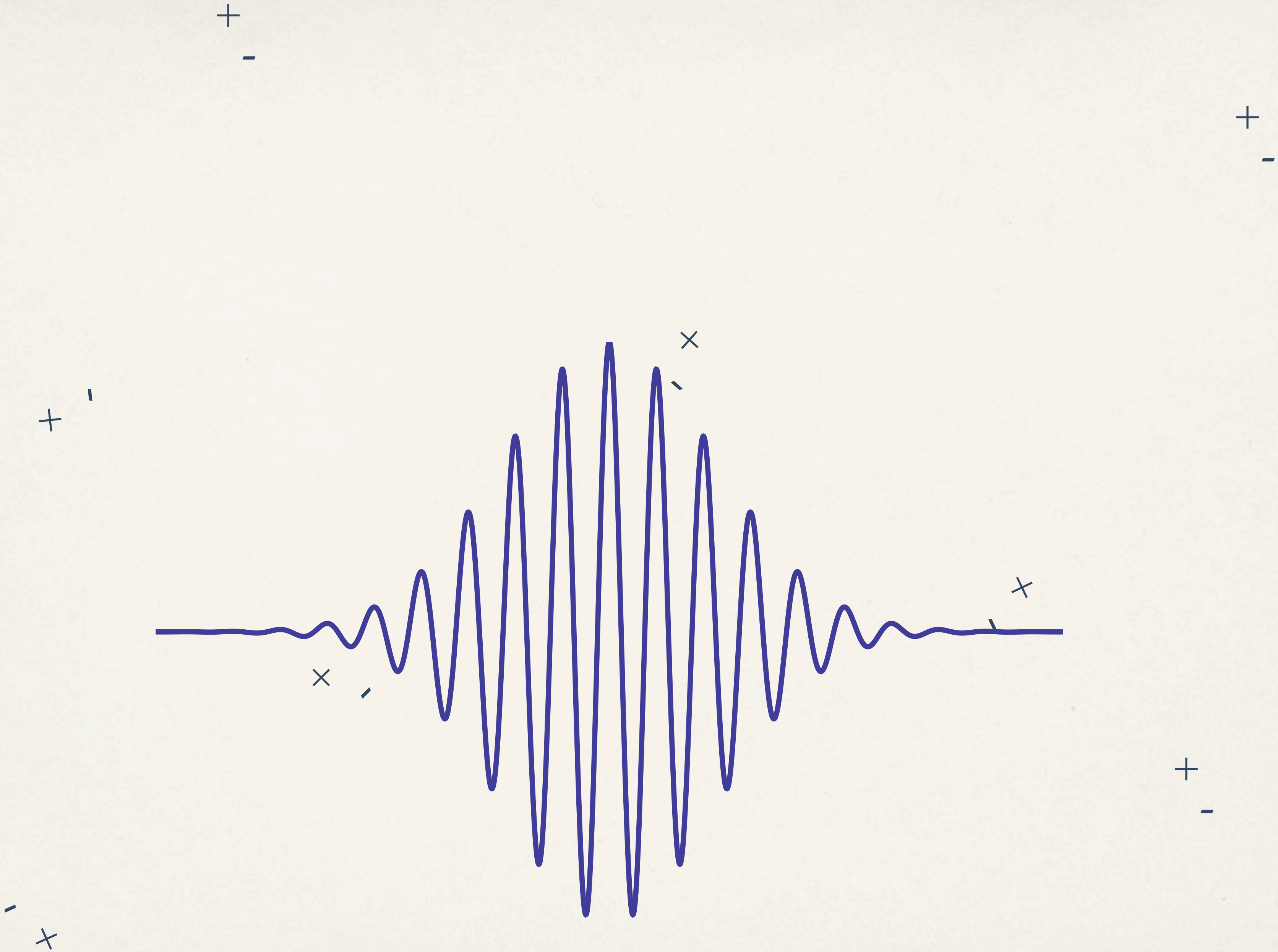
(Received 12 February 2010; published 16 August 2010)



Theoretical Aspects

why the Schwinger effect is such an interesting,
and difficult, QFT problem

we think we understand QED, but:
ultra-intense fields, medium effects, back-reaction, non-equilibrium, ...



QED vacuum polarization

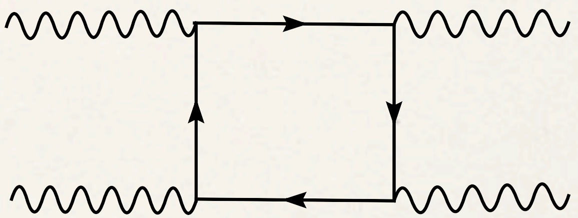
QED effective action

encodes nonlinear properties of QED due to vacuum polarization

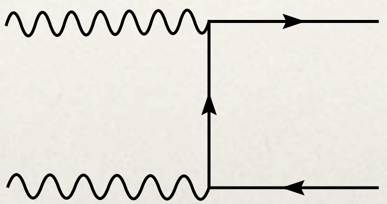
$$\langle O_{\text{out}} \,|\, O_{\text{in}} \rangle \equiv \exp\left(\frac{i}{\hbar}\,\Gamma[A]\right)$$

$$= \exp\left(\frac{i}{\hbar}\,\{\text{Re}(\Gamma) + i\,\text{Im}(\Gamma)\}\right)$$

$\text{Re}\,(\Gamma)$ dispersive effects: e.g. vacuum birefringence



$\text{Im}\,(\Gamma)$ absorptive effects: e.g. vacuum pair production



vacuum pair production

vacuum persistence probability

$$|\langle O_{\rm out} \,|\, O_{\rm in}\rangle|^2 = \exp\left(-\frac{2}{\hbar}\,{\rm Im}\,(\Gamma)\right)$$

$$\approx 1 - \frac{2}{\hbar}\,{\rm Im}\,(\Gamma)$$

probability of pair production $\approx \dfrac{2}{\hbar}\,{\rm Im}\,(\Gamma)$

vacuum pair production

vacuum persistence probability

$$|\langle O_{\rm out} \,|\, O_{\rm in}\rangle|^2 = \exp\left(-\frac{2}{\hbar}\,{\rm Im}\,(\Gamma)\right)$$

$$\approx 1 - \frac{2}{\hbar}\,{\rm Im}\,(\Gamma)$$

probability of pair production $\approx \frac{2}{\hbar}\,{\rm Im}\,(\Gamma)$

relativistic analogue of familiar QM:

$$\Psi(x,t) = \psi(x) e^{-\frac{i}{\hbar} E\, t} \longrightarrow \left|\Psi(x,t)\right|^2 \sim e^{-\frac{2}{\hbar}{\rm Im}(-E)\, t}$$

Heisenberg & Euler

Euler-Heisenberg Effective Action

vacuum polarization due to slowly varying [constant] fields

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{hc}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B}) \cdot \frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

($\mathfrak{E}$, $\mathfrak{B}$ Kraft auf das Elektron.

$$|\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\,\frac{e}{(e^2/mc^2)^2} = \text{„Kritische Feldstärke“}.)$$

the proper-time formalism

Stückelberg, Feynman, Schwinger, Nambu, Fock, ...

Feynman's worldline representation

aim: extend non-relativistic QM path integral to relativistic QED

"We try to represent the amplitude for a particle to get from one point to another as a sum over all trajectories of an amplitude exp(i S) where S is the classical action for a given trajectory. To maintain the relativistic invariance in evidence the idea suggests itself of describing a trajectory in space-time by giving the four variables $x_\mu(u)$ as functions of some fifth parameter u … (somewhat analogous to proper time) …"

PHYSICAL REVIEW VOLUME 76, NUMBER 6 SEPTEMBER 15, 1949

The Theory of Positrons

R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York

(Received April 8, 1949)

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but some paths go backwards in time ?!?

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R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York
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Progress of Theoretical Physics, Vol. V, No. 1, Jan.~Feb., 1950.

The Use of the Proper Time in Quantum Electrodynamics I.

Yôichirô NAMBU

*Department of Physics, University of City Osaka**

(Received November 8, 1949)

whole of it at glance. The time itself loses sense as the indicator of the development of phenomena; there are particles which flow down as well as up the stream of time; the eventual creation and annihilation of pairs that may occur now and then, is no creation nor annihilation, but only a change of directions of moving particles, from past to future, or from future to past; a virtual pair, which, according to the ordinary view, is foredoomed to exist only for a limited interval of time, may also be regarded as a single particle that is circulating round a closed orbit in the four-dimensional theatre; a real particle is then a particle whose orbit is not closed but reaches to infinity. . .

QFT approach: the QED effective action

Schwinger, 1950-1954

expressed the QED effective action in terms of functional determinants

$$\Gamma = \ln \det (i\not{D} + m) \qquad D_\mu = \partial_\mu - i\frac{e}{\hbar c}A_\mu$$

``Incidentally, the probability of actual pair creation is obtained from the imaginary part of the electromagnetic field action integral.''

PHYSICAL REVIEW VOLUME 82, NUMBER 5 JUNE 1, 1951

On Gauge Invariance and Vacuum Polarization

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received December 22, 1950)

QED effective action

PHYSICAL REVIEW VOLUME 90, NUMBER 4 MAY 15, 1593

Fredholm Theory of Scattering in a Given Time-Dependent Field

ABDUS SALAM, *St. John's College, Cambridge, England, and Government College, Lahore, Pakistan*

AND

P. T. MATTHEWS,* *Cavendish Laboratory, Cambridge, England*

(Received October 27, 1952)

It is shown that Feynman's relativistic solution for the scattering of an electron (or pair creation) by a given external field is the Fredholm resolvent of the related integral equation and is thus the unique and absolutely convergent solution for any strength of field.

PHYSICAL REVIEW VOLUME 93, NUMBER 3 FEBRUARY 1, 1954

The Theory of Quantized Fields. V

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received October 26, 1953)

QED effective action

PHYSICAL REVIEW VOLUME 90, NUMBER 4 MAY 15, 1593

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QFT problem: compute non-perturbatively $\mathrm{Im}\,\Gamma[A]$
for a gauge field $A_\mu(x)$ corresponding to a realistic laser pulse

extremely difficult

- semiclassical methods: WKB scattering (1 dim)
- quantum kinetic equation (Bogoliubov transformation): numerical (1 dim)
- worldline path integral: numerical and semiclassical (1 dim and >1 dim)
- Dirac-Heisenberg-Wigner method: numerical (1 dim and >1 dim)
- numerical Dirac equation and dispersion relations

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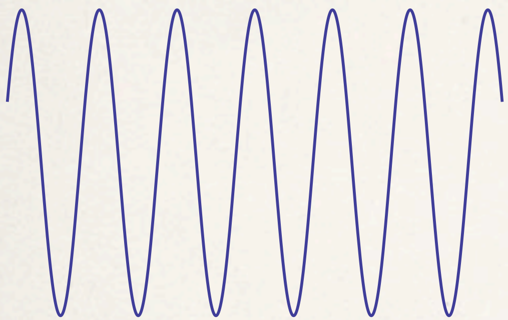
- numerical Dirac equation and dispersion relations

full optimization problem: find $A_\mu(x)$ that maximizes $\mathrm{Im}\,\Gamma[A]$

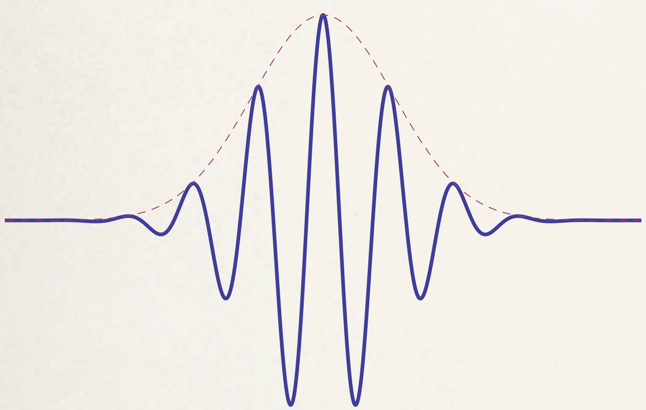
so far, prohibitively difficult

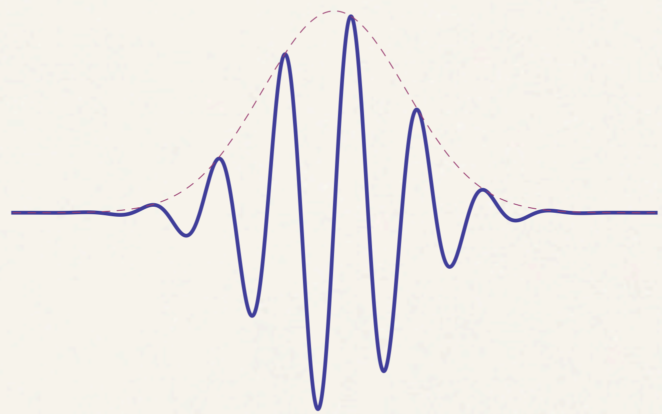
- optimize within an ansatz
- explicit optimal quantum control algorithms
- physical intuition from semiclassical studies of quantum interference

constant E field

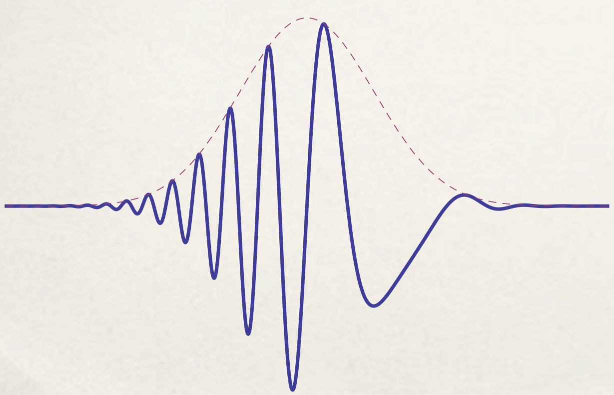


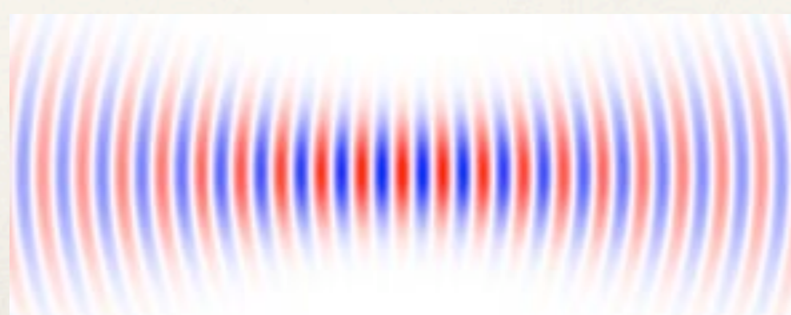
monochromatic
or single pulse





pulse with
sub-cycle
structure;
carrier phase
effect





chirped pulse,
Gaussian beam, ...

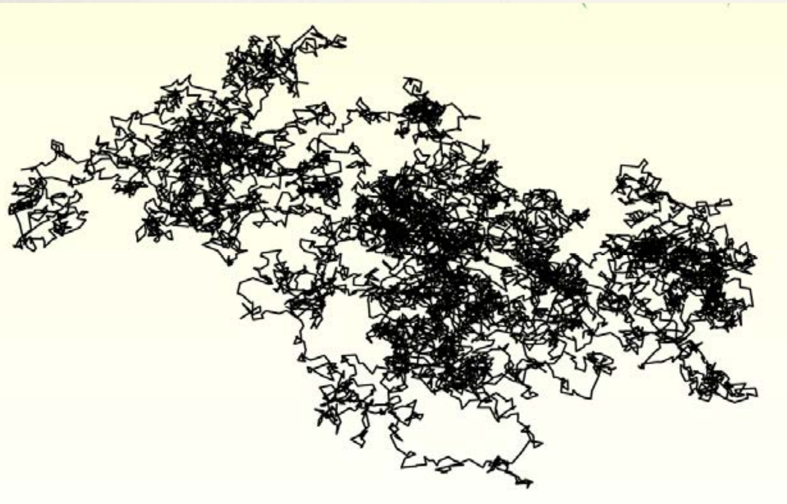
beyond uniform fields

numerical worldline Monte Carlo

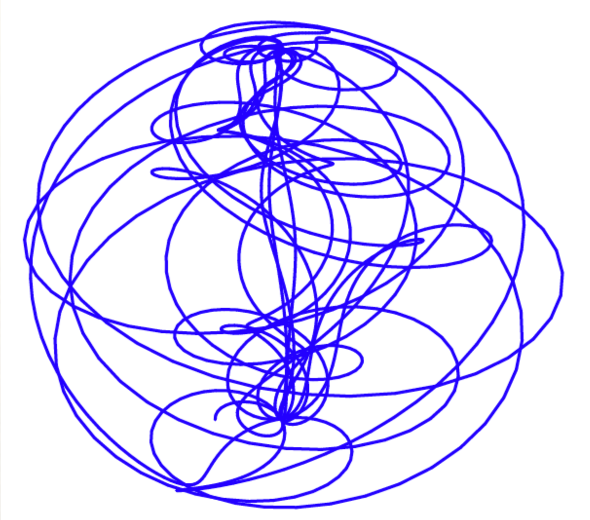
Gies/Klingmüller 2005

$$\Gamma[A] = -\int_0^\infty \frac{dT}{T}\, e^{-m^2 T} \int d^4x \int_{x(T)=x(0)=x} \mathcal{D}x \; e^{-S[x]}$$

- ensemble of closed spacetime loops: weight $e^{-\frac{1}{4}\int_0^T d\tau \dot{x}^2}$
- probe with Wilson loop operator $e^{-\int_0^T d\tau \dot{x}_\mu A_\mu(x)}$
- ensemble independent of form of $A_\mu(x)$



imaginary part?
exponentially small?



worldline instantons

GD, Schubert 2005
GD, Gies, Schubert, Wang, 2006

$$\Gamma[A] = -\int_0^\infty \frac{dT}{T}\, e^{-m^2 T} \int d^4x \int_{x(T)=x(0)=x} \mathcal{D}x\; e^{-S[x]}$$

semiclassical approximation : "instanton dominance"

classical Euclidean equations of motion

$$\ddot{x}_\mu = F_{\mu\nu}(x)\, \dot{x}_\nu$$

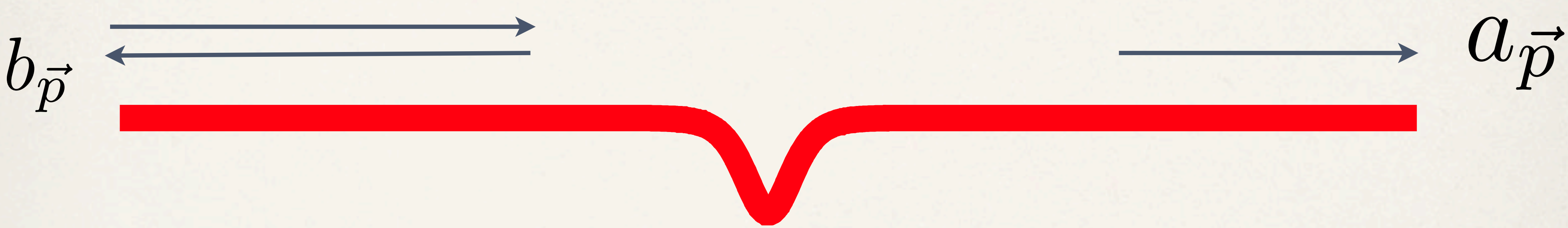
periodic (closed loop) solution = "worldline instanton"

technically difficult for multi-dimensional fields : complex instantons

$$\Gamma[A] = \ln \det \left(i \not{D} + m \right)$$

one-dimensional inhomogeneities $\Leftrightarrow$ one-dim. QM scattering problem

$$-\ddot{\Phi} - \left(p_3 - eA_3(t)\right)^2 \Phi = \left(m^2 + p_\perp^2\right) \Phi$$



temporal variation: over-the-barrier reflection

spatial variation: through-the-barrier transmission

computational simplification : scalar QED realistic laser pulses have structure

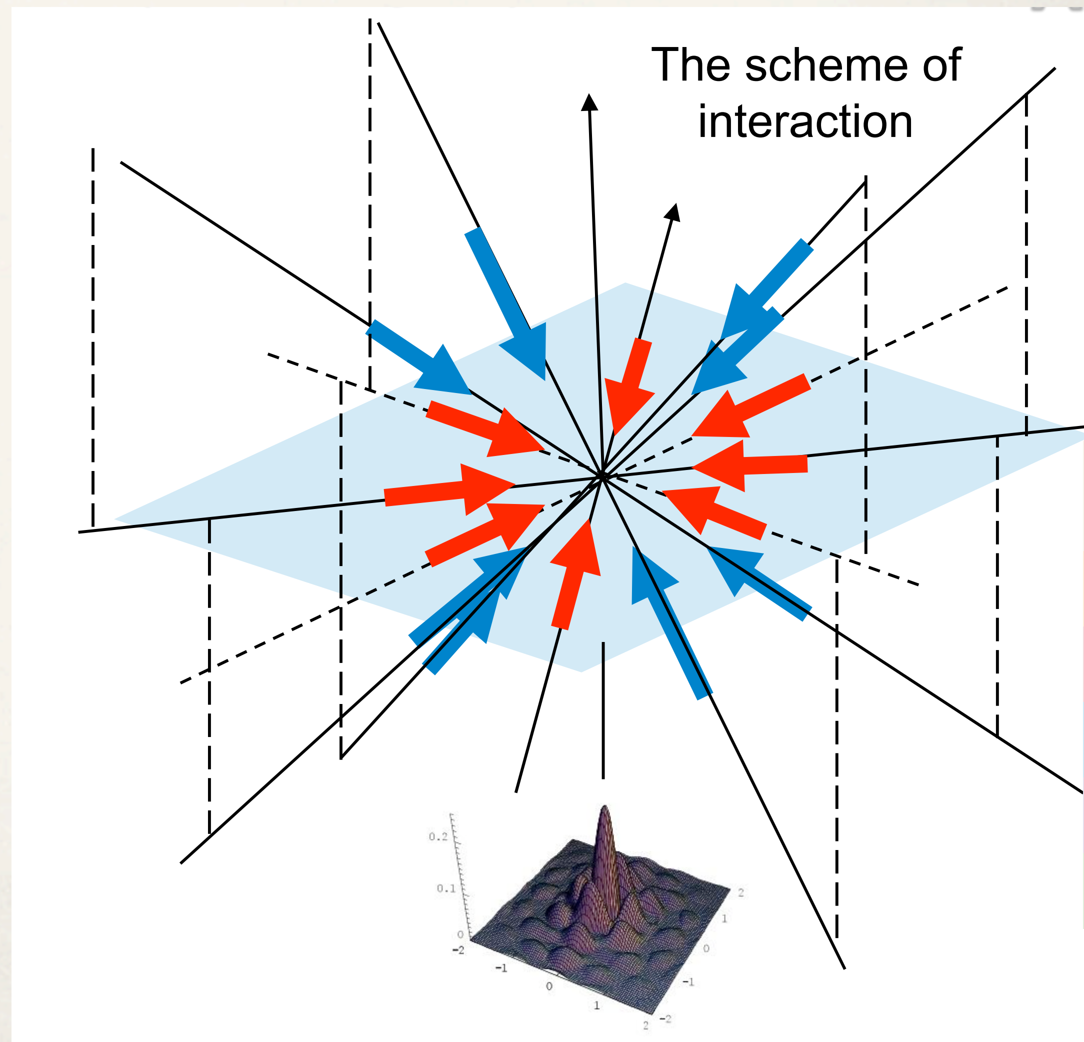
$\Rightarrow$ quantum interference

we can take advantage of this to <u>enhance</u> the Schwinger effect

Multiple Colliding Electromagnetic Pulses: A Way to Lower the Threshold of e^+e^- Pair Production from Vacuum

S. S. Bulanov,[1,2] V. D. Mur,[3] N. B. Narozhny,[3] J. Nees,[1] and V. S. Popov[2]
[1]*FOCUS Center and Center for Ultrafast Optical Science, University of Michigan, Ann Arbor, Michigan 48109, USA*
[2]*Institute of Theoretical and Experimental Physics, Moscow 117218, Russia*
[3]*National Research Nuclear University MEPhI, 115409 Moscow, Russia*
(Received 2 March 2010; published 1 June 2010)

n	$N_{e^+e^-}$ at $W = 10$ kJ	W_{th}, kJ ($N_{e^+e^-} \sim 1$)
2	0[a]	40
4	0[b]	20
8	4.0	10
16	1.8×10^3	8
24	4.2×10^6	5.1



spot size prefactor is important

colliding beams enhance effect

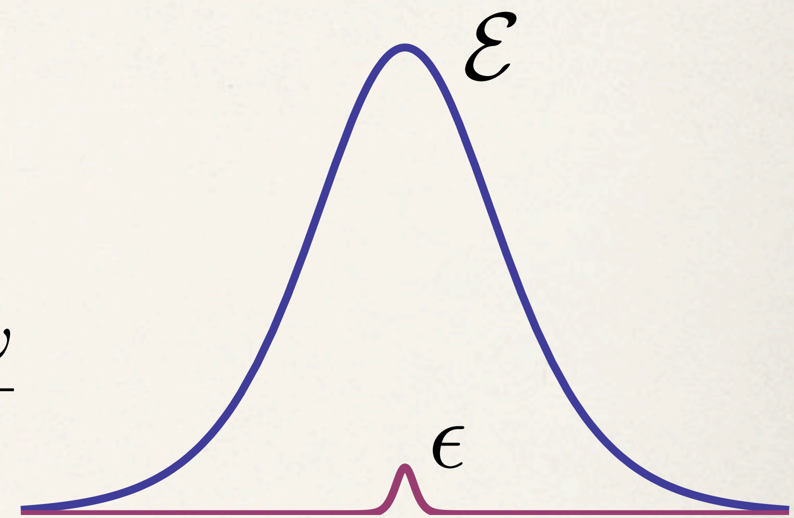
PRL 101, 130404 (2008) PHYSICAL REVIEW LETTERS week ending 26 SEPTEMBER 2008

Dynamically Assisted Schwinger Mechanism

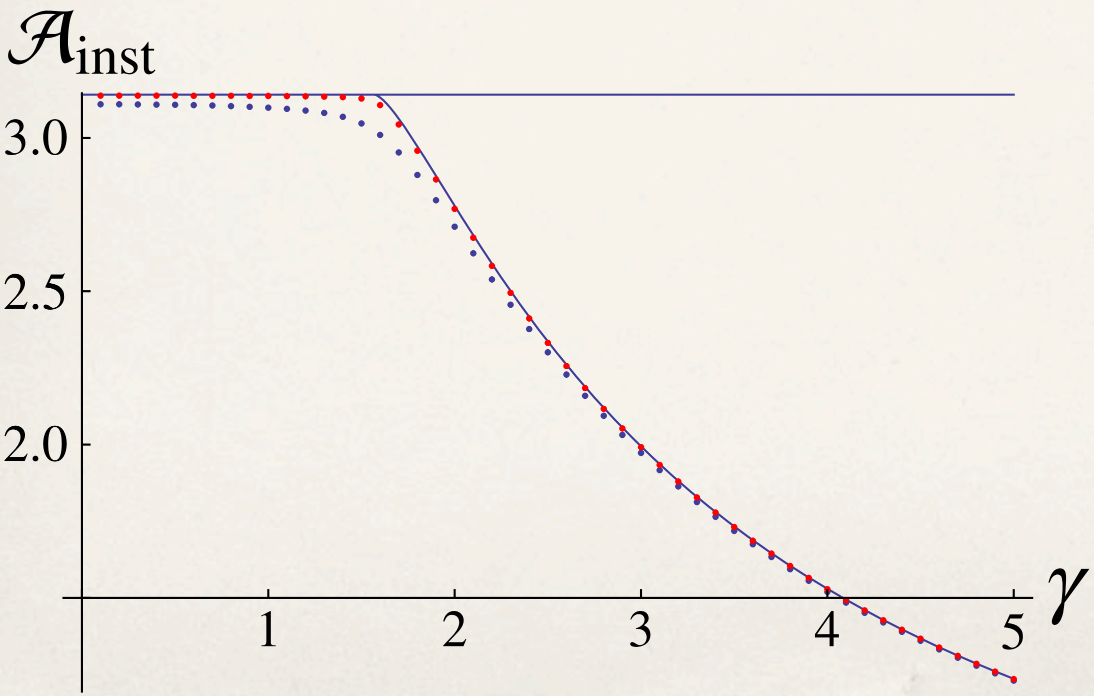
Ralf Schützhold,[1,2] Holger Gies,[3,4] and Gerald Dunne[5]

strong, slow field plus weak, fast field

$$\mathcal{E}(t) = \mathcal{E}\,\text{sech}^2(\Omega\, t) + \epsilon\,\text{sech}^2(\omega\, t)$$



"mixed" Keldysh parameter $\gamma = \dfrac{m\,\omega}{e\,\mathcal{E}}$

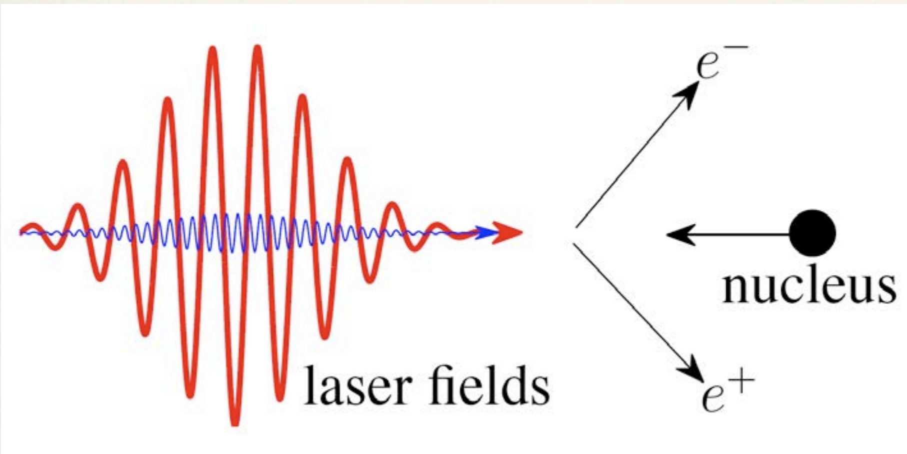


significant enhancement of $e^{-\mathcal{A}_{\text{inst}}}$

large effective γ, but still nonperturbative

Barrier Control in Tunneling e^+-e^- Photoproduction

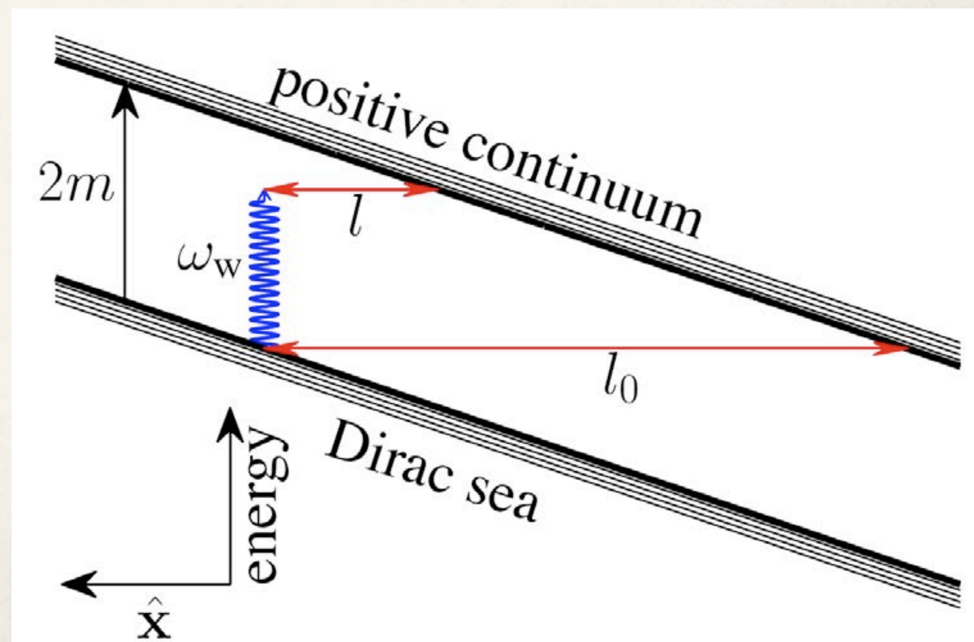
A. Di Piazza,[1,*] E. Lötstedt,[1] A. I. Milstein,[1,2] and C. H. Keitel[1]
[1]*Max-Planck-Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany*
[2]*Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia*



dynamically assisted Schwinger mechanism

strong, slow pulse + weak, fast pulse

e absorbs weak high ω photon, lowering the effective tunnel barrier

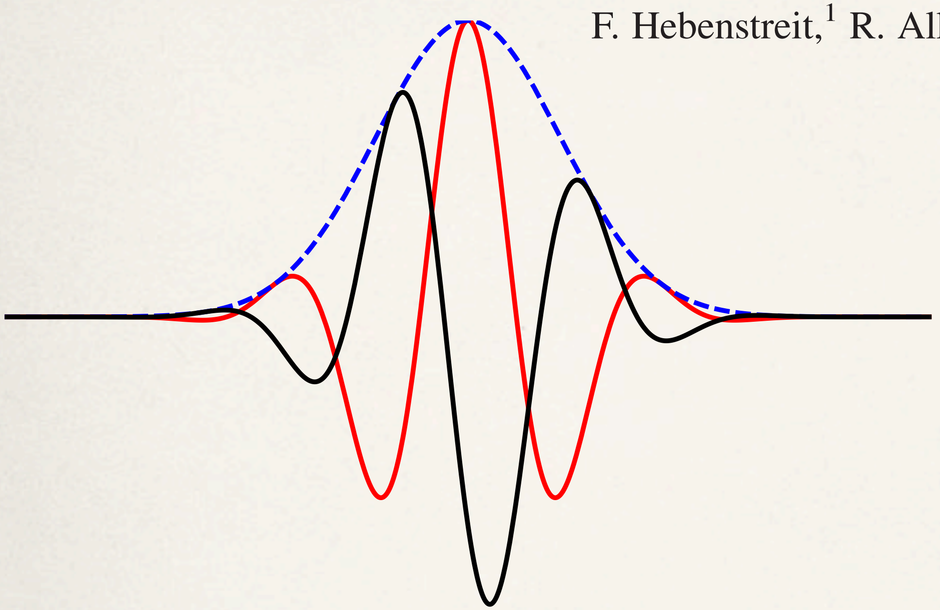


pulse design $\Longrightarrow$ Schwinger critical field can be lowered by 2 - 3 orders of magnitude!

quantitative physical explanation: quantum interference

Momentum Signatures for Schwinger Pair Production in Short Laser Pulses with a Subcycle Structure

F. Hebenstreit,[1] R. Alkofer,[1] G. V. Dunne,[2] and H. Gies[3]



$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t + \phi) \exp\left(-\frac{t^2}{2\tau^2}\right)$$

φ : carrier phase

PRL 102, 150404 (2009) PHYSICAL REVIEW LETTERS week ending 17 APRIL 2009

Momentum Signatures for Schwinger Pair Production in Short Laser Pulses with a Subcycle Structure

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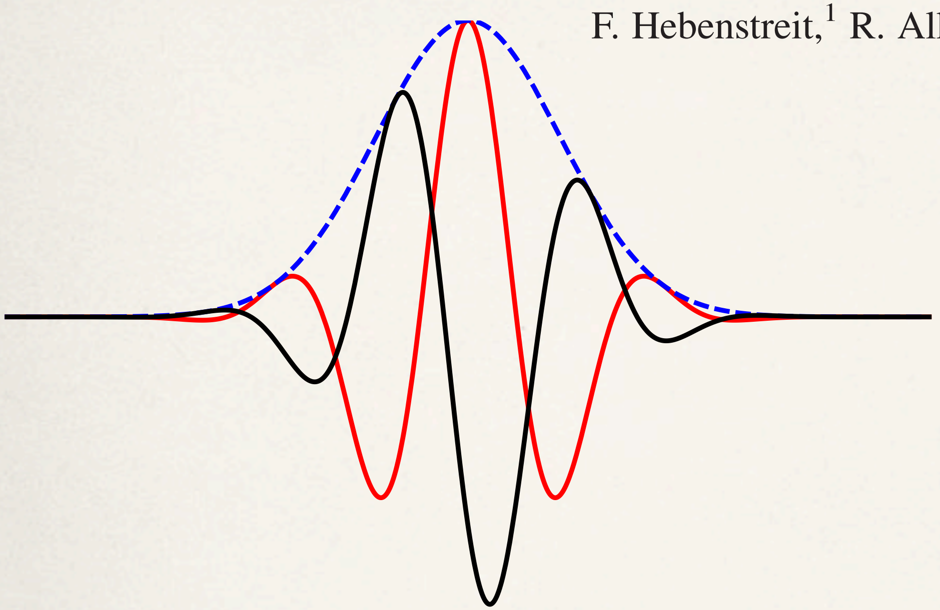

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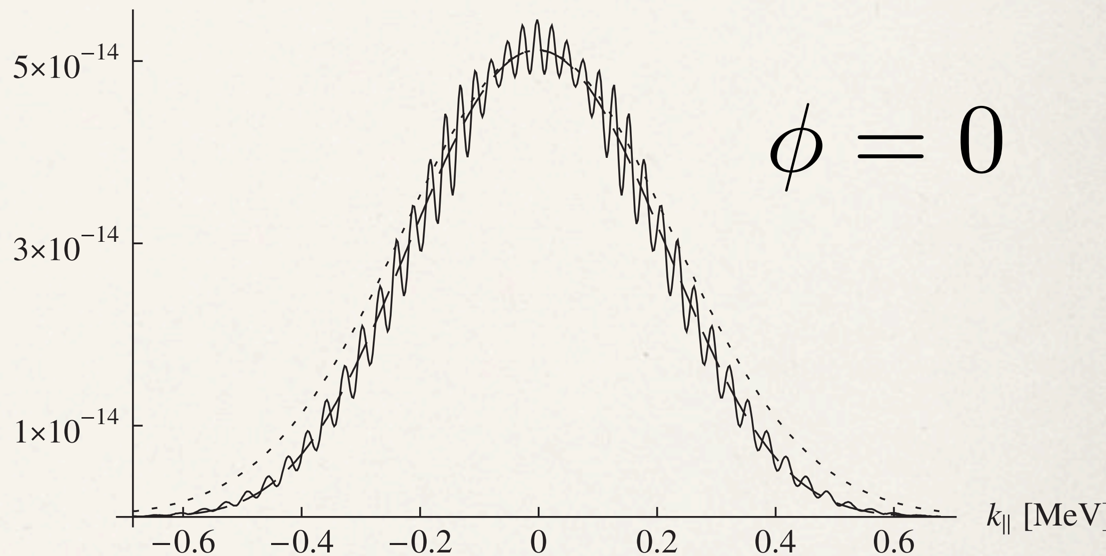
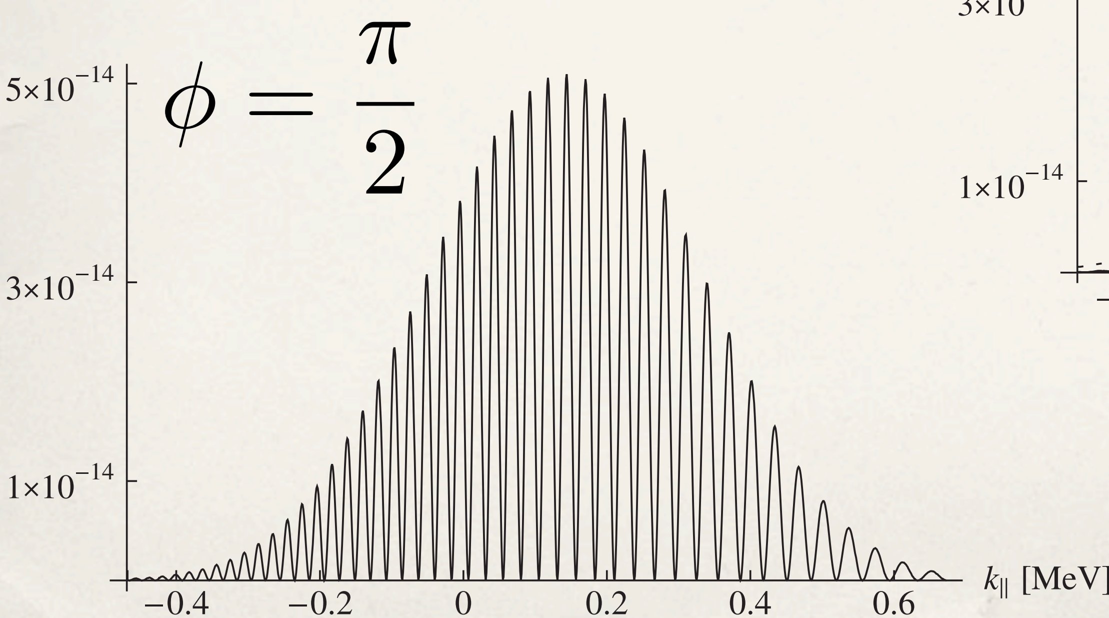




oscillations due to quantum interference

The Stokes Phenomenon

VI. *On the Discontinuity of Arbitrary Constants which appear in Divergent Developments.* *By* G. G. STOKES, M.A., D.C.L., Sec. R.S., *Fellow of Pembroke College, and Lucasian Professor of Mathematics in the University of Cambridge.*

[Read *May* 11, 1857.]

$$\hbar^2 \psi'' + Q\,\psi = 0 \qquad\qquad \psi_\pm = \frac{1}{Q^{1/4}}\, e^{\pm \frac{i}{\hbar} \int^z Q^{1/2}}$$

Stokes: "WKB" solutions are multivalued, even if true solution is not; only LOCAL

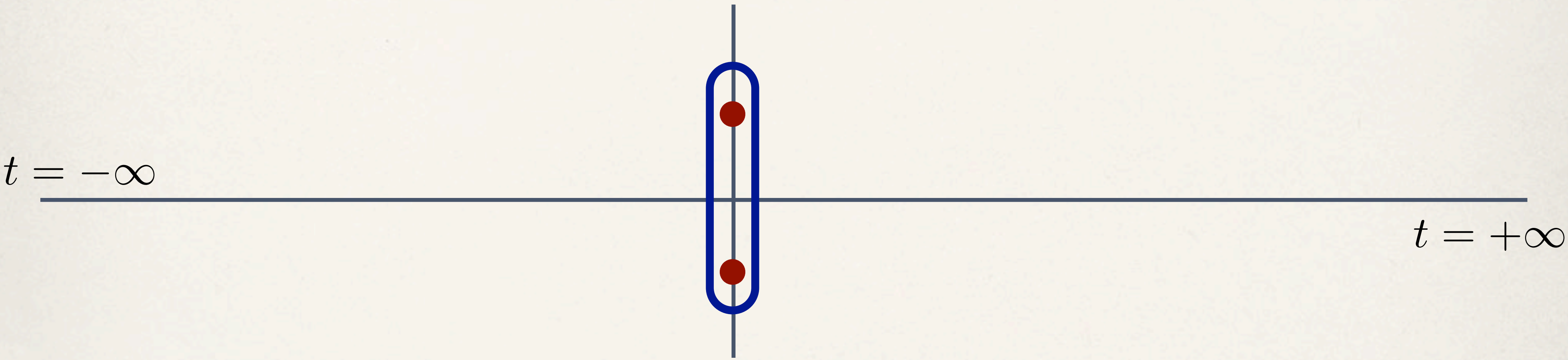
for many applications we need GLOBAL information

PRL 104, 250402 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

Stokes Phenomenon and Schwinger Vacuum Pair Production in Time-Dependent Laser Pulses

Cesim K. Dumlu and Gerald V. Dunne
Department of Physics, University of Connecticut, Storrs, Connecticut 06269-3046, USA
(Received 14 April 2010; published 24 June 2010)

$$\ddot{\Phi} + \omega^2(t)\Phi = 0 \qquad \omega^2(t) = m^2 + p_\perp^2 + (p_\parallel - A(t))^2$$



local ! we need global information

PRL 104, 250402 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

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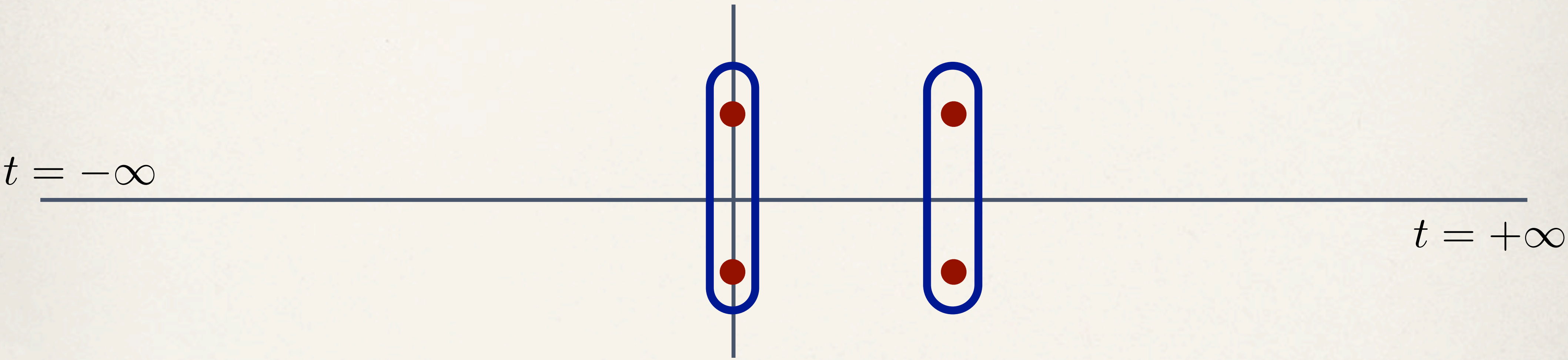

local ! we need global information

PRL **104,** 250402 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

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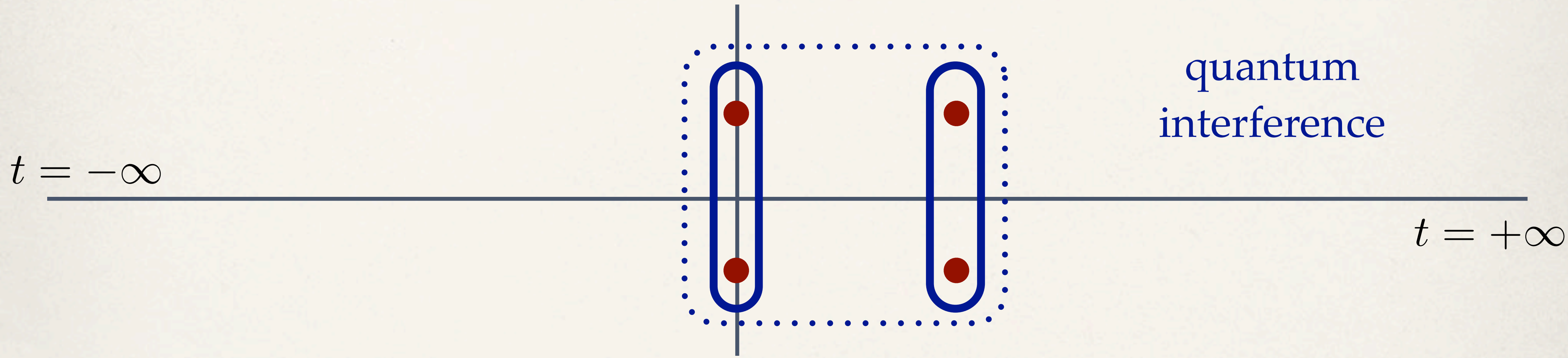
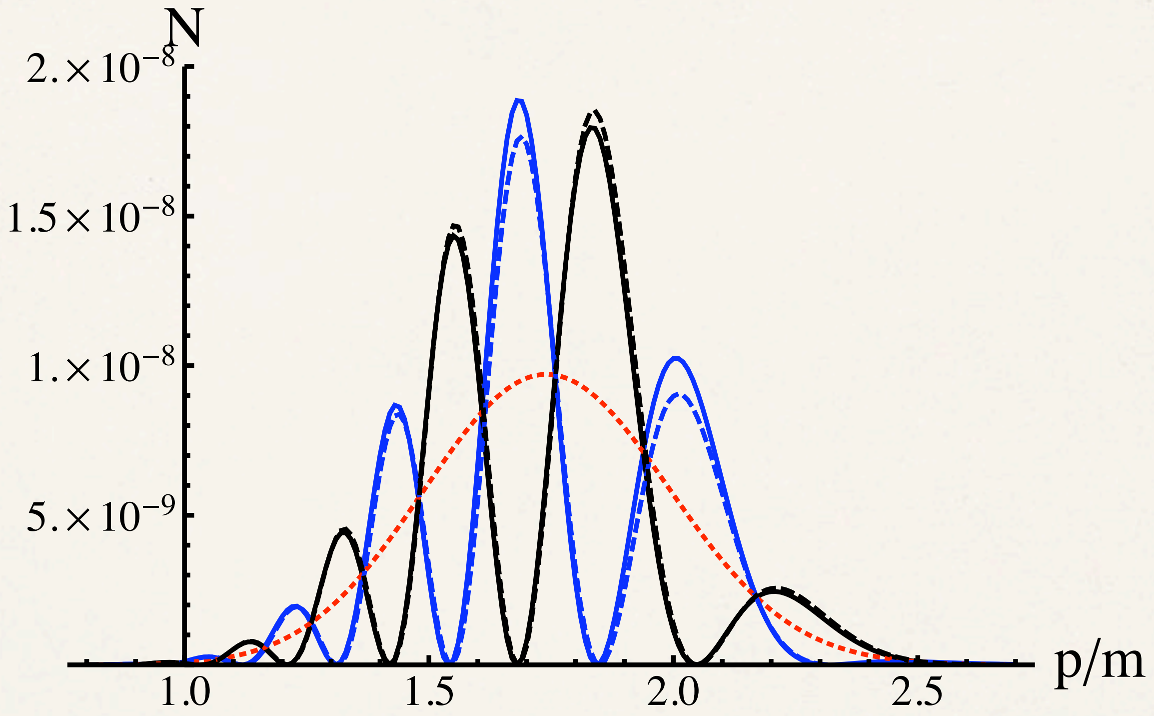


local ! we need global information

quantum interference and quantum statistics

oscillations due to interference effects between pairs of complex turning points:
"Stokes phenomenon"

$$\phi = \frac{\pi}{2}$$

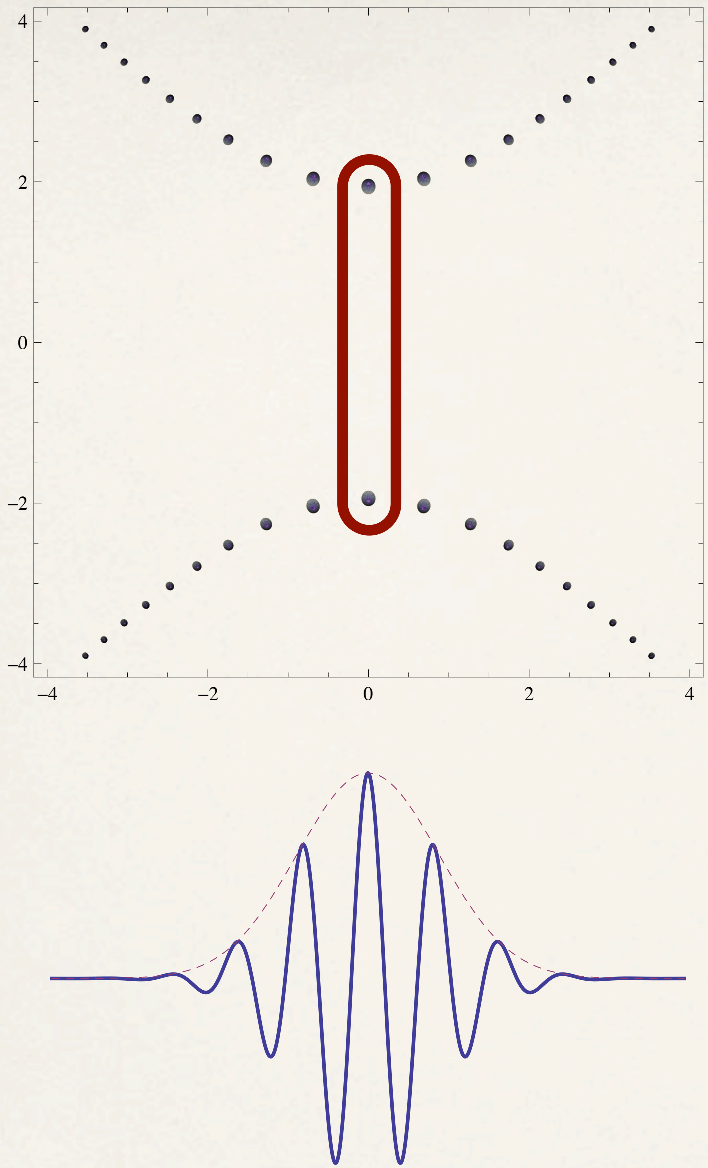


scalar QED

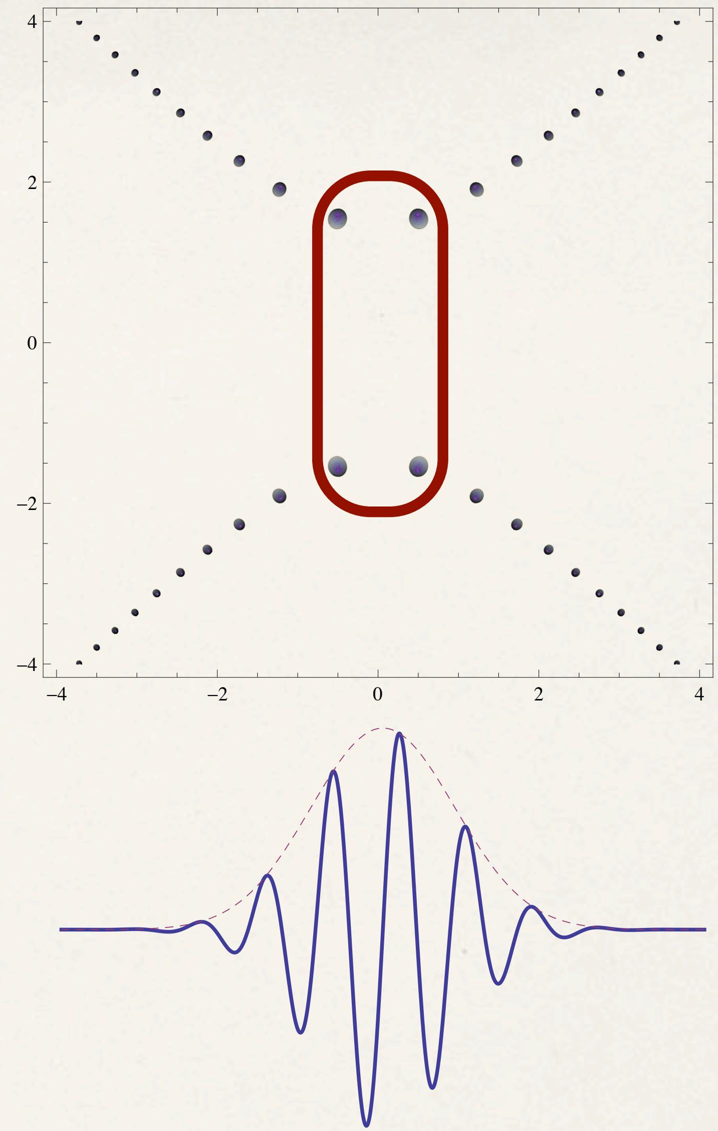
spinor QED

naive WKB

$$|R|^2 \approx e^{-2W_1} + e^{-2W_2} \pm 2\cos(2\alpha)e^{-W_1-W_2}$$



$$\mathcal{E}(t) = \mathcal{E}_0 \, \cos(\omega t) \, e^{-\frac{t^2}{2\tau^2}}$$

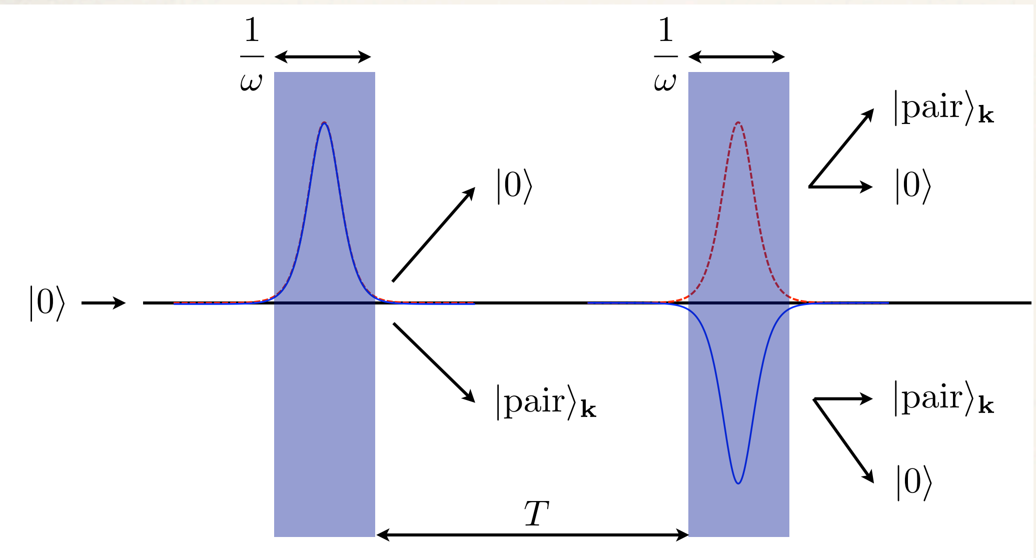


$$\mathcal{E}(t) = \mathcal{E}_0 \, \sin(\omega t) \, e^{-\frac{t^2}{2\tau^2}}$$

PRL **108,** 030401 (2012) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2012

Ramsey Fringes and Time-Domain Multiple-Slit Interference from Vacuum

Eric Akkermans[1] and Gerald V. Dunne[2]



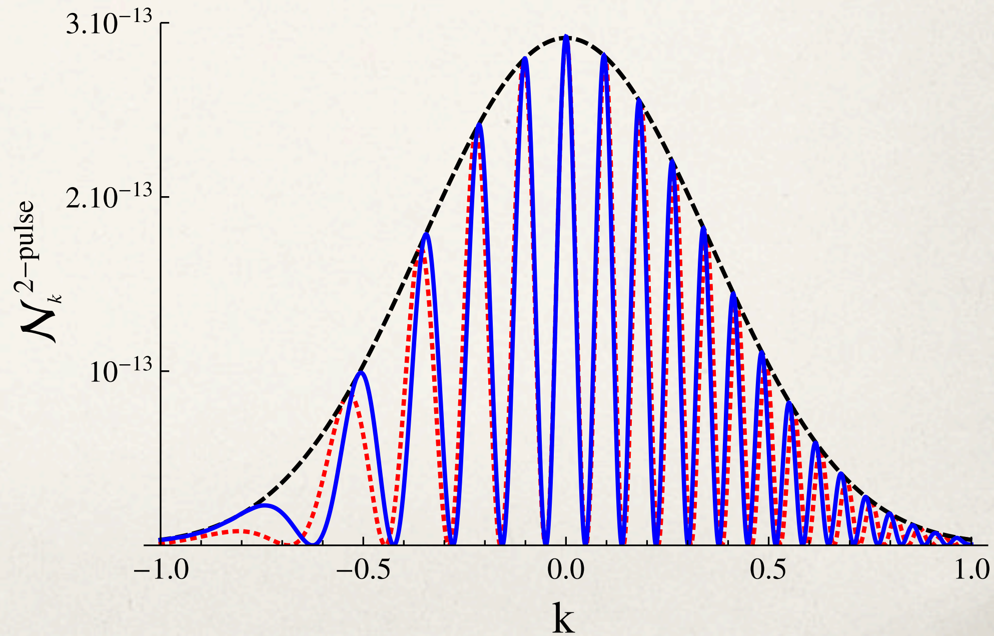
time domain multiple-slit quantum interference effect

$$\mathcal{N}_{\mathbf{k}}^{N-\text{pulse}} \approx \begin{cases} \mathcal{N}_{\mathbf{k}}^{1-\text{pulse}} \sin^2[N\theta_{\mathbf{k}}]/\cos^2[\theta_{\mathbf{k}}], & N \text{ even} \\ \mathcal{N}_{\mathbf{k}}^{1-\text{pulse}} \cos^2[N\theta_{\mathbf{k}}]/\cos^2[\theta_{\mathbf{k}}], & N \text{ odd} \end{cases}$$

N alternating sign pulses

⇒

coherent N² enhancement of certain modes



PRL 108, 030401 (2012) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2012

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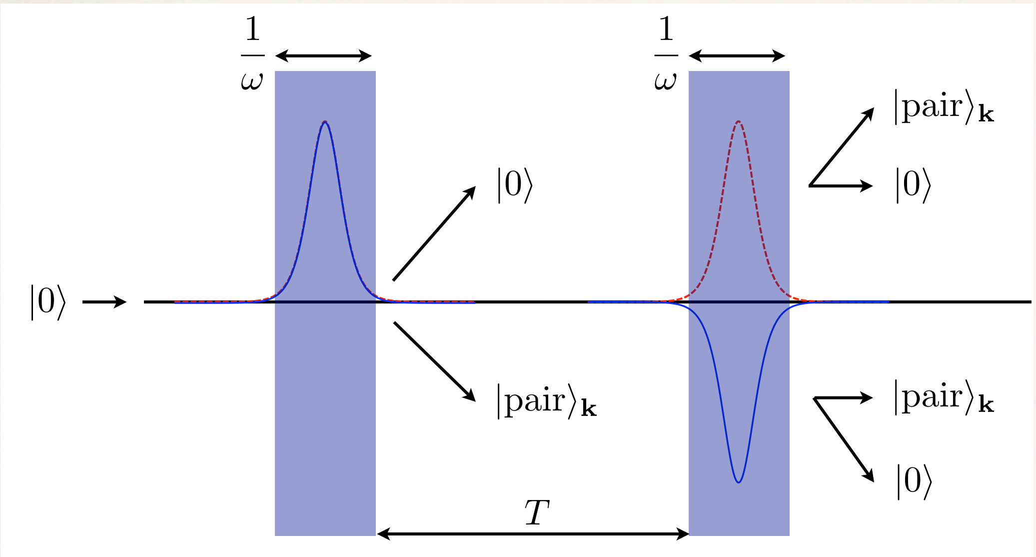
Eric Akkermans[1] and Gerald V. Dunne[2]



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N alternating sign pulses

$\Rightarrow$

coherent N² enhancement of certain modes

PRL **108,** 030401 (2012) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2012

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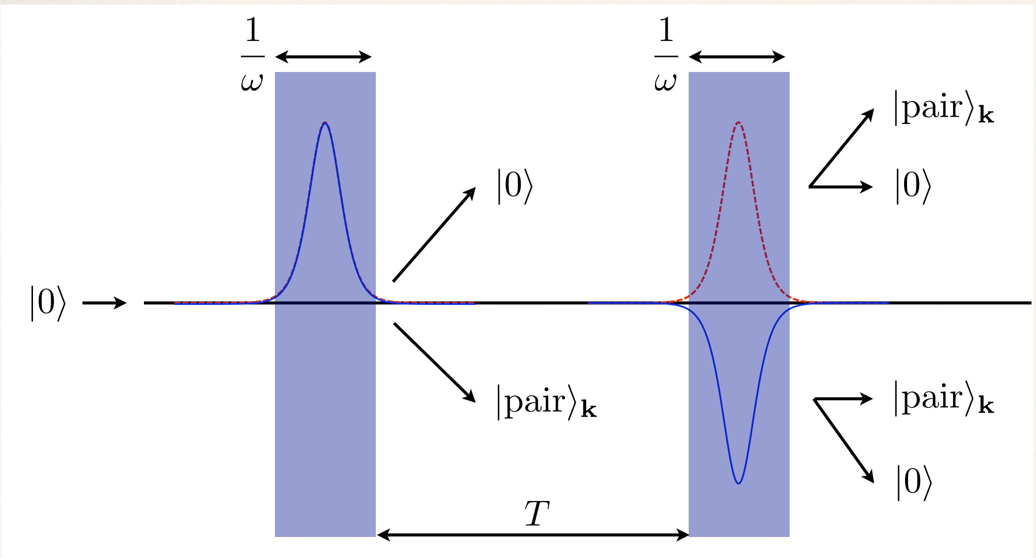
Eric Akkermans[1] and Gerald V. Dunne[2]



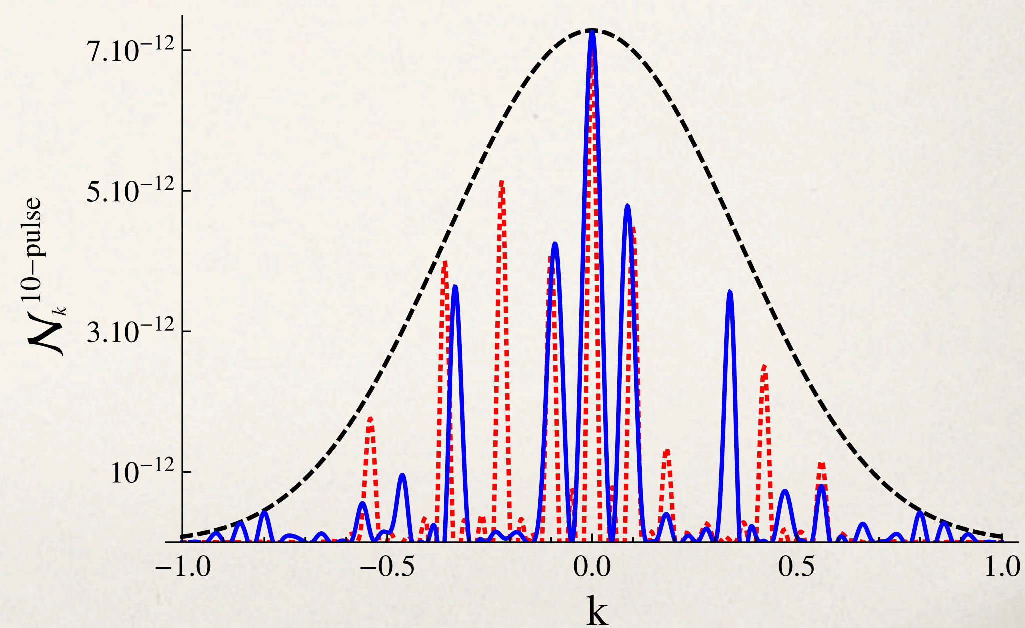
time domain multiple-slit quantum interference effect

$$\mathcal{N}_{\mathbf{k}}^{N-\text{pulse}} \approx \begin{cases} \mathcal{N}_{\mathbf{k}}^{1-\text{pulse}} \sin^2[N\theta_{\mathbf{k}}]/\cos^2[\theta_{\mathbf{k}}], & N \text{ even} \\ \mathcal{N}_{\mathbf{k}}^{1-\text{pulse}} \cos^2[N\theta_{\mathbf{k}}]/\cos^2[\theta_{\mathbf{k}}], & N \text{ odd} \end{cases}$$

N alternating sign pulses

$\Rightarrow$

coherent N^2 enhancement of certain modes

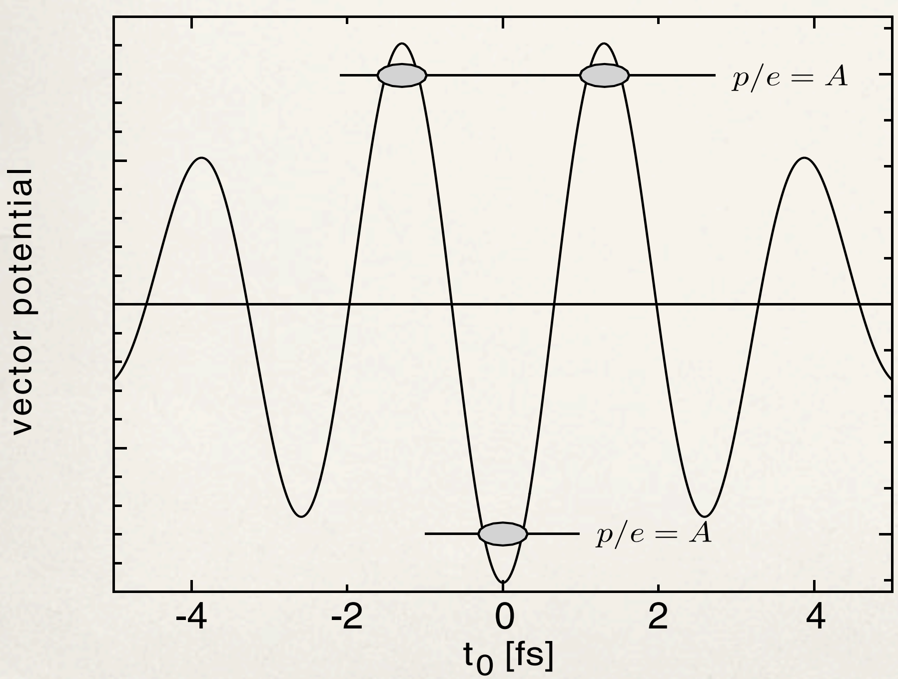


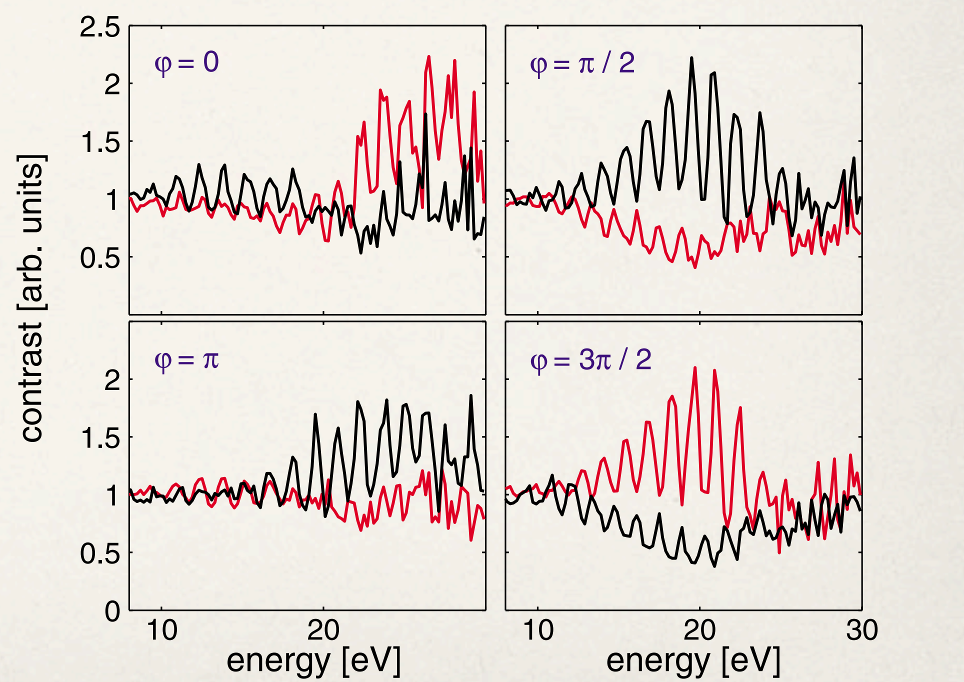
PRL **95,** 040401 (2005) PHYSICAL REVIEW LETTERS week ending 22 JULY 2005

Attosecond Double-Slit Experiment

F. Lindner,[1] M. G. Schätzel,[1] H. Walther,[1,2] A. Baltuška,[1] E. Goulielmakis,[1] F. Krausz,[1,2,3] D. B. Milošević,[4] D. Bauer,[5] W. Becker,[6] and G. G. Paulus[1,2,7]

A new scheme for a double-slit experiment in the time domain is presented. Phase-stabilized few-cycle laser pulses open one to two windows (slits) of attosecond duration for photoionization. Fringes in the angle-resolved energy spectrum of varying visibility depending on the degree of which-way information are measured. A situation in which one and the same electron encounters a single and a double slit at the same time is observed. The investigation of the fringes makes possible interferometry on the attosecond time scale. From the number of visible fringes, for example, one derives that the slits are extended over about 500 as.





optimization

optimal quantum control: find the optimal pulse shape

shaped ultra-short pulses: tune time-dependent amplitudes and phases to match characteristic frequencies of quantum system

e.g. NMR, selective molecular transformations, ...

J. Chem. Phys. **108** (5), 1 February 1998

Rapidly convergent iteration methods for quantum optimal control of population

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Much attention has recently been focused on optimal control of quantum systems, and extensive theoretical and numerical work has been performed.[1–5] An important case is the desire to achieve a large transition probability from a specific initial state into a final target state by means of a controlling external laser field[5] while minimizing the laser energy.

THE JOURNAL OF CHEMICAL PHYSICS **130**, 214702 (2009)

Toward adaptive control of coherent electron transport in semiconductors

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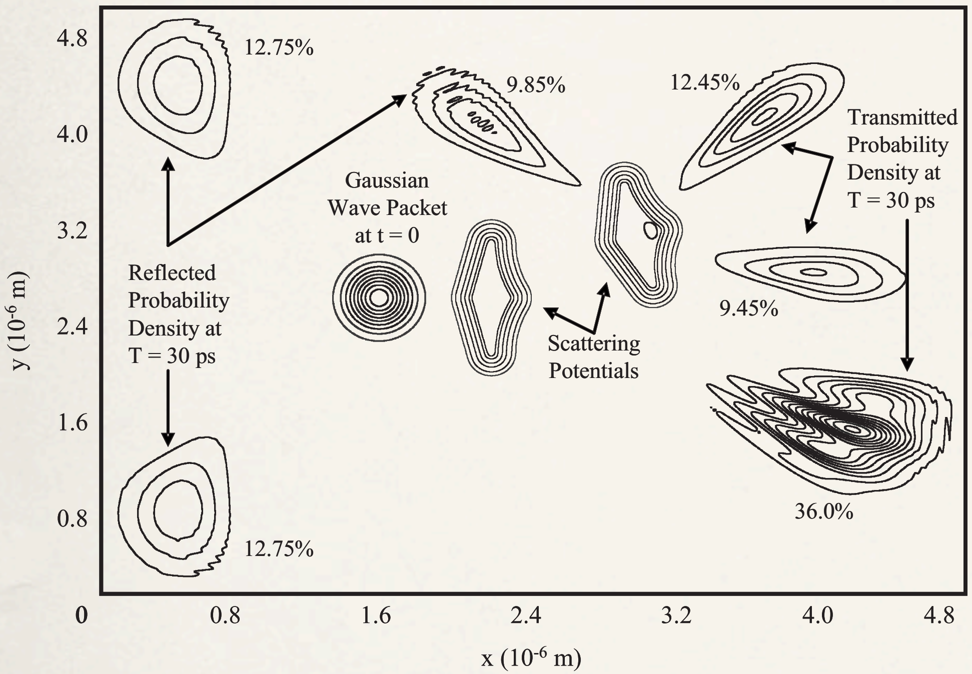
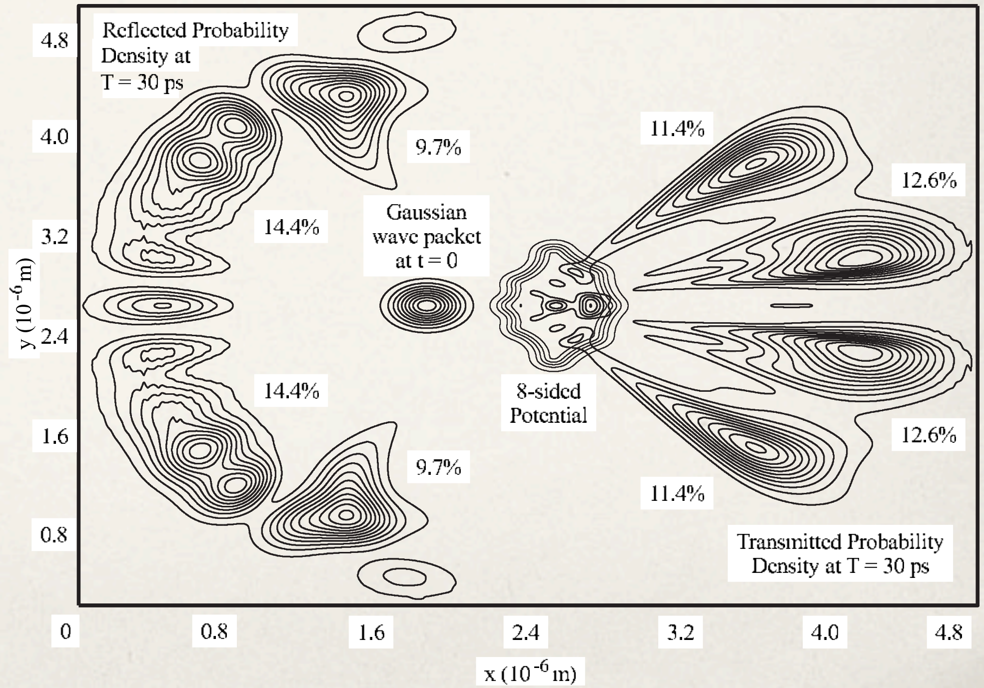


designer potentials for prescribed 2d scattering



Optimal Control of Dynamically Assisted Schwinger Pair Production

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Gordon conference 2011

- OCT has been widely studied for N-level population transfer problems
- need: ultra-relativistic extension, QFT formulation: worldline
- quantum interference provides guiding principle for optimization

Optimizing the pulse shape for Schwinger pair production

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(Dated: December 7, 2012)

perhaps the most interesting theoretical puzzle ... back-reaction

back-reaction and non-equilibrium processes

- created pairs act back on the external electric field
- inherently non-equilibrium process
- go beyond 1-loop effective action picture
- important for heavy ion physics; condensed matter & AMO analogues

PHYSICAL REVIEW D, VOLUME 58, 125015

Quantum Vlasov equation and its Markov limit

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(Received 20 March 1998; published 16 November 1998)

PHYSICAL REVIEW D **87,** 105006 (2013)

Simulating fermion production in 1 + 1 dimensional QED

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QED cascades and ultimate electric field?

back reaction & radiation reaction; polarization effects

PRL **101,** 200403 (2008) PHYSICAL REVIEW LETTERS week ending 14 NOVEMBER 2008

Possibility of Prolific Pair Production with High-Power Lasers

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(Received 8 August 2008; published 11 November 2008)

Prolific electron-positron pair production is possible at laser intensities approaching 10^{24} W cm^{-2} at a wavelength of 1 μm. An analysis of electron trajectories and interactions at the nodes ($B = 0$) of two counterpropagating, circularly polarized laser beams shows that a cascade of γ rays and pairs develops. The geometry is generalized qualitatively to linear polarization and laser beams incident on a solid target.

PRL **105,** 080402 (2010) PHYSICAL REVIEW LETTERS week ending 20 AUGUST 2010

Limitations on the Attainable Intensity of High Power Lasers

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QED cascades and ultimate electric field?

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **14**, 054401 (2011)

QED cascades induced by circularly polarized laser fields

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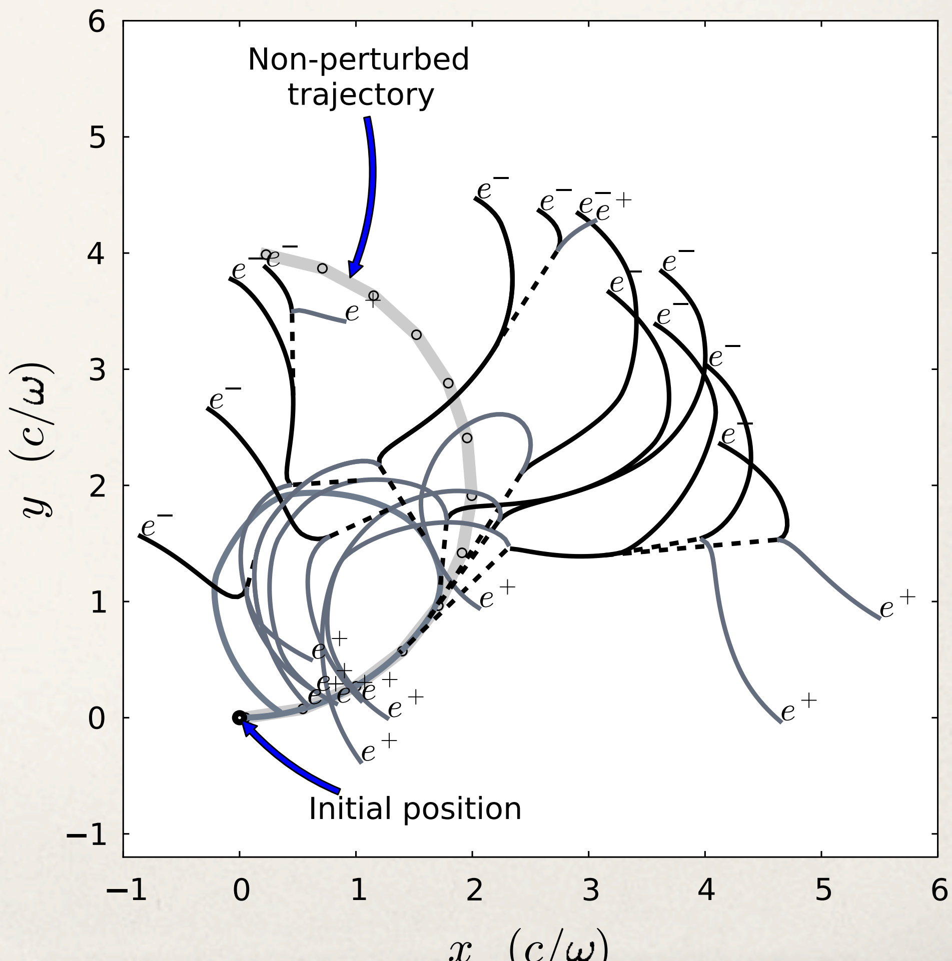
Monte-Carlo simulations

includes radiation friction

fully quantum treatment needed



conclusions

- the ``Schwinger limit'' is not necessarily a sharp limit
- something very interesting is going to happen around 10^{24} - 10^{25} W/cm^2 ...
- experimental challenges : higher intensity, focussing, optics, pulse engineering, plasma effects in intense fields, control schemes, ...
- theoretical challenges : optimal pulse design, non-equilibrium effects, plasma effects in intense fields, ...
- quantum interference is significant; combining e beams with lasers, ...
- conceptual and computational problems: non-equilibrium QFT, back-reaction, cascading, cosmological and gravitational analogues, ...
- a new field of high-intensity laser/particle physics is forming