

A photograph of the University of Bern's main building, a large Gothic-style stone structure with multiple spires and towers, set against a backdrop of green hills and a clear sky. The building is the central focus of the image.

Andreas Crivellin

**Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics
University of Bern**

**The 2HDM of type III:
The MSSM in the Decoupling Limit**

Outline:

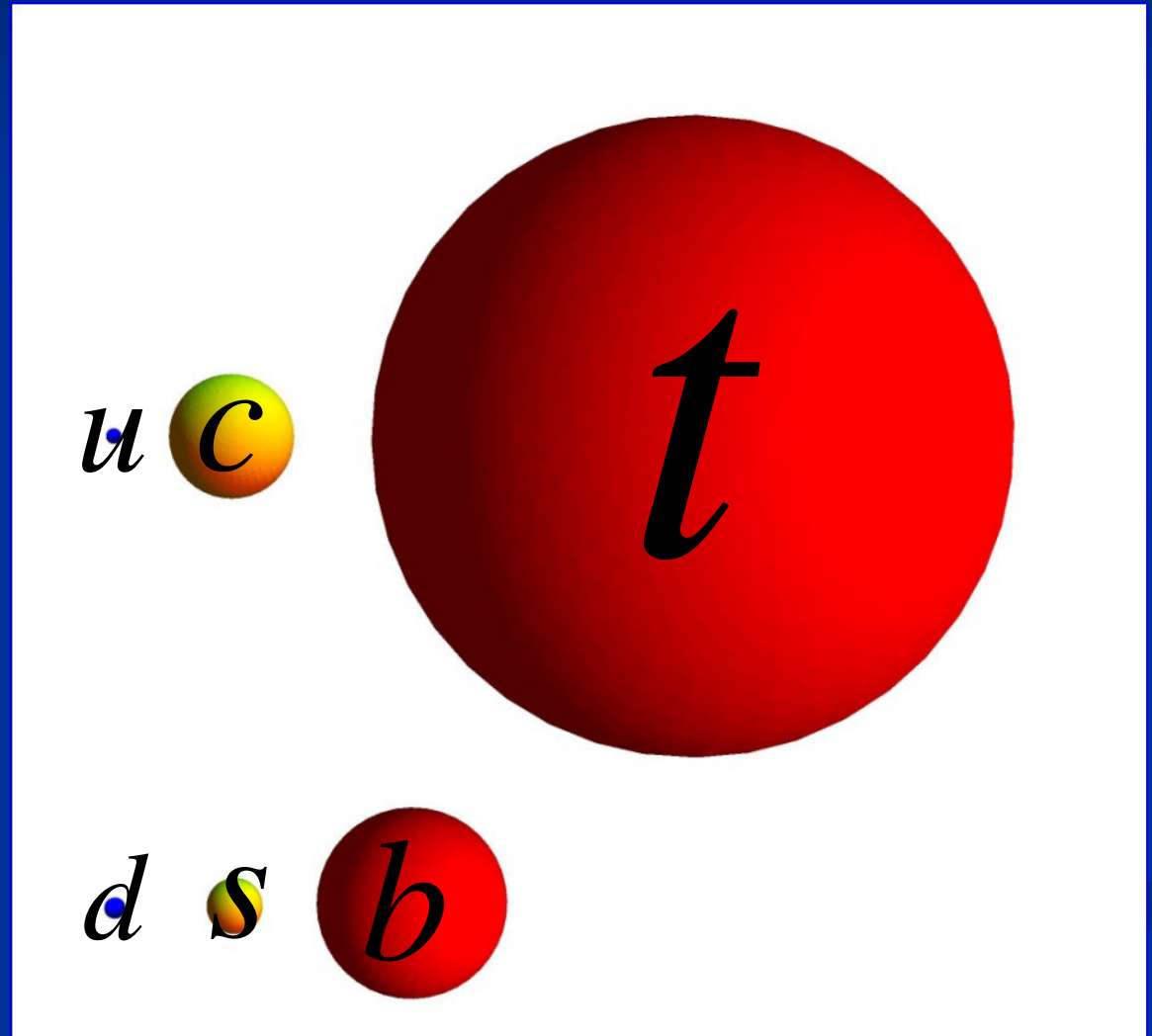
- Introduction
- Matching on the MSSM on the 2HDM
 - Resummation and effective Higgs-quark vertices
 - 2-loop corrections
- Flavour-phenomenology of 2HDMs with generic flavour-structure
 - Constraints from FCNC processes
 - Tauonic B decays
 - Limits on LFV processes
- Conclusions

Introduction

Sources of flavour violation in the MSSM

Quark masses

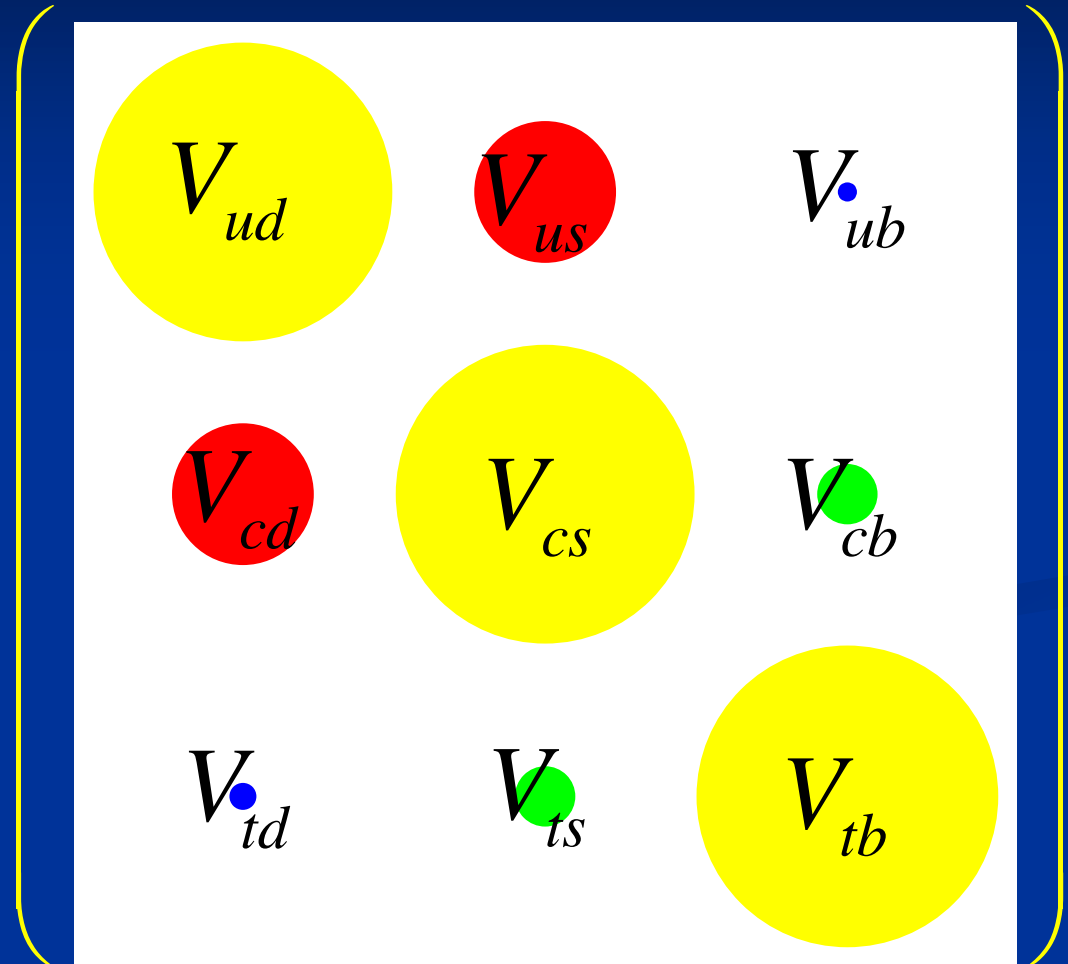
- Top quark is very heavy: $m_t \approx v$
- Bottom quark rather light, but Y^b can be big at large $\tan(\beta)$
- All other quark masses are very small
➔ sensitive to radiative corrections



CKM matrix

- CKM matrix is the only source of flavor and CP violation in the SM.
- No tree-level FCNCs
- Off-diagonal CKM elements are small

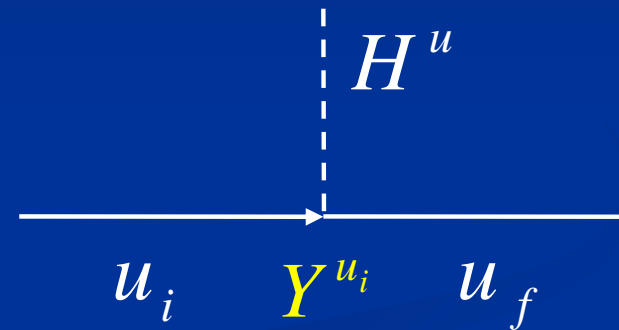
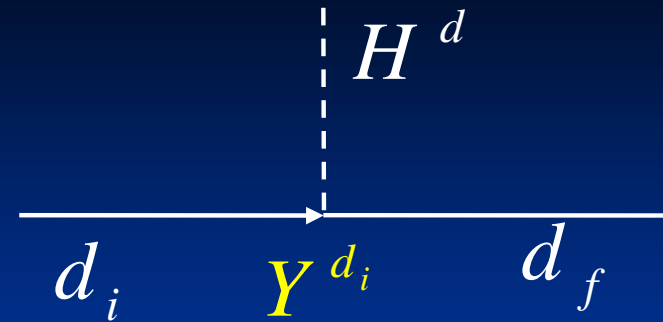
$$V_{\text{CKM}} =$$



➔ Flavor-violation is suppressed in the Standard Model.

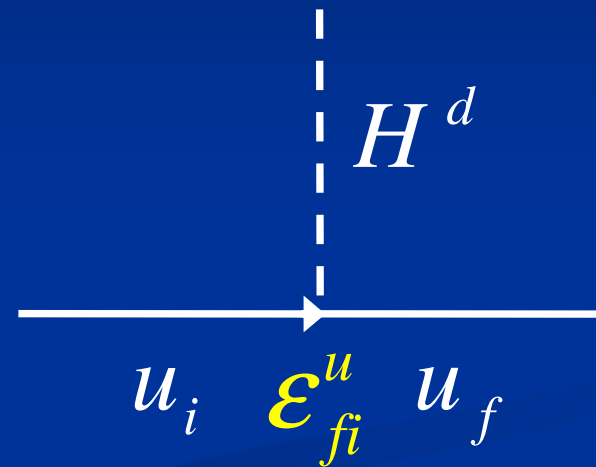
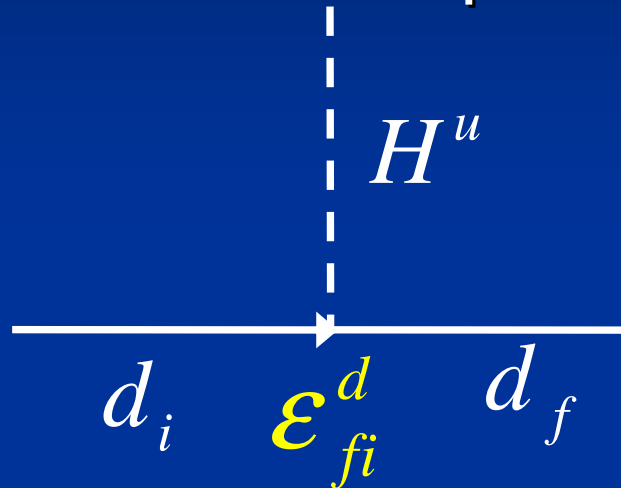
MSSM at tree-level: 2HDM of type II

- One Higgs doublet couples only to down quarks the other Higgs doublet only to up-quarks.
- 2 additional free parameters: $\tan(\beta)=v_u/v_d$ and the heavy Higgs mass M_H
- Neutral Higgs-quark couplings are flavour-conserving.



2HDM of type III

- Both Higgs doublets couple simultaneously to up and down quarks.



$$m_{ij}^d = v_d Y_{ij}^d + v_u \epsilon_{ij}^d$$

$$m_{ij}^u = v_u Y_{ij}^u + v_d \epsilon_{ij}^u$$

- The parameters $\epsilon_{ij}^{u,d}$ describe flavor-changing neutral Higgs interactions
- In the MSSM, $\epsilon_{ij}^{u,d}$ are induced via loops

Squark mass matrix

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{LL}^{\tilde{q}2} & \Delta^{\tilde{q}LR} \\ \Delta^{\tilde{q}LR\dagger} & M_{RR}^{\tilde{q}2} \end{pmatrix}$$

hermitian: $\longrightarrow W^{\tilde{q}\dagger} M_{\tilde{q}}^2 W^{\tilde{q}} = M_{\tilde{q}}^{2(D)}$

$M_{LL,RR}^{\tilde{q}2}$ involves only bilinear terms (in the decoupling limit)

The chirality-changing elements are proportional to a vev

$$\Delta_{ij}^{dLR} = -v_d \left(\mu \tan(\beta) Y_i^d \delta_{ij} + A_{ij}^d \right)$$

$$\Delta_{ij}^{uLR} = -v_u \left(\mu \cot(\beta) Y_i^u \delta_{ij} + A_{ij}^u \right)$$

$$\tan(\beta) = \frac{v_u}{v_d}$$

Squark-Higgs couplings

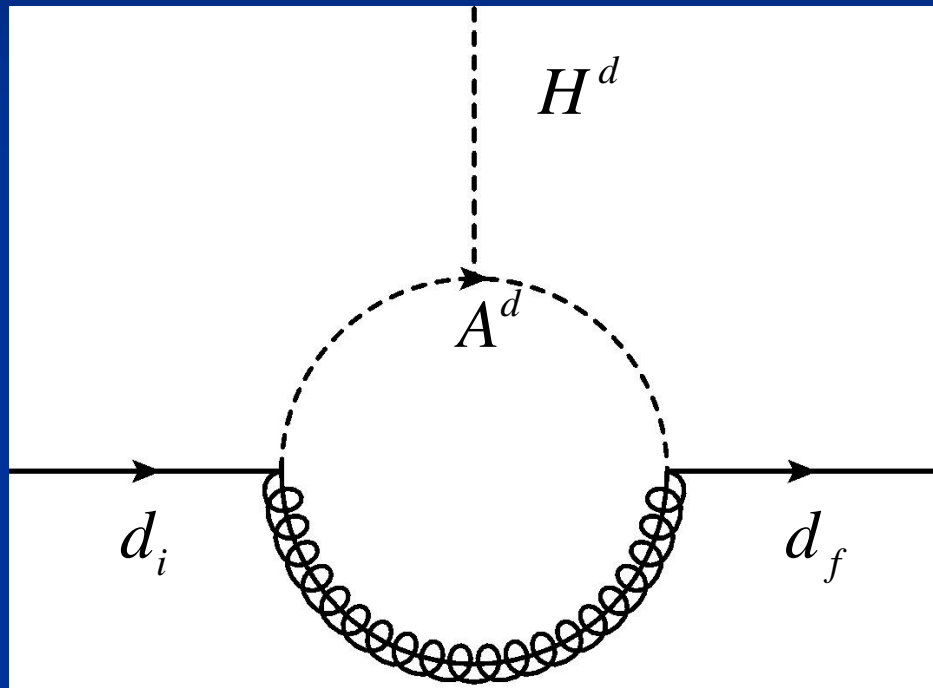
- The off-diagonal elements $\Delta_{ij}^{\tilde{q}LR}$ originate from squark-Higgs couplings

$$\begin{aligned}
 & \text{Diagram 1: } \tilde{d}_i^R \xrightarrow{-iA_{fi}^d} \tilde{d}_f^L \text{ with a vertex } \times \text{ and a vertical dashed line } \text{---} V_d \text{---} \\
 & \text{Diagram 2: } \tilde{d}_i^R \xrightarrow{-i\mu Y^{d_i} \delta_{fi}} \tilde{d}_f^L \text{ with a vertex } \times \text{ and a vertical dashed line } \text{---} V_u \text{---} \\
 & \text{Sum: } \tilde{d}_i^R \xrightarrow{-i\Delta_{fi}^{dLR}} \tilde{d}_f^L \text{ with a vertex } \times \text{---} \\
 & \hat{=} \text{ Mass insertion approximation} \\
 & \delta_{fi}^{\tilde{q}LR} \equiv \frac{\Delta_{fi}^{\tilde{q}LR}}{\hat{m}_{\tilde{q}}^2} \quad \hat{m}_{\tilde{q}}^2 \text{ average squark mass}
 \end{aligned}$$

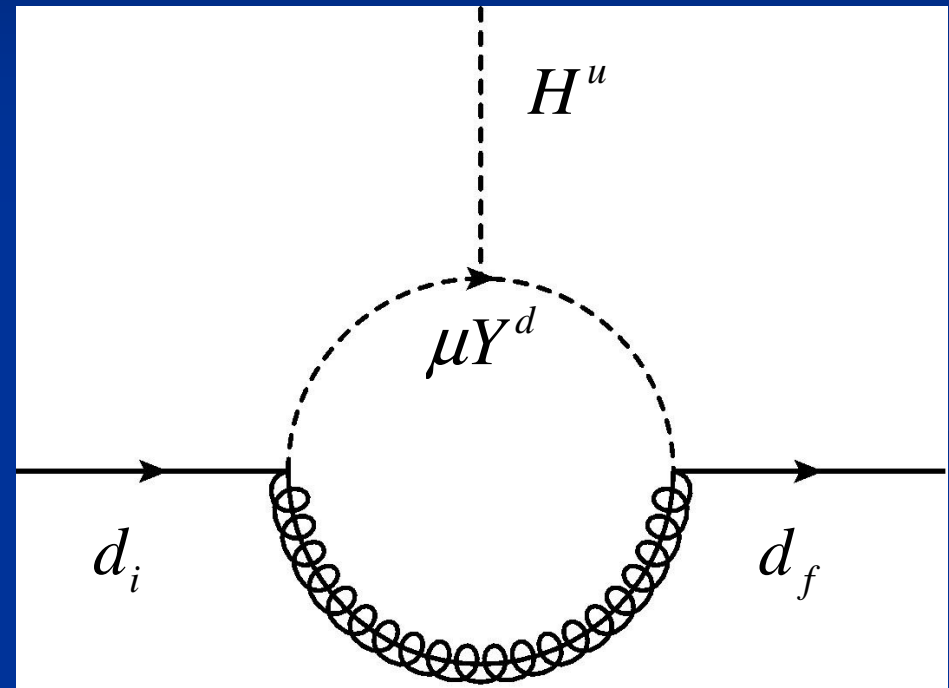
Higgs-quark couplings and quark self-energies

Loop corrections to Higgs quark couplings

- Before electroweak symmetry breaking



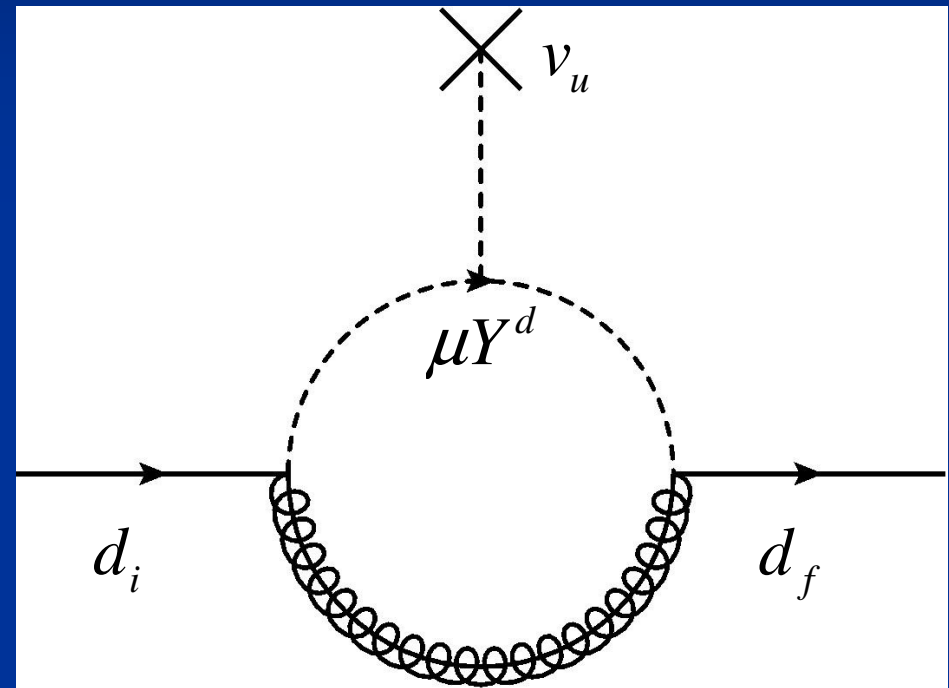
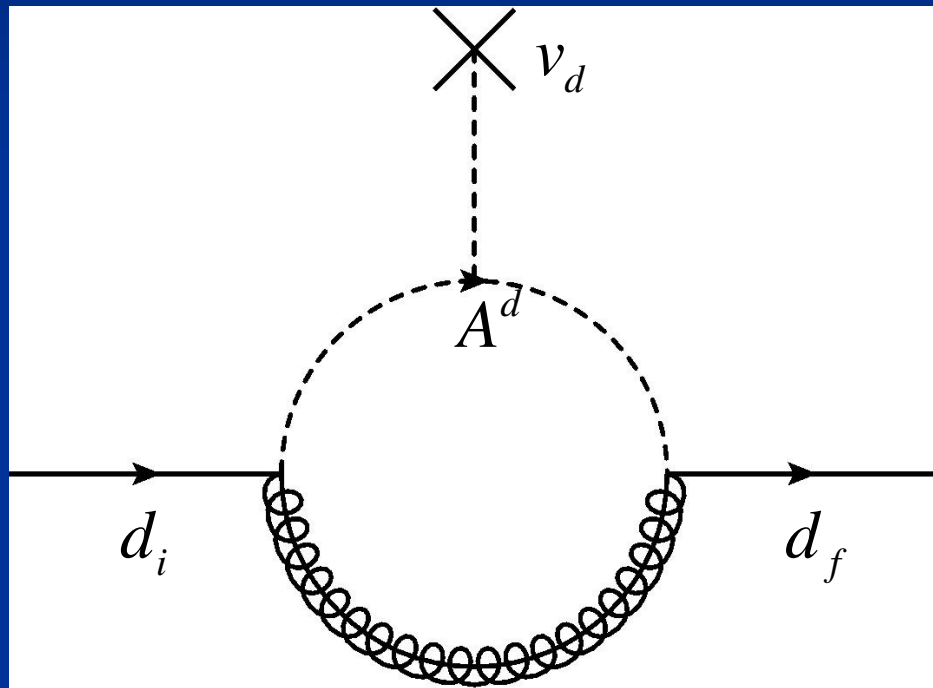
$$\Gamma_{d_f d_i}^{H^d}$$



$$\Gamma_{d_f d_i}^{H^u}$$

Loop corrections to Higgs quark couplings

- After electroweak symmetry breaking



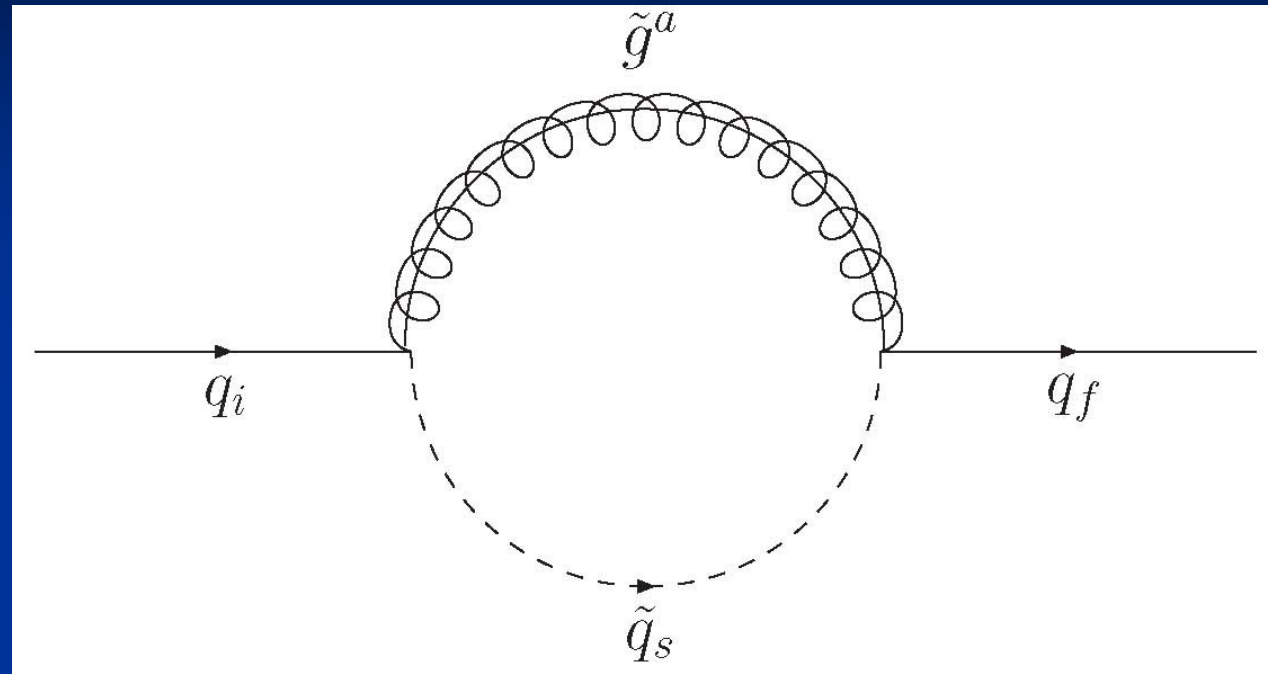
$$\sum_{fi A}^{d LR} = v_d \Gamma_{d_f d_i}^{H^d}$$

$$\sum_{fi Y}^{d LR} = v_u \Gamma_{d_f d_i}^{H^u}$$

➔ One-to-one correspondence between Higgs-quark couplings and chirality changing self-energies. (In the decoupling limit)

SQCD self-energy:

$$-i\Sigma(0)_{fi}^{qLR} =$$



$$\Sigma_{fi}^{qLR} = \alpha_s \frac{2}{3\pi} m_{\tilde{g}} W_{fs} W_{i+3,s}^* B_0(m_{\tilde{g}}^2, m_{\tilde{q}_s}^2)$$

Finite and proportional to at least one power of Δ_{fi}^{qLR}

$$\Sigma_{fi}^{qLR} = \alpha_s \frac{2}{3\pi} m_{\tilde{g}} \left(\Lambda^{qLL} \Delta^{qLR} \Lambda^{qRR} \right)_{fi} C_0(m_{\tilde{g}}^2, m_{\tilde{q}}^2, m_{\tilde{q}}^2)$$

decoupling limit

Decomposition of the self-energy

Decompose the self-energy

$$\sum_{ii}^{\text{d LR}} = \sum_{ii A}^{\text{d LR}} + \sum_{ii Y}^{\text{d LR}}$$

into a holomorphic part proportional to an A-term

$$\sum_{fi A}^{\text{d LR}} = -v_d \alpha_s \frac{2}{3\pi} m_{\tilde{g}} \left(\Lambda^{\text{d LL}} \mathbf{A}^q \Lambda^{\text{d RR}} \right)_{fi} C_0 \left(m_{\tilde{g}}^2, m_{\tilde{q}}^2, m_{\tilde{q}}^2 \right)$$

non-holomorphic part proportional to a Yukawa

$$\sum_{fi Y}^{\text{d LR}} = -v_u \mu \alpha_s \frac{2}{3\pi} m_{\tilde{g}} \mu \left(\Lambda^{\text{d LL}} \mathbf{Y}^d \Lambda^{\text{d RR}} \right)_{fi} C_0 \left(m_{\tilde{g}}^2, m_{\tilde{q}}^2, m_{\tilde{q}}^2 \right)$$

Define dimensionless quantity $\mathcal{E}_i^{\text{d}} = \sum_{ii Y}^{\text{d LR}} / v_u Y^{d_i}$

which is independent of a Yukawa coupling

Threshold corrections

and resummation of chirally enhanced corrections

Determination of the MSSM Yukawa coupling

- All corrections are finite and are non-decoupling

Matching condition:

$$\begin{aligned} m_{d_i} &= v_d Y^{d_i} + \sum_{ii}^{d LR} \\ &= v_d Y^{d_i} + \sum_{ii A}^{q LR} + v_d \tan(\beta) Y^{d_i} \epsilon_{d_i} \end{aligned}$$

➔
$$Y^{d_i} = \frac{m_{d_i} - \sum_{ii A}^{q LR}}{v_d (1 + \tan(\beta) \epsilon_i^d)}$$

- $\tan(\beta)$ is automatically resummed to all orders

Chiral enhancement

$$\Sigma_{fi}^{dLR} \approx \frac{1}{50} \frac{\Delta_{fi}^{qLR}}{M_{SUSY}} = \frac{-v_d}{50} \left(\tan(\beta) Y_i^d \delta_{ij} + \frac{A_{ij}^d}{M_{SUSY}} \right)$$

- For the bottom quark only the term proportional to $\tan(\beta)$ is important.
➔ **$\tan(\beta)$ enhancement**

$$\Sigma_{33Y}^{dLR} = \frac{-1}{100} v_d \tan(\beta) Y^b \sim m_b$$

$$O\left(\frac{\tan(\beta)}{100}\right)$$

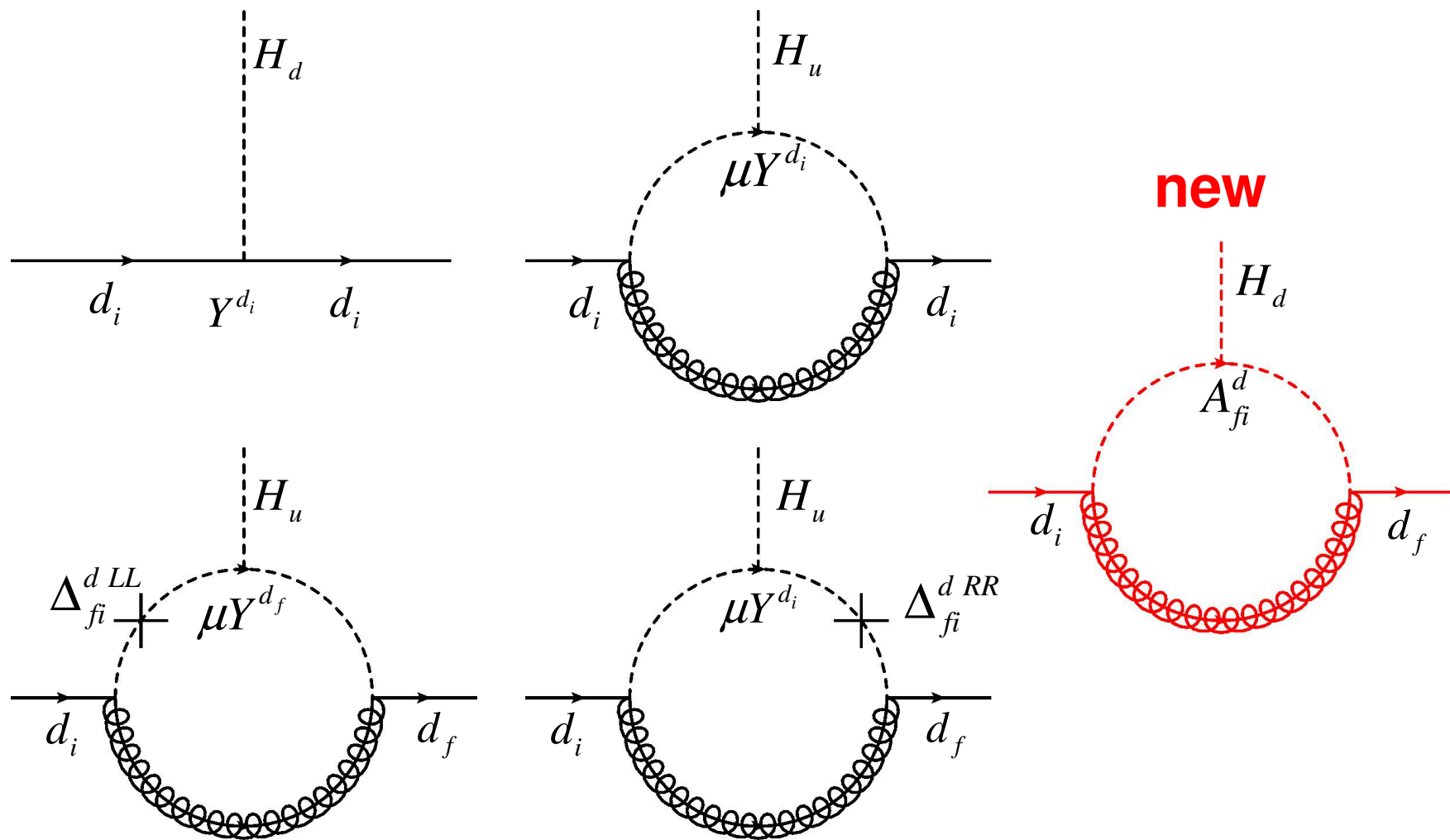
- For the light quarks also the part proportional to the A-term is relevant.

$$\Sigma_{22A}^{dLR} = O(1) \hat{=} A_{22}^d \approx M_{SUSY}$$

$$\Sigma_{11A}^{dLR} = O(1) \hat{=} A_{11}^d \approx \frac{1}{50} M_{SUSY}$$

Effective Higgs-quark vertices

Higgs vertices in the EFT I



Higgs vertices in the EFT II

$$\mathcal{L}_Y^{\text{eff}} = \bar{Q}_{fL}^a \left(\left(Y_i^d \delta_{fi} + E_{fi}^d \right) \varepsilon_{ba} H_d^b + E_{fi}^{\prime d} H_u^{a*} \right) d_{iR}$$

- Non-holomorphic corrections $E_{fi}^{\prime d} = \sum_{fiY}^{dLR} / v_u$
- Holomorphic corrections $E_{fi}^d = \sum_{fiA}^{dLR} / v_d$
- The quark mass matrix $m_{fi}^d = v_d \left(Y^{d_i} \delta_{fi} + E_{fi}^d \right) + v_u E_{fi}^{\prime d}$ is no longer diagonal in the same basis as the Yukawa coupling

 Flavor-changing neutral Higgs couplings

Effective Yukawa couplings

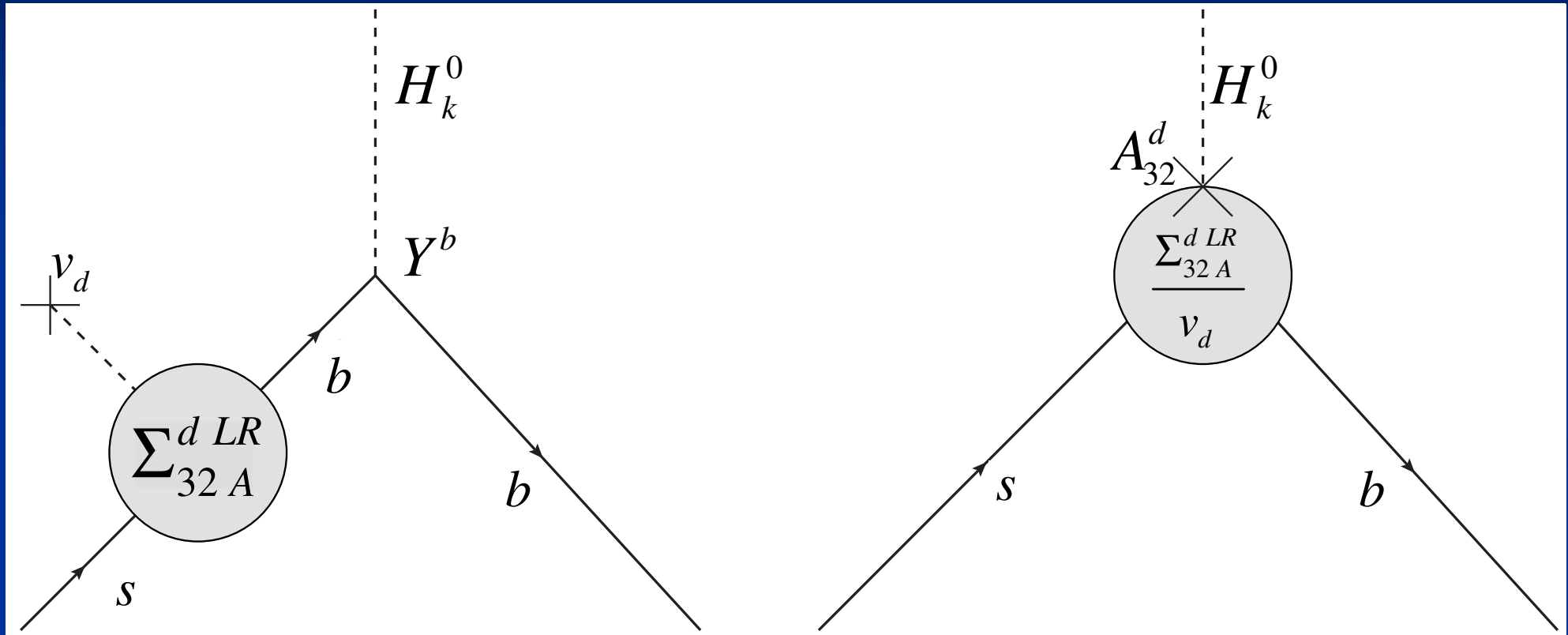
- Final result: $\epsilon_{ij}^d = \frac{1}{v_d} \left(m_{d_i} \delta_{ij} - \tilde{\Sigma}_{ij Y}^{d LR} \right)$ with

$$\tilde{\Sigma}_{jk Y}^{d LR} = U_{jf}^{d L*} \Sigma_{jk Y}^{d LR} U_{ki}^{d R}$$

$$\approx \Sigma_{fi Y}^{d LR} - \left(\begin{array}{ccc} 0 & \frac{\Sigma_{22 Y}^{d LR}}{m_{d_2}} \Sigma_{12}^{d LR} & \frac{\Sigma_{33 Y}^{d LR}}{m_{d_3}} \Sigma_{13}^{d LR} \\ \frac{\Sigma_{22 Y}^{d LR}}{m_{d_2}} \Sigma_{21}^{d LR} & 0 & \frac{\Sigma_{33 Y}^{d LR}}{m_{q_3}} \Sigma_{23}^{d LR} \\ \frac{\Sigma_{33 Y}^{d LR}}{m_{d_3}} \Sigma_{31}^{d LR} & \frac{\Sigma_{33 Y}^{d LR}}{m_{q_3}} \Sigma_{32}^{d LR} & 0 \end{array} \right)$$

Diagrammatic explanation in the full theory:

Higgs vertices in the full theory



- Cancellation incomplete since $v_d Y^b \neq m_b$
Part proportional to $\sum_{33 Y}^{d LR}$ is left over.

➡ A-terms generate flavor-changing Higgs couplings

SUSY_FLAVOR 2.0

A.C., J. Rosiek et al, arXiv:1203.5023

Calculates a large set of flavour observables including the complete resummation of all chirally enhanced corrections and the effective Higgs vertices.

Observable	Most stringent constraints on	Experiment
$\Delta F = 0$		
$\frac{1}{2}(g-2)_e$	$\text{Re} \left[\delta_{11}^{\ell LR, RL} \right]$	$(1159652188.4 \pm 4.3) \times 10^{-12}$
$\frac{1}{2}(g-2)_\mu$	$\text{Re} \left[\delta_{22}^{\ell LR, RL} \right]$	$(11659208.7 \pm 8.7) \times 10^{-10}$
$\frac{1}{2}(g-2)_\tau$	$\text{Re} \left[\delta_{33}^{\ell LR, RL} \right]$	$< 1.1 \times 10^{-3}$
$ d_e (\text{ecm})$	$\text{Im} \left[\delta_{11}^{\ell LR, RL} \right]$	$< 1.6 \times 10^{-27}$
$ d_\mu (\text{ecm})$	$\text{Im} \left[\delta_{22}^{\ell LR, RL} \right]$	$< 2.8 \times 10^{-19}$
$ d_\tau (\text{ecm})$	$\text{Im} \left[\delta_{33}^{\ell LR, RL} \right]$	$< 1.1 \times 10^{-17}$
$ d_n (\text{ecm})$	$\text{Im} \left[\delta_{11}^{d LR, RL} \right], \text{Im} \left[\delta_{11}^{u LR, RL} \right]$	$< 2.9 \times 10^{-26}$
$\Delta F = 1$		
$\text{Br}(\mu \rightarrow e\gamma)$	$\delta_{12,21}^{\ell LR, RL}, \delta_{12}^{\ell LL, RR}$	$< 2.8 \times 10^{-11}$
$\text{Br}(\tau \rightarrow e\gamma)$	$\delta_{13,31}^{\ell LR, RL}, \delta_{13}^{\ell LL, RR}$	$< 3.3 \times 10^{-8}$
$\text{Br}(\tau \rightarrow \mu\gamma)$	$\delta_{23,32}^{\ell LR, RL}, \delta_{23}^{\ell LL, RR}$	$< 4.4 \times 10^{-8}$
$\text{Br}(K_L \rightarrow \pi^0 \nu\nu)$	$\delta_{23}^{u LR}, \delta_{13}^{u LR} \times \delta_{23}^{u LR}$	$< 6.7 \times 10^{-8}$
$\text{Br}(K^+ \rightarrow \pi^+ \nu\nu)$	sensitive to $\delta_{13}^{u LR} \times \delta_{23}^{u LR}$	$17.3_{-10.5}^{+11.5} \times 10^{-11}$
$\text{Br}(B_d \rightarrow ee)$	$\delta_{13}^{d LL, RR}$	$< 1.13 \times 10^{-7}$
$\text{Br}(B_d \rightarrow \mu\mu)$	$\delta_{13}^{d LL, RR}$	$< 1.8 \times 10^{-8}$
$\text{Br}(B_d \rightarrow \tau\tau)$	$\delta_{13}^{d LL, RR}$	$< 4.1 \times 10^{-3}$
$\text{Br}(B_s \rightarrow ee)$	$\delta_{23}^{d LL, RR}$	$< 7.0 \times 10^{-5}$
$\text{Br}(B_s \rightarrow \mu\mu)$	$\delta_{23}^{d LL, RR}$	$< 1.08 \times 10^{-8}$
$\text{Br}(B_s \rightarrow \tau\tau)$	$\delta_{23}^{d LL, RR}$	—
$\text{Br}(B_s \rightarrow \mu e)$	$\delta_{23}^{d LL, RR} \times \delta_{12}^{\ell LL, RR}$	$< 2.0 \times 10^{-7}$
$\text{Br}(B_s \rightarrow \tau e)$	$\delta_{23}^{d LL, RR} \times \delta_{13}^{\ell LL, RR}$	$< 2.8 \times 10^{-5}$
$\text{Br}(B_s \rightarrow \mu\tau)$	$\delta_{23}^{d LL, RR} \times \delta_{23}^{\ell LL, RR}$	$< 2.2 \times 10^{-5}$
$\text{Br}(B^+ \rightarrow \tau^+ \nu)$	—	$(1.65 \pm 0.34) \times 10^{-4}$
$\text{Br}(B_d \rightarrow D\tau\nu)/\text{Br}(B_d \rightarrow D\nu)$	—	$(0.407 \pm 0.12 \pm 0.049)$
$\text{Br}(B \rightarrow X_s \gamma)$	$\delta_{23}^{d LL, RR}$ for large $\tan\beta$, $\delta_{23,32}^{d LR}$	$(3.52 \pm 0.25) \times 10^{-4}$
$\Delta F = 2$		
$ \epsilon_K $	$\text{Im} \left[(\delta_{12}^{d LL, RR})^2 \right], \text{Im} \left[(\delta_{12,21}^{d LR})^2 \right]$	$(2.229 \pm 0.010) \times 10^{-3}$
ΔM_K	$\delta_{12}^{d LL, RR}, \delta_{12,21}^{d LR}$	$(5.292 \pm 0.009) \times 10^{-3} \text{ ps}^{-1}$
ΔM_D	$\delta_{12}^{u LL, RR}, \delta_{12,21}^{u LR}$	$(2.37_{-0.71}^{+0.66}) \times 10^{-2} \text{ ps}^{-1}$
ΔM_{B_d}	$\delta_{13}^{d LL, RR}, \delta_{13,31}^{d LR}$	$(0.507 \pm 0.005) \text{ ps}^{-1}$
ΔM_{B_s}	$\delta_{23}^{d LL, RR}, \delta_{23,32}^{d LR}$	$(17.77 \pm 0.12) \text{ ps}^{-1}$

The SQCD quark self-energy at two-loop

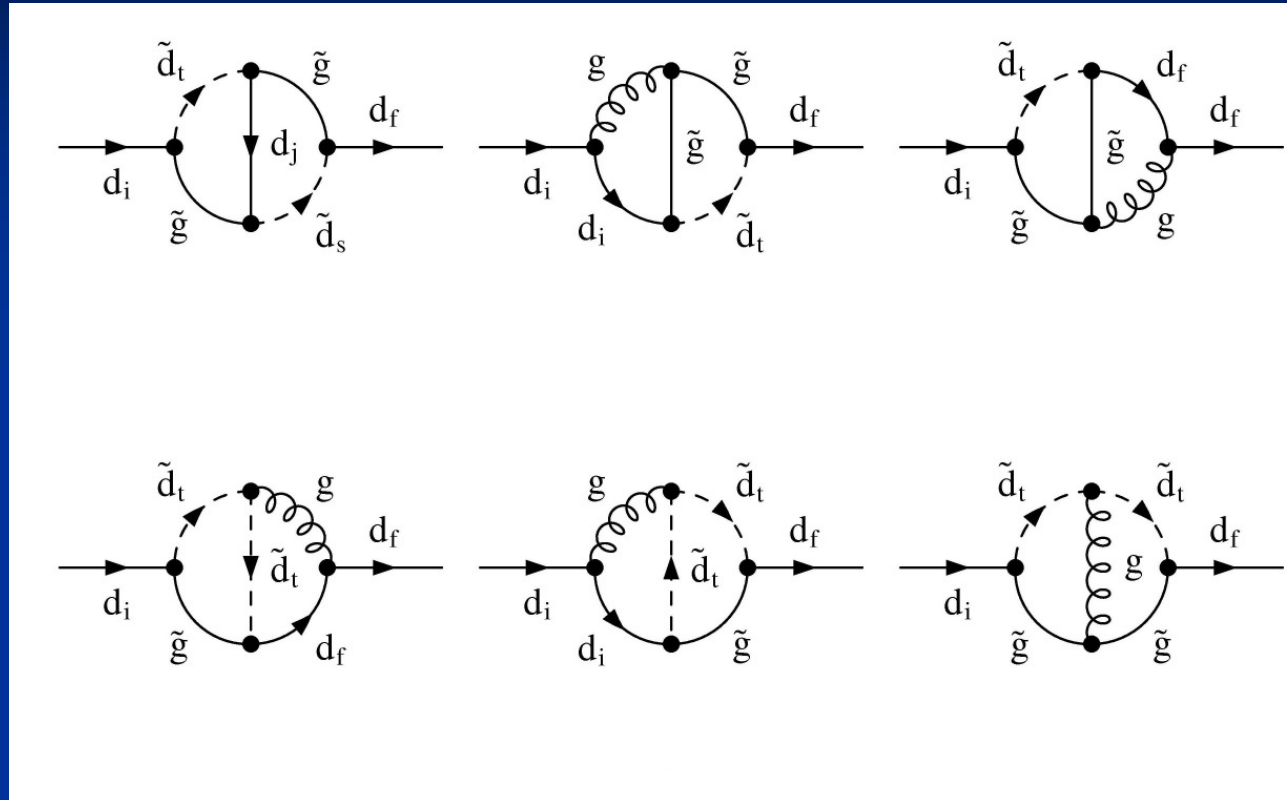
NLO calculation of the quark self-energies

NLO calculation is important for:

- Computation of effective Higgs-quark vertices.
- Determination of the Yukawa couplings of the MSSM superpotential (needed for the study of Yukawa unification in GUTs).
- NLO calculation of FCNC processes in the MSSM at large $\tan(\beta)$.

Reduction of the matching scale dependence

NLO calculation

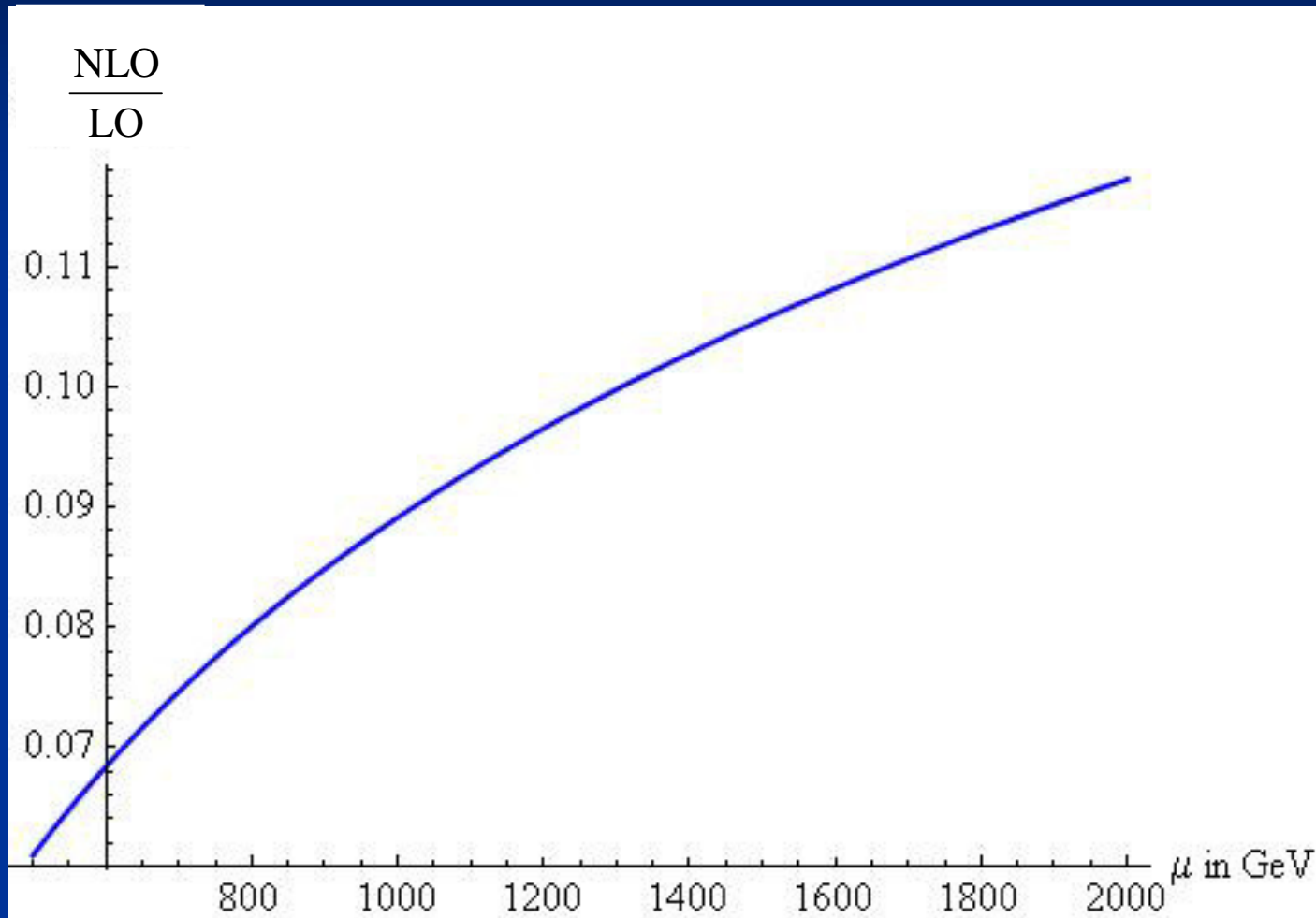


Examples of 2-loop diagrams

- NLO calculation includes analytic results and $\tan(\beta)$ resummation in the generic MSSM.

Δ_b at order α_s^2

NLO results



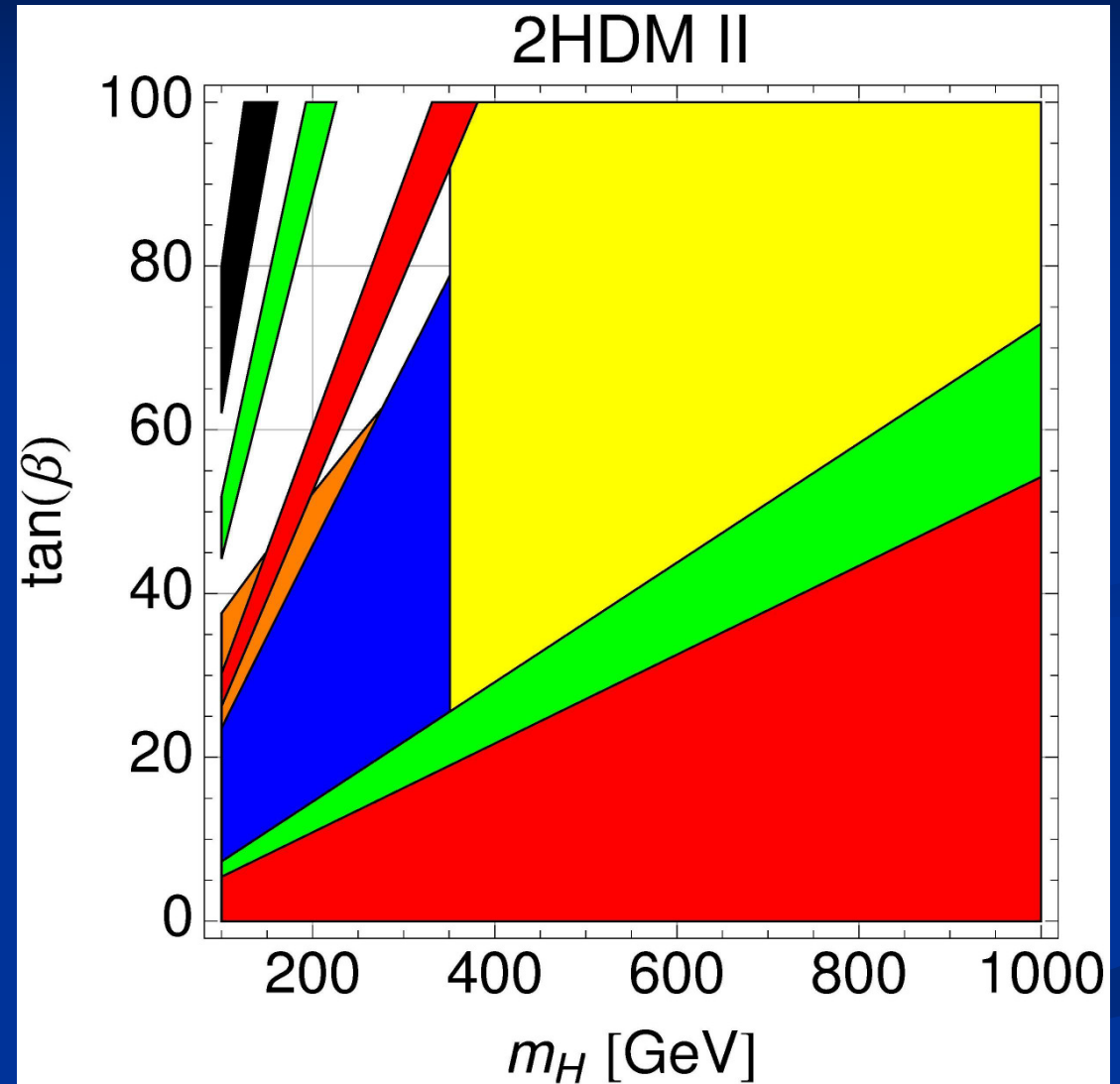
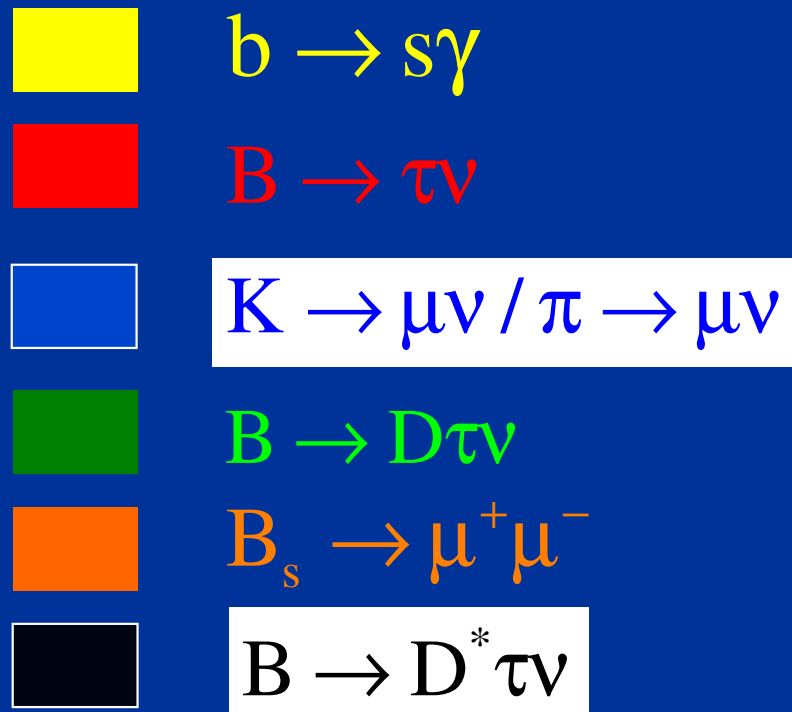
Relative importance of the 2-loop corrections
approximately 9%

Flavour Phenomenology of the 2HDM of Type III

Type-II 2HDM

Allowed

2 σ regions from:
(superimposed)



 Tension from $B \rightarrow D^*\tau\nu$

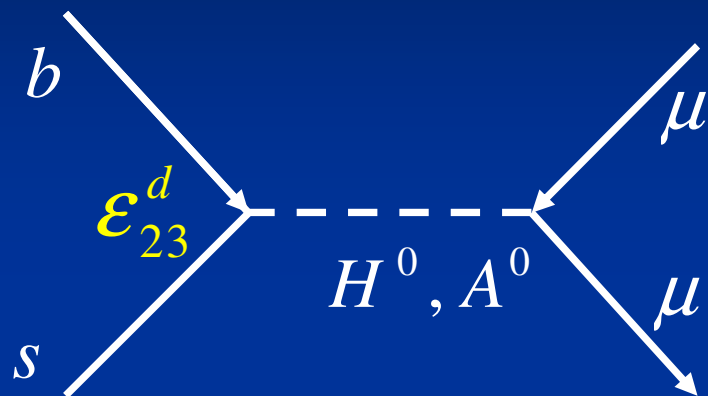
Type-III: constraints from $M \rightarrow \mu^+ \mu^-$

$$\tan(\beta) = 50$$

■ $m_H = 700 \text{ GeV}$

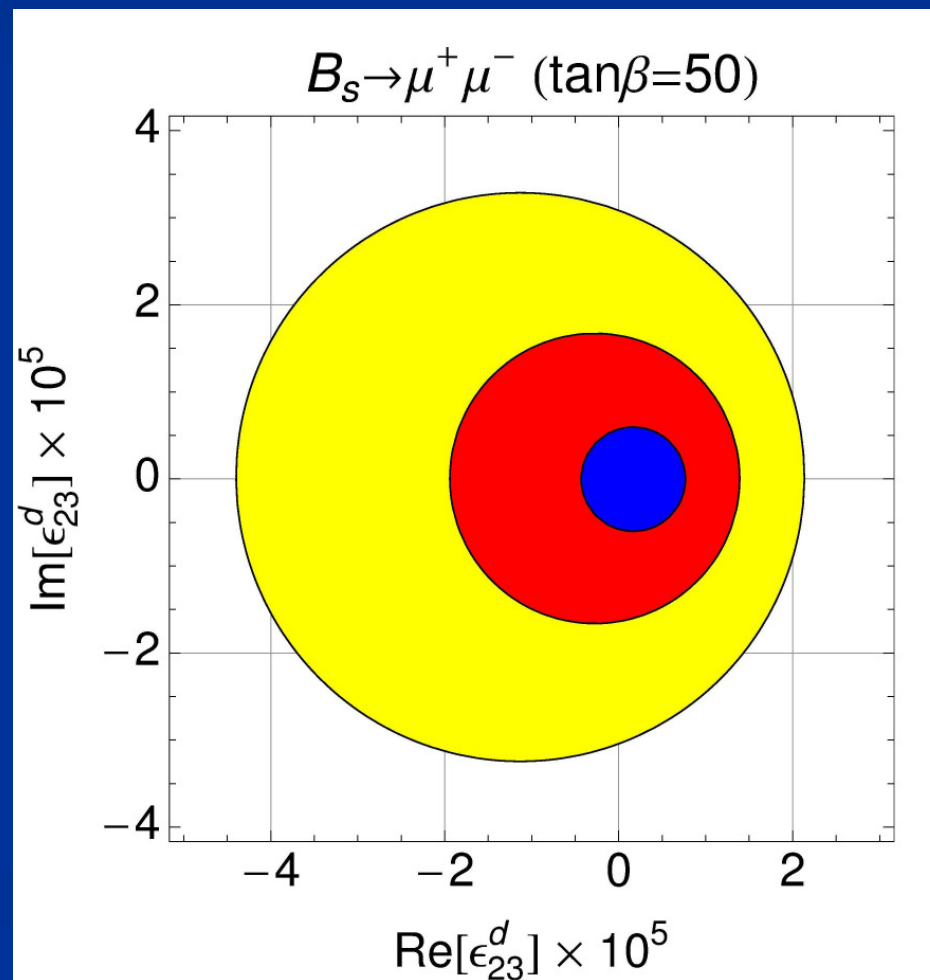
■ $m_H = 500 \text{ GeV}$

■ $m_H = 300 \text{ GeV}$



- $B \rightarrow \mu^+ \mu^-$ constrains $\epsilon_{13,31}^d$
- $B_s \rightarrow \mu^+ \mu^-$ constrains $\epsilon_{23,32}^d$
- $K_L \rightarrow \mu^+ \mu^-$ constrains $\epsilon_{12,21}^d$
- $D \rightarrow \mu^+ \mu^-$ constrains $\epsilon_{12,21}^u$

➔ $\epsilon_{32,23}^u$ and $\epsilon_{13,31}^u$ unconstrained
from tree-level FCNCs




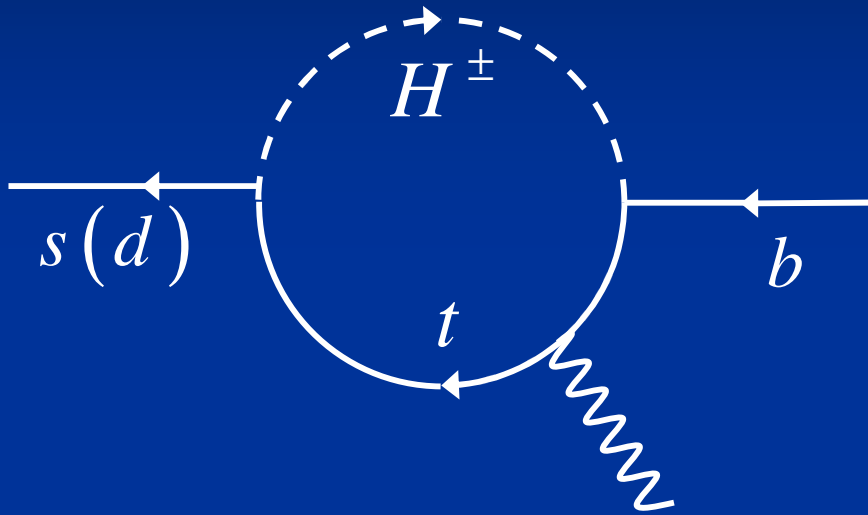
Type-III: Constraints from $b \rightarrow s(d) \gamma$

$$\tan(\beta) = 50$$

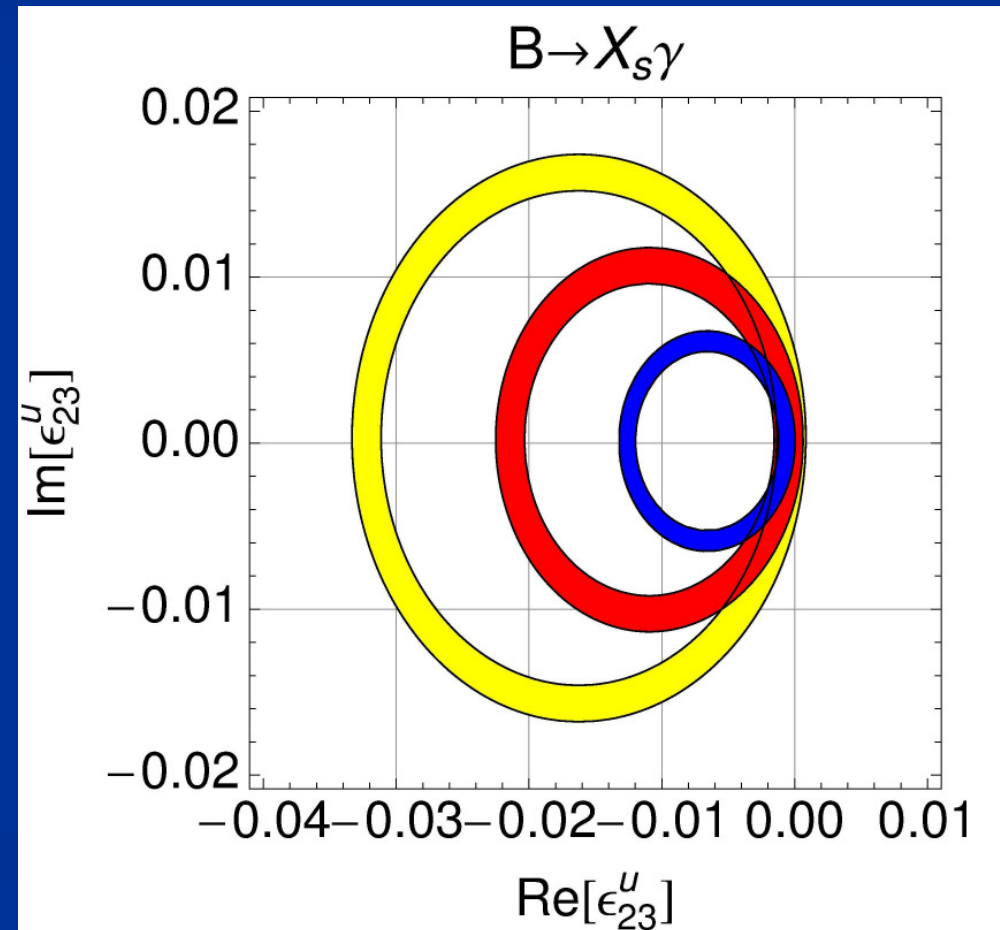
 $m_H = 700 \text{ GeV}$

 $m_H = 500 \text{ GeV}$

 $m_H = 300 \text{ GeV}$



- $b \rightarrow s \gamma$ constrains ϵ_{23}^u
- $b \rightarrow d \gamma$ constrains ϵ_{13}^u
- $\epsilon_{31,32}^u$ still unconstrained



Tauonic B decays

- Tree-level decays in the SM via W-boson
- Sensitive to a charged Higgs due to the heavy tau lepton in the final state.

Observable	SM	Experiment	Significance
$\text{Br}[B \rightarrow \tau \nu]$	$(0.719^{+0.115}_{-0.076}) \times 10^{-4}$	$(1.15 \pm 0.23) \times 10^{-4}$	1.6σ
$\text{Br}[B \rightarrow D \tau \nu] / \text{Br}[B \rightarrow D \ell \nu]$	0.297 ± 0.017	0.440 ± 0.072	2.0σ
$\text{Br}[B \rightarrow D^* \tau \nu] / \text{Br}[B \rightarrow D^* \ell \nu]$	0.252 ± 0.003	0.332 ± 0.030	2.7σ

➡ All three observables are above the SM prediction

$B \rightarrow \tau \nu$

$$Br[B \rightarrow \tau \nu] = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 f_B^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right) \tau_B \left|1 + \frac{m_B^2}{m_b m_t} \frac{C_R^{ub} - C_L^{ub}}{C_{SM}^{ub}}\right|^2$$

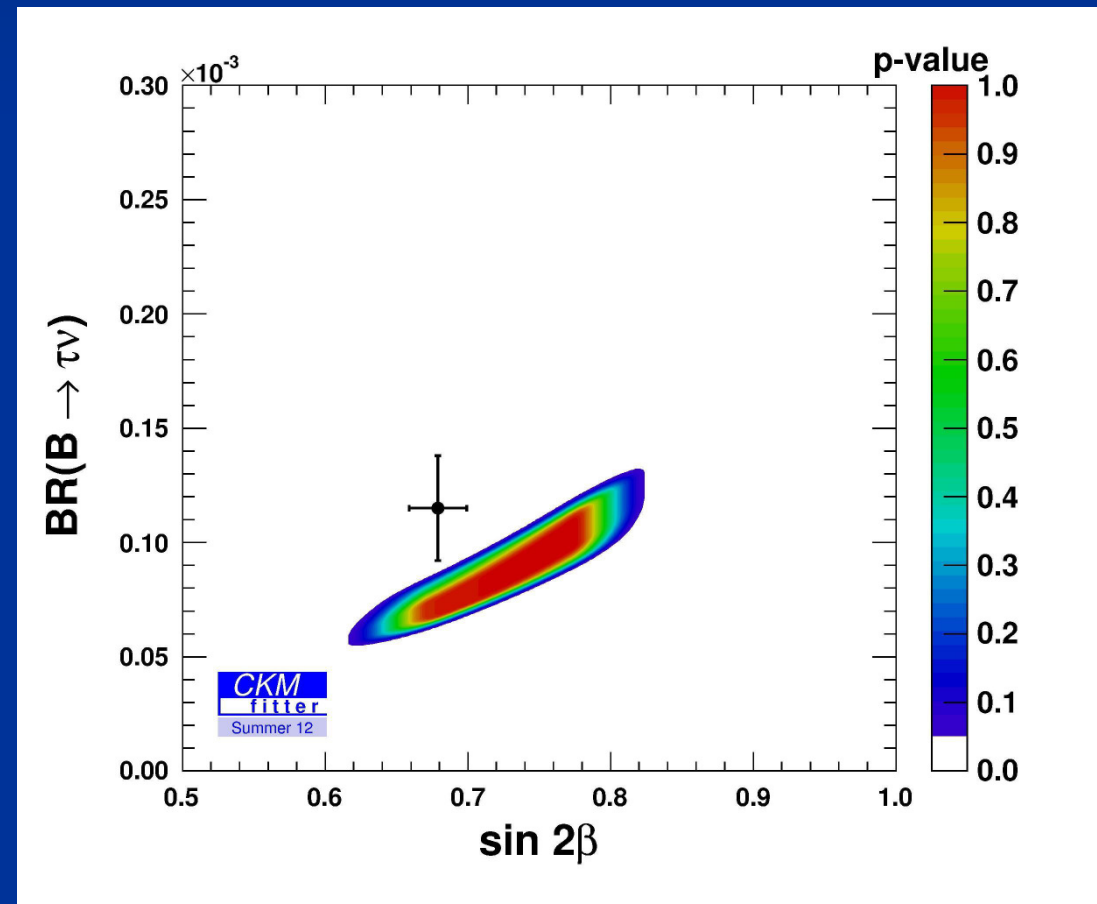
V_{ub} can be

determined from

- $B \rightarrow \pi \ell \nu$
- inclusive decay
- Global fit to the CKM matrix

Different determinations
do not agree

➔ V_{ub} problem

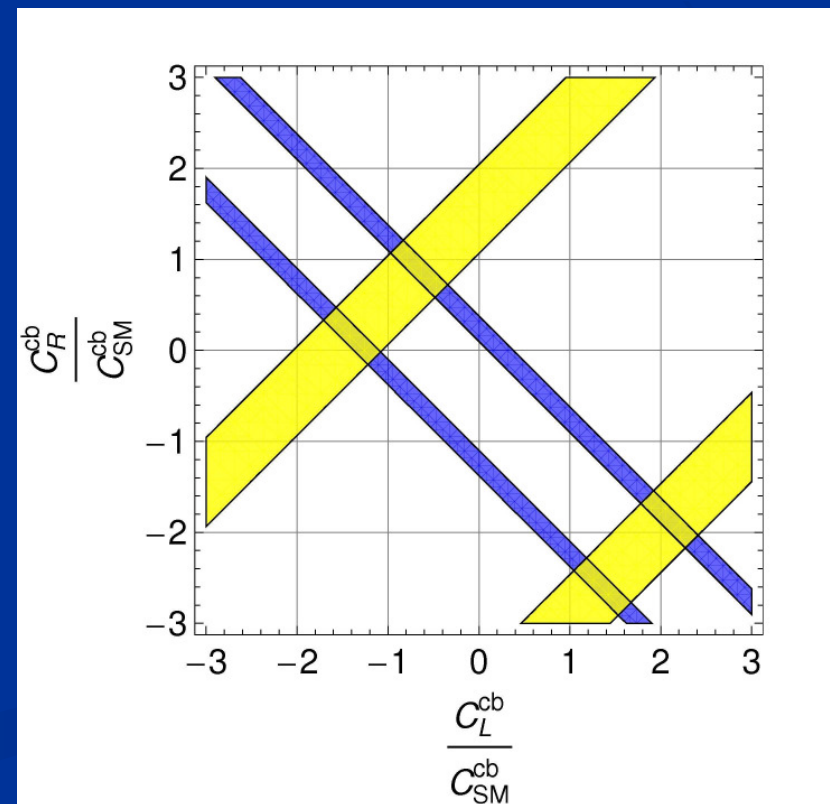


$B \rightarrow D^{(*)} \tau \nu$

$$R(D) = \frac{\text{Br}[B \rightarrow D \tau \nu]}{\text{Br}[B \rightarrow D \ell \nu]} = R_{SM}(D) \left(1 + 1.5 \text{Re} \left[\frac{C_R^{cb} + C_L^{cb}}{C_{SM}^{cb}} \right] + 1.0 \left| \frac{C_R^{cb} + C_L^{cb}}{C_{SM}^{cb}} \right|^2 \right)$$

$$R(D^*) = \frac{\text{Br}[B \rightarrow D^* \tau \nu]}{\text{Br}[B \rightarrow D^* \ell \nu]} = R_{SM}(D^*) \left(1 + 0.12 \text{Re} \left[\frac{C_R^{cb} - C_L^{cb}}{C_{SM}^{cb}} \right] + 0.05 \left| \frac{C_R^{cb} - C_L^{cb}}{C_{SM}^{cb}} \right|^2 \right)$$

- Form factors uncertainties drop out to a large extent in the ratios $R(D)$ and $R(D^*)$.
- $R(D^*)$ less sensitive to NP
- C_R cannot explain $R(D)$ and $R(D^*)$ simultaneously but C_L can.



Tauonic B decays in the 2HDM II

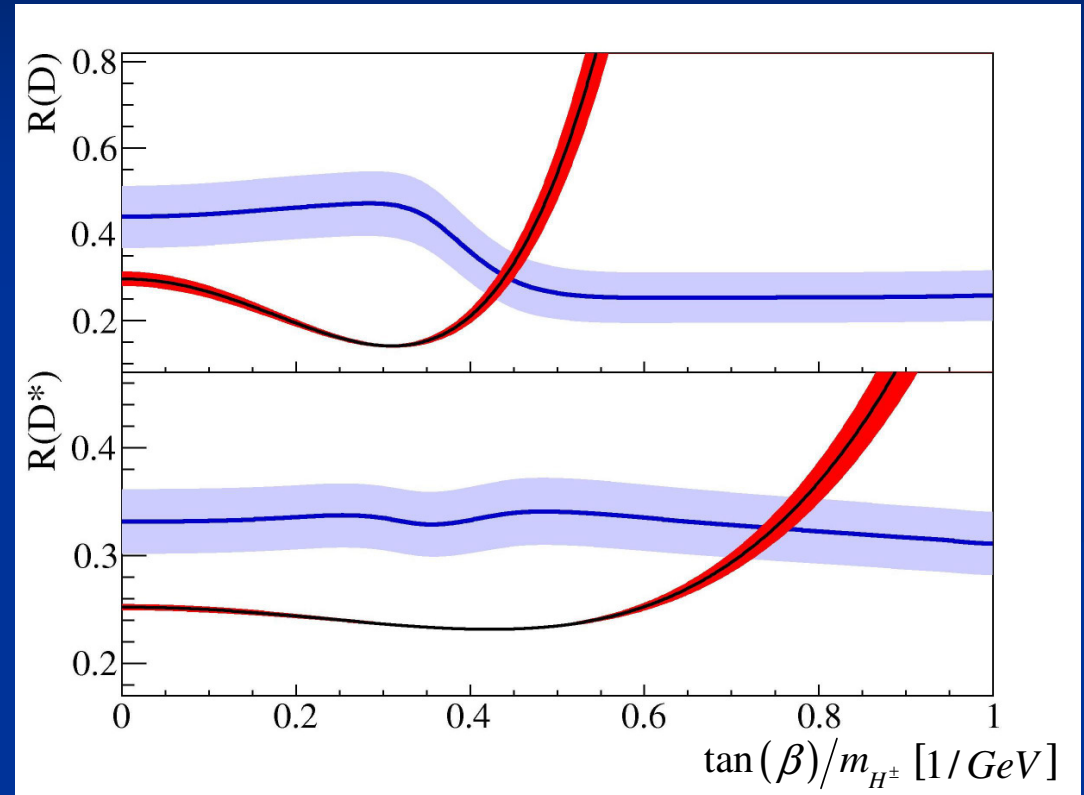
$$C_R^{qb} = \frac{-1}{m_{H^\pm}^2} V_{qb} \frac{m_b m_\tau}{v^2} \tan^2(\beta)$$

$$C_L^{qb} \approx 0$$

- Contribution to $B \rightarrow \tau \nu$ necessarily destructive.
- $\tan(\beta)/m_{H^\pm}$ needed for $R(D^*)$ too large.
- Cannot explain $B \rightarrow D^{(*)} \tau \nu$ and $B \rightarrow D \tau \nu$ simultaneously.

BaBar collaboration 1205.5442

➔ Disfavored by current data



arXiv:1205.5442



measurement

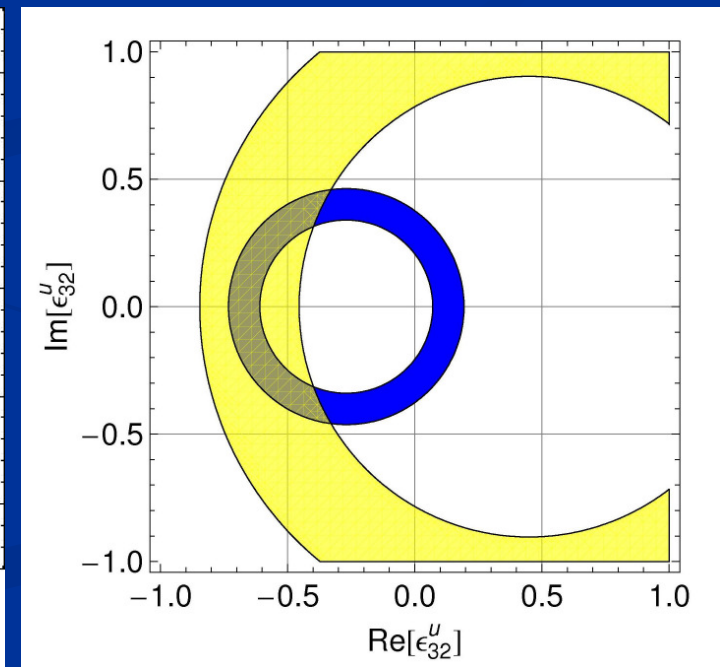
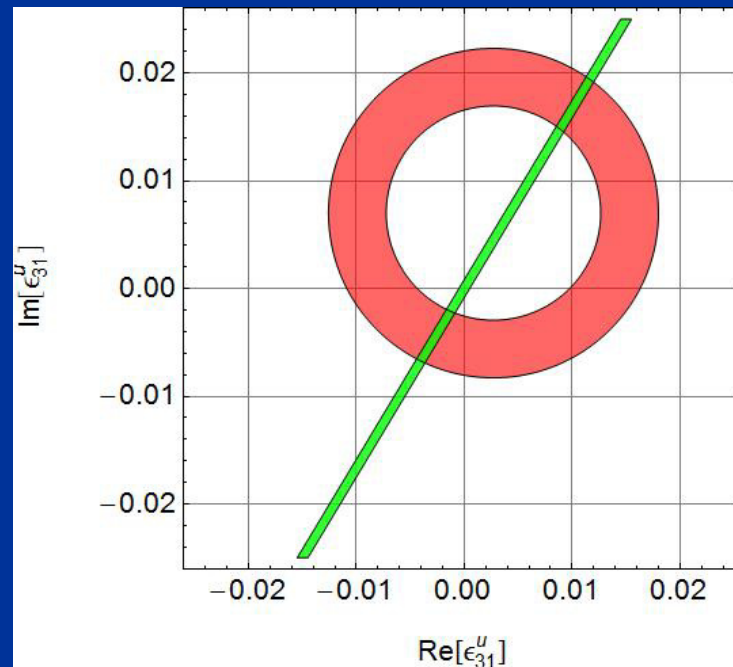
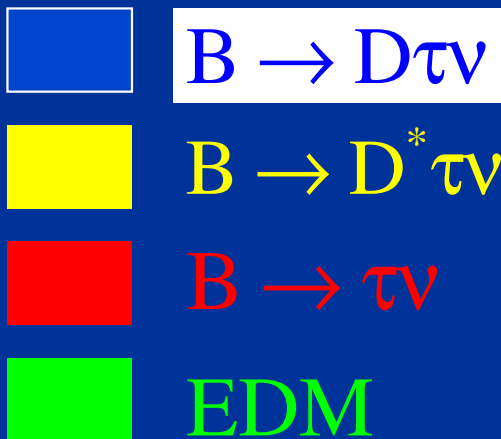


theory prediction

2HDM of type III with flavour-violation in the up-sector

- Constructive contribution to $B \rightarrow \tau \nu$ using ϵ_{31}^u is possible.
- $B \rightarrow D^{(*)} \tau \nu$ and $B \rightarrow D \tau \nu$ can be explained simultaneously using ϵ_{32}^u . **→ Check model via $H^0, A^0 \rightarrow \bar{t}c$**

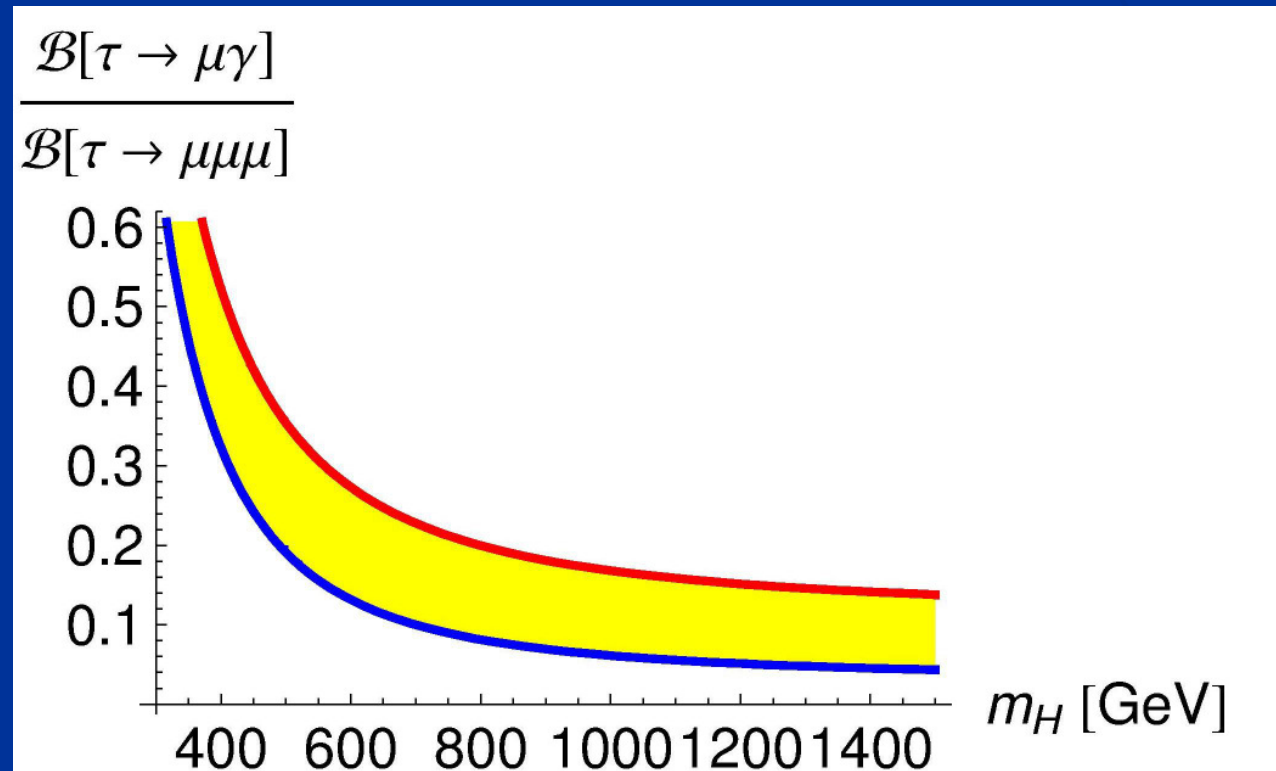
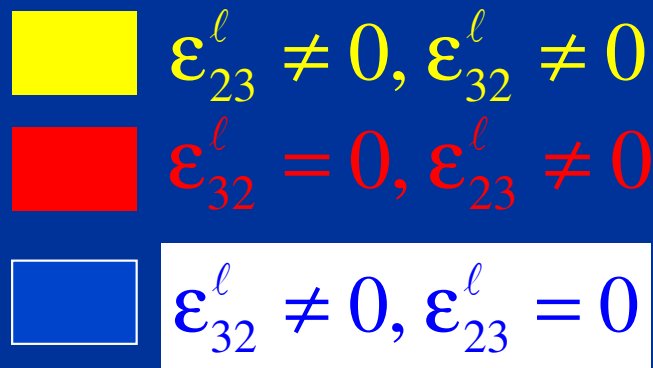
Allowed regions from:



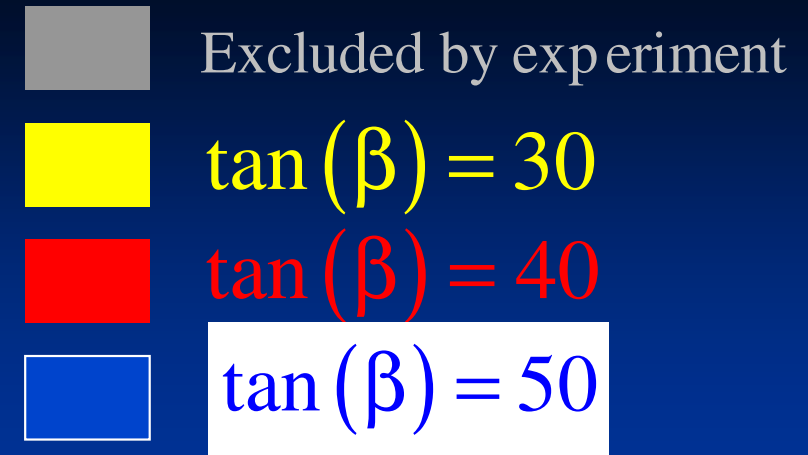
Lepton Flavor violation

- Correlations between $\tau \rightarrow \mu\mu\mu$ and $\tau \rightarrow \mu\gamma$

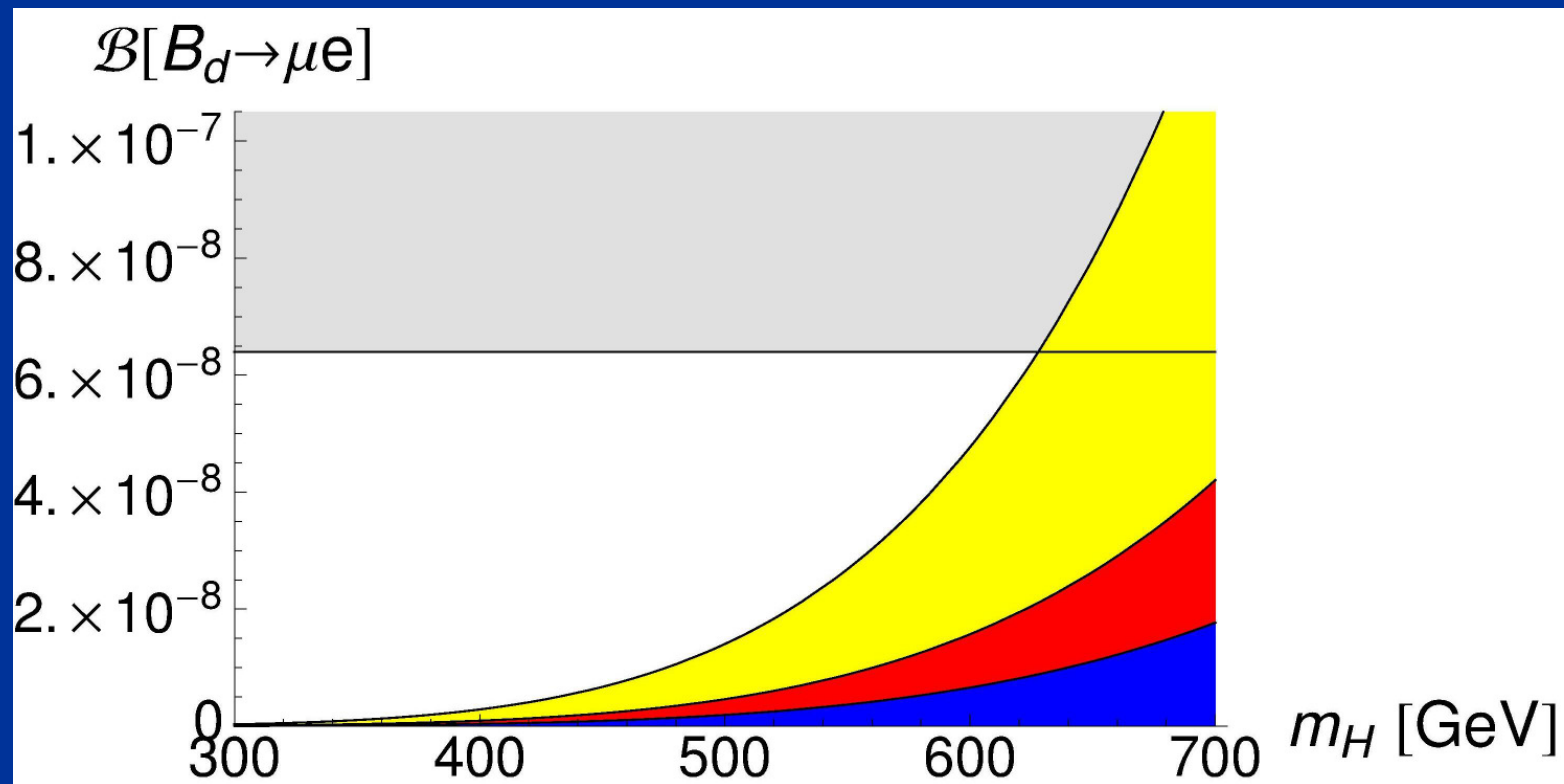
Predicted ratio in the 2HDM of type III



Upper limits on lepton flavour violating B decays



Allowed regions respecting the constraints from $\mu \rightarrow e\gamma$ and $B_d \rightarrow \mu^+\mu^-$



Conclusions

- In the MSSM self-energies generate threshold correction which can be of order one.
- A-terms generate flavor-changing neutral Higgs couplings.
- SUSY_FLAVOR 2.0 is a useful tool for calculating flavour observables in the generic MSSM.
- 2-loop calculation of Higgs-quark couplings significantly reduces the matching scale dependence.
- A 2HDM of type III with flavour violation in the up-sector can explain $B \rightarrow \tau \nu$, $B \rightarrow D \tau \nu$ and $B \rightarrow D^* \tau \nu$ despite the stringent constraints from FCNC processes.
- Interesting correlations among lepton flavour violating observables in the 2HDM with generic flavour structure.